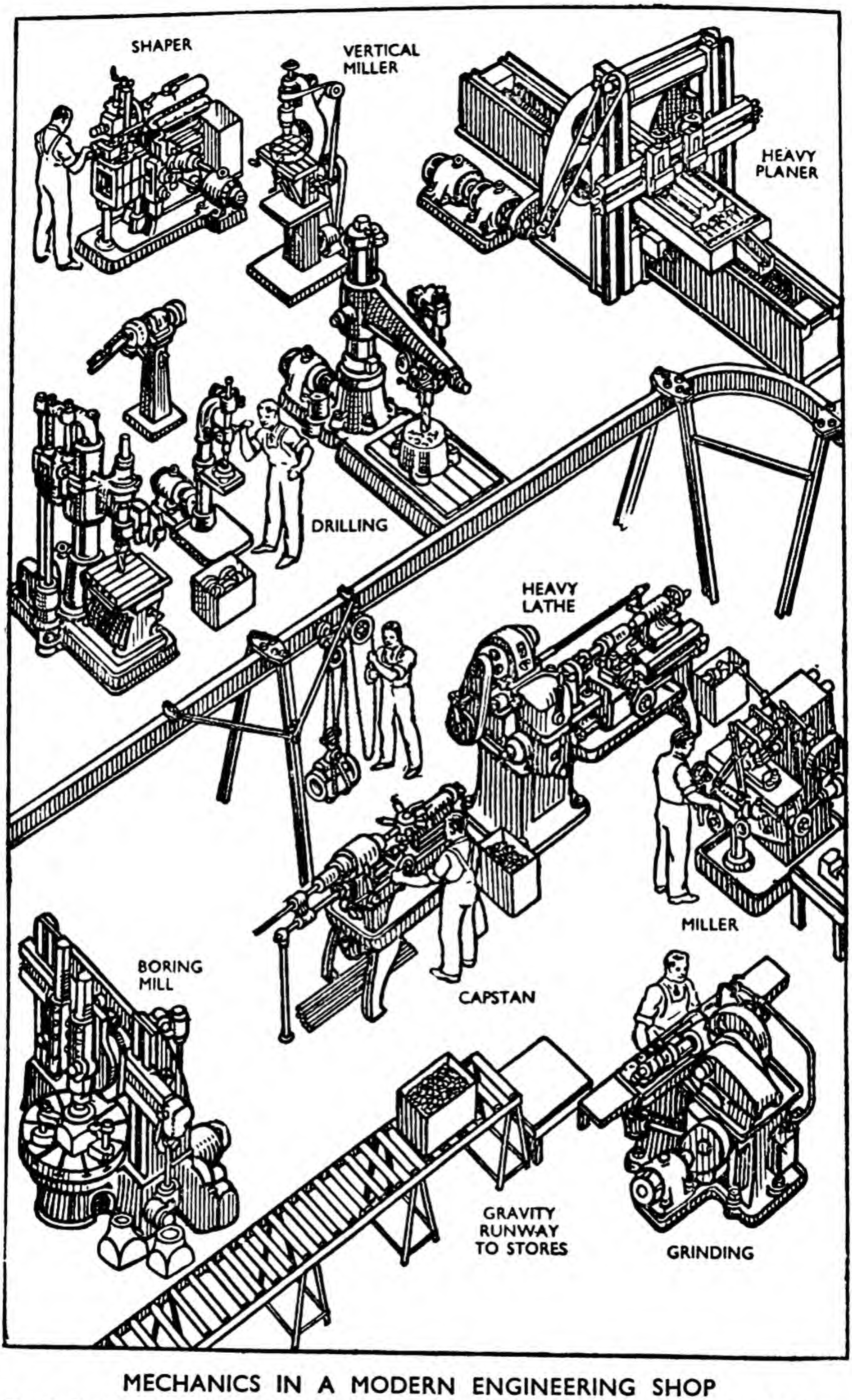
PRACTICAL MECHANICS FOR ALL

ILLUSTRATED

Edited by

LEROY A. BEAUFOY
M.Sc., Ph.D., A.K.C., A.M.Inst.C.E., M.I.Mech.E.

ASIA PUBLISHING HOUSE
BOMBAY—CALCUTTA—NEW DELHI—MADRAS



MECHANICS IN A MODERN ENGINEERING SHOP

Fig. 1. Equipment in engineering workshops embodies countless mechanical devices.

CHAPTER 1

THIS MECHANICAL WORLD

BRANCHES OF MECHANICS. GENERAL PRINCIPLES. SPECIAL DEVICES. MECHANICAL PRINCIPLES AT HOME AND AT WORK. MECHANICAL LAWS AND NATURAL PHENOMENA. HOW WHIRLPOOLS ARE FORMED. VARIATIONS IN TIDES. MOVEMENTS OF THE PLANETS. WORK OF GALILEO. NEWTON'S LAWS. CONSERVATION OF ENERGY. OVERCOMING RESISTANCE. PRINCIPLE OF MOMENTS. STREAMLINING. INTERNAL FRICTION OF LIQUIDS. FASCINATION OF KNOWLEDGE. UNIVERSALITY OF MECHANICS.

MECHANICS is concerned with the action of forces on bodies and with the motions that they produce. A glance at the contents page of this book will give some idea of the scope of the subject, of which one important branch of mechanics is that dealing with forces that balance or are, as it is said, in equilibrium. From the very earliest times, the principles involved in this branch of mechanics were understood, and were the basis, as, of course, they remain today, of much building and structural work. The same principles apply in connexion with the forces due to a pressure of water, or with the equilibrium of a floating ship.

Well-known Laws

Then there is the branch of mechanics that is concerned with motion in its relation to force, and the well-known laws of motion associated with the name of Newton constitute a statement of this relationship. Movement generally, together with the resultant expenditure of energy, and the problem of overcoming frictional and other resistances, implying as it usually does the provision of suitable lubrication—all these are aspects of this second branch of mechanics, which are exemplified in practice in innumerable ways. The mechanics of hydraulic machinery, as well as of flight, are examples which are dealt with in some detail in this book.

Designing a Dam

The civil engineer designing a dam is concerned with the water pressure at the face of the dam, and he must be sure that the resultant of this force and the weight of the dam falls within a certain part of the base of the dam, if essential conditions of equilibrium and of strength are to be satisfied.

Equally, the mechanical engineer must bear such principles in mind, when, for example, selecting a suitable spring for use with a spring-operated safety valve intended to blow at a predetermined steam pressure.

In the workshop (Fig. 1) the labourer's task in slinging a heavy weight so that it may be safely hoisted into the air and moved by crane from one part of the shop to another, calls for an understanding, which may be either conscious or instinctive, of the significance of a centre of gravity. The housewife,

in placing a loaded tea-tray on a restricted space on the table, faces the same problem.

The highway engineer who builds a road specially banked round the curve does so because it provides the means of counteracting the sideways thrust on a vehicle travelling round that curve. He is applying his knowledge of the mechanics of movement to the problem, and is able to decide the exact angle of banking required for a vehicle moving at a chosen speed.

Out-of-balance Forces

Similarly, the locomotive engineer arranges for special balance weights to be fitted to the driving wheel of a locomotive in order to reduce vibrations due to out-of-balance forces set up when the locomotive is in motion, and he is able, beforehand, to calculate very closely what balance weights will be needed for a perfect balance.

Finally, there is the branch of mechanics which deals with motion without relation to forces. A knowledge of this enables us to understand mechanisms, those convenient devices whereby desired movements are faithfully reproduced.

Practical Applications

To the engineer or the physicist, mechanics may be said to be fundamental in the fullest sense, since much of their work is dependent on an application of these principles. To everyone, however, applications of mechanics are matters of everyday experience. The man or woman in the workshop, in the office, or in the home, or anywhere in fact, is constantly encountering such applications, and it is to enable them to appreciate and to understand the principles involved more fully that this book has been prepared.

This is, to a greater extent than most of us perhaps realize, a mechanical world. The evidences of this are all round us. A very long time ago, man was mainly dependent on his own exertions for transport from place to place. Today he may travel by such mechanical means as the car, the train, the ship or the aeroplane, to mention only a few of the possibilities open to him. Whereas, formerly, his sources of light and heat were crude in the extreme, today, the public utilities are highly organized, and have been reduced to a closely supervised sequence of mechanical operations.

Domestic Appliances

The amenities of life in our homes have been greatly increased as a result of an ever developing mechanization. We have constant supplies of hot and cold water readily available, and washing, cooking, and cleaning, to mention only a few common household operations, are greatly facilitated by mechanical devices such as the washing machine, gas and electric stoves, and vacuum cleaner.

Offices, with their telephones, typewriters, duplicators, comptometers, dictaphones, pneumatic message-transmission systems, and so on, have undergone a revolution in recent times involving a mechanization of many of the processes carried out in them.

Special Workshop Devices

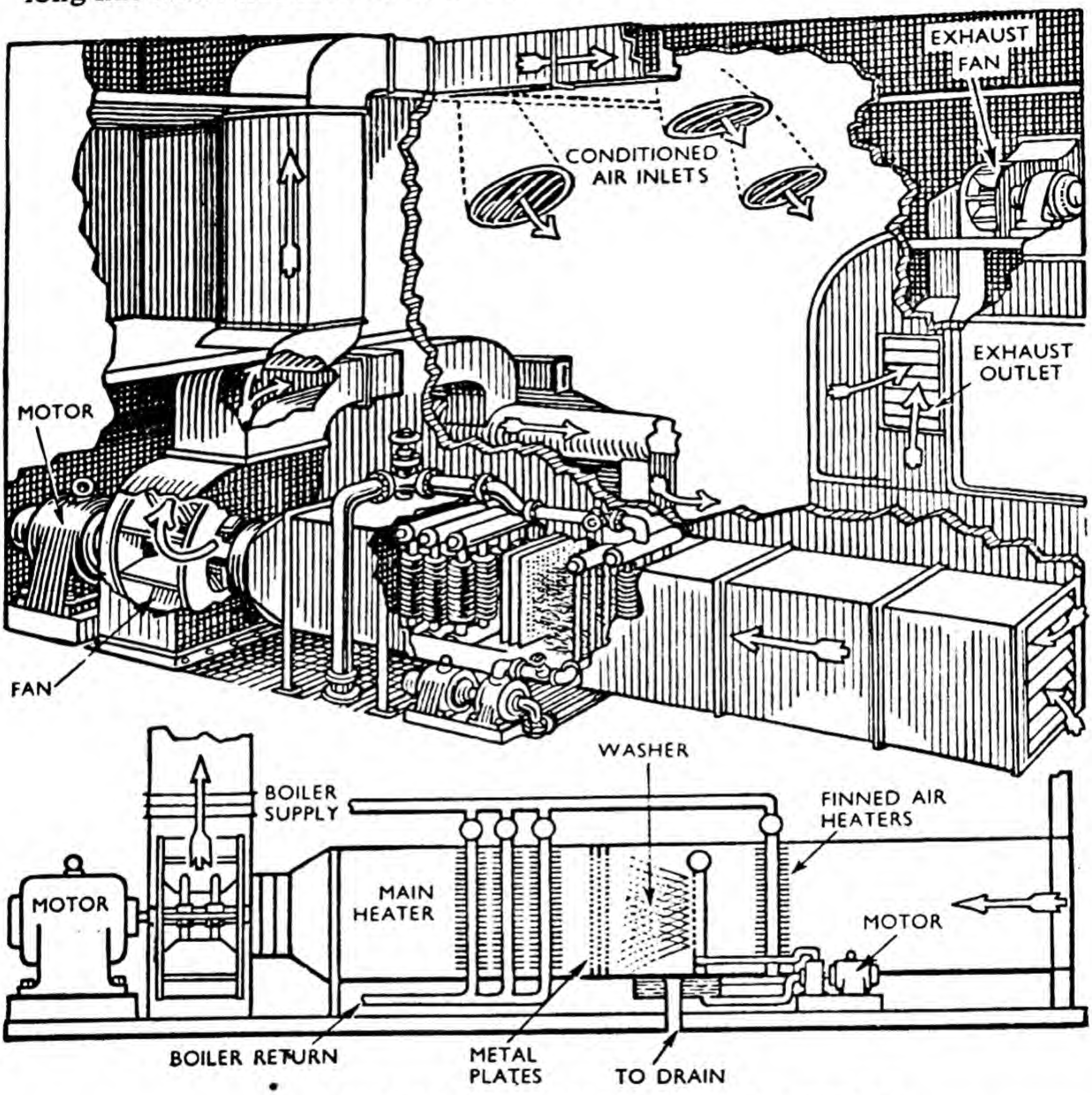
It will be found that even in a workshop many operations which would have been performed manually as recently as ten or twenty years ago, are today achieved

mechanically by special devices or special machines.

The farmer, pitting his skill against the vagaries of nature, for long has cultivated his soil with but the crudest of implements, but agricultural machinery is today available which enormously increases the speed of his work. The tractor, the thresher and the combine harvester are instances of such machinery.

The story is endless. In whichever direction we turn, there are evidences that this is a mechanical world. And the more closely any particular piece of evidence is examined, the more evidence is found.

It is a hot day and we turn



AN AIR-CONDITIONING PLANT AT WORK

Fig. 2. The fan is a predominant feature of the air-conditioning plant shown above. It draws fresh air through the inlet, past the finned air heaters of the apparatus, through the washer, and past the main heaters. The fan then directs the air through distributing pipes to the various air inlets in the building. Finally, a second fan disperses the vitiated air, drawing it through the exhaust outlet and discharging it into the atmosphere.

gratefully to the fan which is at least keeping the air in circulation around us. We might well pause and contemplate just this one mechanical device in a world of mechanical devices. At the least it is replacing the rudimentary palm leaf, but it can be used for a wide variety of other purposes.

Many Uses of the Fan

Let us consider some of these. On land, apart from its application in ventilation, the fan is fundamental to systems of air conditioning, where it is used for driving cleaned and humidified air at chosen speeds in selected directions to given places (Fig. 2). It is used in saw-mills for extracting sawdust, in workshops for extracting abrasive and other material from grinding operations, in laboratories and workshops for extracting noxious fumes from the atmosphere, and in households, in the capacity of the vacuum cleaner, for extracting dust from carpets. A further land application of the fan is its use to provide mechanical draught for boilers, thereby avoiding the fluctuating conditions which would ensue if only natural draught were available. Below ground, the fan makes the life of the miner more endurable by ventilating the passages in which he has to work.

At sea, the fan serves to provide ventilation for both passenger and crew accommodation, and for the engine room. In addition, it provides mechanical draught for the boilers, and is an important link in the process of cargo refrigeration.

If the structure of the fan itself is examined more minutely, further evidence of its mechanical aspect will be found. The fan impeller is carried on a carefully manufactured steel shaft supported in journal- or ball- or roller-bearings, depending on the size of the fan.

Delicately Balanced

The fan must also be delicately balanced in its bearings so that no untoward forces shall come into play when it is rotating at high speed. It is also clear that, to achieve this balance, to machine the shaft, to manufacture the ball-bearings and so on, a high degree of mechanical application is demanded.

But it is no part of our present purpose to consider the details of manufacturing operations. Let it suffice that the contemplation of a fan has led our thoughts to the common applications of such an appliance, and a realization that the manufacture of the parts making up the fan has a predominantly mechanical aspect. It will be found that the more such appliances as the fan are considered in detail, the greater the evidence of this mechanical aspect.

All this is true of so many things around us—of the artificial or manufactured things of life—that it will, on reflection, be seen that nearly all of the amenities we enjoy are due to applications of mechanical principles. Furthermore, applications of these principles are being extended in all sorts of ways, and, in this respect, the future holds out very important and interesting possibilities.

Natural Phenomena

But, although we are surrounded by mechanisms and devices of a mechanical character, it is not only in connexion with the artificial

things of life that we observe the evidence of the extent to which this is a mechanical world.

It appears that many natural phenomena are also governed by definite mechanical laws. We know, for example, that a stone falling from a height always falls in a vertical direction. It is also known that water flowing into a valley eventually finds its own level, that is, it eventually settles into such a position that its free surface is everywhere horizontal. Both of these natural phenomena are simply obeying the law of gravitation.

Look now a little further afield and consider some other cases of natural phenomena conforming to mechanical laws. There is, for example, the case of the whirlpool, or free vortex, which forms when the plug is pulled out of a bath containing water. The water, in running away, takes up a spiral movement, and it is a matter of common observation that, as the particles of water get closer and closer to the centre of rotation, viz., the discharge outlet from the bath, their speed gets greater and greater. In the outlet itself, this speed is so high that it forces the particles of water to keep to the outer boundary of the pipe.

Centrifugal Force

The effect on the water is the same as that on the racing car which is travelling round a curved banked track ; the centrifugal force, or force exerted in a direction away from the centre due to the rotary motion, operates to move the car farther and farther towards the outer edge of the track.

So, in the case of the bath outlet, the rotational speed of the particles of water becomes so high by the time that they have reached the outlet itself, that the centrifugal force set up is high enough to maintain the flow on the outer boundary of the outlet only, so that there is a column of air at the centre. The result is the familiar funnel-shaped depression in the free surface of the water, with a gurgling sound coming from the bath as air and water together finally plunge down the outlet.

Spiral Motion of Water

What we have to realize here is that the phenomenon of the free vortex depends upon the fact that, in its spiral motion, the water, getting ever more and more close to the exit pipe, acquires greater and greater speed, thereby satisfying the fundamental mechanical principle, embodied in the words : conservation of angular momentum.

The same phenomenon may be seen on a more spectacular scale when a swimming-pool is in the process of being emptied ; the greater rush of the water near the discharge pipe is more marked, and the bell-mouthed depression of the water surface is clearly visible.

There are occasions in the course of certain large-scale civil engineering constructional works when a free vortex is produced with vastly greater impressive effect. Such occurrences arise when, for one reason or another, the contents of colossal mountain lakes are drained away by discharge through submerged orifices, thereby creating gigantic whirlpools.

The question of the direction of rotation of the discharging water has long been a subject of controversy, and it is sometimes asserted

that this is determined by the influence of the rotation of the earth, which would mean that in our hemisphere the direction of a free vortex would be counter-clockwise, while in the southern hemisphere it would be clockwise.

Direction of Rotation

In the case of some large whirlpools this has been observed to be the case, but in the more usual smaller ones, the effect of accidental circumstances such as the direction of rotation imparted by the hand in the act of pulling out the plug of the wash basin, is of greater importance than the rotation of the earth.

From the very earliest days, observations have been made of the movements of bodies in the solar system, and of heavenly bodies in general, and it is now known that these movements follow certain definite mechanical laws. In fact, the *Nautical Almanac*, which is published some years in advance, predicts these movements with such unfailing accuracy that one is led to conclude that these natural

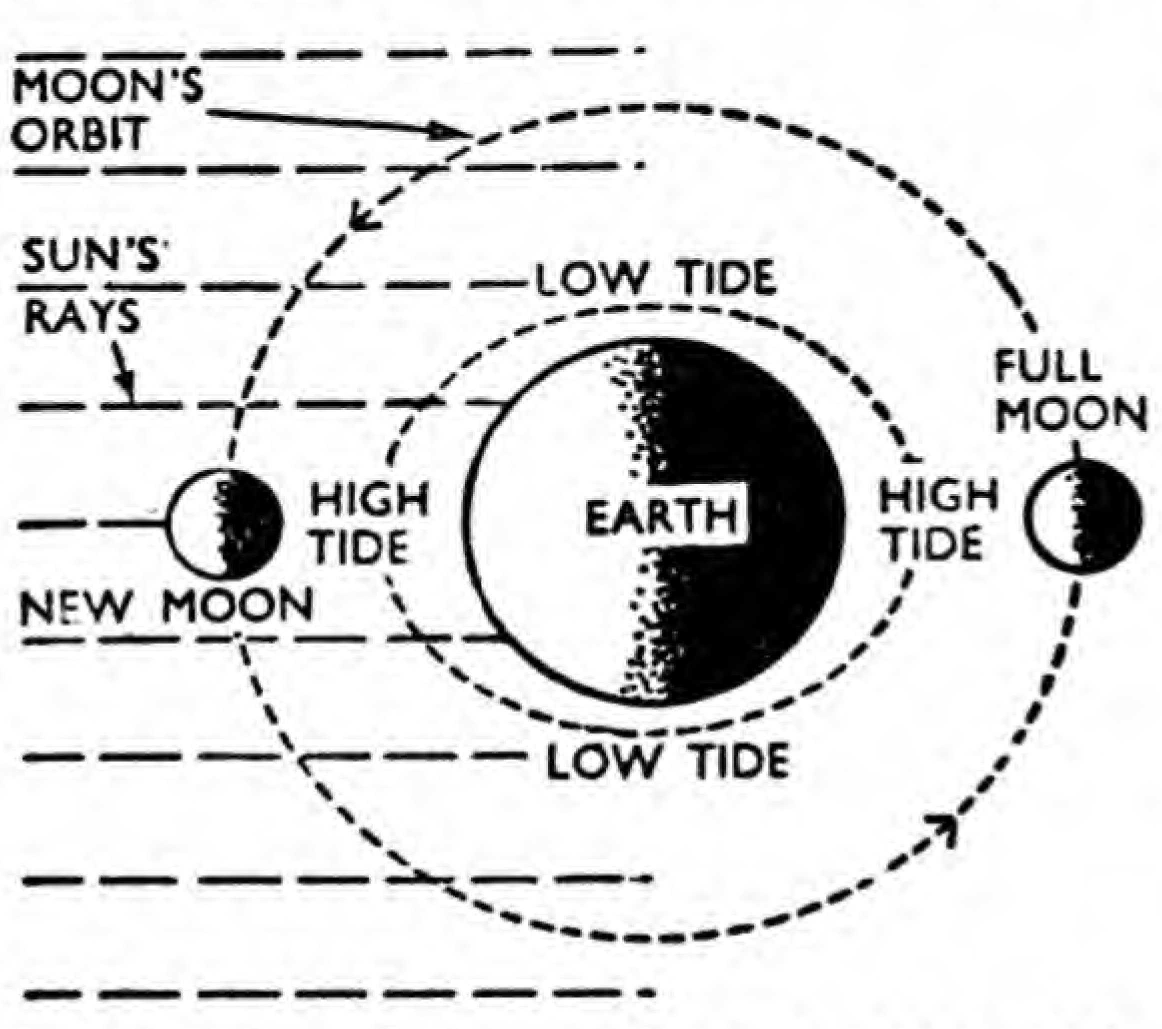


Fig. 3. The highest tides are produced when the moon is new or full. The attractive force exerted by the sun is then in the same line as that exerted by the moon. This is the condition which is known as spring tides.

phenomena do obey the mechanical laws used in the calculations from which the tables in the *Nautical Almanac* are prepared.

Here, then, is another example of natural phenomena confirming the suspicion that this is, from one point of view, basically a mechanical world. The starry heavens are not only the mariner's guide and the surveyor's and map-maker's tool, they are evidence of underlying mechanical principles which seem to be fundamental to this world.

Therefore, it is not surprising to find that such other natural phenomena, as, for example, the waves, appear to be subject to mechanical laws. The spectacle of a line of coast with the breakers rolling in can be a beautiful, and, at times, an awe-inspiring sight. The forces of nature are at work moving vast masses of water before them, and it is not at once apparent that the waves are conforming to a mechanical pattern.

But close observation again has shown that there is a pattern, that waves form under given conditions and break under others, that some waves are waves of oscillation only, while others are waves of translation, and so on. The civil engineer, intent upon preserving the coast line from the fury of the waves, building breakwaters and harbour works, learns much about such things, and applies what he learns to his construction in order that a sounder job shall result.

Cause of Tides

Another familiar natural phenomenon is that of the tides, which has been explained in terms of the mutual attraction exerted between the earth and the moon, combined

with that between the earth and the sun. The effect of this force of attraction exerted on a vast free surface of water, such as the oceans, is to cause the water surface to heap up on the side facing the moon.

The sun, of course, exercises a similar effect, although, because of the enormously greater distance of the sun as compared with that of the moon, the measure of the effect in this case is not so great. A combination of the two effects produces the familiar tidal movement of the water.

It is found, as a matter of common observation, that there is a variation in the character of the tides with the phases of the moon. What are called spring tides occur at new or full moon, when the attractive force exerted by the moon is in the same line as that exerted by the sun. The result of this combined attraction is that the tide rises to its maximum height. The illustration (Fig. 3) shows the positions of the earth and the moon in relation to the sun under these conditions.

The neap tides, on the other hand, form when the moon is in the first or last quarters, and its attractive force is exerted by it in a direction at right angles to the force of the sun. In this case, as Fig. 4 shows, the combined attraction again produces high tide on the side facing the moon, but, as the forces exerted by the sun and moon are not in the same line, high tide in this case does not reach the same level as at new or full moon.

Centre of Gravity

It must not be forgotten that both earth and moon are meantime constantly moving, and another point will be understood if their move-

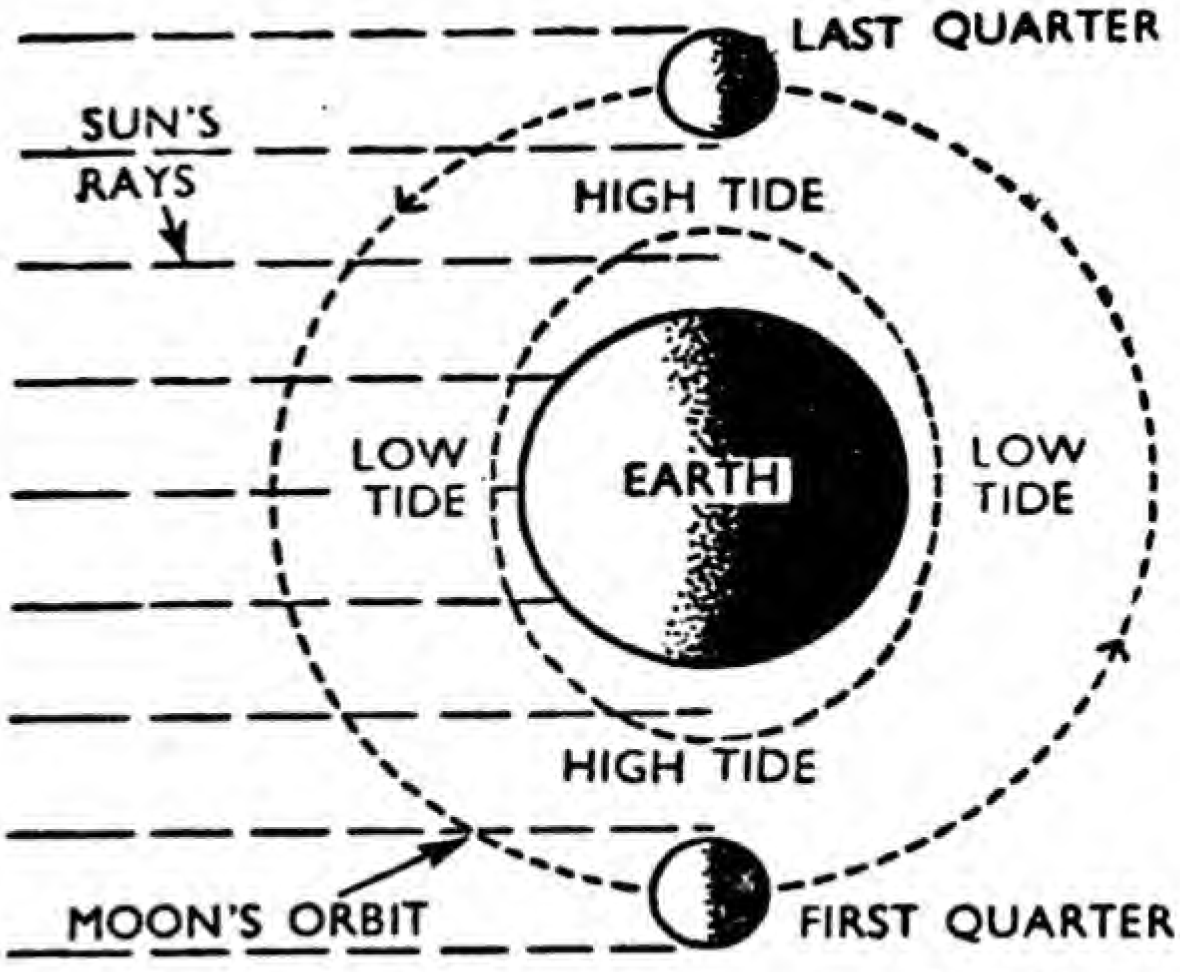


Fig. 4. When the moon is at the first or last quarter, the attractive force exerted by the sun is perpendicular to that exerted by the moon, giving rise to the neap tides, which do not reach the same level as spring tides.

ments are considered in relationship one to the other. Both bodies are rotating about a common centre of gravity and from the relative masses of earth and moon it can be shown that this centre of gravity lies between them, at a depth of just over 1,000 miles below the earth's surface.

Centrifugal Force is Set Up

A result of rotation about this point is the setting up of a centrifugal force, which has the effect of heaping up the water on the side of the earth away from the moon, where the distance from the common centre of gravity is a maximum, and the centrifugal force, therefore, is greatest. The tides may thus be seen as a natural result of the operation of mechanical laws.

As has already been said, the movements of bodies in the solar system have been closely observed and recorded from very early days. From time to time, various attempts were made to derive some rational laws from them, but without any success until about three

centuries ago, when, as a result of close study of all the observations, the following laws were enunciated.

First, that every planet moves in the path of an ellipse with the sun at one of the foci. Second, that the line joining the sun to the planet sweeps over equal areas in equal times. Third, that the square of the time taken by a planet to complete a full revolution round the sun is proportional to the cube of its mean distance from the sun. Here, at last, was a basis from which might be tested the suspicion, so long felt, that the natural movements of the heavenly bodies conformed to a mechanical pattern.

Newton's Laws

Later, Newton was able to deduce from these laws and other phenomena the law of gravitation. Before the enunciation of the above three laws, however, much valuable work was done by Galileo, who concluded, from experiments conducted from the famous Leaning Tower of Pisa, that the acceleration of a falling body was constant, whatever its weight. He also discovered the first two of three laws now known as Newton's laws of motion, and there is reason to believe that the third of these laws was also known by other people before Newton presented them to the world in his famous *Principia.*

The great significance of Newton's achievement, however, was that he gave crystalline form to ideas which had recently developed concerning the movement of bodies.

These laws of motion are fundamental to the whole science of mechanics, which has indeed been built up round them, and to which other mechanical principles, which also conform to all human experience, have now been added. One now regards as axiomatic, for example, the principle of the conservation of energy, by which is meant the principle stating that energy can neither be created nor destroyed but only changed in its form. In other words, if a body subjected to certain forces loses a known amount of a particular form of energy, one looks for the appearance of the same amount of energy in another form.

For example, water in a mountain lake possesses energy in virtue of its position high above a neighbouring mountain village. If this water is released from the lake and conducted by pipe down to a power station in the village, it loses the energy it had due to its position, but it is known that it acquires energy in virtue of its velocity, and, in fact, its velocity, on emerging from the pipe, is sufficient to drive power plant for the generation of electricity. The energy of the still mountain water has been converted into electrical power.

Wasteful Conversions

Not all such conversions, however, are so useful. The man who, by hammering a piece of lead, succeeds in fashioning it into a thin sheet, finds that the sheet has become hot in the process ; a part of the energy which he has expended in hammering the lead has been converted into heat in the sheet, and this is gradually dissipated by transfer to the air, being apparently lost to the worker in the process.

The principle of work, which states that the work put into a machine is equivalent to the useful work taken out of it, plus the work

lost in frictional resistances is, therefore, simply a special case of the principle of the conservation of energy, but is expressed in a form which makes it immediately applicable to the machine.

This principle, which does indeed appear obvious, means nothing more than that, in order to get a certain amount of work out of a machine, we must put into it the same amount of work, *plus* something extra for overcoming the resistance in the machine, and that, in its turn, means that more energy will never be got out of the machine than has been put into it.

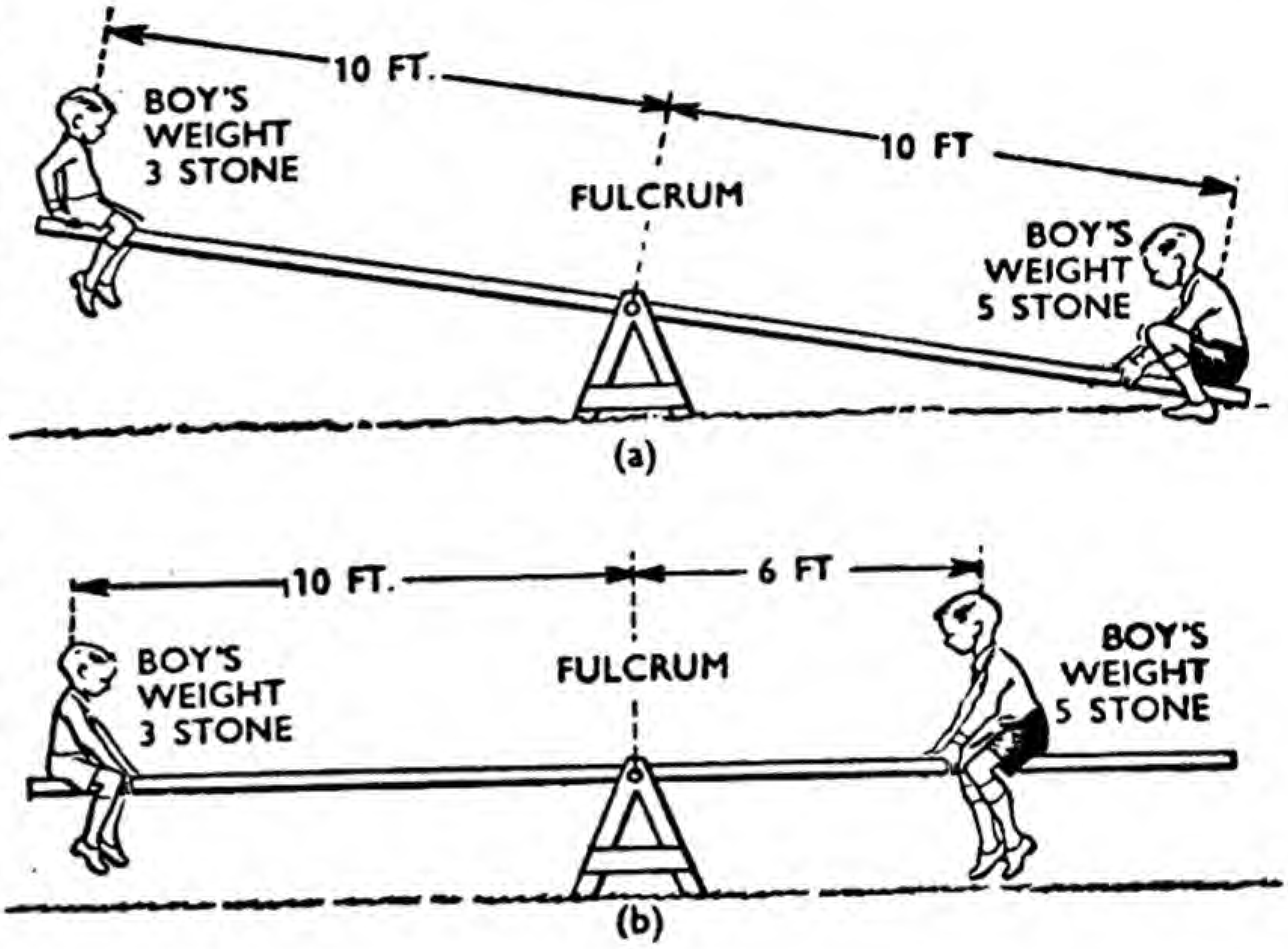


Fig. 5 (right). Two boys on a see-saw automatically satisfy the requirements of the principle of moments. (*a*) Shows that when the boy of 5 stone is the same distance (10 ft.) from the fulcrum as the boy of 3 stone, the see-saw will not balance in a horizontal position because the moments exerted by the weights will not be the same. Balance is achieved (*b*) by the heavier boy moving to a point 6 ft. from the fulcrum, so making the moments about the fulcrum equal.

Conservation of Momentum

Then there is the principle of the conservation of momentum. Momentum means impetus possessed by a moving body, as expressed by the product of its mass and its velocity. This principle is that the sum of the momenta of two moving bodies remains the same, after they have collided, as it was before.

Many of the things which happen on the billiard table become much clearer when the implication of this principle has been understood. For example, when a player pots his opponent's ball he has to avoid dropping his own ball in the pocket as well, which is sometimes done by striking his ball in such a manner that, after the impact, it rapidly comes to rest. His opponent's ball then moves off with a greater speed than it would have if it were sharing the total momentum with a following ball.

The recoil of a gun is similarly dependent on this principle. If the muzzle velocity of the shell is known, as well as the masses of the shell and the gun, the velocity of recoil of the gun follows from the fact that the backward momentum of the gun must be the same as the forward momentum of the shell.

Principle of Moments

Children playing on a see-saw instinctively satisfy the requirements of the principle of moments. The heavier child automatically gets closer to the fulcrum than his

fellow, in order that the moments exerted by their weights about the fulcrum shall be the same (Fig. 5). The equation between clockwise moments and anti-clockwise moments, which is required by the principle of moments, is not achieved here by a laborious mathematical process, but by a simple automatic adjustment.

Striking a Balance

In the same way, a man carrying a heavy weight in one hand, instinctively throws his own weight to the other side, in order to strike a balance between the two moments which these weights will exert about his point of contact with the ground.

These, then, are some of the mechanical principles which experience has shown are commonly followed in our everyday life. At this stage, we may ask what is the importance of having knowledge of these laws of mechanics. The answer must be, first, that they explain many of the things which are happening around us and to us, and, second, that they enable us to use or modify phenomena for our own purposes.

Inertia Effect

Consider, then, the first of these points: that a knowledge of mechanical principles explains things happening around us and to us. A passenger in a bus finds that he is jolted violently backward when the bus starts abruptly from rest. This is called an inertia effect. The passenger's mass resists the change from a state of rest to one of movement forward.

On the other hand, when the bus stops abruptly, the passenger is jolted forward; his momentum tends to carry him forward after the bus has come to rest, his mass resisting a change this time from a state of forward motion to one of rest. Again, when the bus is travelling round a bend in the road the passenger finds himself leaning sideways: the effect of centrifugal force is to tend to move him towards the outer side of the curve.

What about the peculiar feeling in the pit of the stomach caused when travelling in a fast lift? When the lift is stationary, or, for that matter, travelling at a steady speed, the weight of a passenger is resisted by an equal and opposite reaction provided by the floor. But when the lift is accelerating upward, for example, the reaction at the floor has not only to support the passenger's weight, but has also to provide an additional force to accelerate him upward.

Pull of Sun and Moon

We have already seen that there is a mechanical explanation for the occurrence of the tides. It is known that the heaping up of the waters is due to the combined effect, on the water surface, of the pulls exerted by the sun and the moon, and it is known, too, that it is, in part, due to the combined rotational movement of the earth and moon, regarded as a pair, moving round a common centre of gravity.

Now consider another common occurrence around us: the flight of birds. For a very long time now man has been immensely fascinated by the ability of the birds to move through the air, with no other support than the lift with which they provide themselves by means of their wings. The full secret of the mechanism of the process by which they do this still eludes him, but he has progressed far along the

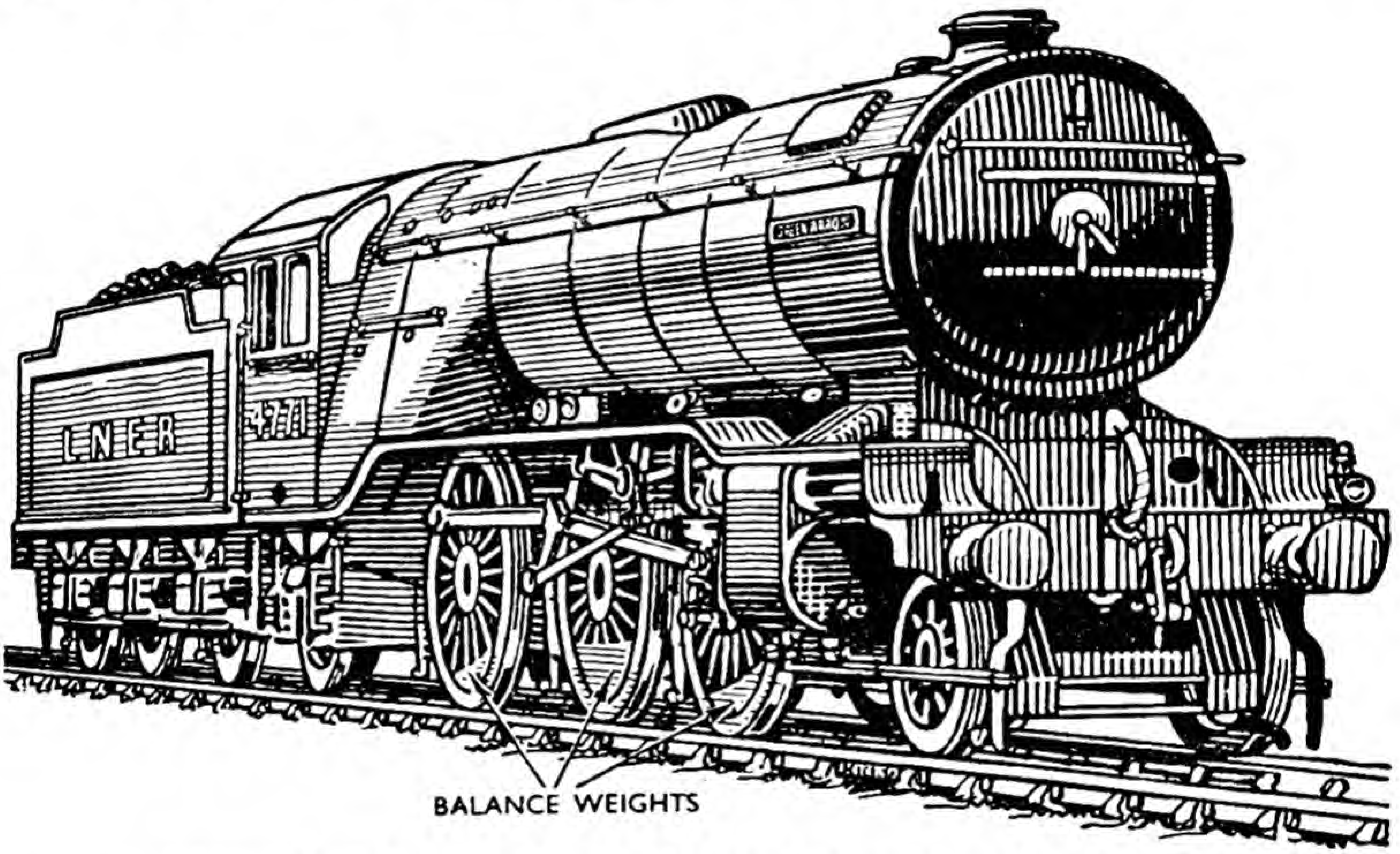
road towards this secret, and flight through the air is becoming a common experience for him, although so far it is flight in a machine, rather than flight without one, which he has achieved.

A great deal is already known about the mechanics associated with stream-lining, for example, and enlightenment has thereby been reached as to the form of the bodies and wings of birds, which are shaped by nature to satisfy the same requirements that are artificially satisfied by stream-lining.

It should be noted, however, that contrary to popular opinion, much stream-lining in practice, for example, of motor cars and locomotives, is done not so much with a view to reducing air resistance as to satisfying æsthetic requirements.

One of the locomotive engineer's problems is that of hammer blow, which is set up by out-of-balance forces on the rotating and reciprocating parts of the locomotive. To reduce, as far as possible, the inertia forces set up when the mechanism of the engine is moving, large balance weights are cast with the driving wheels (Fig. 6). It is not, however, possible to achieve a perfect balance for all conditions, and the result is that an out-of-balance vertical force is set up, during each revolution of the driving wheels, which has the same effect as a downward blow on the rails.

At another part of the revolution, the force acts vertically upward, and tends to lift the wheels off the rails. This causes slip to



COUNTERBALANCING LOCOMOTIVE DRIVING WHEELS

Fig. 6. Crescent-shaped balance weights are added to locomotive driving wheels in order to counterbalance the forces resulting from motion of the locomotive. These forces are of two kinds ; those due to the revolving parts, and those due to the reciprocating parts. The balance weights are adjusted so that complete counterbalancing is achieved in respect of the revolving parts ; only partial counterbalancing is attempted in respect of the reciprocating parts.

occur between wheels and rails, and increases the wear on both. Out-of-balance forces are not only experienced in the vertical direction. In the horizontal direction they introduce impulsive effects on the drawbar. Furthermore, these unbalanced forces, by acting alternately on opposite sides of the engine, introduce a sinuous motion due to the horizontal components, and a rolling motion due to the vertical components.

Water Hammer

Another hammer effect, which most people have experienced, may be produced in a water-pipe line by suddenly shutting off the tap. When the tap was open, water was flowing from it, and the water in the pipe possessed what we have already called impetus or momentum. The rapid closing of the tap causes a sudden destruction of this momentum, and very large pressures may be set up, large enough in some cases to burst the pipe.

Those who, from time to time, have occasion to watch the movement of barges along canals, will have observed that there is often some difficulty in keeping the barge to the centre of the canal, and that this is particularly noticeable when a narrow, shallow canal is being negotiated at some speed. There is, it seems, some natural force acting on the barge which is constantly at work trying to push it over to one side of the canal. Why should this be? Fig. 7 will help to answer this question. The effect of pulling a barge rapidly through a narrow canal is to force the water to rush past it at the sides, in order to fill the space just vacated by the barge. As a result, a wave is formed at the bow of the barge, where the water tends to build up, prior to its rush past the sides.

Just in front of the barge the water will be substantially stationary, so

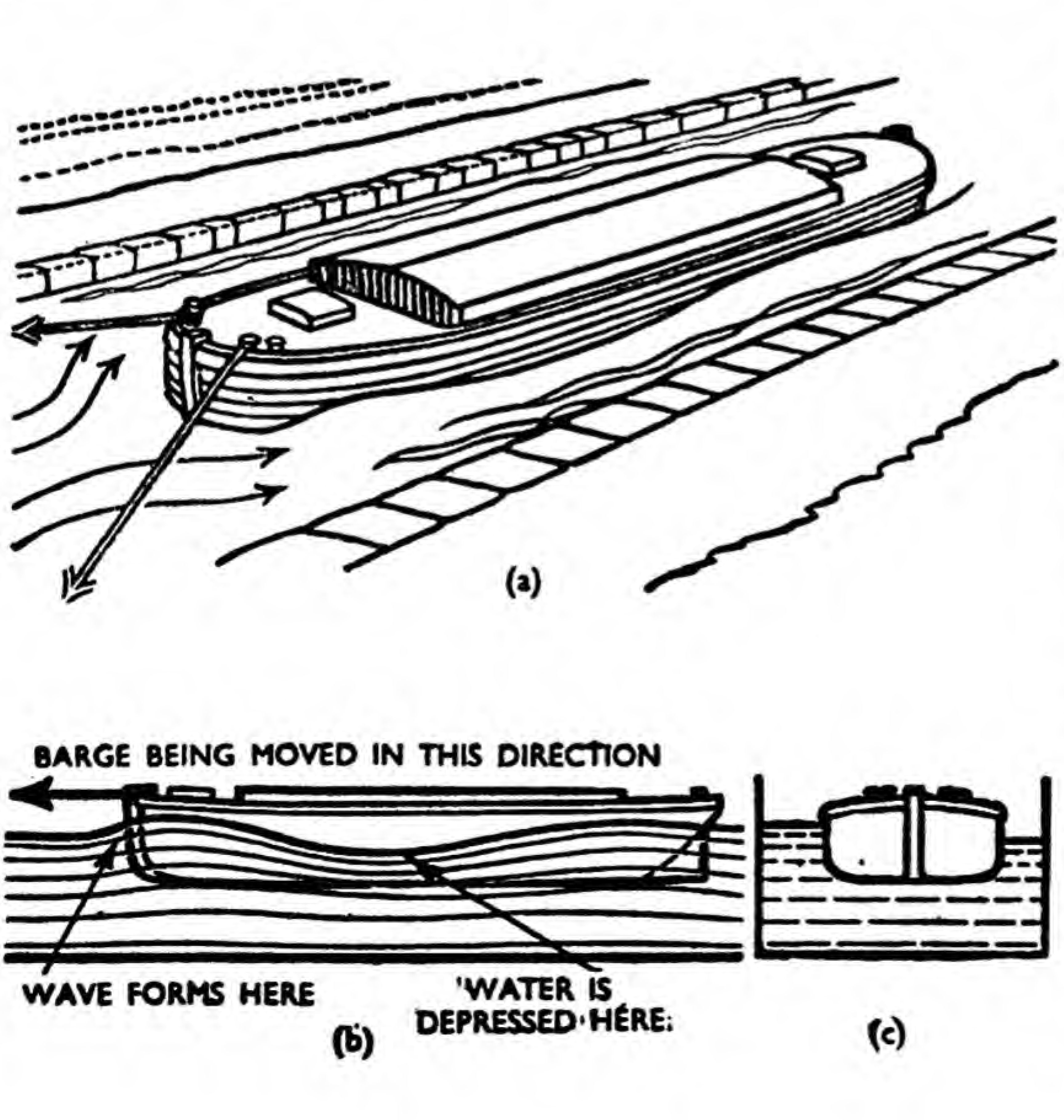


Fig. 7 (left). (*a*) As the barge is pulled forward, the water rushes past at the sides to fill the space left by the barge. (*b*) Illustrates how this causes the water to build up a wave at the front, while the water is depressed to a lower level further back in accordance with Bernoulli's theorem. (*c*) If the barge is closer to one side of the canal than the other, the water is depressed more on the narrower side. As a result, the pressure exerted by the higher level of water on the one side forces the barge towards the other side.

that its energy is possessed entirely in the form of pressure due to its depth. The same water rushing past the sides of the barge, however, possesses energy in virtue of its velocity, as well as pressure energy. And here is seen an application of the principle of the conservation of energy.

Because the water at the sides of the barge possesses velocity energy, its pressure energy, or depth, must, therefore, be less than the depth at the bow. And that is what, in fact, is seen. The water level at the sides is depressed to a lower level, in other words, the water surface dips down. And if the barge is already closer to one side than the other, the water will be depressed more on the narrower side, where the velocity is higher.

As a result, pressure is exerted by the higher level of water on the one side, and the barge is forced towards the other side ; in other words, if it is already closer to one side, the natural forces set up will tend to push it still further to the same side. The difficulty of steering may thus be explained in terms of the mechanics involved.

And so the number of examples of the way in which a knowledge of mechanical principles will provide an explanation of many things happening around us could be multiplied indefinitely.

Modifying Phenomena

But turn now to a consideration of the use that can be made of mechanics to enable us to use, or modify, phenomena to suit our own purposes.

One fundamental mechanical law is that embodied in the words ' action and reaction are equal and opposite,' or ' to every action there is an equal and opposite reaction.' We make use of this law in the ordinary garden sprinkler which is connected to the water tap (Fig. 8).

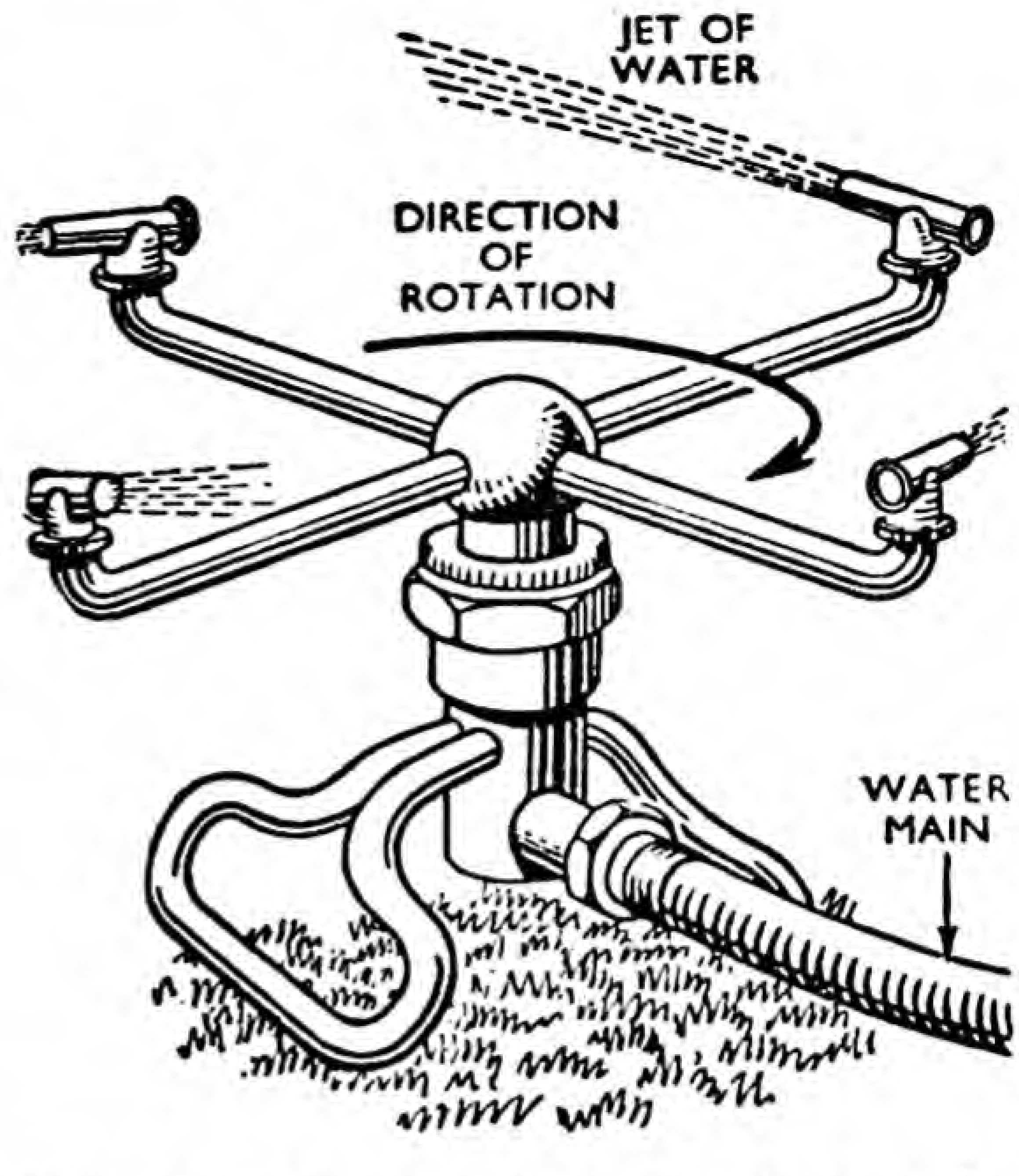


Fig. 8. This mechanical device for watering the lawn is driven by the excess of hydrostatic pressure on the side of the radial arms opposite to the discharge. This causes the upper part of the sprinkler, consisting of the arms and the central boss out of which they spring, to rotate in a direction opposite to that of the discharge. It is interesting to note that the fundamental principle in the jet propulsion of aircraft is similar to that used in this sprinkler.

Automatic Rotation

The sprinkler itself consists of a small revolving head with a number of short open radial pipes bent at their ends at a right angle. They are all turned in the same direction. When the water is turned on, it forces its way out through the radial pipes. But this force of discharge gives rise to an equal and opposite force, causing the whole head of the sprinkler to rotate.

Therefore, no separate power has to be provided to cause the sprinkler to rotate. It takes the power it needs from the water with which it

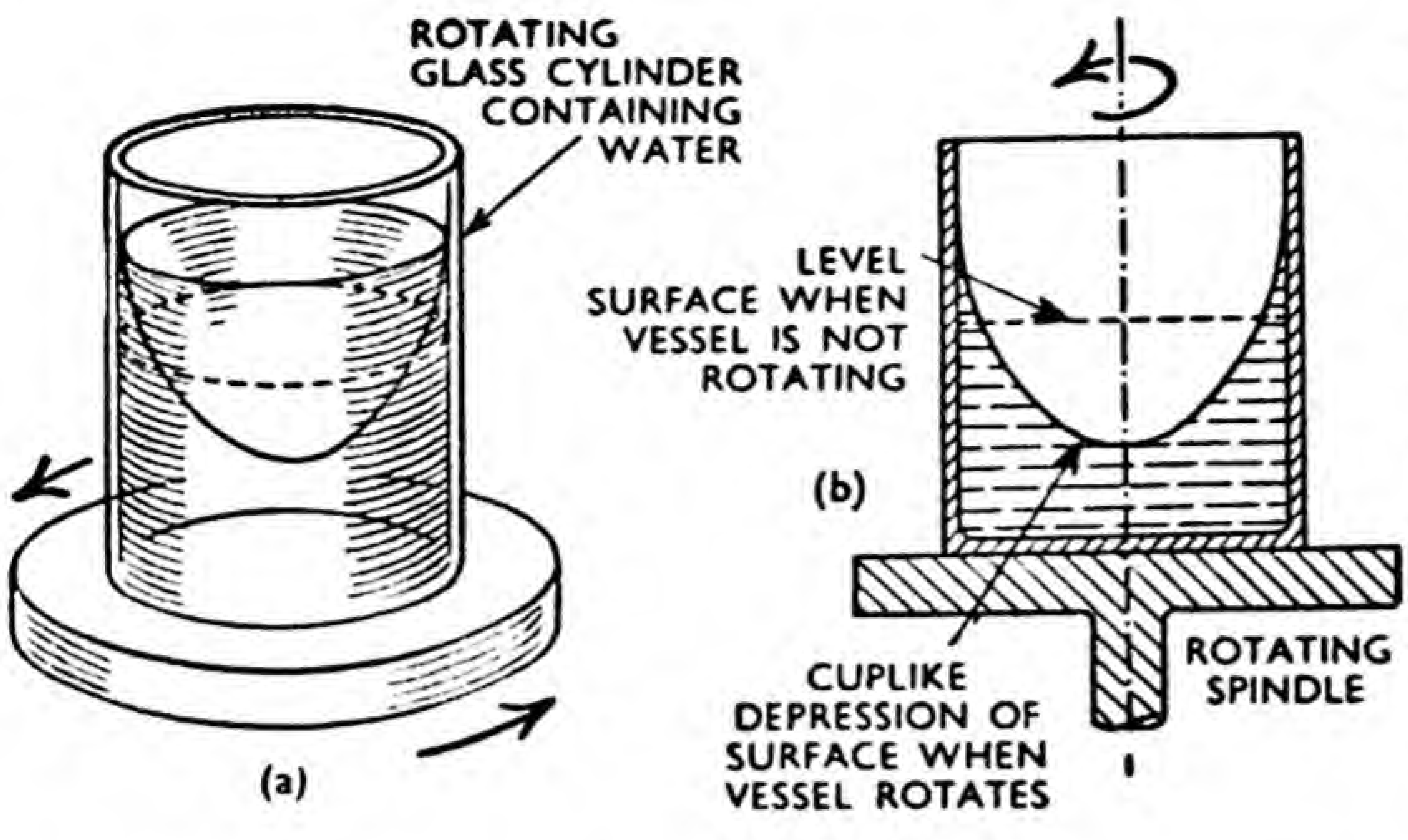


Fig. 9 (left). The speed of revolution of a vertical spindle may be found by a novel form of speed indicator. It consists of an open glass cylinder containing water. The cylinder rotates at the same speed as the spindle, and the resulting depression of the water surface is measured. As this is related directly to the speed of rotation, the latter is at once known.

is supplied, and, so long as the supply of water under pressure is continued, it will rotate, and, in so doing, will distribute the particles of water over the garden.

Forced Vortex

The formation of a free vortex or whirlpool has already been discussed. But this is not the only type of vortex motion known. When we stir our tea, it moves bodily round in the teacup, and its free surface takes up a basin-like

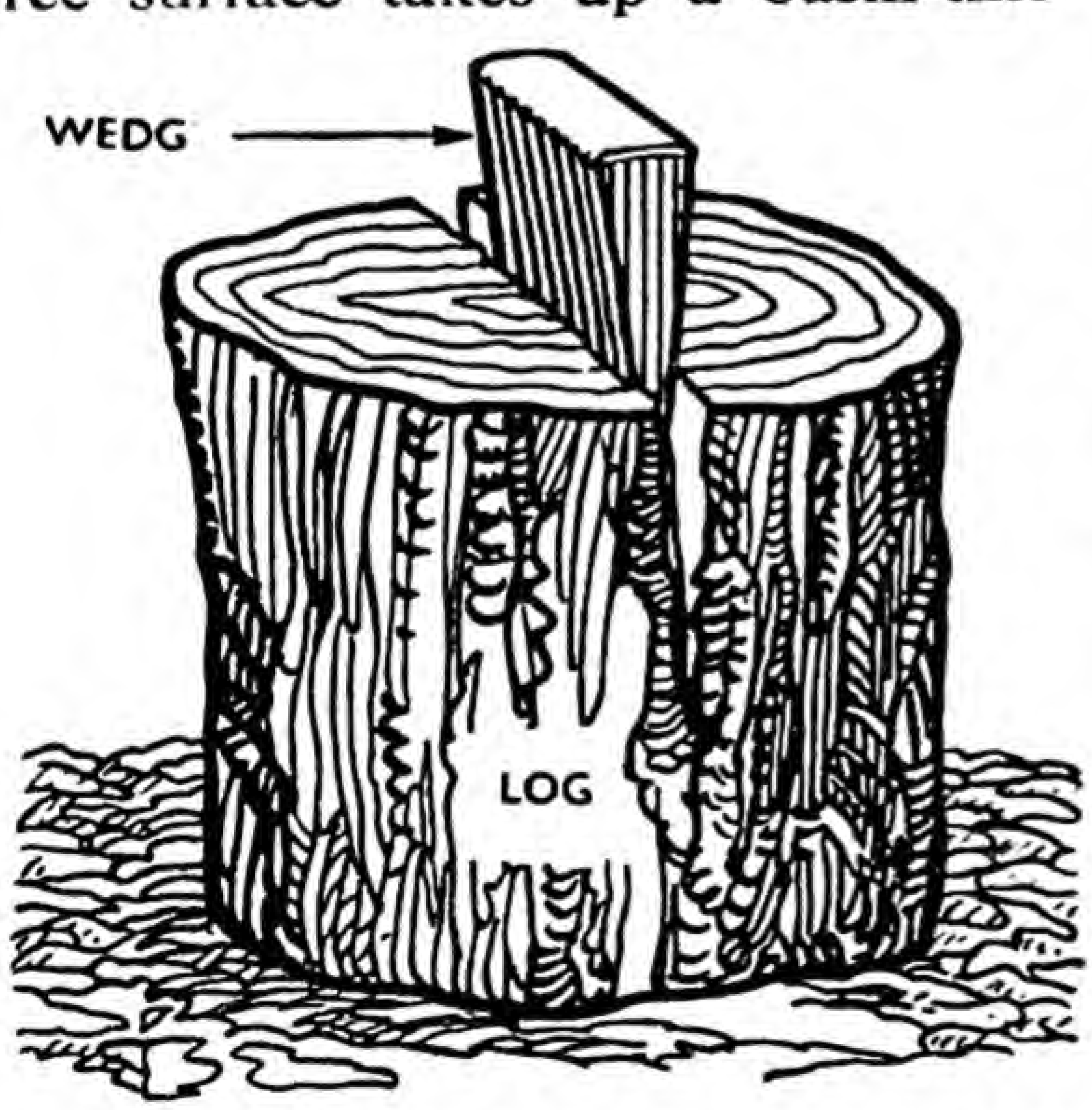


Fig. 10. In the above diagram we see a wedge being used to split a log. It should be noted that driving a wedge in the direction of its length, against a resistance on its slant sides, causes a strong bursting pressure to be set up, exercised against the resistance.

shape, the tea piling up at the outer edges and being depressed at the centre. This type of motion is called the forced vortex, and the basin-like shape of the surface is, in fact, a paraboloid, whose geometrical properties are well known.

The faster the stirring, the deeper the depression of the surface at the centre, and the higher the paraboloid. Therefore, the speed of stirring can be related to this height. In other words, we have a means of measuring the speed of rotation by observing the height of the paraboloid.

Now, it may be objected that nobody wants to know the rate of rotation of the tea in a cup, but the principle can be applied to the measurement of the speed of revolution of a vertical shaft. For this purpose, a glass cylinder containing, say, water, is mounted on the axis for which the speed of rotation is required (Fig. 9).

Internal Friction

The effect of rotation is to cause the water surface to take up the paraboloid form, the physical rotation of the containing vessel being conveyed through the body of the liquid by internal friction, the property known as viscosity, which

now undertakes the work of the spoon in the teacup. The height of the paraboloid is measured, and this gives us at once the corresponding speed of revolution

Sailor's Clear Vision at Sea

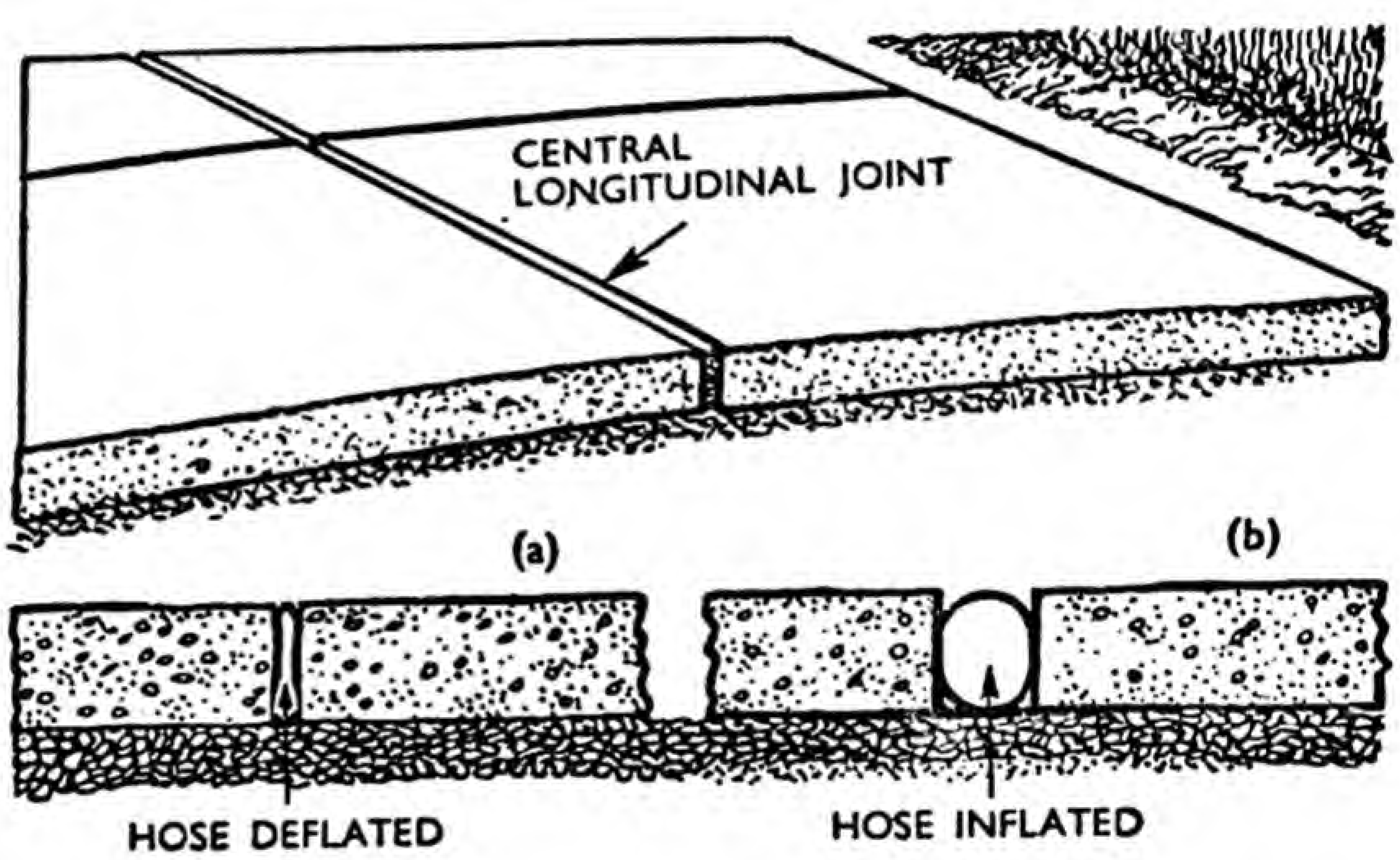
One of the problems of the sailor is to secure a clear view from the bridge under the worst possible conditions of rain and heavy weather. His conditions may be much worse than those of the motorist, who can rely upon a flimsy windscreen wiper, and there must be no uncertainty for him in manœuvring his enormous vessel, which is capable of exerting such tremendous forces. And so, his view through the bridge windows must be clear and unrestricted.

The simple mechanical principle of centrifugal force is brought to his aid for this purpose. If a glass disk is rotated on its axis at high speed, any rain or water thrown against it is immediately flung off to its outer edges, leaving a clear field of vision through the glass disk. This is the basis of the Clear View Screen, which is fitted into the windows of the bridge wherever clear vision is required. Small electric motors are used for rotating the disks, which are made of optically true glass in order to ensure that there shall be no distortion of vision.

The principle of the wedge finds many applications. The effect of driving a wedge in the direction of its length against a resistance on its slant sides, is to set up a strong bursting pressure exercised against the resistance. Hence its familiar use for splitting logs (Fig. 10).

The quarry worker, too, faced with the problem of having to break down large masses of solid rock into smaller pieces, harnesses the power of the wedge. A series of holes are drilled in line in the surface of the rock, and inserted in these are tapered plugs, which are then tapped in until the rock splits along the line of the holes.

Fig. 11 (right). This method of introducing a bursting pressure has been used for moving a concrete road bodily sideways. (*a*) Bituminous mixture is raked out of the central longitudinal joint. Hose is inserted in a deflated condition. (*b*) Air pumped into hose. Pressure exerted laterally against sides of road surfaces, which gradually move over bodily, enlarging gap to required distance.



Moving a Concrete Road

A variation of this method of separating adjacent masses was recently successfully applied on a road job (Fig. 11). The problem was to convert a single-carriageway concrete road into one with dual carriageways, separated by a pedestrian island down the middle. The entire road was moved bodily

several feet, to make room for the island, in the following way.

First, the bituminous mixture was raked out of the central longitudinal joint in the concrete road. This left room for a length of air hose, in its deflated condition, to be inserted. Air was then pumped into this length of hose. The pressure of the air was exerted laterally against the edges of the road surfaces, which gradually moved over bodily, enlarging the gap in the joint. When the gap had been made as large as was permitted by the size of the hose, the operation was repeated, after distance pieces had been inserted to reduce the gap, until sufficient room was left for the pedestrian island to be built in.

Harnessing the Tides

Proposals to harness the power of the tides in order to generate electrical power have frequently been made. Basically, the idea is to convert the potential energy possessed by tidal water at one level, into velocity energy at another level, and to utilize this velocity energy for driving turbines to generate electrical power.

In this country, these proposals are associated with a scheme for the River Severn, where the tidal range is a maximum. As it can be shown that the power obtainable from the tides is, in fact, roughly proportional to the square of the range, this is the most favourable site in this country.

The Severn offers the further advantage that there would be comparative freedom from silting up, due to deposition of solid matter carried in suspension in the water.

There is a tendency for the layman to associate engines and machines with mention of the word mechanics, and he does not always realize that the fundamental principles of the science of mechanics are of universal application. That is why, in this chapter, such miscellaneous matters as whirlpools, the flight of birds, tidal power, and so on, have been discussed, as well as such evidently mechanical things as wedges and fans. Later chapters in the book will deal, in some detail, with the individual mechanical principles in turn, and with their applications, but at the start, the great thing is to appreciate the universality of these principles.

It is an interesting, and should be a sobering, thought that many of these fundamental principles have not so far been formally proved in the sense that a scientist speaks of proof. They are based on evidence derived from experiment, and all the evidence goes to demonstrate their truth and their universality.

Possibly it is the fact that no general proofs have yet been given of their truth, that accounts for the numerous attempts made, from time to time, to produce devices involving perpetual motion. It should be realized that such devices, if they are to be successful, involve the creation of energy. In other words, it must be possible to get more work out of them than is put into them. This would be a violation of the law of the conservation of energy, and no device has yet been made which can be described as successful in that sense.

It follows, then, that if we accept the truth and universality of these laws, we may not break them. Consequently, if we are to avoid futile, although they may be unconscious, attempts to break these laws, we must understand them.

CHAPTER 2

MECHANICS OF EQUILIBRIUM

EQUILIBRIUM AND FORCE. TENSION AND COMPRESSION. VECTORS. PARALLELOGRAM AND TRIANGLE OF FORCES. RESULTANTS AND EQUILIBRANTS. COMPONENTS. POLYGON OF FORCES. MOMENTS. LEVERS. THE LEVER SAFETY-VALVE. THE STEELYARD. RESULTANT OF PARALLEL FORCES. REACTIONS OF BEAMS AND CANTILEVERS. COUPLES. CENTRE OF GRAVITY. CENTROID. CRITERION FOR STABILITY. STABILITY OF VEHICLES. STABLE, UNSTABLE AND NEUTRAL EQUILIBRIUM.

A BODY is said to be in equilibrium when all the forces acting on it are balanced, i.e., there is no resultant or effective force acting. The term statics is often used, quite appropriately, to signify that part of the subject of mechanics which deals with equilibrium. All bodies are subjected to forces, and in statics we are concerned with the conditions of equilibrium of stationary bodies, or of bodies which are moving with constant speed.

Even for those who are not engineers, architects or builders, an elementary knowledge of statics is of great assistance in the everyday tasks of home and business. More important, however, is the fact that such knowledge enables us to understand something of the forces which play their part, often without visible evidence, in the complicated pattern of the world around us.

Vital Principles

Since the stability and safety of buildings and structures depend on the principles of statics, it is understandable that this branch of applied science, both experimental and theoretical, has been the subject of much study from the earliest times. Prehistoric man, in his efforts to make a safer and more comfortable environment, was forced to rely upon methods of trial and error. When building his primitive structures for protection against the elements, wild animals and human enemies, he lacked the advantage of even the most elementary theory and relied upon repeated trials. Undoubtedly errors were frequent and led to practical knowledge through bitter experience.

There is evidence, however, that the Egyptians and Babylonians were able to apply some of the fundamental principles of statics to the construction of their monumental works. The more general theories in use today were largely developed in the sixteenth century and are, therefore, ancient compared with the other branches of mechanics.

Now, if an unbalanced force acts on a body it will cause it to move, or if the body is already moving it will change its speed or direction or both, depending upon the direction in which the force acts.

Thus force is defined as an action which changes, or tends to change, the state of rest or the state of uniform motion of a body. A force

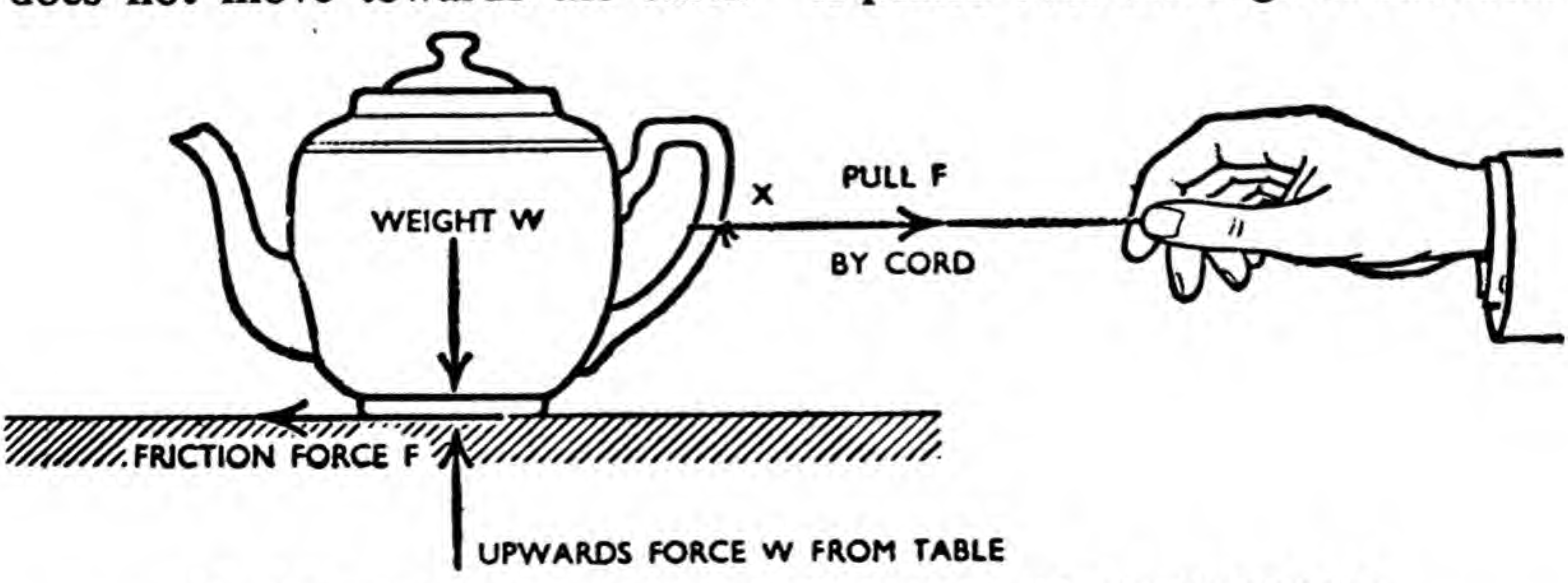
can be exerted by muscular action. For example, a body lying on a table may be pushed or pulled along the surface by a force exerted by the hand and arm. A very important action in statics is the force due to the attraction of gravity which the earth exerts on all bodies, and this force is called weight. A pendant hanging from the ceiling is pulled towards the ground by a force equal to its own weight. Under the action of this force it would fall downward vertically were it not sustained by an equal and opposite force exerted by the supporting cord or chain.

Sources of Force

Magnetic attraction, atmospheric pressure, and wind pressure are other sources of force and all tend to cause motion. In this chapter, however, we are considering bodies which are in equilibrium, i.e., the applied force is balanced by another force or system of forces which is exactly equal and opposite in effect.

Consider, for example, a teapot (Fig. 1) standing on a table. It is attracted to the earth with a force equal to its own weight W; it does not move towards the earth because the table exerts an equal and opposite force W, upward. This may not be an easy point for the reader to grasp, but let us look at it this way. Supposing, in a crowd, one man pushes hard against another man. If this other man requires to remain stationary, he must exert an equal and opposite shove. In fact, he must push back with exactly the same force as is being applied to him. Now supposing further that the first man pushed against a wall instead of another man. The wall doesn't move and the man doesn't move, so we say that the wall is exerting an equal and opposite force to that which is being applied by the man in his endeavour to move it.

Consider again our teapot on the table. If we attach a cord to the handle and pull with a moderate force F, the teapot still does not move because an equal and opposite force F is provided by the friction between the base of the teapot and the table, and the body thus remains in equilibrium. But when the pull on the cord exceeds the frictional resistance, the equilibrium is destroyed, and the teapot then moves. If the teapot is pulled over the edge of the table



TEAPOT AT REST ON THE TABLE IS IN EQUILIBRIUM

Fig. 1. It is pushed upward by the table with a force W equal to the weight of the teapot. Similarly, the horizontal force F in the cord attached to the handle is resisted by an equal and opposite force due to friction against the table.

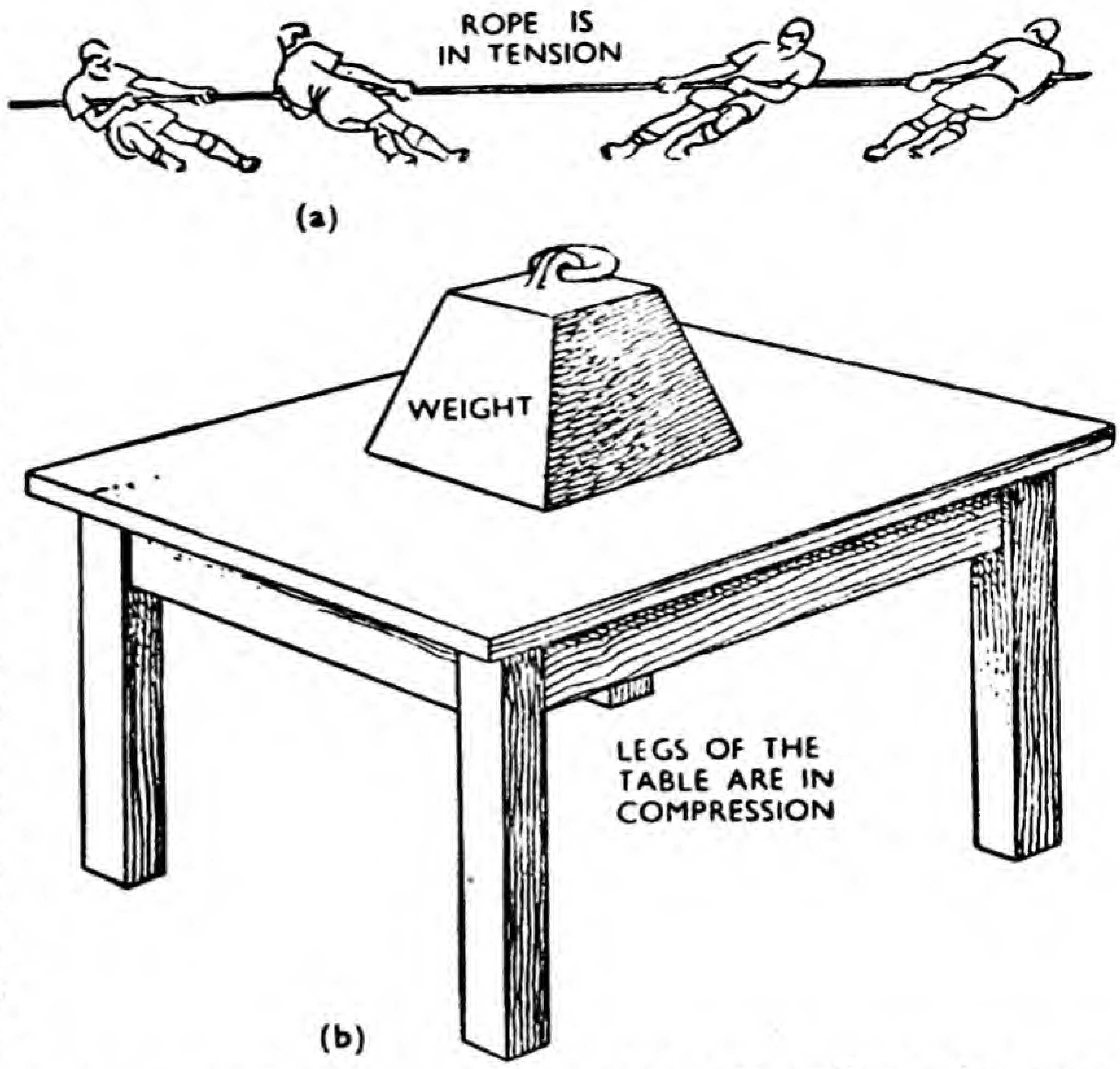


Fig. 2. (*a*) In a tug-of-war, each team pulls on the rope, which tends to become longer; the rope is thus in tension. (*b*) The legs of the table, however, tend to become shorter under the weight, and are in compression.

so that the supporting force provided by the table is no longer available, the downward force W is no longer balanced and the teapot falls to the ground. The motion of a body under such conditions is studied in a later chapter dealing with the mechanics of movement.

Let us, for the time being, take as the unit of force the pound (1 lb.), a unit familiar in everyday life. This is the weight, in London, of a carefully guarded cylinder of platinum which is used as a standard of reference which does not alter with time. Actually, the attraction which the earth exerts on this reference body varies with its position on the earth, but the variation is very small and, for the purpose of this chapter, it may be disregarded.

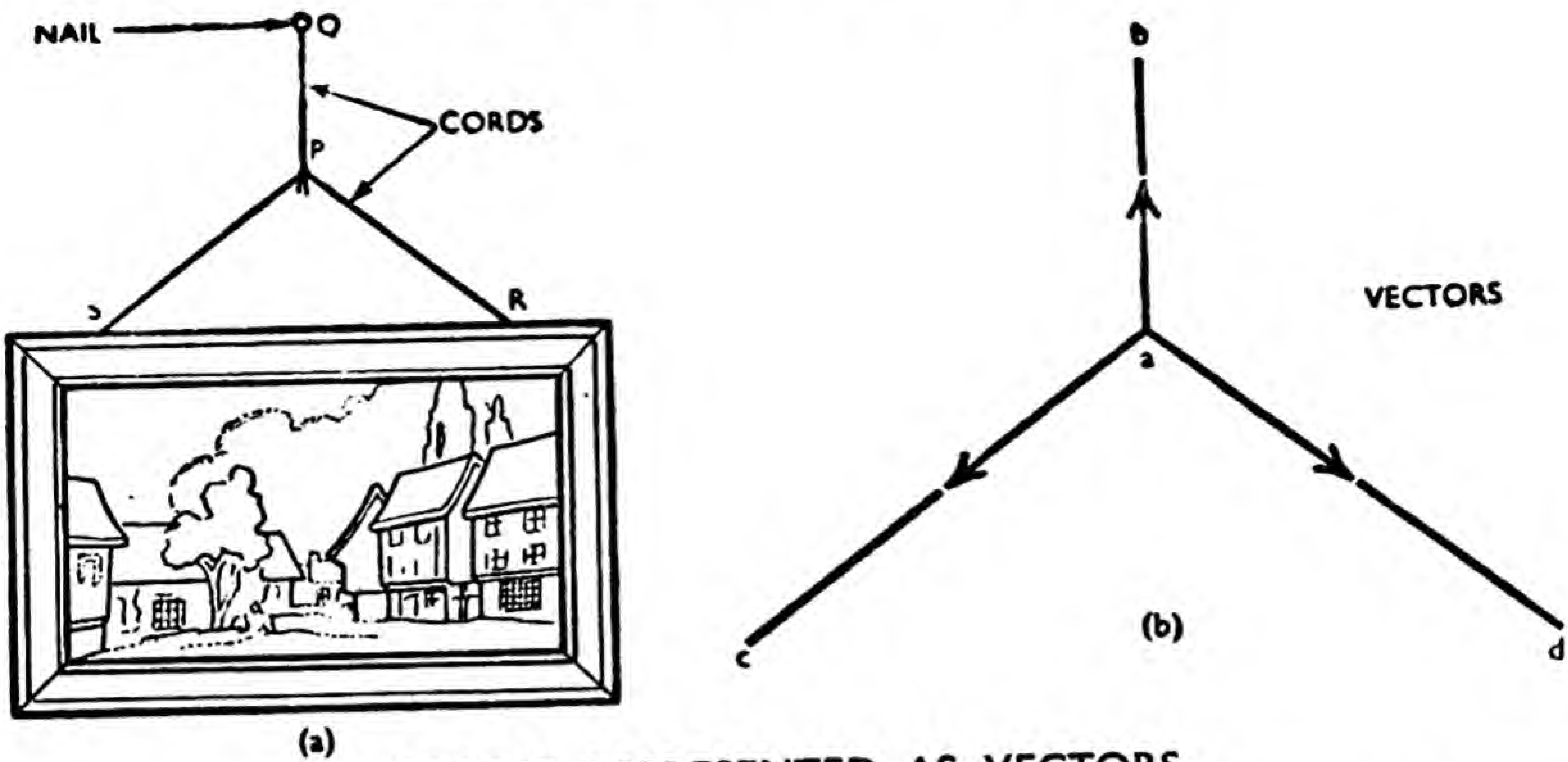
To be strictly correct, we should speak of applying a pull of, say, fifty pounds *weight* to a garden roller, but usually we contract this to 50 lb. It is understood that the force exerted on the roller is equivalent to the weight of fifty of the standard cylinders of platinum. Fractions or multiples of the pound give rise to other units of force such as ounces, stones, hundredweights and tons. In continental practice and in most scientific work the unit of weight is the kilogram, which is equivalent to approximately $2\frac{1}{5}$ lb.

If a force which is applied to a body tends to make it longer and to pull it apart, we say that the body is in tension and that the force is tensile. Conversely, a force which tends to crush and shorten a body is compressive and the body is in compression.

Tension and Compression

Fig. 2 illustrates simple examples of tension and compression, and many others will, no doubt, occur to the reader. If we lean on a walking-stick, the force in it is compressive, but if we hook it round a branch of a tree and pull, then the force is tensile. The reader should study everyday objects such as the clothes-line and prop, chains, pillars, etc., to decide whether they are in tension or compression.

To define a force completely, we must know (1) its magnitude, (2) its direction, (3) its sense and



FORCES REPRESENTED AS VECTORS

Fig. 3. (*a*) In this illustration a picture frame is shown hanging from a nail at Q by three cords meeting at a point. (*b*) Shows how the forces in these cords are represented graphically as vectors.

(4) its point of application. For example, considering Fig. 1, the applied force *F* is completely defined if we say that its magnitude is 4 oz., and its direction is parallel to the top of the table. The sense of the force is a pull in a direction away from the spout, and its point of application is the point *X* in the handle of the teapot.

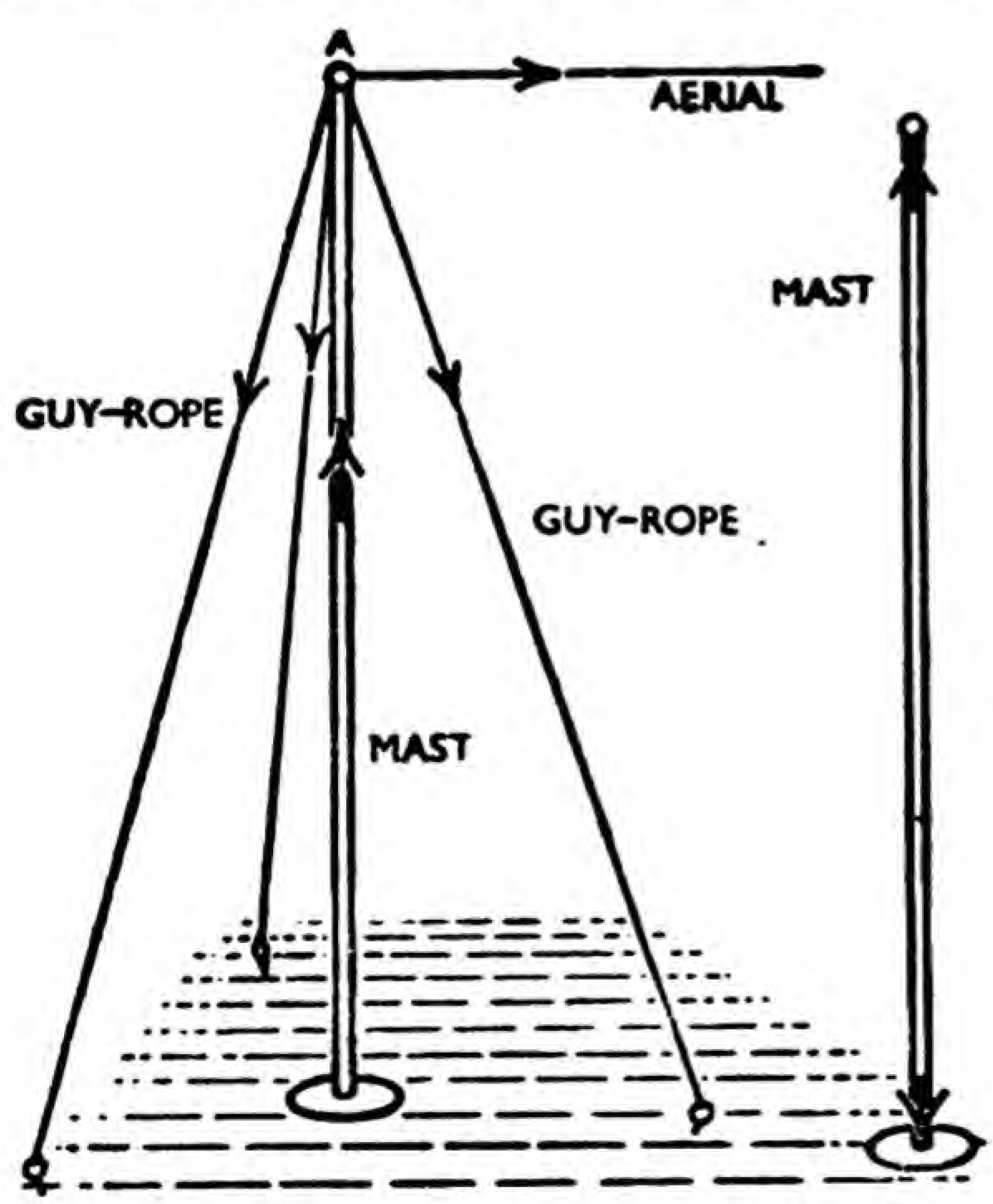


Fig. 4. Conventional method of representing the sense of forces in a wireless mast, supporting guy ropes, and aerial.

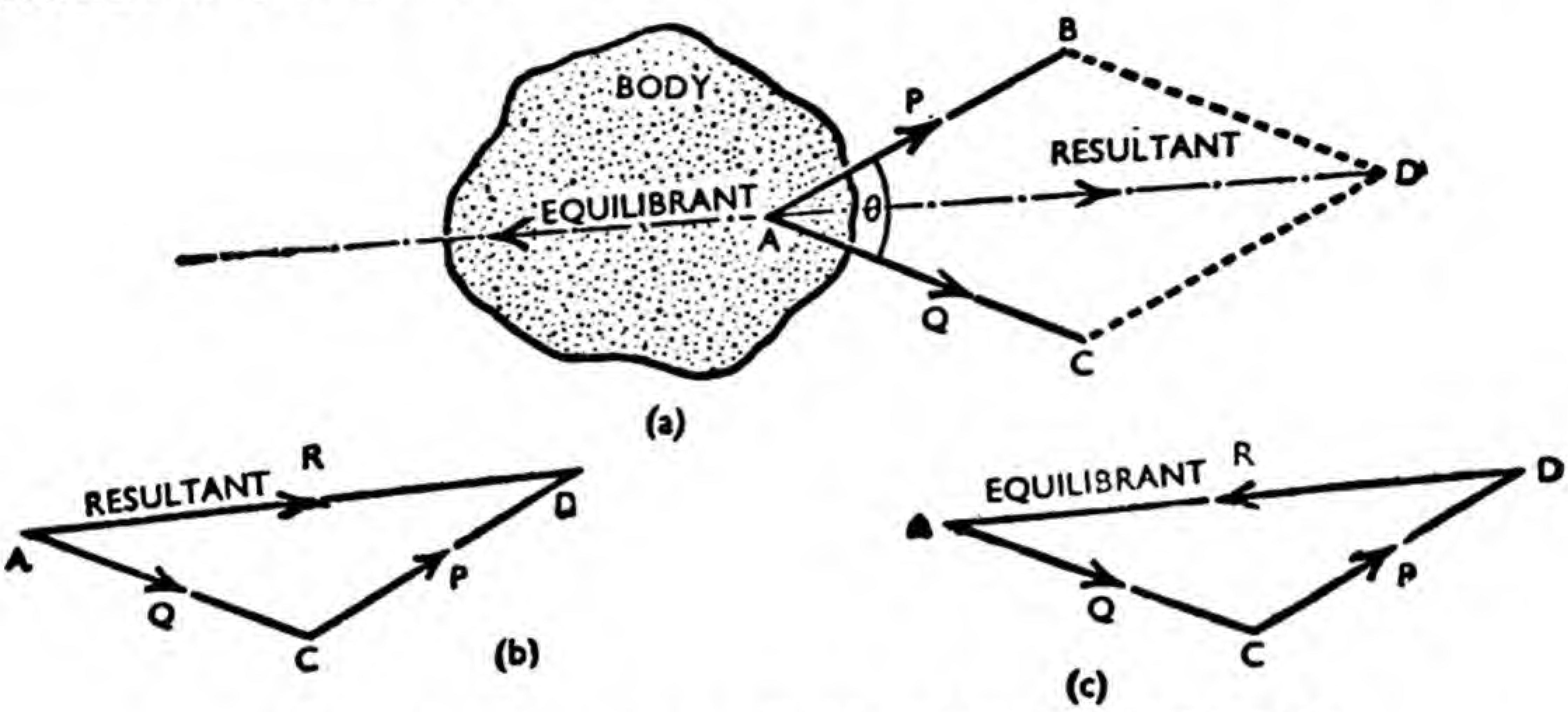
Vectors Explained

These qualities can be conveniently represented in graphic form by a straight line called a vector. Fig. 3(a) shows a picture and frame suspended from a nail at *Q* by three cords meeting at the knot *P*. The cord *PQ* pulls upward from *P* with a force equal to the weight of the picture, and this is indicated by the line *ab* in Fig. 3(b), which is of such a length, drawn to scale, that it represents (1), the magnitude of the force. The line *ab* is drawn parallel to *PQ*, thus representing (2), the direction of force. The sense is indicated by the arrowhead on *ab* showing that the force is pulling upward and away from *P*. Such a line is known as a vector. The point of application (4), is not indicated in the vector diagram, but it is evidently the point *P* in Fig. 3(a). In the same manner, the vectors *ac* and *ad* represent the forces in the cords *PS* and *PR*, and

we shall see later how the magnitudes of these forces can be determined.

Care must be taken when drawing each arrowhead to remember that it indicates the sense of the force relative to the point of application. Thus, the tension in *PQ* is pulling upward from *P*, and since the vector diagram represents the forces acting at this point, the arrowhead points upward. But points downward, as shown in the separate diagram of the mast.

Considering this separate diagram, the arrowheads drawn on it might suggest to the unwary that it was being pulled at each end, and was, therefore, in tension. We can avoid this error if we always remember that each arrowhead shows the sense of the force in relation to the point at which it acts, and does not refer to the body through



FINDING THE RESULTANT OR EQUILIBRANT

Fig. 5. Resultant or equilibrant of two forces may be found by completing a parallelogram having, as adjacent sides, vectors representing the forces. Alternatively, a triangle of forces will be found to give the same result.

this same force is pulling downward from *Q* and, if the vector referred to this point, the arrowhead would be pointing downward.

Sense of Forces

Consider, for example, the forces acting at point *A*, the top of a guyed mast for a wireless aerial (Fig. 4). The aerial and guy-ropes are pulling away from *A* with tensile forces, so the arrowheads on them point away from *A*. On the other hand, the mast is clearly in compression, so its arrowhead points towards *A*. At its base the mast presses on the ground ; in this case the arrowhead representing the sense of this force which it is applied. Stress is laid on this because it is important in the study of many structures ; the arrowheads are used to determine the nature of the forces, whether compressive or tensile, in the parts of the structure.

Confining our attention to forces in one plane and meeting at a point, we shall now consider several important results.

If two forces *P* and *Q*, represented by the vectors *AB* and *AC* in Fig. 5(a), act on a body at a point *A*, their combined effect is equivalent to a single force, represented by the vector *AD*, which is obtained by completing the parallelogram having *AB* and *AC* as

adjacent sides. This equivalent force is called the resultant of the two original forces, and the diagonal vector represents it in magnitude, direction and sense, with the arrowhead as shown in Fig. 5. If no other forces were acting, the body would move in the direction AD as if the single force AD were alone acting on it.

Trigonometry Avoided

The magnitude of the resultant R may be readily calculated with the aid of simple trigonometry. For example, if the angle between the given forces is θ, then,

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}.$$

Sufficient accuracy for most practical purposes is, however, attained by drawing the figure to scale and measuring the resultant directly. Therefore, those readers who do not know anything about trigonometry need not fear that they have come to a dead stop where, for the sake of those who are able to understand it, simple trigonometry is introduced.

Obviously, it is unnecessary to draw the whole parallelogram, since we can draw the vectors AC and CD, as in Fig. 5(b), when AD is obtained as the resultant. If the body is imagined to be free of all forces other than P and Q, it would move in the direction of the resultant. To maintain equilibrium, therefore, a force having the same direction and magnitude as the resultant, but opposite in sense, would be required. This is obtained by drawing the same triangle but with the arrowhead pointing from D to A, as shown in Fig. 5(c). The force DA required to balance P and Q is appropriately known as the equilibrant.

We are now in a position to make one of the most important fundamental statements in statics. If three forces acting at a point are in equilibrium, they can be represented vectorially by the sides of a triangle, with the arrowheads pointing in the same direction around that triangle. It is equally true to say that, if a body is acted upon by three forces for which the vectors form a triangle with the arrowheads pointing in the same direction around that triangle, then the body is in equilibrium.

These figures are known as the parallelogram and the triangle of forces,

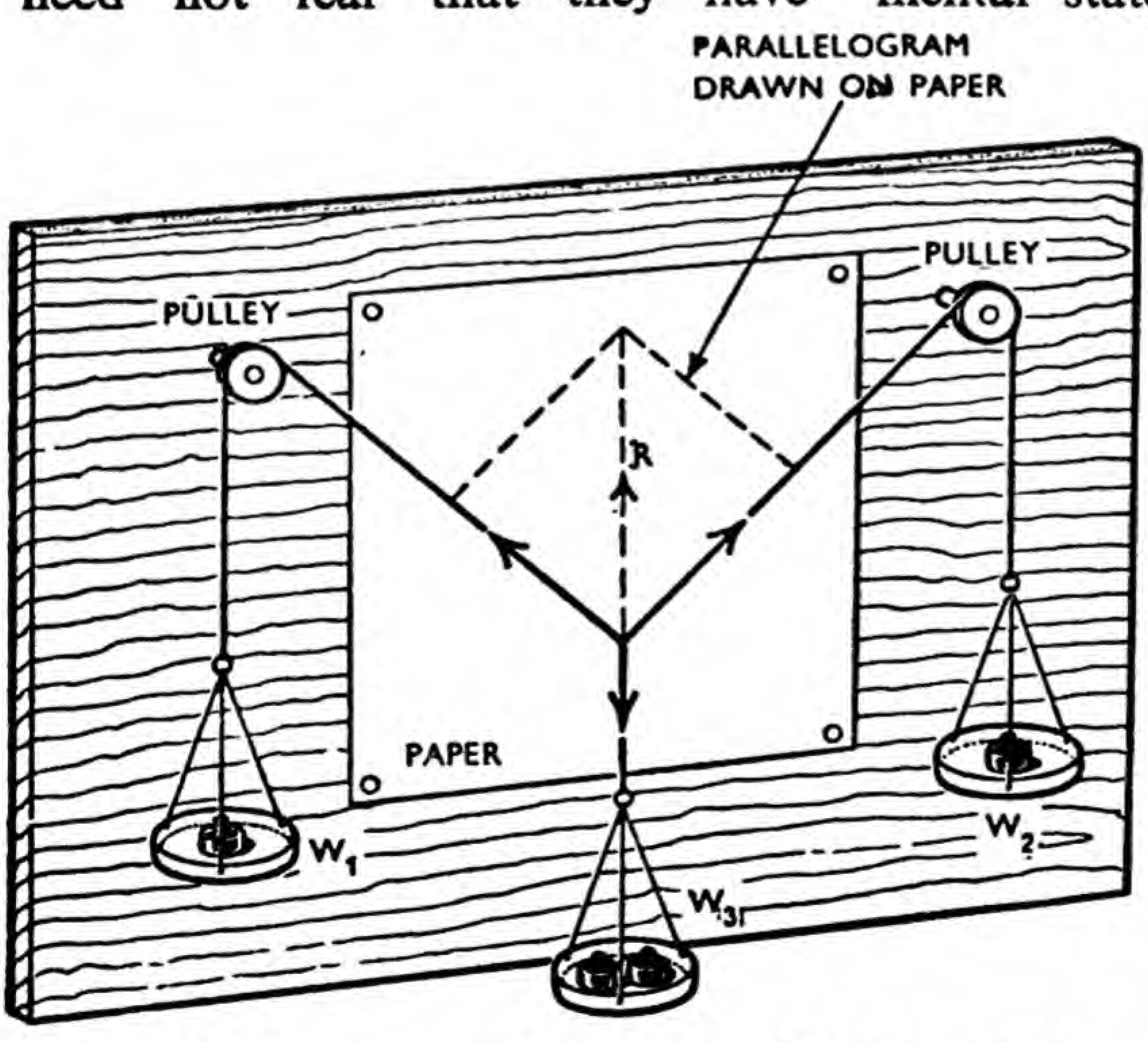


Fig. 6. This apparatus is used to verify the parallelogram of forces. Completing the parallelogram with vectors representing W_1 and W_2 as adjacent sides, we find the diagonal R represents a force equal and opposite to W_3, which is the equilibrant of W_1 and W_2.

and the results can readily be verified with the aid of the apparatus shown in Fig. 6.

Cords carrying weights W_1 and W_2 pass over light pulleys and join in a knot from which is suspended a third weight W_3. After suitable weights have been placed in the scale-pans, the system will oscillate to some extent, but will quickly attain its equilibrium position. Provided the frictional resistance of the pulleys is small, we may assume that the tensions in the three cords are W_1, W_2 and W_3. The point of application of these forces is the knot, which is also the body upon which they act.

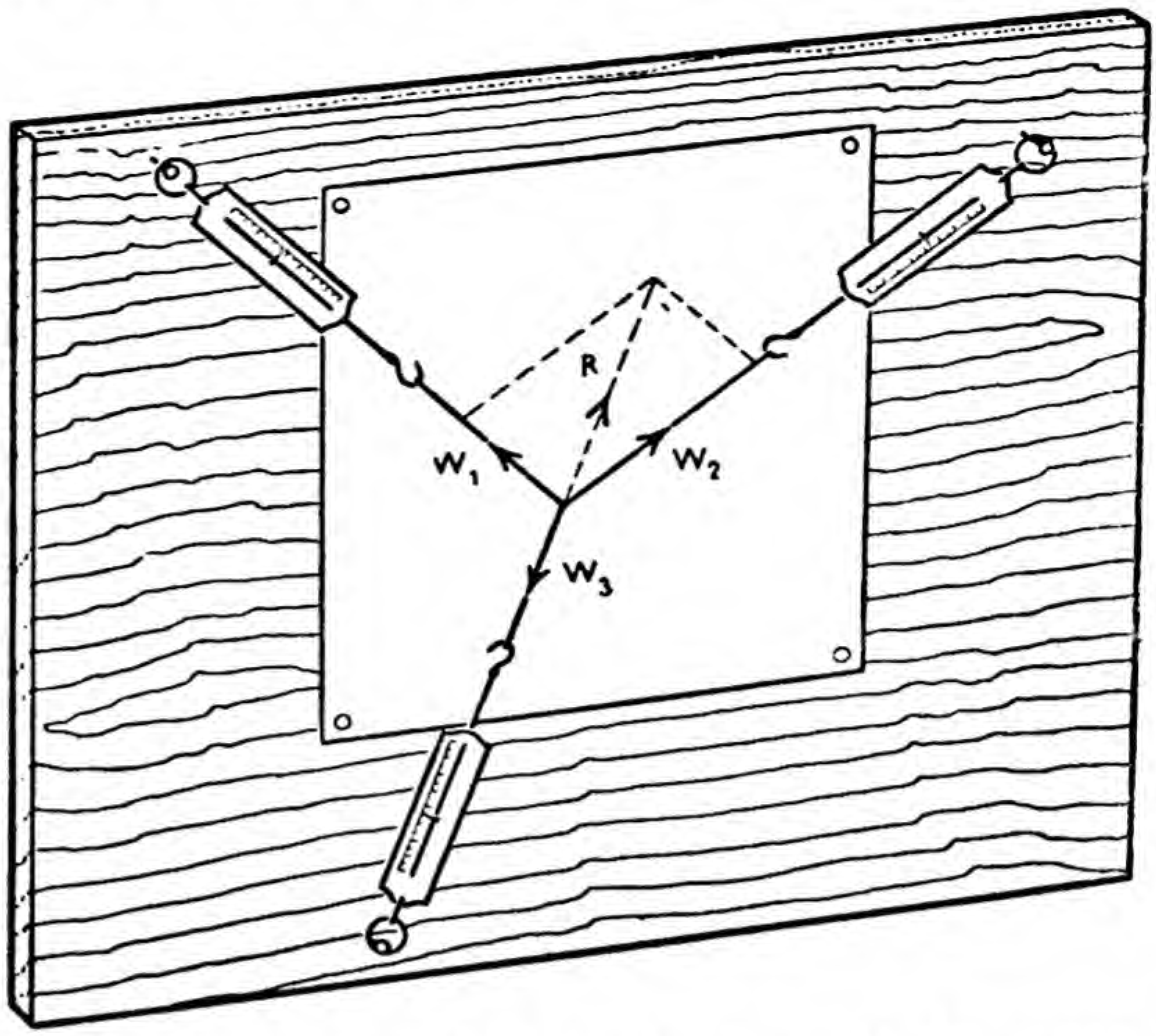


Fig. 7. Parallelogram of forces can be verified by using spring balances instead of weights and pulleys.

On a piece of paper pinned to the board behind the cords, we draw, in the appropriate directions and to scale, vectors representing W_1 and W_2, and complete the parallelogram of forces.

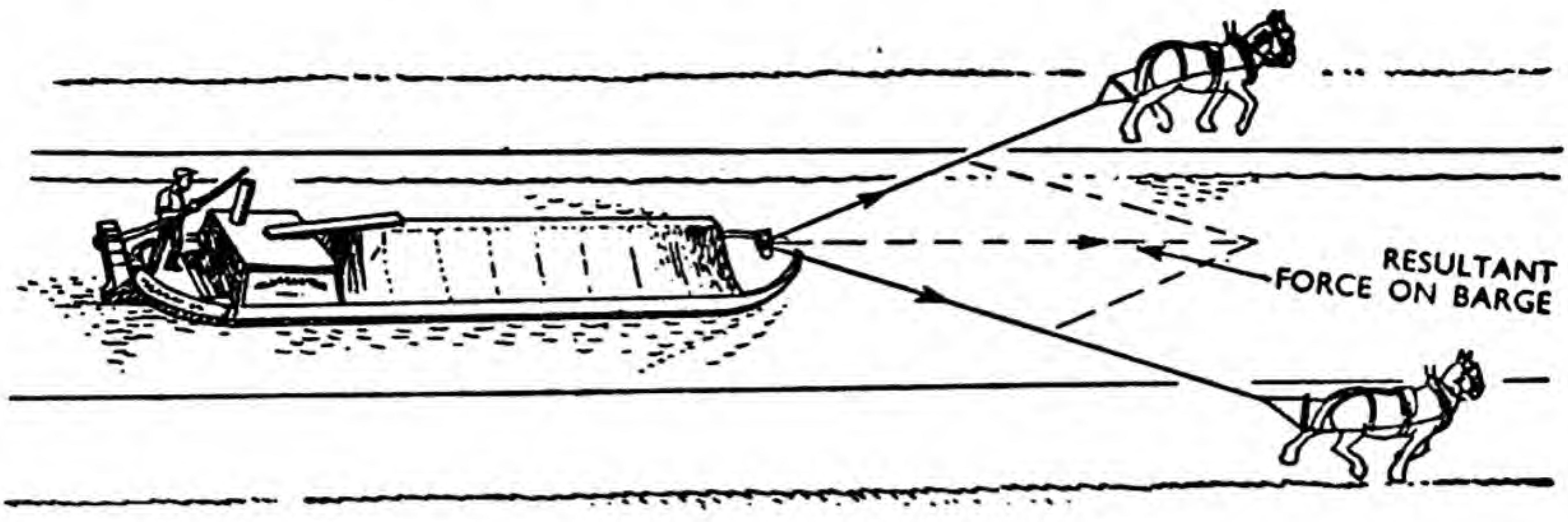
The diagonal R of the parallelogram is drawn, and whatever the magnitudes of the forces, it will always be found that R has the same magnitude and direction (vertical) as W_3. Since the forces are in equilibrium, W_3 is the equilibrant of W_1 and W_2, and since R is found to be equal and opposite to the equilibrant, we conclude that the diagonal represents the resultant of W_1 and W_2. Of course, the loads must be such that equilibrium is possible; for example, the demonstration obviously fails if W_3 is greater than $W_1 + W_2$.

Alternatively, spring balances may be used, as shown in Fig. 7, instead of weights and pulleys. Notice that, in this case, W_3 and R are not necessarily vertical but they will be found to be always equal and opposite as before.

Useful Applications

The parallelogram and triangle of forces find many useful applications, and an example of this is shown in Fig. 8. If each horse is pulling with a force of, say, 200 lb., it does not mean that the force moving the barge forward is 400 lb. Clearly, to find the effective force we must find the resultant of the two forces as indicated in the figure.

To obtain a numerical value in pounds for the resultant, we should draw to scale two lines, representing the forces exerted by the horses, at an angle equal to that between the tow-ropes. We should then complete the parallelogram, and the



EFFECTIVE DRIVING FORCE

Fig. 8. When a barge is drawn by two horses on opposite tow-paths of a canal, the effective driving force is the resultant of the forces in the two ropes. This resultant can be found by drawing a parallelogram or triangle of forces.

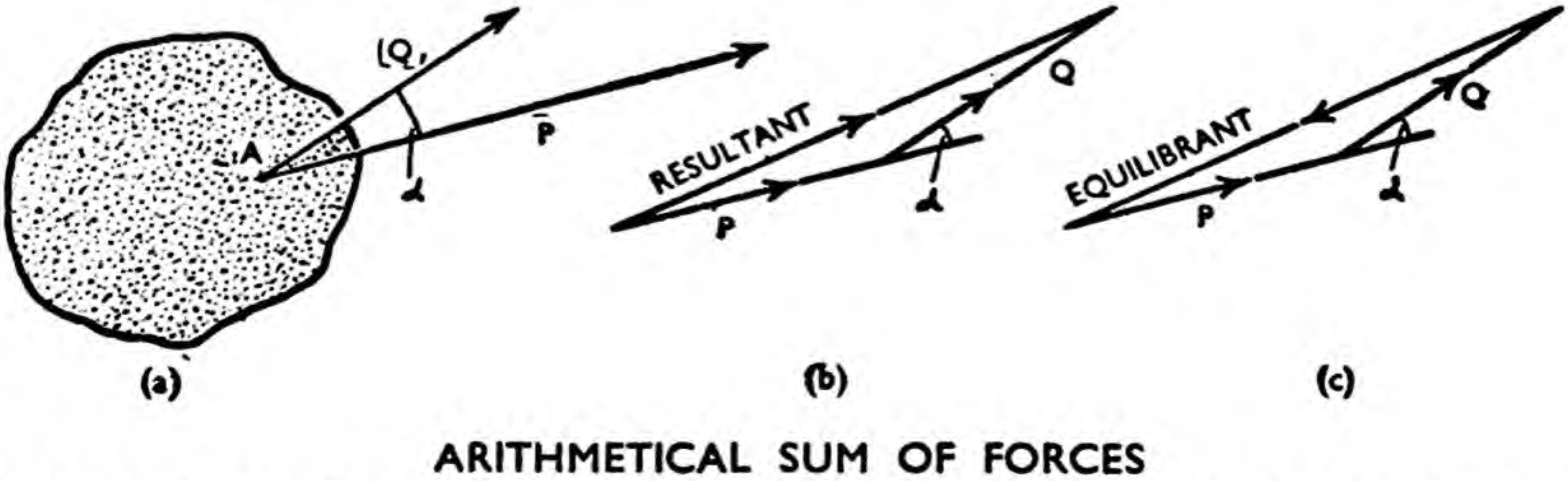
diagonal would represent the resultant in magnitude and direction. The sense and point of application are known, so that the resultant is completely defined. Alternatively, the triangle of forces may be used with the same result.

If two forces P and Q in Fig. 9(a) act on a body at a point A, and make a very small angle α with each other, we can obtain their resultant (b) or equilibrant (c) by drawing the triangle of forces.

Now if the angle α is decreased until it becomes nil, it is clear that the triangle becomes a straight line, and the resultant then equals the arithmetical sum of the forces, and acts in the same direction as the forces. If the two forces act in the same direction, but in opposite senses, the resultant is equal to their difference and has the sense of the larger force.

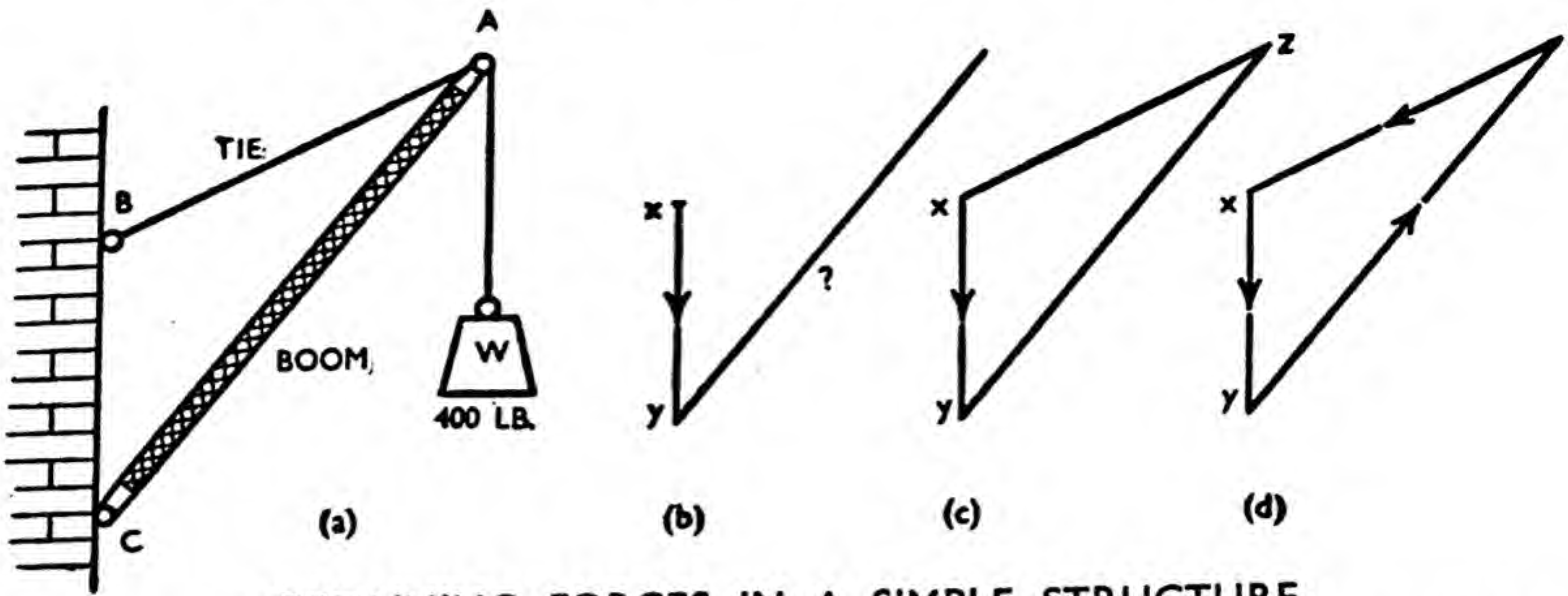
Components

Just as the triangle of forces is used to determine resultants and equilibrants, so it may be used to solve the converse problem. If we are given an equilibrant or resultant and the directions of the two original forces are known, their magnitudes can then be determined. Two such forces, which are equivalent to a single force, are known as the components of the single force. A vector is drawn to represent the



ARITHMETICAL SUM OF FORCES

Fig. 9. When the angle between two forces becomes smaller and smaller, the triangle of forces becomes more and more acute until eventually, when the angle is nil, the triangle becomes a straight line. The resultant or equilibrant of forces having the same direction and sense is thus the arithmetical sum of the forces.



OBTAINING FORCES IN A SIMPLE STRUCTURE

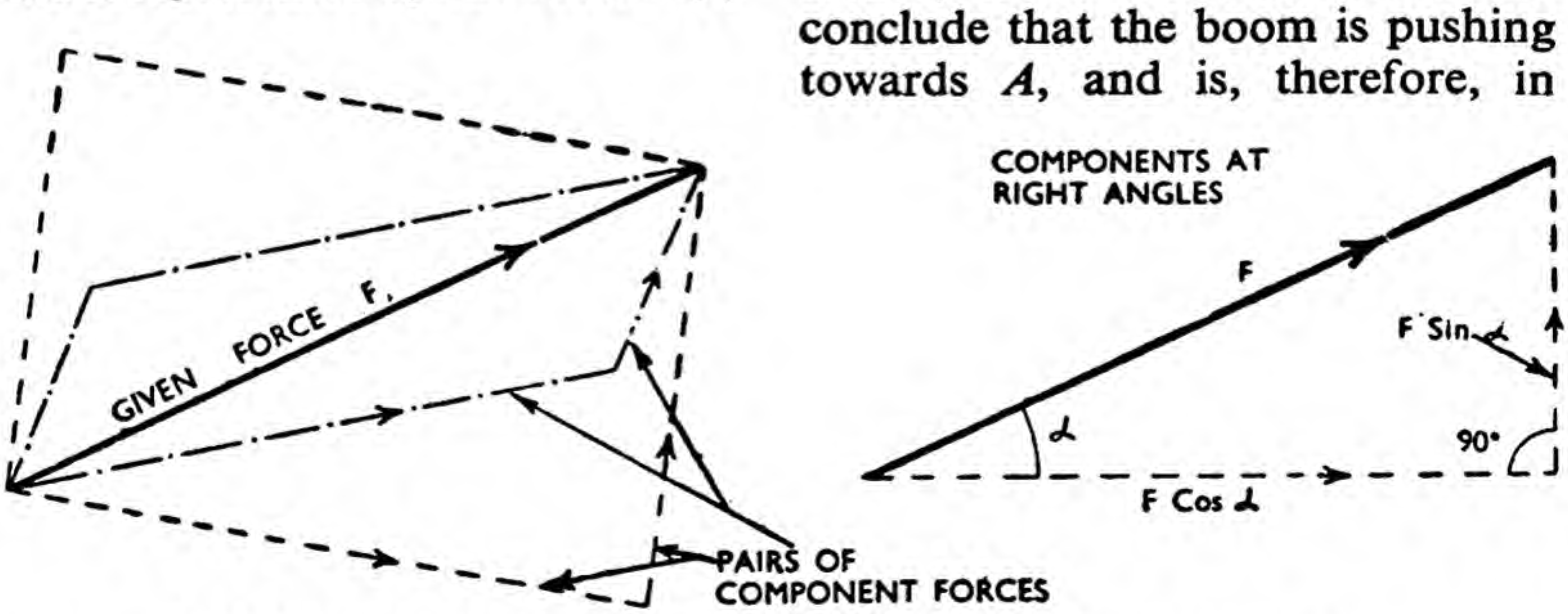
Fig. 10. When a load is suspended from a wall-crane, the forces in the tie and the boom can be found by drawing the triangle of forces for the point *A*.

known single force, the triangle is completed by drawing parallels to the directions of the components, and then the magnitudes of these two unknowns can be scaled from the drawing. This is best illustrated by a simple example.

Practical Example

Let us imagine that a weight of 400 lb. is suspended from the wall-crane shown in Fig. 10(a), and we wish to find the forces in the boom *AC*, and the tie *AB*. We consider the equilibrium of the point *A*. The force in the cable *AW* is 400 lb. acting vertically downward, and this is represented by the vector *xy*, drawn vertically as in Fig. 10(b). If we use a scale of, say, 1 in. = 100 lb., then *xy* would be made 4 in. to represent 400 lb. The direction of the force in the boom is known, and so through *y* we draw a parallel to *AC*.

Similarly, through *x* a line is drawn parallel to *AB*, and we obtain the intersection *z* in Fig. 10(c). By measuring the lengths of *yz* and *zx*, we know the magnitudes of the forces in *AC* and *AB*. But the point *A* is in equilibrium ; therefore, the arrowheads must point in the same direction around the triangle as shown in Fig. 10(d). Hence, from the arrowheads we conclude that the boom is pushing towards *A*, and is, therefore, in



RESOLVING A FORCE INTO PAIRS OF COMPONENTS

Fig. 11. A force may be resolved into an infinite number of pairs of components depending upon the angles of the parallelogram. The most important case occurs when the components are at right angles to each other.

compression. The tie AB is pulling from A and is in tension in a corresponding manner. Note that W is the equilibrant of the component forces in AB and AC. Equally, of course, any one of the three forces is the equilibrant of the other two.

Resolution of Forces

We can always find two components which are equivalent to a given force F, by drawing a parallelogram of forces (Fig. 11). With the same diagonal, an infinite number of pairs of components can be obtained depending upon the angles of the parallelogram, and this process is called the resolution of a force. The most important case of resolution occurs when the components are at right angles to each other. The components then have magnitudes of $F \sin \alpha$ and $F \cos \alpha$, as shown.

We may now consider a practical example. Suppose a railway wagon (Fig. 12) is pulled by means of a rope with a force of 100 lb. The rope is horizontal but inclined to the direction of the rails at an angle of 30 deg. What is the force tending to pull the wagon forward?

To answer this question, we resolve the force of 100 lb. into two components, one, at right angles to the rails, of magnitude 100 sin 30° = 50 lb., and one in the direction of the rails, of magnitude 100 cos 30° = 86·6 lb. The latter tends to move the wagon forward, while the former merely presses the wheel flanges against the rail. Readers who are not familiar with trigonometry can obtain these results by drawing.

Sometimes we use the triangle of forces to determine the direction of forces of known magnitude. For example, Fig. 13 shows a simple experiment with pulleys, cords and weights. What will be the sag in the cord under the twelve-pound weight? In this case we cannot draw parallels since the

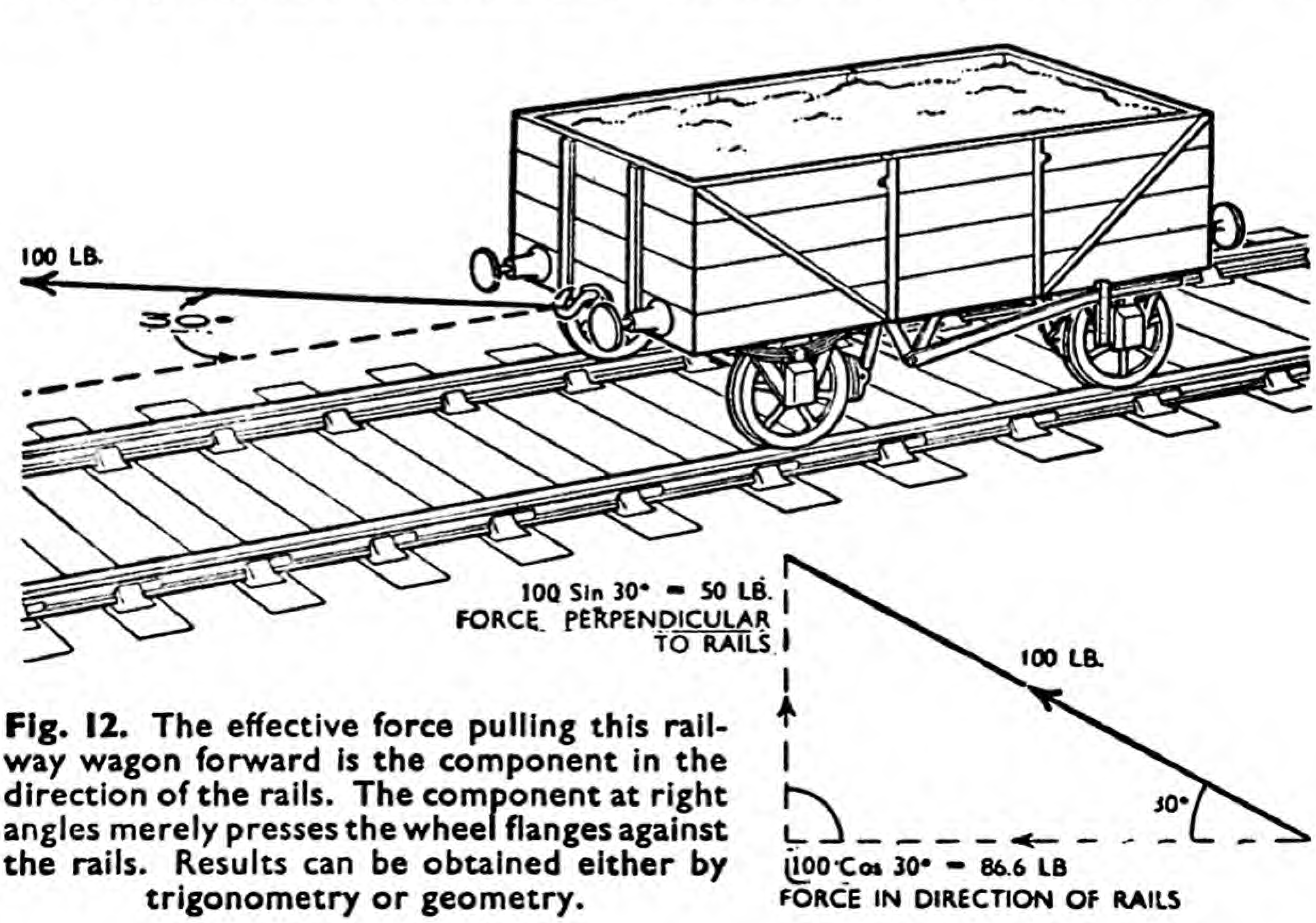
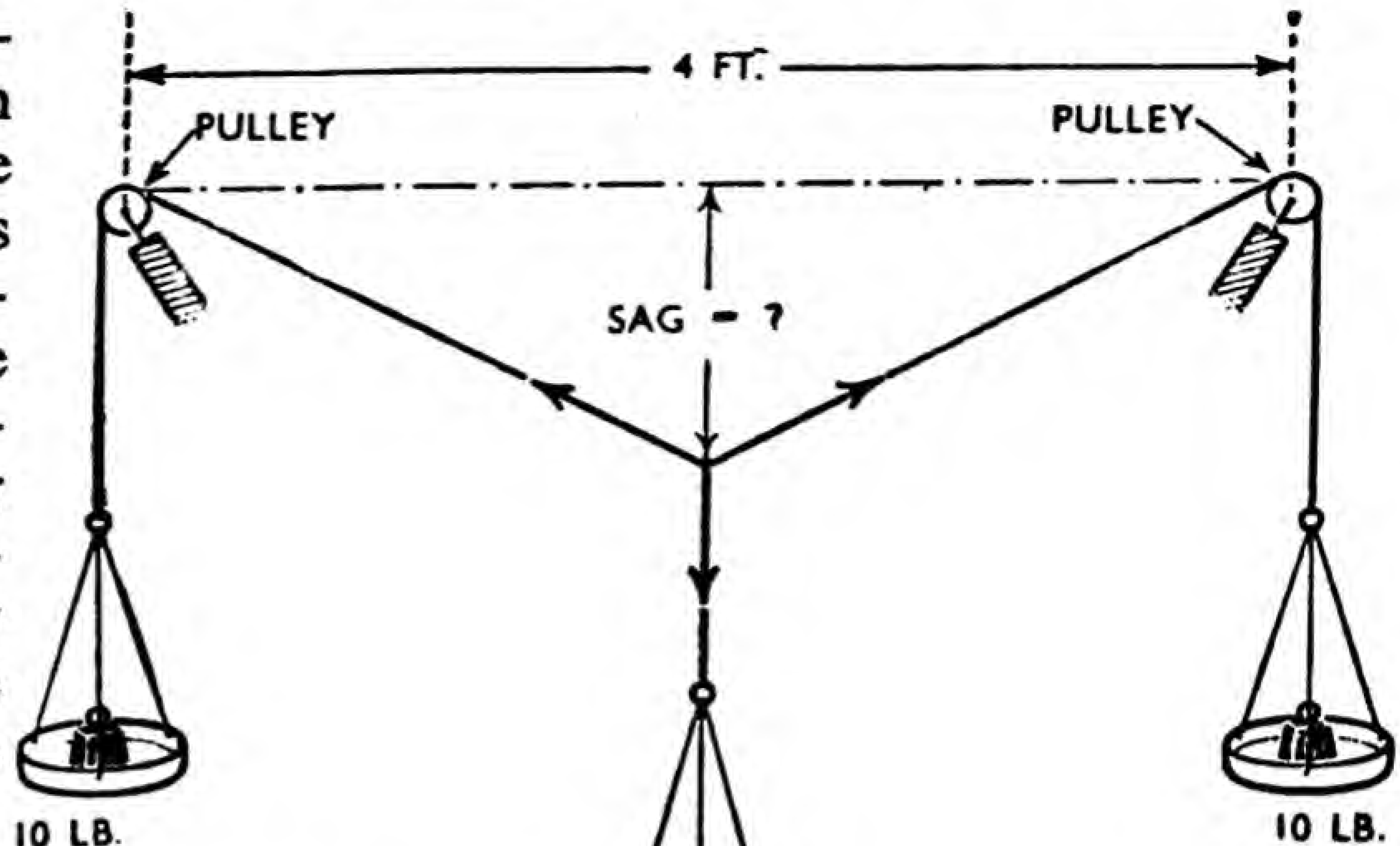


Fig. 12. The effective force pulling this railway wagon forward is the component in the direction of the rails. The component at right angles merely presses the wheel flanges against the rails. Results can be obtained either by trigonometry or geometry.

directions are unknown, but we can complete the triangle by intersecting arcs as shown. The directions of all three forces are then obtained, and the reader should have no difficulty in finding, by drawing or by simple geometry, that the sag is 1½ ft.



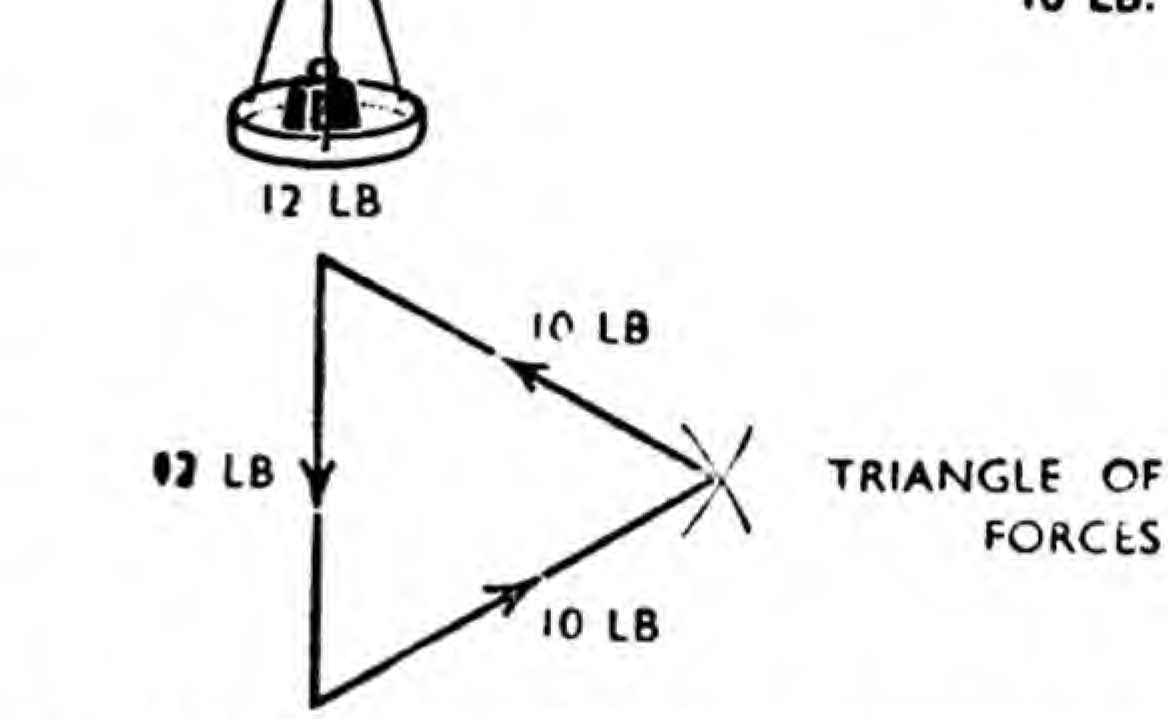


Fig. 13. In this example, we require to find the sag in the cord. Knowing the magnitudes of all three forces we draw the triangle of forces by intersecting arcs. Then, by proportion, the sag is found to be 1½ ft.

For a final application of the triangle of forces, we are able to express the statement in another form. If a body under the action of three forces is in equilibrium, then the lines of action of these forces meet at a point. This is a useful form which in many cases leads to a simple, direct solution of a problem.

Another Example

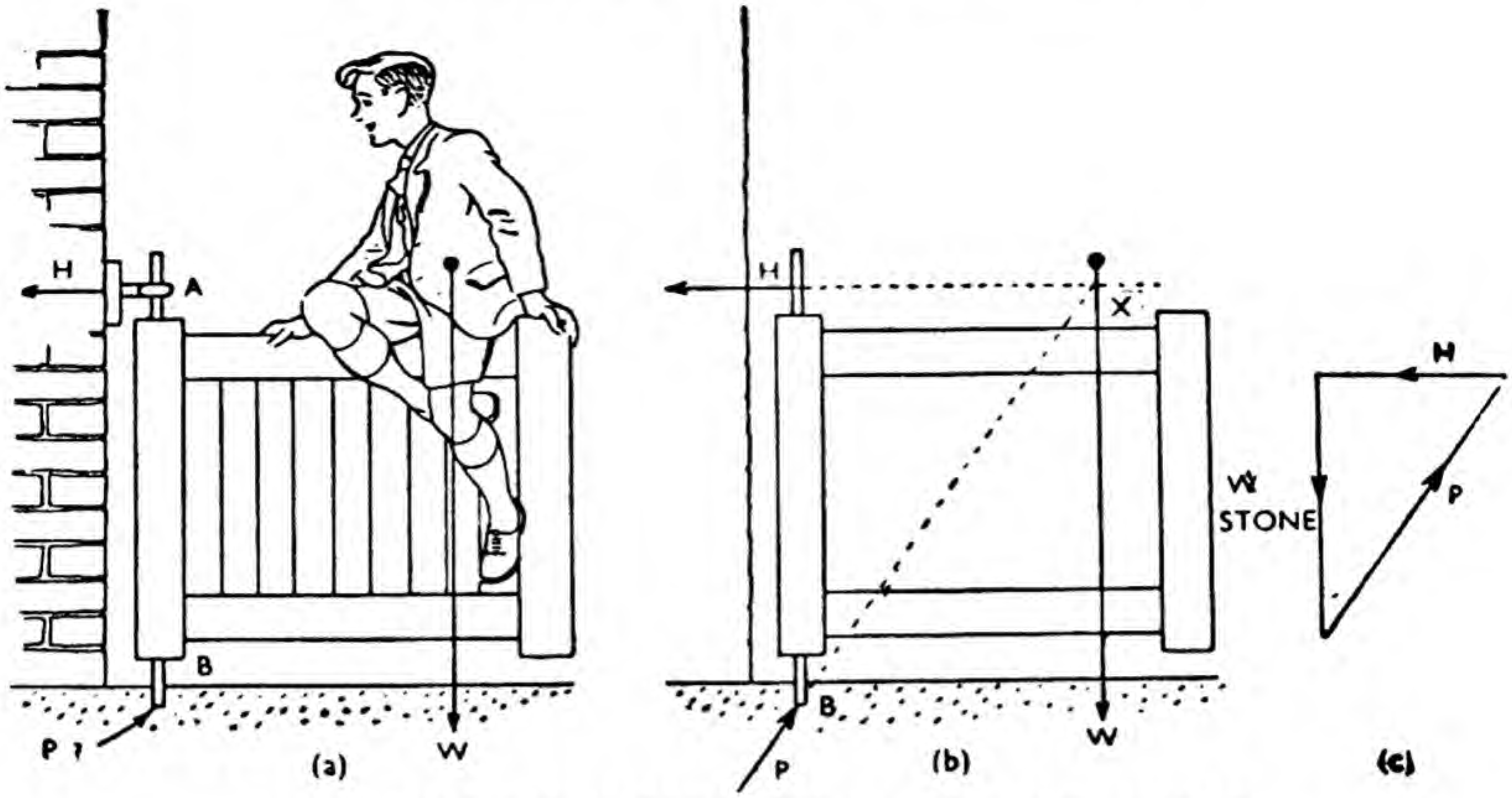
Consider the case shown in Fig. 14(a) of a boy sitting on a gate. His weight W, say, five stone, acts downward vertically. The ring at A produces a force H on the gate which can only have a horizontal direction. If the reader cannot accept this, let him imagine the gate tied at A to the wall by a strong horizontal cord; this can then only provide a horizontal force; the result is the same.

At B, however, there is a force on the gate, the direction of which is unknown. But, since the gate is in equilibrium, we know that the lines of action of W, H and P must meet at a point. The lines of action of W and H intersect at point X in Fig. 14(b). Thus, by joining BX, we now know the direction of P, and it only remains then to draw the triangle of forces, as shown in Fig. 14(c), with W = five stone, drawn to scale, and we may determine H and P completely. The forces supporting the davit, shown in Fig. 15, may be determined by the same method.

Three Parallel Forces

It should be noted that the triangle of forces cannot be applied when the three forces are parallel. A body may be in equilibrium under the action of three such forces, but clearly we cannot draw a vector triangle for these, nor can we find a common point of intersection. This case will be dealt with later when the equilibrium of parallel forces is considered.

If more than two forces meeting



THREE FORCES INTERSECTING AT ONE POINT

Fig. 14. (*a*) Gate is in equilibrium under the action of three forces, of which the direction of two, *W* and *H*, are known. (*b*) The lines of action of these three forces must intersect at one point, and so the direction of *P* is found, and (*c*) by a triangle of forces, *H* and *P* are determined completely.

at a point act on a body, we can find the resultant or equilibrant by similar methods. Thus, in Fig. 16(a) let us suppose that four forces F_a, F_b, F_c and F_d acting at the point O, are represented by the vectors OA, OB, OC and OD. The resultant of F_a and F_b is obtained as the diagonal OX of the parallelogram $OAXB$. Now OX, replacing F_a and F_b, is combined with F_c by completing the parallelogram $OXYC$, and we obtain OY as the resultant of F_a, F_b and F_c. Finally, OY and OD are combined, giving us OZ as the resultant of the original four forces.

Polygon of Forces

Once again we see that it is unnecessary to complete each parallelogram since, by drawing pq and qr in Fig. 16(b), we obtain pr as the resultant of F_a and F_b, although it is not even necessary to draw this line. By drawing rs to represent F_c, we obtain ps as the resultant of the first three forces and so on. We may now

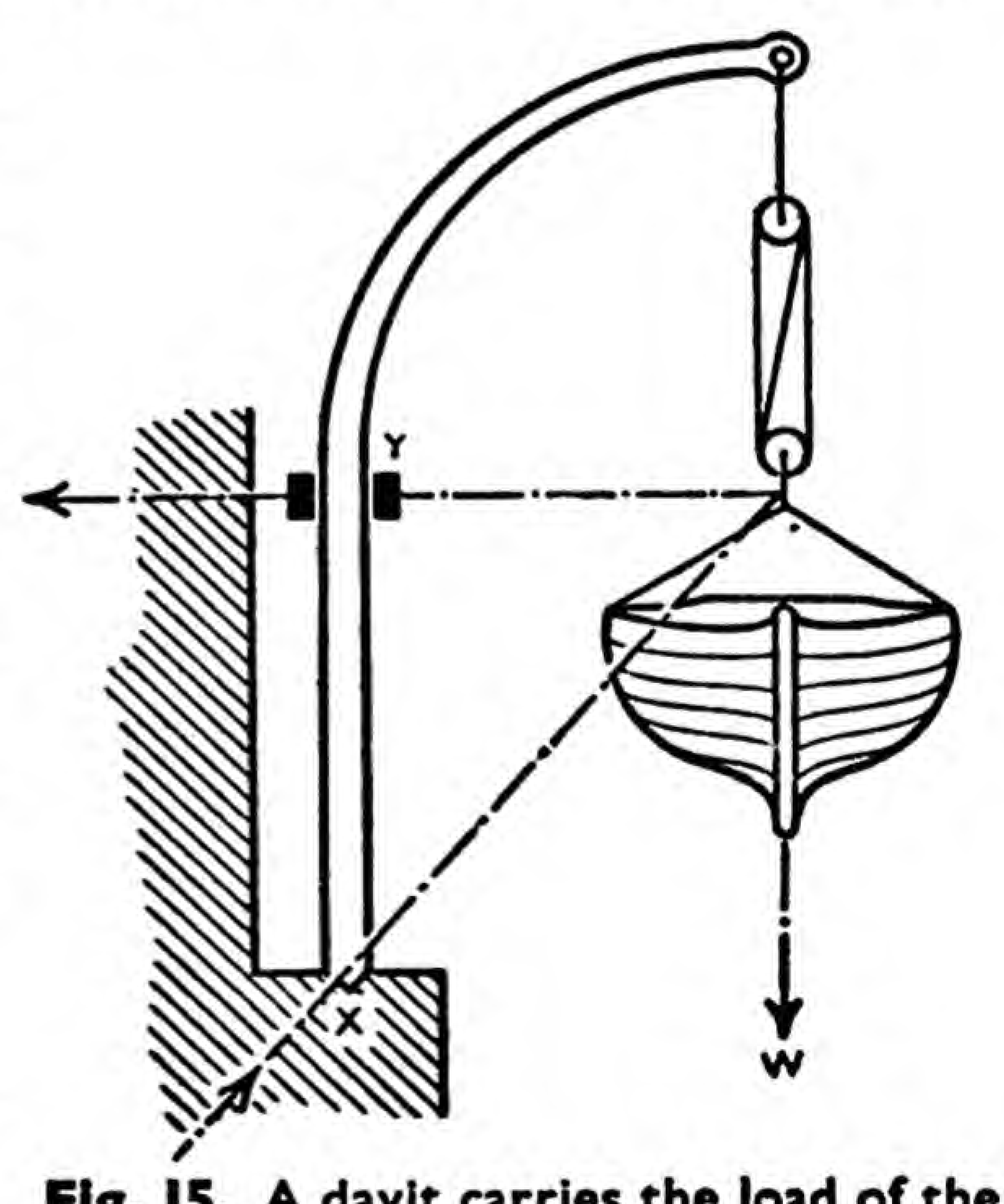
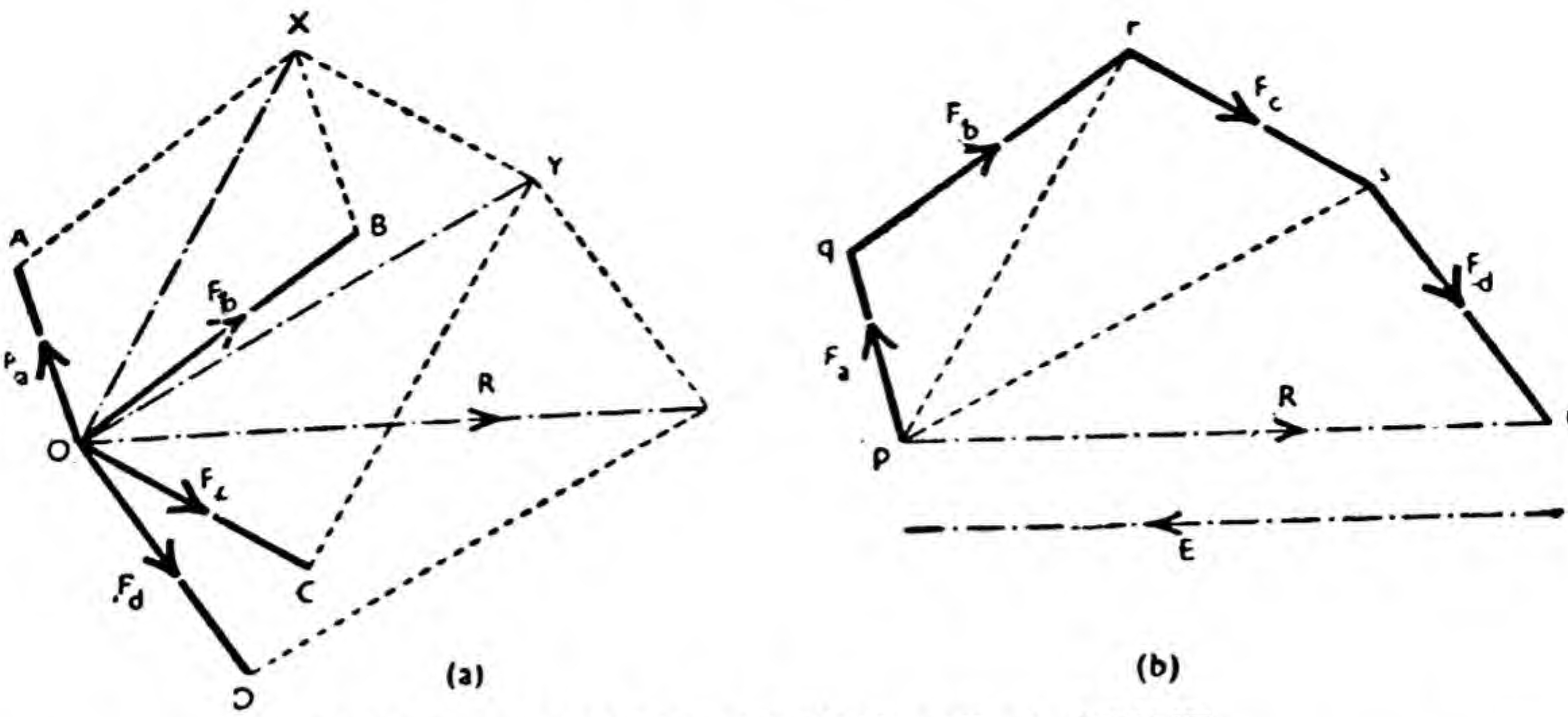


Fig. 15. A davit carries the load of the lifeboat and is supported by reactions at *X* and *Y*, the latter being horizontal. The lines of action of these three forces must meet in a point; thus, the directions and hence the magnitudes of the reactions are determined.



SEVERAL FORCES ACTING AT A POINT

Fig. 16. (*a*) Resultant of a system of several forces acting at a point may be determined by successive applications of the parallelogram of forces. More conveniently, however, the resultant or equilibrant can be obtained by a polygon of forces (*b*) in which vectors are drawn to represent the given forces.

commence from any point *p*, to draw vectors *pq*, *qr*, *rs* and *st*, representing the given forces in magnitude, direction and sense. The resultant *R* is represented by *pt* and, of course, the equilibrant *E* by *tp*. This figure is a polygon of forces, and we note that the triangle of forces is only a special case, viz., a polygon with three sides.

In more complex problems there may be some doubt about the pointing of the arrowhead on the resultant, and it is helpful to imagine that the polygon is a plan of the route taken while walking from *p* to *t*. We may go by a circuitous route, but the net, or resultant, distance moved is from *p* to *t* and, by the reasoning already given, the resultant force is from *p* to *t*.

It should be noticed that if the equilibrant is drawn, we shall have a closed polygon with the arrowheads pointing in the same direction round the figure. Thus, if the vectors representing any system of forces form a closed figure—polygon, triangle or straight line—with the arrowheads pointing in the same direction round the figure, we say that the system is in equilibrium for translation. This means, simply, that the body under the action of these forces will not be moved or translated in any direction. It will be seen later that, if the forces do not meet at a point, while the body may be in equilibrium for translation, it may not be in equilibrium for rotation. This does not arise when the forces meet at one point.

Example of Polygon of Forces

As an example, let us consider the forces meeting at *A* in the lamp-standard shown in Fig. 17(a). The pulley at *A* is assumed to be small, so that the four forces may be considered to meet at one point. Also, we assume the pulley to be frictionless, so that the tension in both parts of the cable supporting the lamp is *W*, the weight of the lamp. The directions oi the forces *P* and *Q* are known, and it is now necessary to find

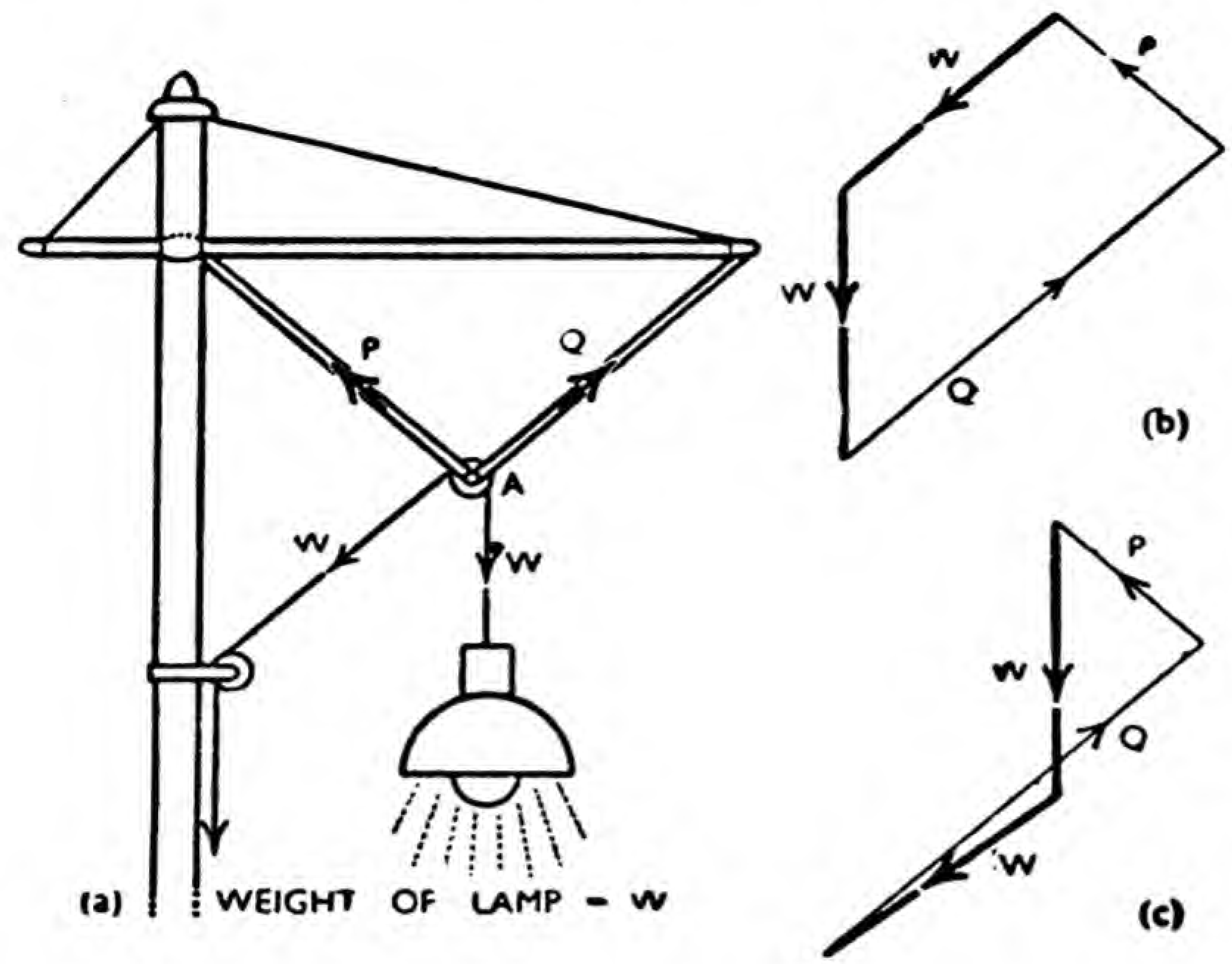


Fig. 17. (*a*) Street lamp suspended by a cable passing over a pulley at *A*. (*b*) The forces *P* and *Q* acting in two supports, can be found by a polygon of forces. (*c*) Sequence in which forces are taken does not matter because, whatever the order in which the vectors are drawn, the same forces are obtained from the polygon.

the magnitudes of these forces.

Therefore, we draw two vectors to represent the forces *W* in the two parts of the cable. These lines are, of course, drawn to scale. For example, if the lamp weighs 50 lb. and we decide to use a scale of 1 in. = 20 lb., then these lines would each be made $2\frac{1}{2}$ in. long and drawn parallel to the directions of the cable. Lines are then drawn parallel to the unknown forces *P* and *Q* until an intersection is obtained, as shown in Fig. 17(b).

Scaling Magnitudes

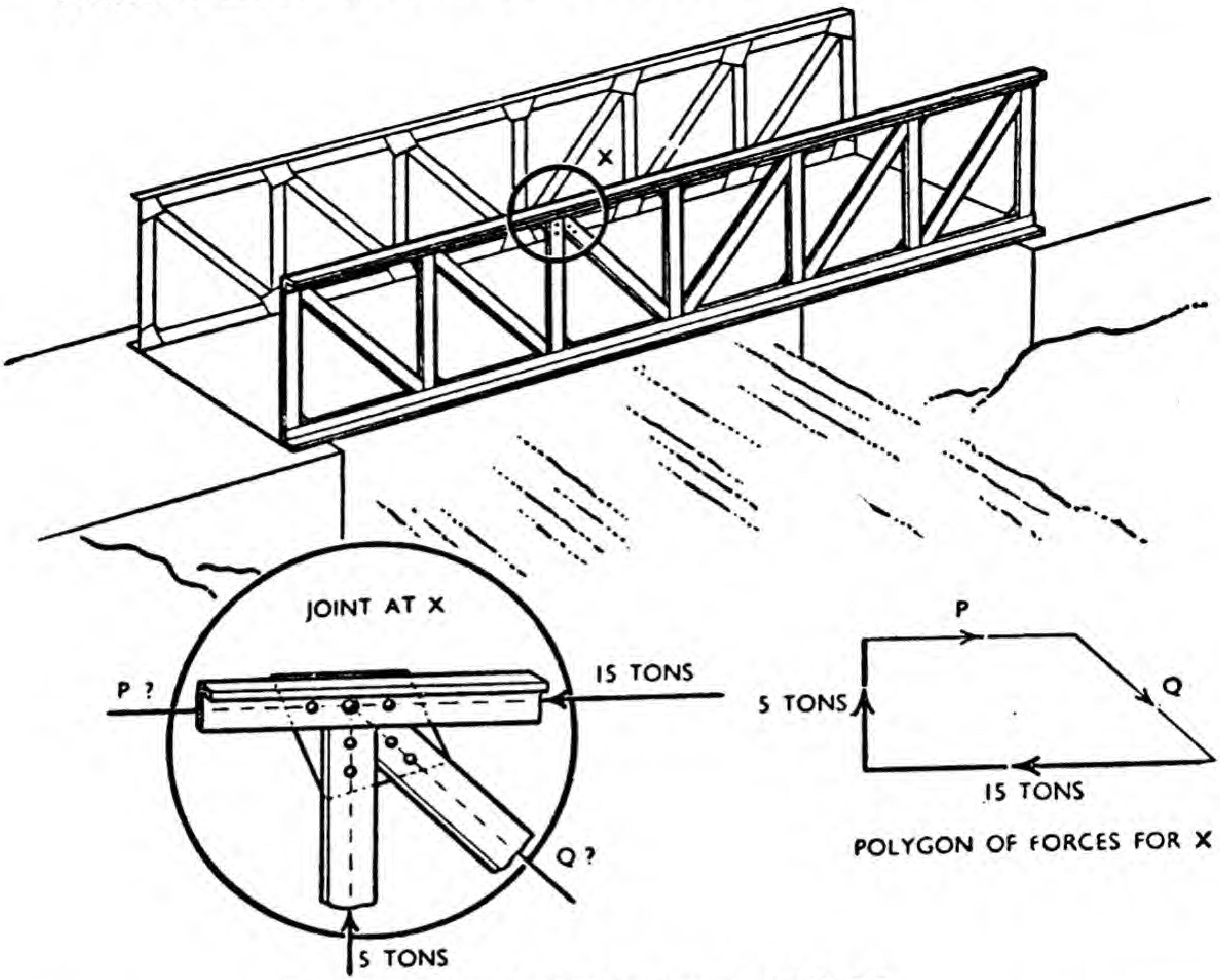
At once we can, from the diagram, scale the magnitudes of *P* and *Q*. But, since the body (the pulley) is in equilibrium, we know that not only is the force polygon a closed figure but, also, the arrowheads must point in the same direction round the figure, as shown. Following the direction of the arrowheads, we deduce that *P* and *Q* are both pulling away from the pulley, and are both, therefore, tensile forces.

The reader might be concerned about the order in which the vectors should be drawn, but this has no effect on the solution of the problem. Thus, in Fig. 17(c), the polygon is drawn with a different sequence, but we obtain the same values for *P* and *Q* as before. In this example, a polygon is drawn to determine the magnitudes of two forces, the directions of which are known, and the reader can very quickly satisfy himself, with pencil and paper, that it is impossible to complete a polygon if there are more than two unknowns.

It may be that we desire to find the directions of two forces of known magnitude, or possibly one direction and one magnitude. Should there be not more than two unknowns, either magnitude or direction, we can complete a closed polygon.

Let us now consider the forces acting at one of the joints of a simple structure. The following chapter deals with the methods of determining the forces in the various parts of a structure in more detail, but we can illustrate now the underlying principle.

At the joint marked *X* of the



FORCES AT A JOINT IN A BRIDGE

Fig. 18. Polygon of forces may be drawn for the joint of a footbridge, as illustrated above. Since the directions of the four forces acting at the joint are known, the unknown magnitudes of two of the forces can be determined.

footbridge shown in Fig. 18, four bars or members of the frame meet. Remembering that we can only complete a polygon if there are not

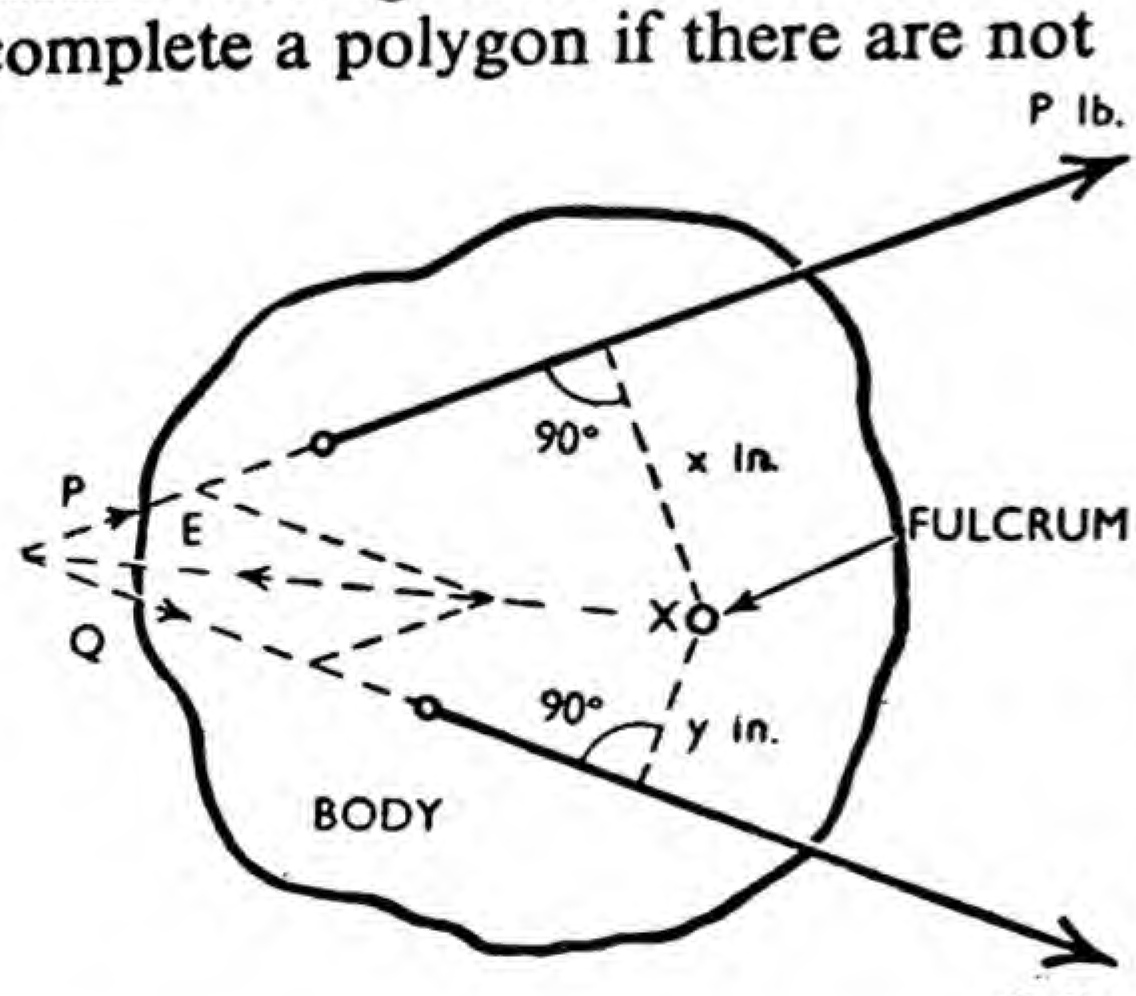


Fig. 19. Moment of force *P* lb. about fulcrum at *X* is *Px* lb.-in., and is said to be clockwise because of the direction in which it tends to rotate the body. Similarly, anticlockwise moment of *Q* about *X* is *Qy* lb.-in.

more than two unknowns, we will assume that forces of 15 tons and 5 tons are acting, as shown, when the bridge is crowded with people. Notice that we know the directions of the unknown forces. What are their magnitudes?

Practical Use of Vectors

To answer this question, we draw to scale vectors representing the forces of 15 tons and 5 tons, with the arrowheads pointing in the correct directions, as shown in the polygon. Then, parallel to the directions of the unknown forces *P* and *Q*, lines are drawn until they intersect. We can then, from the drawing, scale the magnitudes of *P* and *Q*, say about 10 tons and 7 tons respectively. Should we

draw the arrowheads so that they point in the same direction round the polygon, we see that the arrowhead for P points from left to right, indicating that this force is pushing towards the joint, and is, therefore, compressive. Similarly, the arrowhead for Q points away from the joint, showing that Q is a tensile force.

In this example, the reader should not be misled by the fact that, in the detailed diagram, the top horizontal member is shown as a continuous piece. This does not affect the equilibrium of the joint, and it is quite justifiable to draw a polygon of forces as described in the foregoing paragraphs.

Up to the present stage, we have been considering the equilibrium of forces which meet at one point, but many problems in mechanics are concerned with the effect of a force at a point which is not in its line of action. In this case we consider the moment of the force about the point. The term moment is used in the sense of importance just as we speak of things of great moment. It measures the importance of the force in producing rotation of the body around a given point. We define the moment of a force as its turning effect about the point, and it is measured by the product of the force (lb., oz., etc.), and the perpendicular distance (ft., in., etc.) between the line of action of the force and the point. Therefore, the magnitude of a moment must be quoted in composite units, such as lb.-ft., oz.-in., etc.

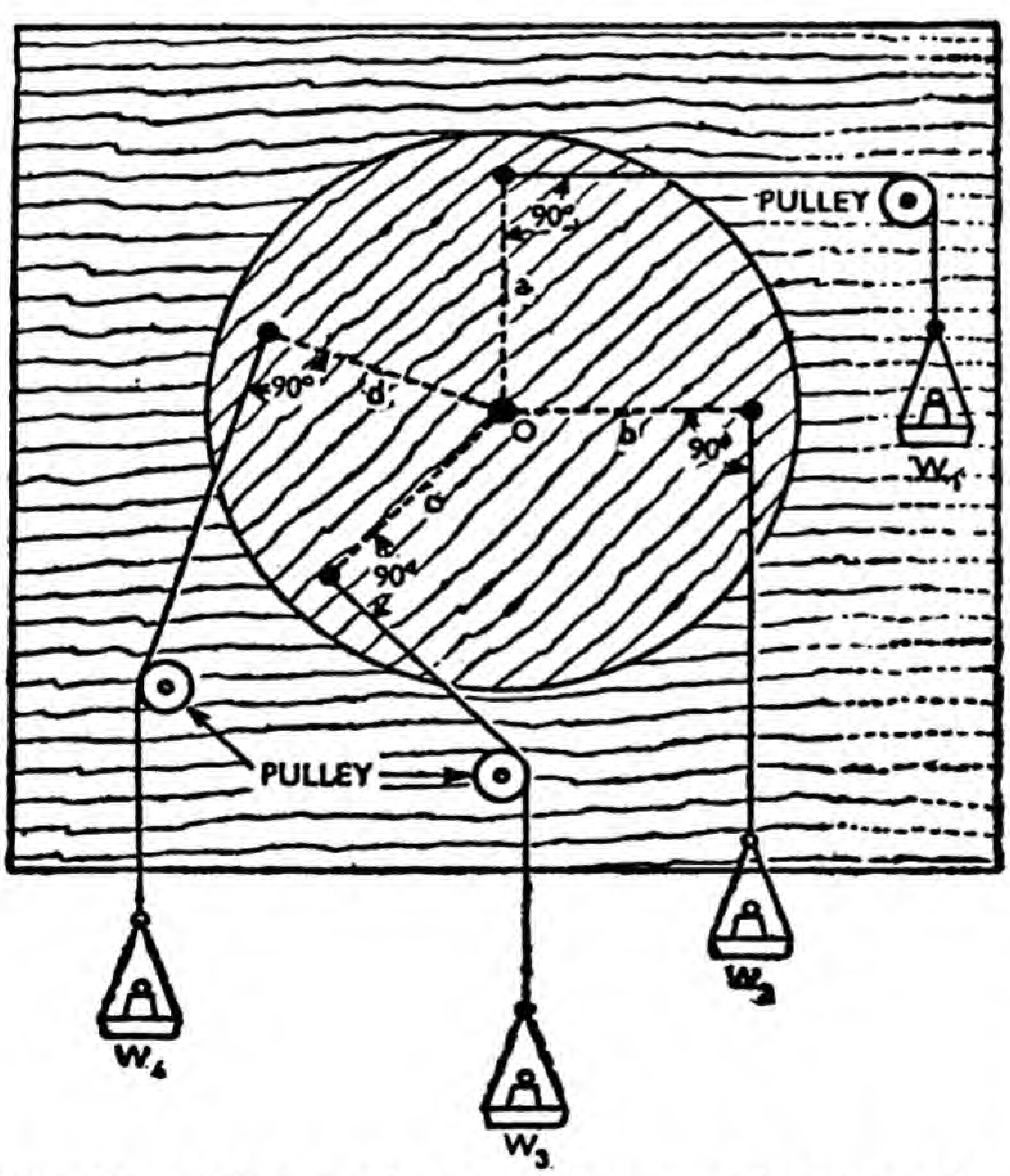


Fig. 20. When various weights are placed in the scalepans, the wheel rotates about the fulcrum at O until equilibrium is established. We then measure the lever arms a, b, etc., and so calculate the moments of the forces about O. It is then found that the sum of the clockwise moments always equals the sum of the anticlockwise moments.

Clockwise Moment

In Fig. 19 the moment of the force P about the point X is Px lb.-in., and, since it tends to rotate the body in a clockwise direction about X, we say it is a clockwise moment. Notice carefully that the distance x is measured in a direction which is perpendicular to the line of action of the force P; this distance is called the lever arm.

Similarly, the anticlockwise moment of Q about X is Qy lb.-in. We saw earlier that a body moves in the direction of an

applied force, unless an equal and opposite force is called into play. In the same way, a body under the action of a moment will rotate, unless prevented by an equal and opposite moment. If the body shown in Fig. 19 is in equilibrium, that is, if it is not rotating, then :—

clockwise moment Px lb.-in. = anticlockwise moment Qy lb.-in.

To prevent translation, the body is physically pivoted about the point X, and such a pivot or hinge is known as a fulcrum. The body cannot move in the general direction of P and Q because of a resisting force provided by the fulcrum. Clearly then, this force must be the equilibrant of P and Q, and its magnitude can readily be found by constructing the parallelogram of forces, as shown in the figure.

For any values of P, Q, x and y, the line of action of the equilibrant will be found to pass through X. This must be so, since the equilibrant is provided by the fulcrum at X. The requirement for equilibrium in rotation can readily be demonstrated by the simple apparatus shown in Fig. 20.

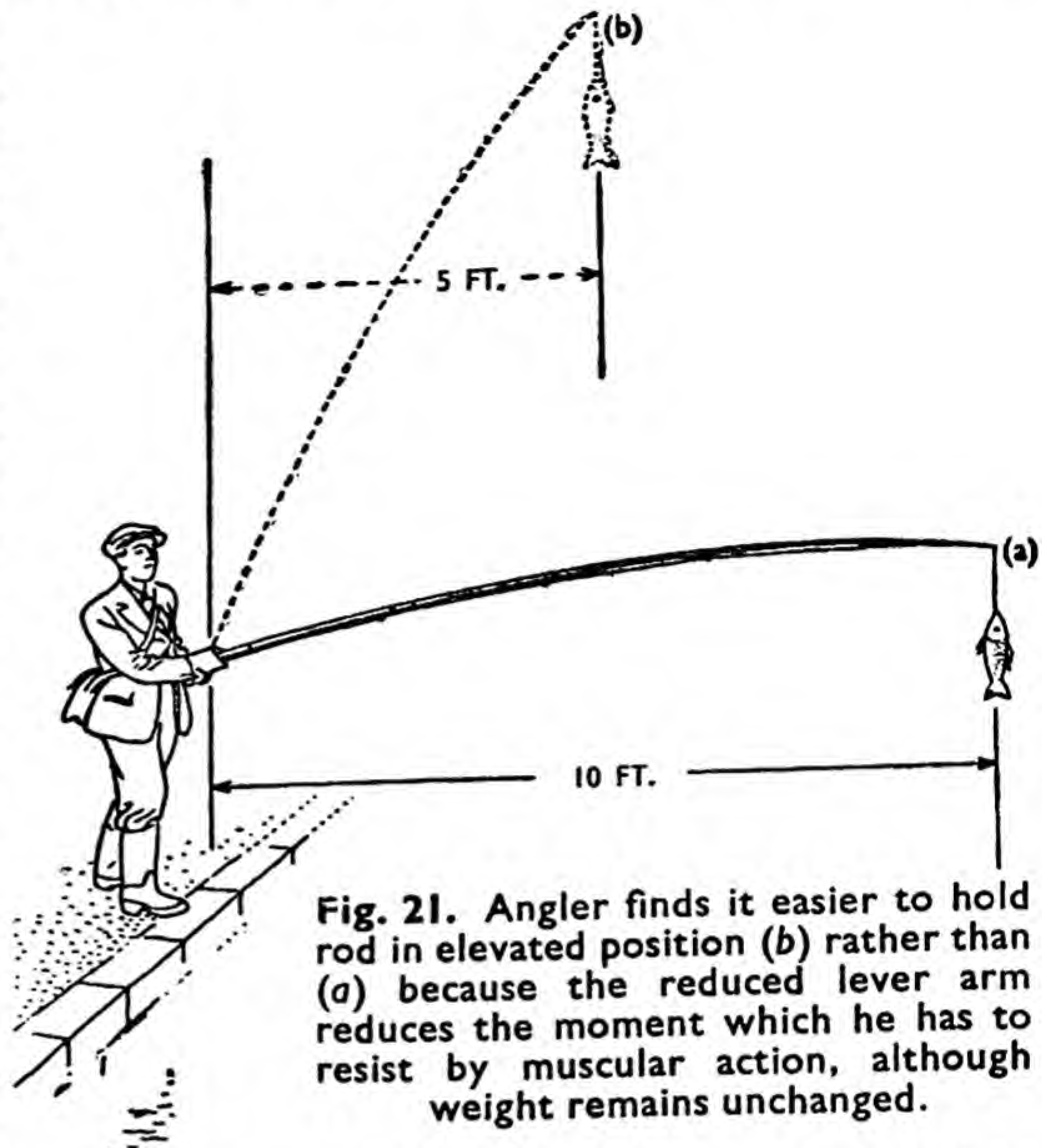


Fig. 21. Angler finds it easier to hold rod in elevated position (*b*) rather than (*a*) because the reduced lever arm reduces the moment which he has to resist by muscular action, although weight remains unchanged.

Informative Demonstration

A wooden wheel is mounted on a central bearing at O, so that it may rotate freely with as little friction as possible. Cords carrying weights W_1, W_2, etc., are attached to various points in the wheel. After setting this up, we release the wheel which will quickly rotate to the position of equilibrium and we can then measure carefully the perpendicular distances a, b, etc., from the cords to the fulcrum O. It will then be found that the sum of the clockwise moments will always equal the sum of the anticlockwise moments. For the conditions shown in Fig. 20, we would find that the moments would agree with the following equation :—

$$aW_1 + bW_2 \text{ (clockwise)}$$
$$= cW_3 + dW_4 \text{ (anticlockwise).}$$

A simple example from common experience will illustrate the nature of a moment, and its variation with the force and perpendicular distance.

Let us suppose that an angler (Fig. 21) has caught a fish

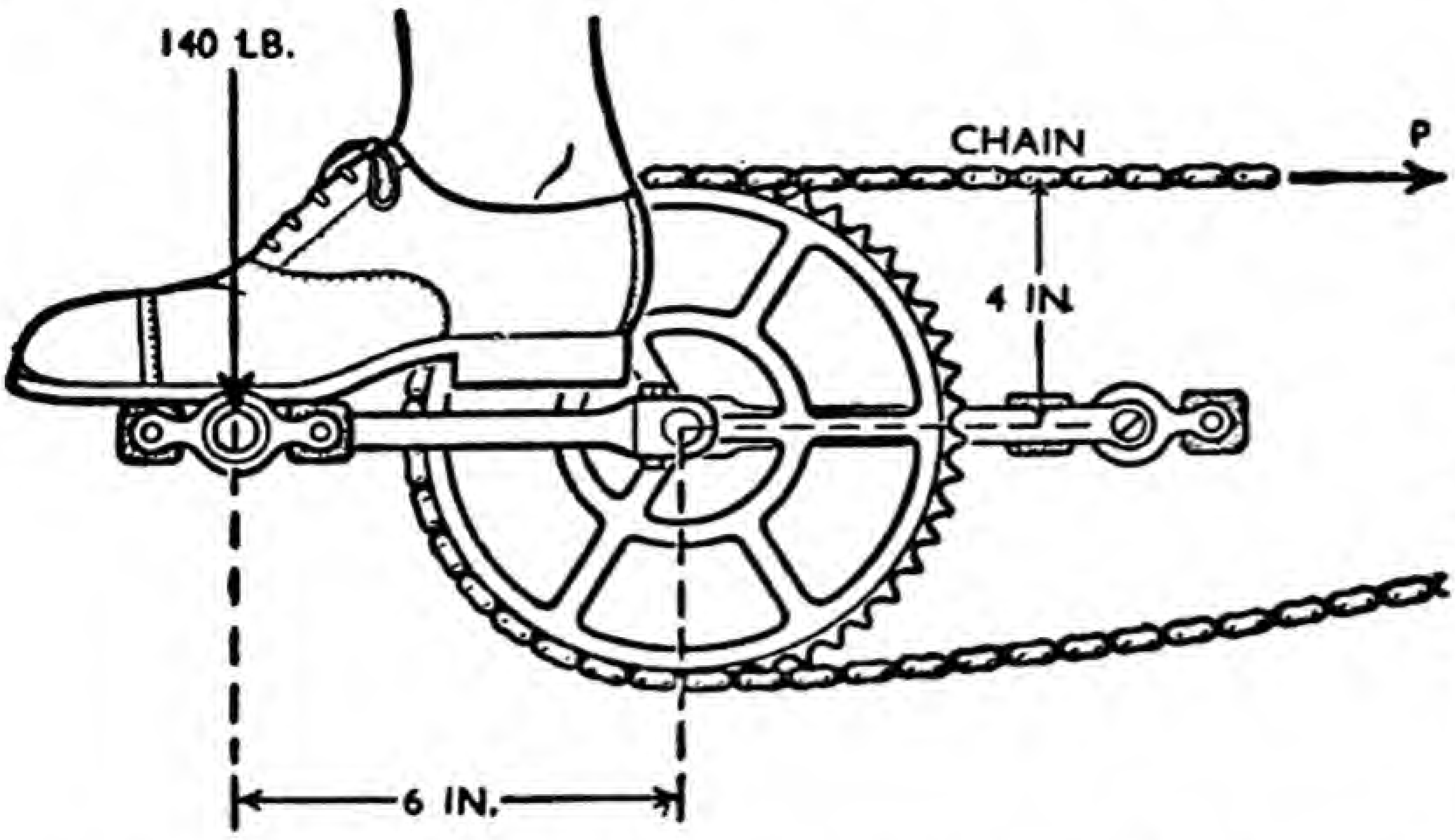


Fig. 22. Cyclist, pressing on the pedal of a bicycle, applies an anticlockwise moment to the chain-wheel. This is resisted by the clockwise moment of tension *P* in the upper part of the chain. *P* is greater than the force on the pedal because its lever arm is smaller.

weighing $\frac{1}{2}$ lb. If he holds his rod and catch in position (a) the moment at his hand is $\frac{1}{2} \times 10 =$ 5 lb.-ft. To maintain this position, he must provide, by muscular action, a moment equal and opposite to this. He raises his rod to position (b) and the moment is now $\frac{1}{2} \times 5 = 2\frac{1}{2}$ lb.-ft.

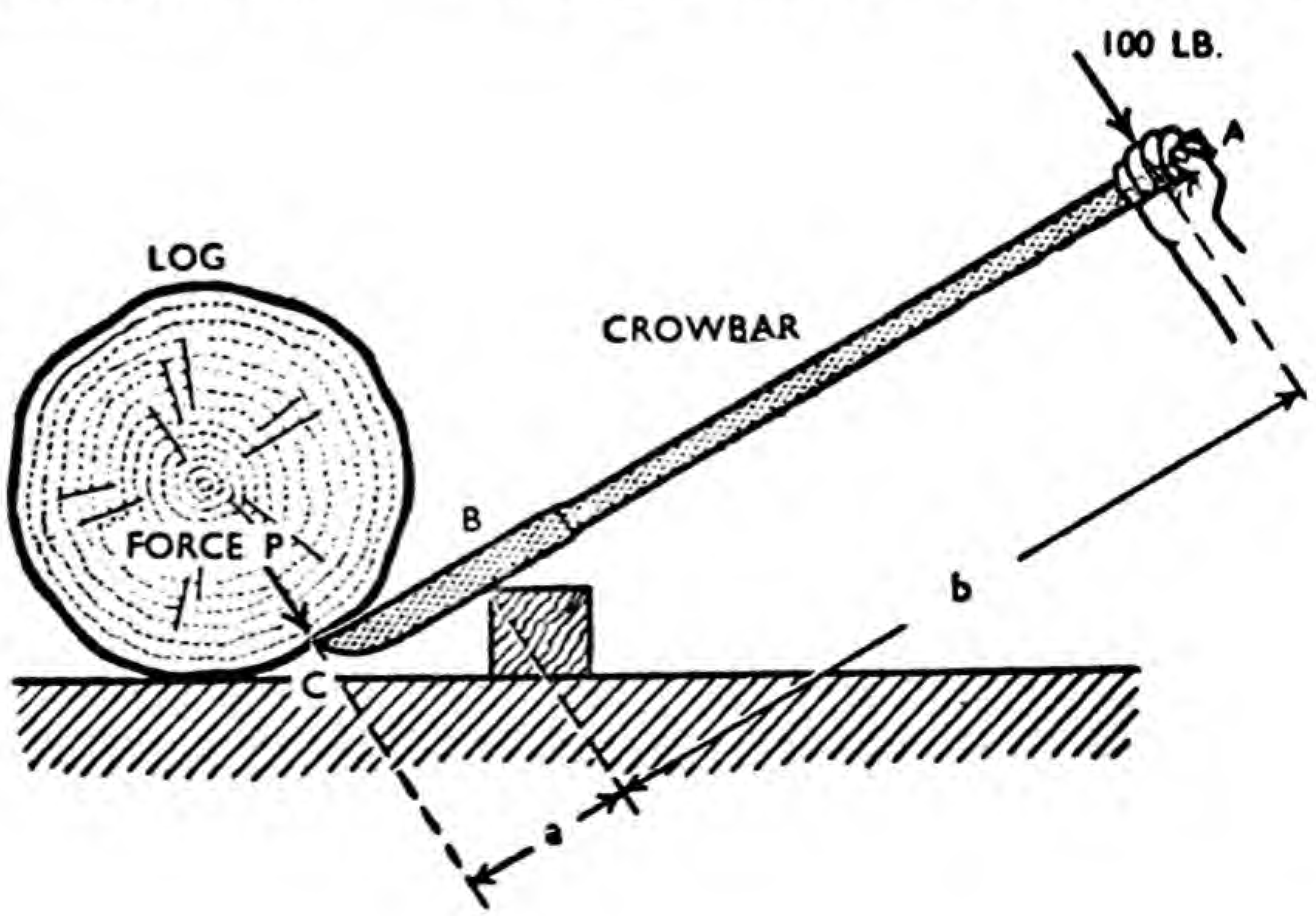
We know, by everyday experience, that position (b) requires less effort to maintain. Notice that, in both cases, the angler has the same weight to sustain, but the moment varies with the perpendicular distance. The angler is now more successful, and his next catch weighs 1 lb. The moment required to maintain position (a) in this instance is 10 lb.-ft., and we are all familiar with the increased effort required to resist the increase in moment due to the additional force.

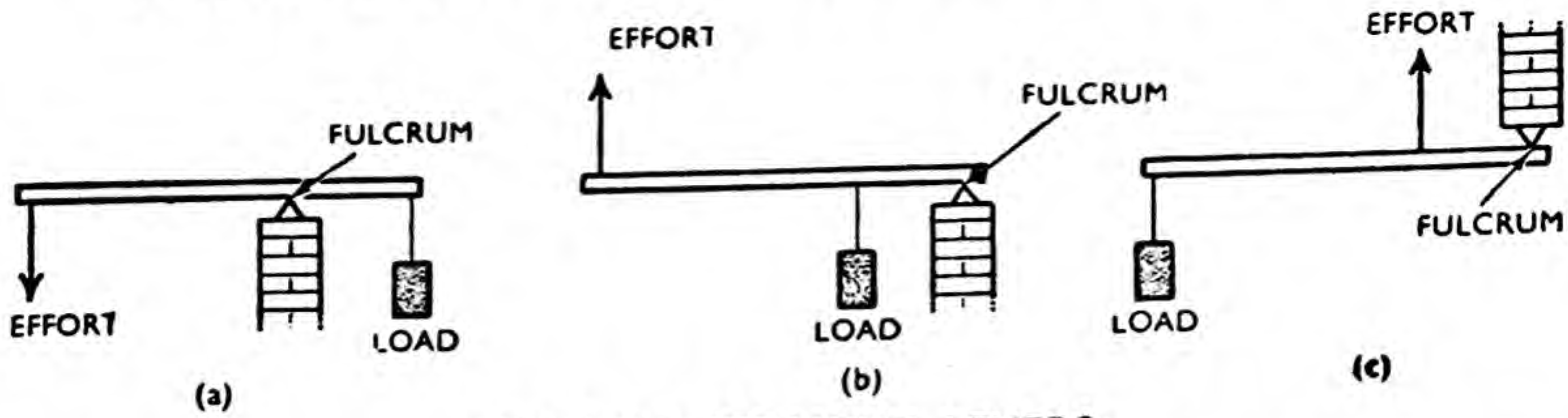
If the rod is held vertically, the perpendicular distance is nil, therefore, the moment is nil, and the muscles have now only to support the weight.

When moving a long ladder, which has been standing against a wall, from one position to another, we instinctively attempt to keep it vertical to avoid the effort that would be required to counteract a larger moment at the base. Those who have witnessed Highland games will recall that the massive caber is skilfully handled in this manner, with the same object in view.

Taking another practical example,

Fig. 23. In order to move a log, a large force can be produced by a moderate force of, say, 100 lb. exerted at one end of a crowbar. This is done by placing the fulcrum suitably, and the lever arm of the 100-lb. force can be made relatively large, giving a large moment.





TYPES OF STRAIGHT LEVERS

Fig. 24. Depending on the relative positions of the effort, the load, and the fulcrum, levers may be classified into three groups, of which examples are illustrated here.

let us suppose that a cyclist is holding his bicycle with the brakes fully applied to prevent motion, and presses the whole of his weight of 140 lb. on the pedal with the crank in the horizontal position, as shown in Fig. 22.

If the crank is 6 in. long, he is applying a moment of $140 \times 6 =$ 840 lb.-in. to the chain-wheel. If unopposed, this moment would rotate the chain-wheel in an anti-clockwise direction, but this is prevented by the moment of the tension P in the upper part of the chain. Since the wheel is in equilibrium, the two moments are equal. Therefore, 840 lb.-in. clockwise $= P \times 4$ lb.-in. anti-clockwise, whence $P = 210$ lb.

Thus, by applying a force of 140 lb. at one point, we obtain a force of 210 lb. at another point, and, obviously, the magnifying ratio can be increased by adjusting the distances from the fulcrum. This is the principle underlying many familiar devices, and we will now consider some of these.

Value of Levers

We are all familiar with the value of a crowbar, or similar lever, for obtaining large forces which we are unable to apply directly.

Fig. 23 shows a crowbar being employed to move a large log. At A, a force of, say, 100 lb. is applied by the hands, and the bar will pivot about the fulcrum at B. At C the bar pushes the log with a force P so that :—

$$Pa \text{ lb.-in.} = 100b \text{ lb.-in.}$$

Now, if b is large compared with a, say 55 in. as compared with 5 in., then $P = 100 \times \frac{55}{5} = 1{,}100$ lb.

We now have the moderate force of 100 lb., exerted by a man at the end of the bar, causing a large force of nearly half a ton to be applied to the log.

Note that although we say the bar applies a force of 1,100 lb. to the log, it is equally true to say that the log applies this force to the bar, which is the condition represented in the diagram (Fig. 23).

Large Force Obtained

It is very necessary to emphasize clearly that it is impossible to obtain more work from the bar at C than is put into it at A. This and other types of lever are actually simple machines which are considered in a subsequent chapter. The large force is, in fact, obtained, but the work done by a force (as will be shown in Chapter 4) is the product of the force and the distance through which its point of application moves. In this case, if the man works by moving point A, the distance through which the

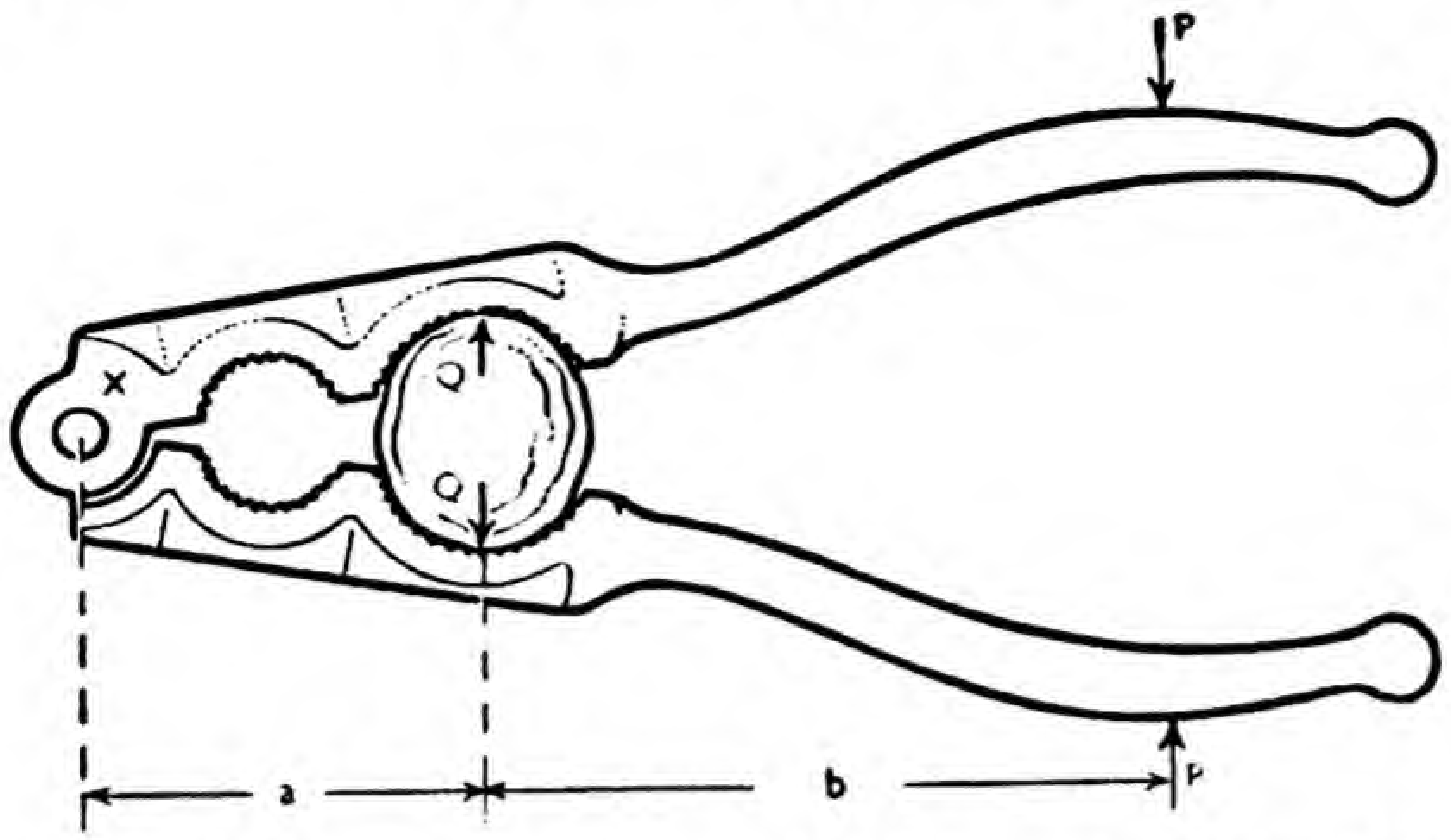


Fig. 25. The pair of nutcrackers illustrated here is a double lever of type (b). Reciprocal forces *P*, applied by hand, produce a compressive force Q in the nut sufficient to crack it, because of the differences in the lever arms about the fulcrum *X*.

point *C* moves is very small, being only 5/55ths or 1/11th of the movement of *A*. We now find the work given out at *C* is not greater than the input at *A*, and is, indeed, always somewhat less because of losses due to friction.

It is helpful to assign names to the two forces involved. The force exerted on the bar by muscular action or other means is termed the effort, and the resistance to be overcome or balanced, whatever its form, is termed the load. Considering straight levers we can divide them into the three classes indicated diagrammatically in Fig. 24.

A crowbar used in the manner shown in Fig. 23 is an example of (a) ; if the tip of the bar rests on the fixed support, which point is also the fulcrum, it is then a lever of type (b).

As another example of type (b), consider the pair of nutcrackers illustrated in Fig. 25.

When cracking a nut we apply a force *P* by the hand, and the moment about the fulcrum at *X* is $P(a + b)$. This is resisted by the moment of the force *Q* in the nut which has a value Qa.

Hence, $Qa = P(a + b)$

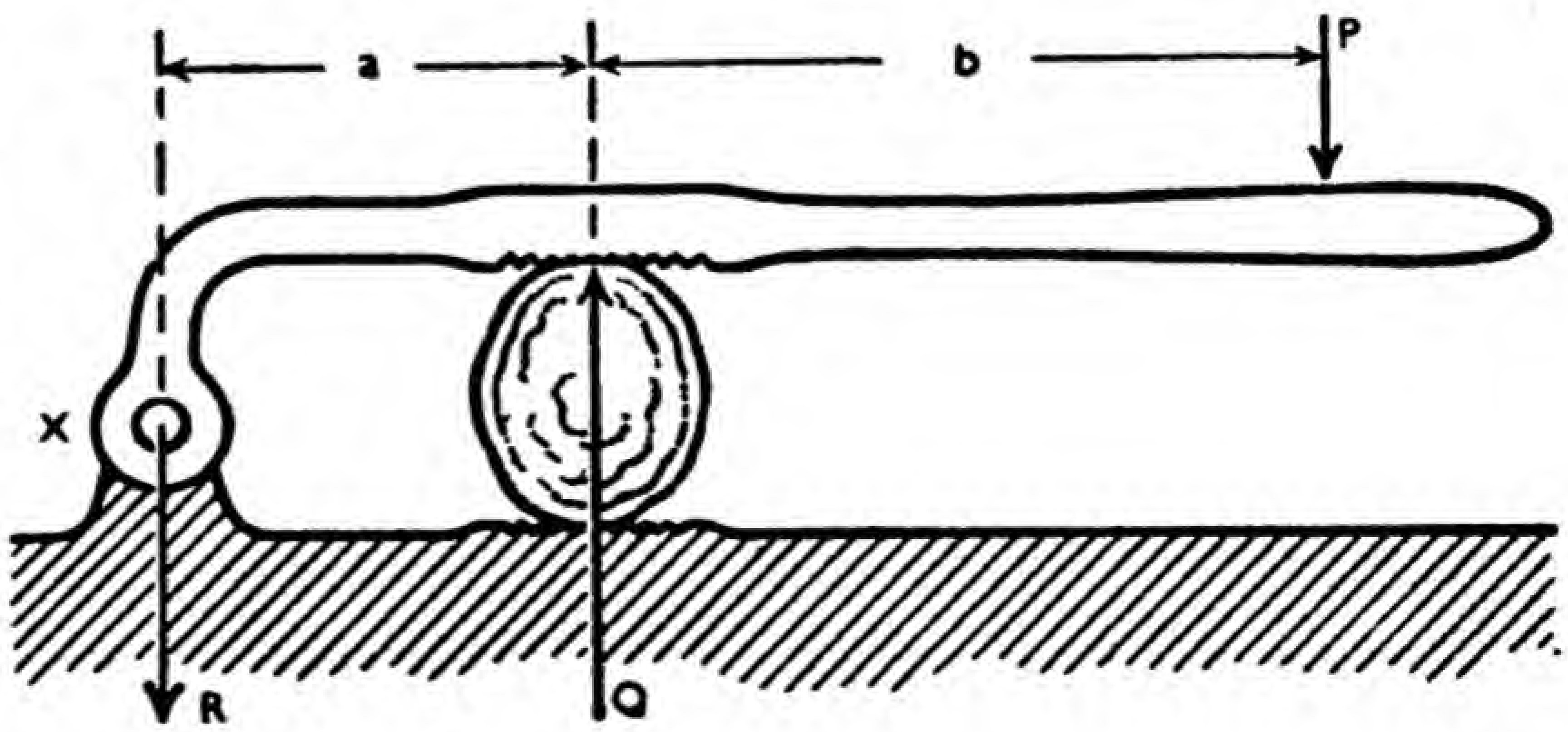
and, $Q = P\left(\dfrac{a+b}{a}\right)$

This, of course, represents the condition before the nut breaks. In actual use, we progressively increase *P* until *Q* reaches such a value that the nut cracks.

Double Lever

This device is sometimes referred to as a double lever, because of the symmetrical duplication of the rigid bars and forces. The reader should not be confused by the fact that two forces *P*, and two forces *Q*, are shown in Fig. 25. The reason

Fig. 26. To avoid confusion in the mechanics of double levers, a single lever with a rigid base, and having the same lever arms is illustrated. This will be found to give the same force in the nut for the same effort.



lies in the fact that the pair of nut-crackers is independent of a reaction provided by the ground or rigid base. Thus, to prevent translation of the whole body, an external force P must be opposed by an equal and opposite force P. To make this point clear, a possible alternative design, using a fixed base, is shown in Fig. 26.

As before, $Q = P\left(\frac{a+b}{a}\right)$, the only difference being that in this case Q is a reaction provided by the fixed base. Notice that the base also produces a downward reactive force R at X, which can also be calculated by moments. By taking moments about the point of contact with the nut, which can now be considered as the fulcrum, we find :—

$$Ra = Pb.$$

$$\therefore \quad R = P\left(\frac{b}{a}\right).$$

It is interesting to find the sum of P and R, since these both act downward vertically.

$$R+P=P\left(\frac{b}{a}\right)+P=P\left(\frac{a+b}{a}\right)=Q.$$

We now find that the sum of the vertical forces acting downward equals the vertical upward force. This example illustrates a fundamental principle in statics which may be stated in the following way :

If a body is in equilibrium under the action of a system of forces, the sum of the forces is nil and the sum of the moments about any point is nil.

It will be understood that we take the algebraic sum of parallel forces, using, say, the + sign for downward forces, and the − sign for upward forces. Similarly, clockwise and anticlockwise moments are allotted differing signs. If the forces are not parallel, the statement is still true provided it is understood that we take the vectorial sum (or resultant) of the forces. This is merely another way of saying that the polygon of the forces acting on a body which is in equilibrium, is a closed figure with the arrowheads pointing in the same direction round the figure. We have already met this requirement for the equilibrium of forces which meet at a point.

The reader will realize that, in this case, the requirement that the sum of the moments must be nil is unnecessary, since the moment of each force about the meeting point is nil, because the perpendicular distance is nil.

Fig. 27. When using a pair of sugar-tongs, the effort we apply by the fingers and thumb is greater than the force which grips the lump of sugar. The lever enables us to produce a sufficient force in a position where it would be inconvenient to apply a force directly.

Effort Greater than Load

Levers are not always employed to obtain a larger force than the applied effort. This is the case with type (c) levers, in which the effort is obviously greater than the load. Such levers are used to produce a force at a point where it would be impossible, or inconvenient, to apply the force directly.

A pair of spring sugar-tongs, illustrated in Fig. 27, is an example of a double lever of this class.

We neglect the small resistance offered by the curved spring at the end, and treat this as a free hinge, or fulcrum. Once again, by taking moments, we can find the force with which the sugar-lump is gripped, if we know the pressure applied by the fingers and thumb. Although this force is smaller than the effort, it enables us to pick up, hygienically, a lump of sugar from, say, an awkward position at the bottom of the bowl.

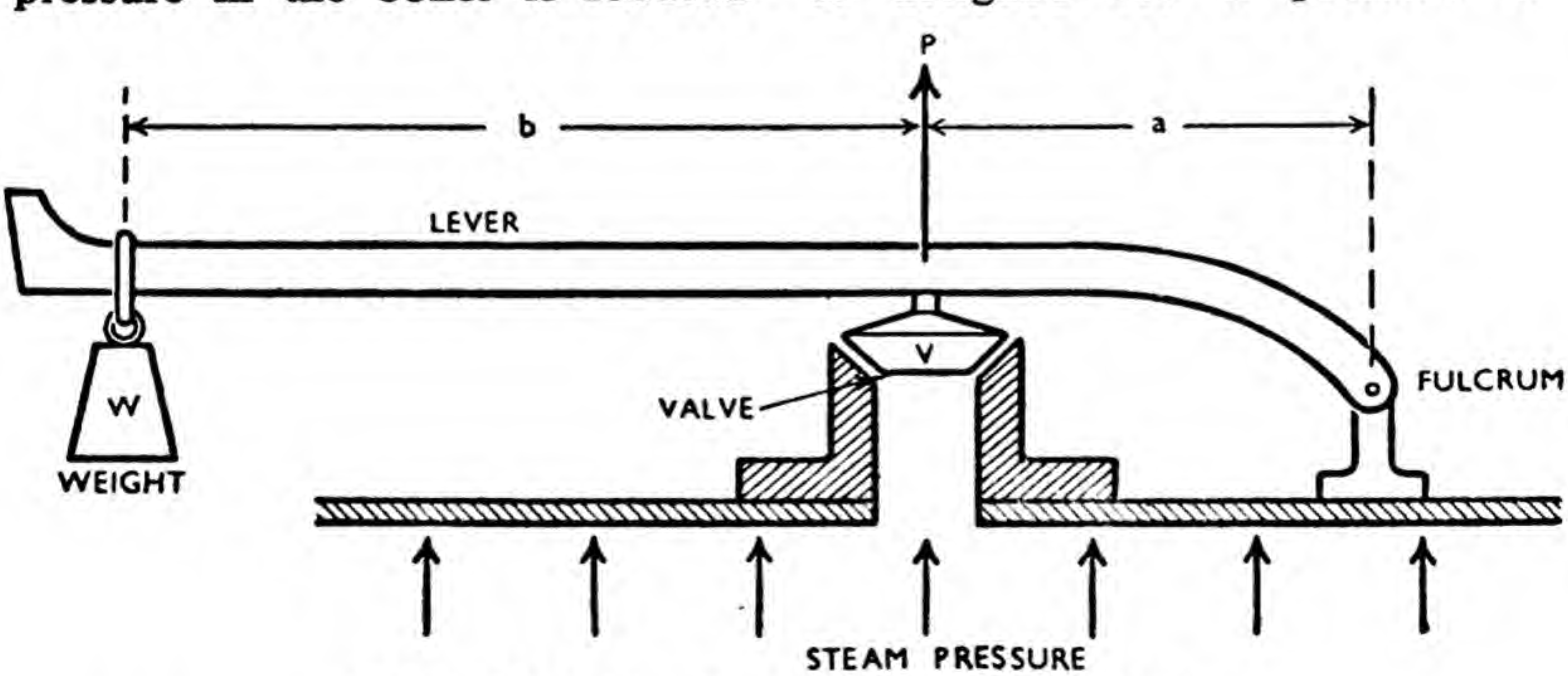
The lever safety-valve provides another example of the useful application of the principle of moments. We all know that when steam is generated in a closed vessel, it exerts a pressure on the sides of the container. It is this pressure, generated in a boiler, which is used to drive locomotives, turbines and steam-engines of all kinds.

A boiler is designed to withstand, safely, a definite maximum pressure, and it is obviously essential to avoid any excess of this pressure, otherwise a disastrous explosion might occur. Now, the pressure in the boiler is reduced by the passage of steam to the cylinders, or turbines, and is increased by supplying more heat from the burning fuel. But it is impossible to regulate the consumption of steam, and the supply of heat, so that the pressure remains constant.

Action of Safety-valve

For example, let us suppose a locomotive is pulling a heavy train, uniformly, at sixty miles per hour, and that the firemen are steadily supplying coal to maintain the supply of heat and steam pressure for this speed. Now imagine the train is halted by an adverse signal. At once the consumption of steam ceases, and, since the fuel already in the combustion chamber continues to burn, the pressure in the boiler rises sharply. In order that such a rise in pressure shall not reach a dangerous figure, the boiler is fitted with a safety-valve of which one type fitted to stationary boilers is illustrated in Fig. 28.

Let us now suppose that a boiler is designed for a pressure of



SAFETY-VALVE TO AVOID EXPLOSIONS

Fig. 28. **This is one type of safety-valve fitted to boilers to prevent explosions. When the pressure in the boiler increases, the force on the valve *V* increases, and when the moment of this force about the fulcrum exceeds the moment of the weight *W*, then the valve rises, steam escapes, and the pressure is relieved.**

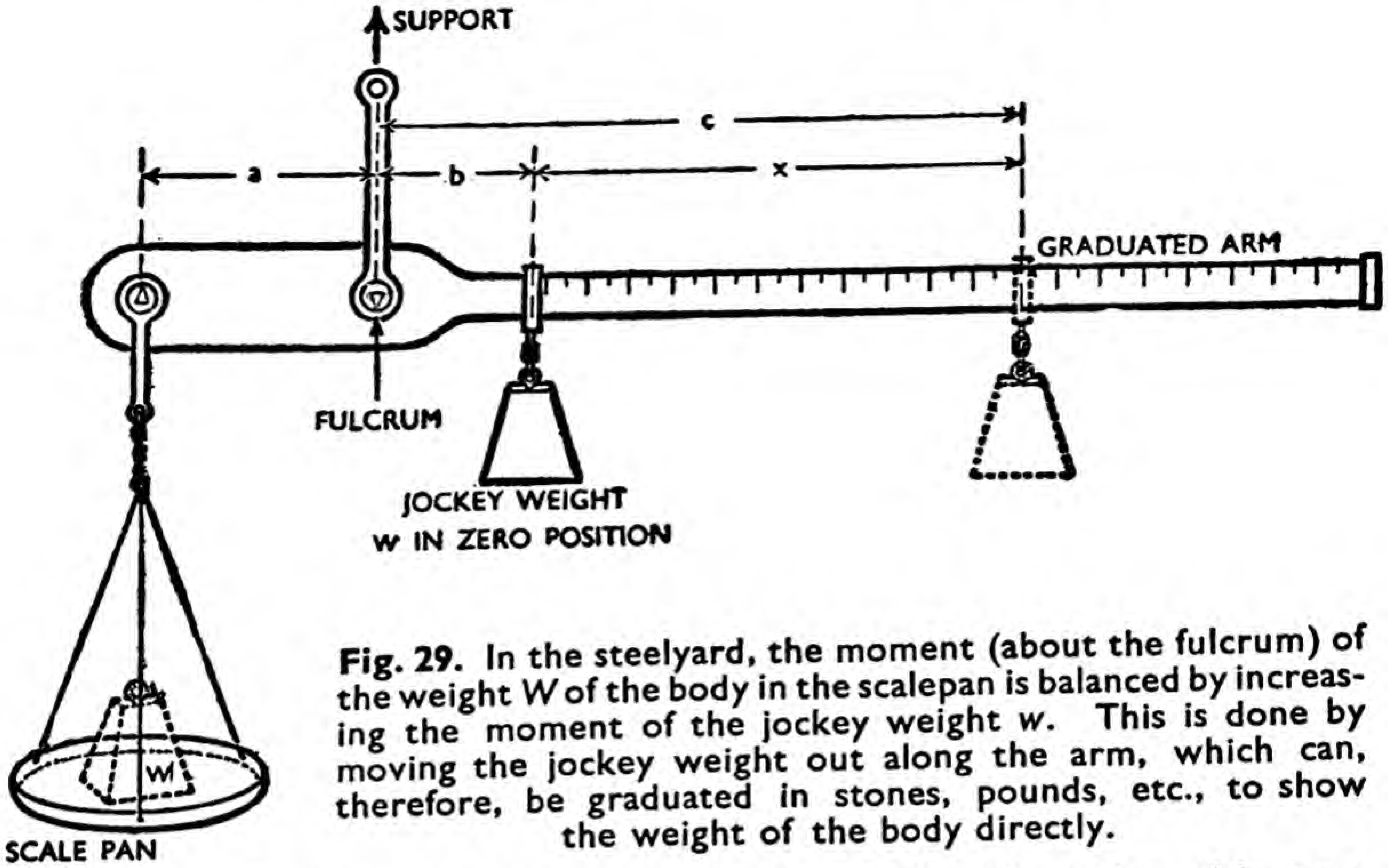


Fig. 29. In the steelyard, the moment (about the fulcrum) of the weight *W* of the body in the scalepan is balanced by increasing the moment of the jockey weight *w*. This is done by moving the jockey weight out along the arm, which can, therefore, be graduated in stones, pounds, etc., to show the weight of the body directly.

200 lb. per sq. in., and that the area of the valve exposed to the pressure is 2 sq. in. Then the other parts should be designed to ensure that the valve will lift when the force on it exceeds $200 \times 2 = 400$ lb., so that steam may escape. This will happen when the clockwise moment of the force of the steam about the fulcrum exceeds the anticlockwise moment of the weight hung from the lever.

Therefore, when $400a$ exceeds $W(a + b)$, the valve rises and the pressure is relieved. It is thus a simple matter to proportion W, a and b, so that the pressure in the boiler can never exceed 200 lb. per sq. in. Actually, the weight of the lever itself has an anticlockwise moment about the fulcrum, but the calculation of this involves a knowledge of centres of gravity, which are discussed later. So, for our present purpose, we assume that the weight of the lever is small, and that its moment about the fulcrum is negligible compared with the moment of W.

As mentioned earlier, this type of safety-valve is fitted to stationary boilers. Any violent movement of a boiler would tend to wobble the lever, and might even unseat the valve. Under such conditions, the load on the valve is maintained by a powerful spring instead of a lever, and the spring is so designed that it deforms under a predetermined load; in the example which is illustrated above, this load would be 400 lb.

The Steelyard

Another lever of great practical importance is the steelyard, which is illustrated in Fig. 29. It consists of an arm suspended from a fulcrum, which is usually a knife-edge in order to reduce friction to a minimum. On opposite sides of the fulcrum there are a scalepan, to accommodate the body to be weighed, and a jockey weight, which can be moved along the graduated arm shown in the figure.

Let us suppose that, with no weight in the scalepan, the arm is

balanced horizontally with the jockey weight at a distance b from the fulcrum. This means that the moments of the weights of the scalepan, and the left-hand part of the arm, are balanced by the moments of the jockey weight and the right-hand part of the beam. Notice that we do not require to know what these moments are; as long as the steelyard is balanced with no load in the scalepan, the position of the jockey weight corresponds to zero load, and accordingly, that point in the arm is marked 0 lb.

Now imagine that a body weighing W lb. is placed in the scalepan, thus applying an anticlockwise moment Wa lb.-in. to the lever. Equilibrium is destroyed, the scalepan descends, and the graduated arm rises. A clockwise moment is, therefore, applied by moving the jockey weight out along the arm until equilibrium in the horizontal position is restored. This takes place when the increase in moment, due to the greater lever arm of the jockey weight, equals the moment of W about the fulcrum, viz., when :—

$$Wa = w(c - b) = wx$$

We can write this equation in the form $W = \left(\frac{w}{a}\right)x$, and the reader will notice that, since w and a do not vary, W is directly proportional to x. This means that the arm may be calibrated with a uniform scale of hundredweights, stones, pounds, etc., from which the weight W can be directly read.

The steelyard is a device of considerable antiquity. This is understandable when we remember that the principle of moments was amongst the first of the basic conceptions in statics to be understood. Archimedes (287-212 B.C.) not only lent drama to scientific discovery in the manner known to every schoolboy, but also formulated the laws of equilibrium of forces acting on a lever.

Early Steelyards

Early Roman steelyards were very similar to that illustrated in Fig. 29, although they were frequently embellished by making the jockey weight in the form of a small bust, presumably representing the contemporary emperor, or other personage. Much later the steelyard came to signify a community of foreign merchants who settled in London in the thirteenth century. This transference of title is explained by the enormous importance of the steelyard in early commerce.

Fig. 30 shows a Danish steelyard, which, although used less in practice, is again a good example of the application of the principle

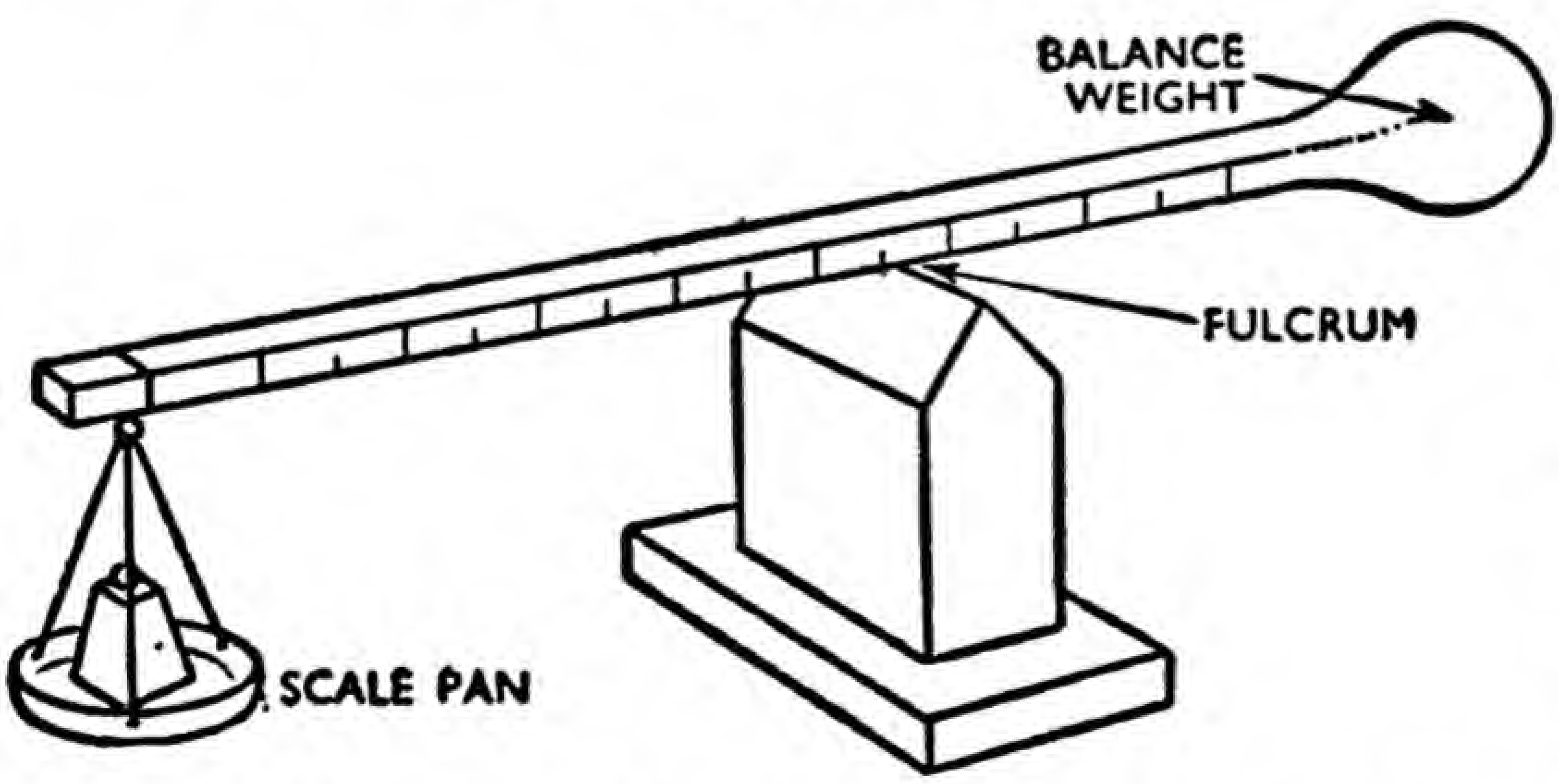


Fig. 30. When weighing a body with the Danish steelyard, equilibrium is obtained by moving the whole lever until the point of balance is found. The weight is then read from the calibration opposite the fulcrum.

of moments. It consists of a lever having a balance weight at one end, and a scalepan at the other end, both of these being fixed in position relative to the lever. In contrast to the common steelyard, equilibrium is established by moving the whole lever until the point of balance on the fulcrum is obtained. Calibrations are marked on the lever so that the weight in the scalepan is read directly from the position of the fulcrum. This has been left as an exercise for the reader to show how these graduations should be placed.

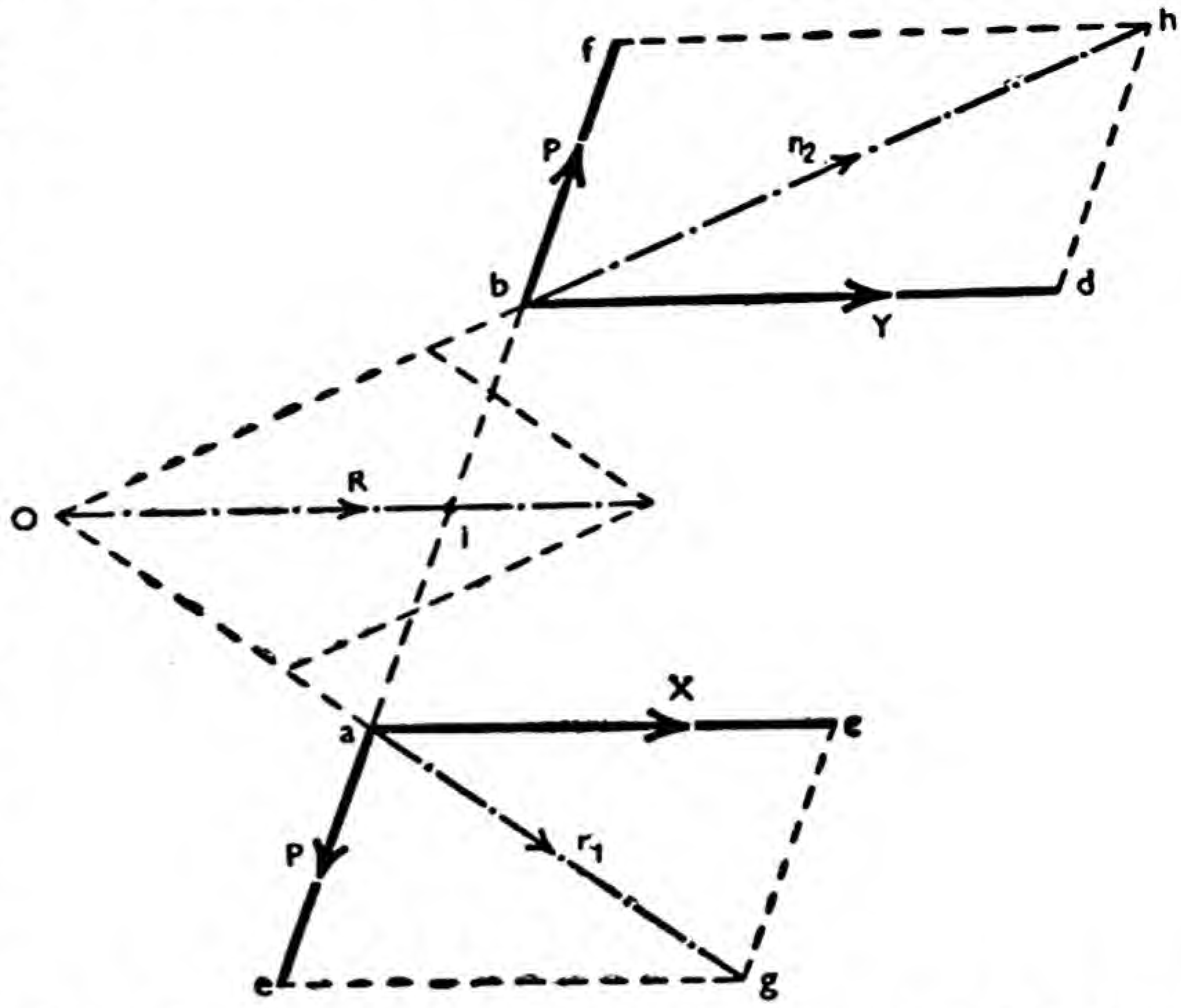


Fig. 31. This diagram enables us to prove that the resultant of two like parallel forces equals the sum of the forces, and that it divides the distance between them in the inverse ratio of their magnitudes.

We saw earlier how to obtain the resultant of two or more forces meeting at a point by drawing a parallelogram, triangle or polygon of forces. Although we have considered parallel forces when dealing with levers, we do not yet know how to find the resultant, or equilibrant, of two or more parallel forces.

If we draw a polygon for parallel forces, it is obviously a straight line, and we at once know the magnitude, direction and sense of the resultant, but we do not know its point of application. This difficulty does not arise with non-parallel forces meeting at a point, because the line of action of the resultant must pass through this point.

'Like' Parallel Forces

When two parallel forces have the same sense, both pushing towards or pulling away from the body, they are said to be like, and we will now find the resultant of two like parallel forces. Fig. 31 shows two like parallel forces *X* and *Y*, represented by the vectors *ac* and *bd*, acting on a body at the points *a* and *b*. Now, as an artificial device, we imagine that two equal and opposite forces *P*, of any convenient magnitude, are applied at *a* and *b* in the line *ab*. Since these forces are equal and opposite, they neutralize each other and have no effect on the equilibrium of the body.

Vectors *ae* and *bf* are drawn to represent the two forces *P*, and we complete the parallelograms to obtain two resultants r_1 and r_2, as shown. These resultants intersect at point *O*, and, therefore, the resultant *R* of the original forces *X* and *Y* must pass through *O*. Now the components of r_1 and r_2 in the direction of the original forces are

X and Y, and, remembering that the opposite forces P cancel out, we deduce that the resultant $R = (X + Y)$ and is parallel to X and Y.

If the reader cannot immediately understand this direct proof, he should check the result graphically with set-square and scale, choosing any convenient values for X, Y and P. Indeed, it may be felt that no proof is necessary to show that $R = (X + Y)$, since X and Y are like parallel forces, but we are especially interested in the position of R, and the construction enables us to find this.

Again, drawing a parallelogram for r_1 and r_2, we find R, the resultant, parallel to X and Y and equal in magnitude to $(X + Y)$. R cuts the line ab in the point i. Now triangles agc and oai are similar.

$$\therefore \quad \frac{oi}{ia} = \frac{ac}{cg} = \frac{X}{P} \quad \ldots\ldots (1)$$

In the same way from the similar triangles bdh and oib :—

$$\frac{oi}{ib} = \frac{bd}{dh} = \frac{Y}{P} \quad \ldots\ldots (2)$$

Dividing (1) by (2),

$$\frac{oi}{ia} \div \frac{oi}{ib} = \frac{X}{P} \div \frac{Y}{P}.$$

$$\frac{\cancel{oi}}{ia} \cdot \frac{ib}{\cancel{oi}} = \frac{X}{\cancel{P}} \cdot \frac{\cancel{P}}{Y}.$$

$$\therefore \quad \frac{ib}{ia} = \frac{X}{Y}.$$

Thus, the point i divides ab internally in the inverse ratio of the like parallel forces.

When two parallel forces are of opposite senses, one pushing and one pulling, by a similar construction it can readily be shown that the resultant equals the difference of the forces and that i divides ab externally in the inverse ratio of their magnitudes. These results are interesting, but they can be stated in a more useful form.

A glance at Fig. 31 should convince the reader that the resultant will also divide the perpendicular distance between the forces in the inverse ratio of their magnitudes. This means that the moment of X about a point in the resultant equals the moment of Y about the same point.

Resultant of Parallel Forces

To sum up, we can say that the resultant of two like parallel forces equals their sum, and is in such a position that the clockwise moment of one force about a point in the line of the resultant, equals the anticlockwise moment of the other force about the same point. The statement relating to moments also holds for unlike parallel forces, but the resultant, in this case, equals the difference of their magnitudes and has the sense of the greater force. The resultant

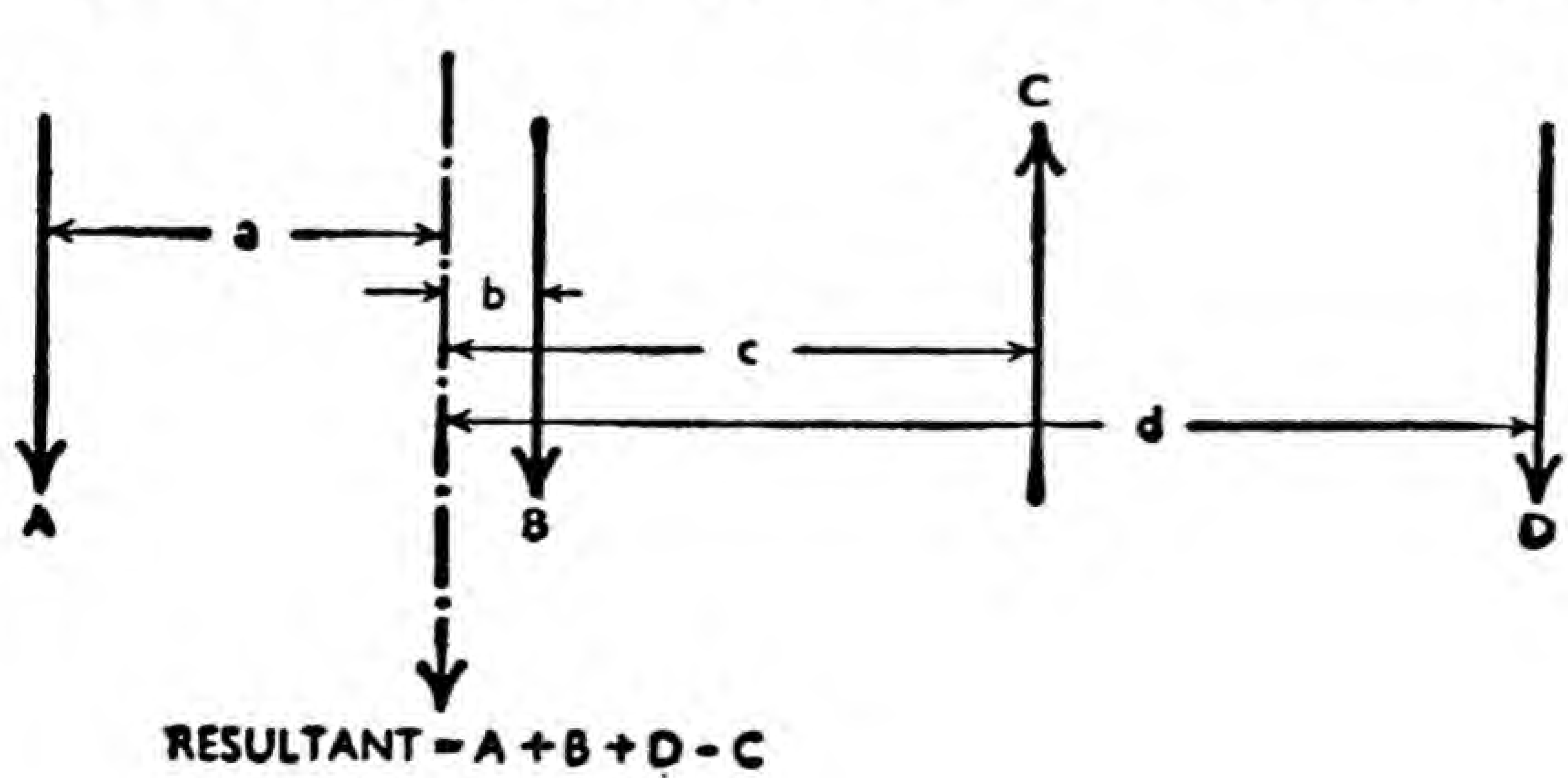


Fig. 32. Resultant of the parallel forces A, B, C and D is in such a position that clockwise moments $Bb + Dd$ equal anticlockwise moments $Aa + Cc$.

of like parallel forces always lies between them, while if the forces are unlike, the resultant will be found to be outside them.

In all of the foregoing, the equilibrant may be considered instead of the resultant, the only difference being in the sense, as indicated by the arrowhead.

The reader is now invited to look back again to the steelyard and the lever safety-valve considered on previous pages. In those cases, clockwise and anticlockwise moments about the fulcrum were equated, but the force exerted at the fulcrum was not considered. It should now be realized that this force is the equilibrant of two parallel forces, like in the case of the steelyard, and unlike in the case of the lever safety-valve.

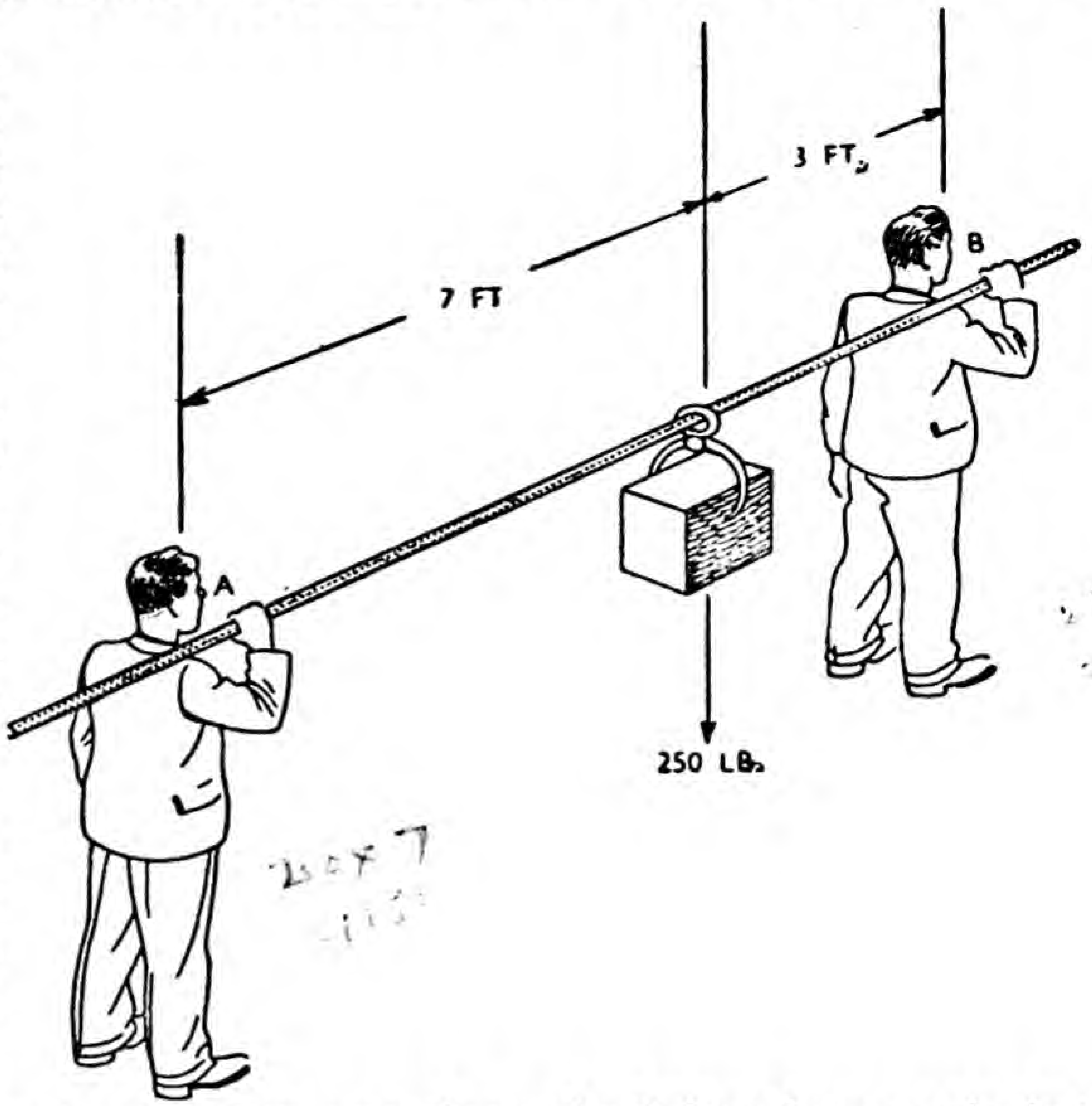


Fig. 33. Two men share a load slung from a pole, but not in equal proportions. The 250-lb. weight is the equilibrant of the upward forces exerted by the men, and these forces can be found by taking moments.

Just as, by successive applications of the parallelogram of forces, we obtained the polygon of forces, and hence the resultant of a system of forces, so we can find the resultant of a number of parallel forces by successive application of the preceding methods. On doing this, we find that the resultant is in such a position that the sum of the clockwise moments about a point in the resultant, equals the sum of the anticlockwise moments about this point. This is illustrated in Fig. 32.

Let us now consider some practical examples of the resultants and equilibrants of parallel forces. Suppose two men are required to carry a stone weighing 250 lb. with the aid of a pole (Fig. 33). One man claims to be stronger than the other and volunteers to carry a greater proportion of the weight. Therefore, the stone is slung from a point in the pole nearer to him than it is to the weaker man, and with this rough justice the load is carried to its destination. But in mechanics we require a more precise result and, in this case, we would ask what actual load each man carried on his shoulder.

Finding Forces by Calculating Moments

Since the pole is in equilibrium, it is clear that the 250-lb. weight is the equilibrant of the two

vertical forces exerted by the men at A and B. We will call these forces a and b respectively, and first write :—

$$a + b = 250 \text{ lb.} \ldots\ldots\ldots (1)$$

Secondly, the moment of a and the moment of b about the equilibrant must be equal.

$$\therefore 7a = 3b \ldots\ldots\ldots\ldots\ldots (2)$$

Solving (1) and (2) gives us $a = 75$ lb. and $b = 175$ lb., results which, no doubt the reader has already obtained by simple proportion. The aim, however, was to emphasize the correct methods to employ, and to remind the reader that equilibrium in translation and rotation should both be considered. The weight of the pole itself has been neglected. Provided the pole was uniform in cross-section and not tapered, this would cause an equal additional burden for each man.

Frequently it is more convenient to consider one of the unknown forces, say, a, as the equilibrant. Then, taking moments about B :—

$$10a = 250 \times 3,$$

whence $a = 75$ lb., as before.

Similarly, taking moments about A :—

$$10b = 250 \times 7,$$

whence $b = 175$ lb., as before.

Alternatively, having obtained $a = 75$ lb., since the pole is in equilibrium for translation, we can write :—

$$b = 250 - 75 = 175 \text{ lb.}$$

More Complex Problems

In more complex problems it is desirable to calculate both the supporting forces by taking moments, and then make sure that the sum of the upward forces equals the sum of the downward forces. This procedure provides a useful check on the arithmetic.

It will be realized that we have taken moments about three different points, the point of loading and the two ends, and the results are the same in each case. In fact, when a body is in equilibrium, the sum of the clockwise moments about any point equals

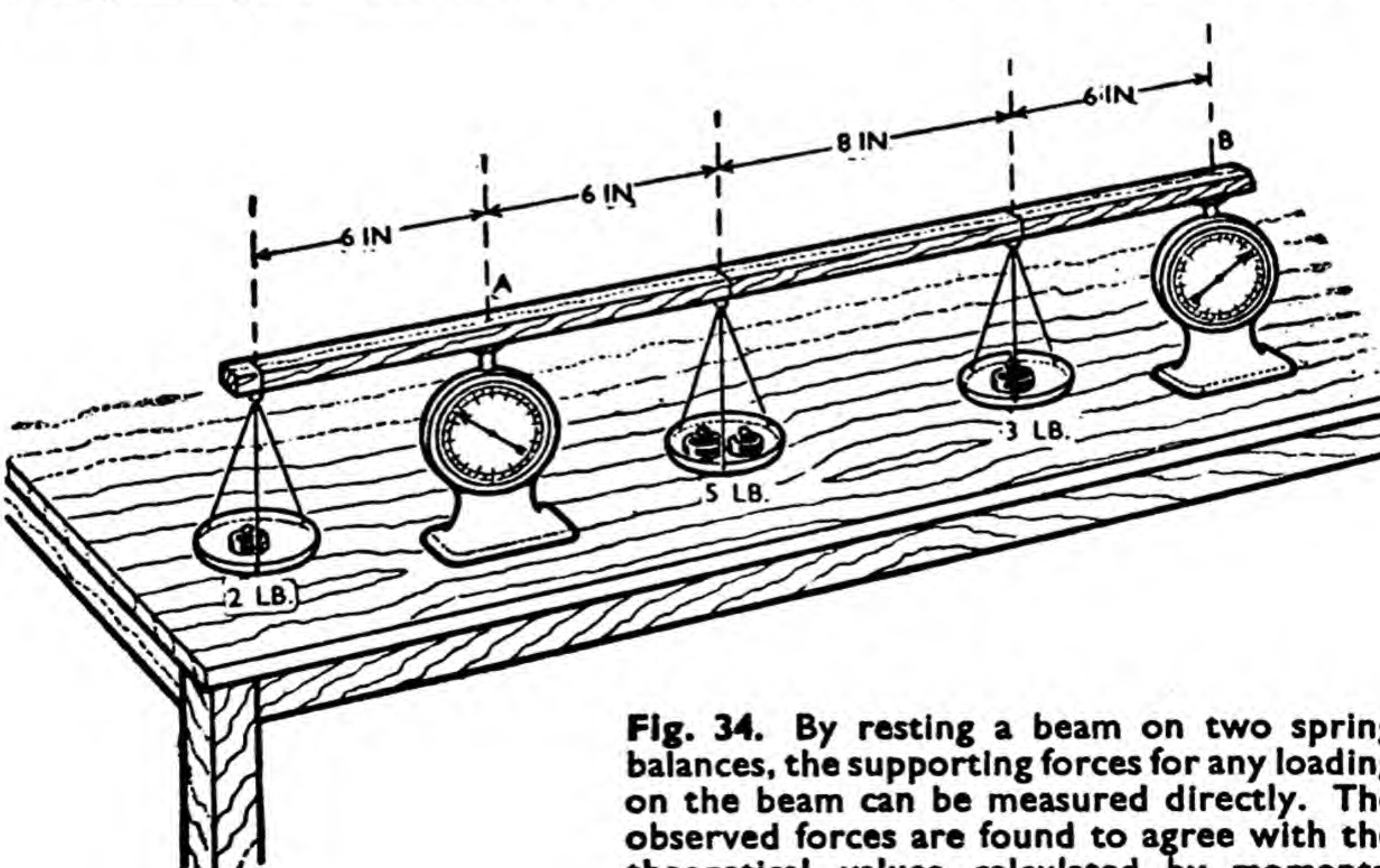
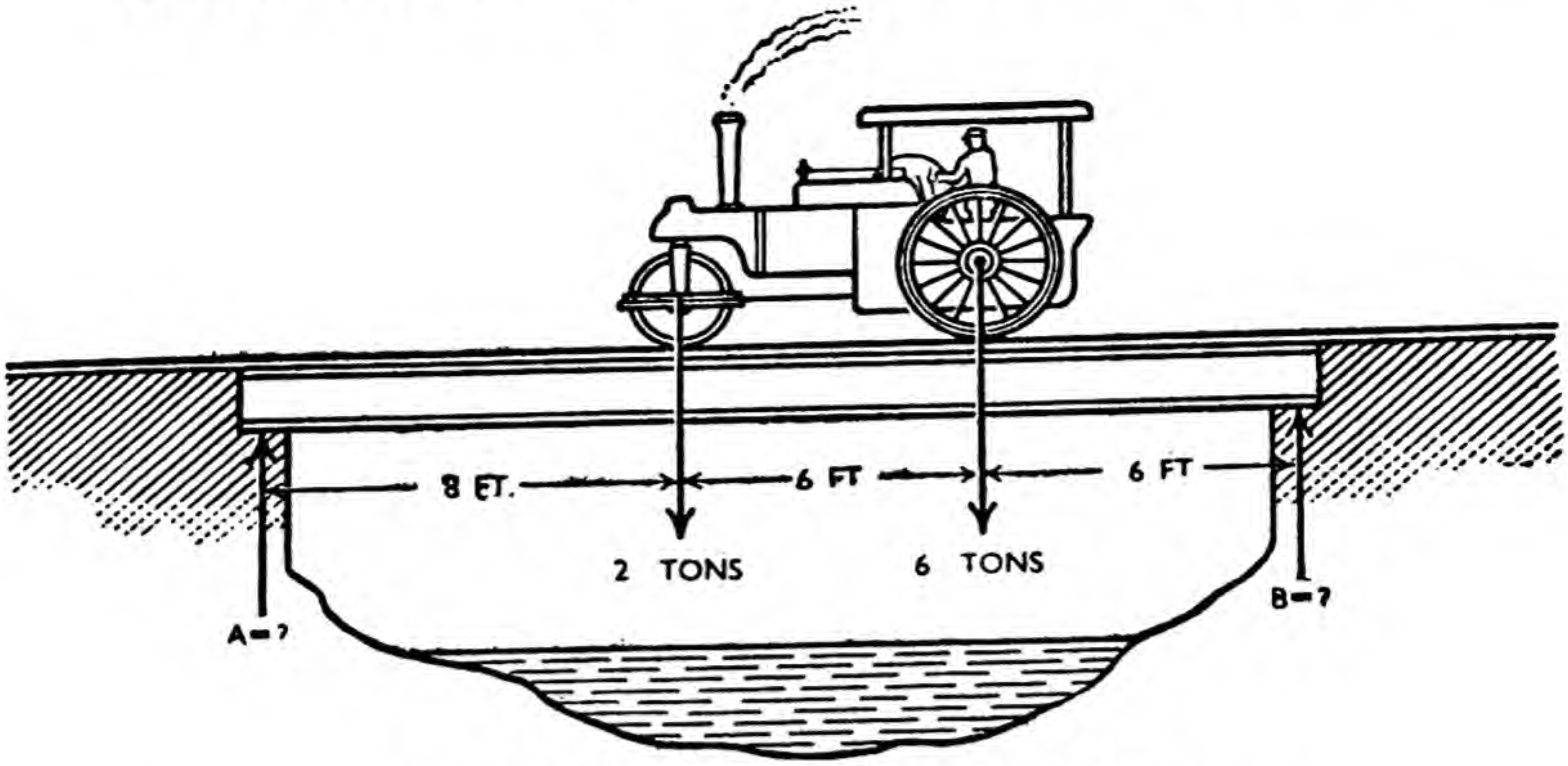


Fig. 34. By resting a beam on two spring balances, the supporting forces for any loading on the beam can be measured directly. The observed forces are found to agree with the theoretical values, calculated by moments.



REACTIONS SUPPORTING VEHICLE

Fig. 35. Steam roller, in this position on a bridge of 20-ft. span, is supported by reactions *A* and *B* which the reader should be able to calculate.

the sum of the anticlockwise moments about that point. We see then that there is no necessity to decide which of the forces is the equilibrant, since any one of them may be considered as such.

Simple Experiment

A simple experiment, which is easily set up on a table, can give definite evidence of the magnitudes of the forces supporting a beam.

In Fig. 34 we see a light wooden beam resting on two spring balances and, from it, various loads are hung.

Since the upward forces supporting the beam are provided by the spring balances, we can read directly from the dials the magnitudes of these forces. The loads may be arranged in any way and be of any convenient magnitude. The arrangement of loads shown in Fig. 34 is complicated slightly by the fact that the beam projects over one of the supports and is, therefore, said to be overhung. This is introduced to emphasize an important aspect of the procedure for calculating the supporting forces. Calling the forces on the balances a and b respectively and equating moments about B, we have :—

(anticlockwise) 2 lb. $\times$ 26 in. + 5 lb. $\times$ 14 in. + 3 lb. $\times$ 6 in. = (clockwise) a lb. $\times$ 20 in.

So that, $140 = 20a$,

whence $a = 7$ lb.

Equating moments about A :—

(clockwise) 5 lb. $\times$ 6 in. + 3 lb. $\times$ 14 in. = (anticlockwise) 2 lb. $\times$ 6 in. + b lb. $\times$ 20 in.

So that, $60 = 20b$,

whence $b = 3$ lb.

Now check the sum of the forces. Upward, we have $a + b = 7 + 3 =$ 10 lb., and downward $2 + 5 + 3 =$ 10 lb., which shows that there has been no error in arithmetic when calculating the forces by taking moments.

Notice that the 2-lb. force has an anticlockwise moment about A, and that in all such cases, clockwise and anticlockwise moments, whether known or unknown, should be carefully grouped on opposite sides of the equation.

If this experiment had been set

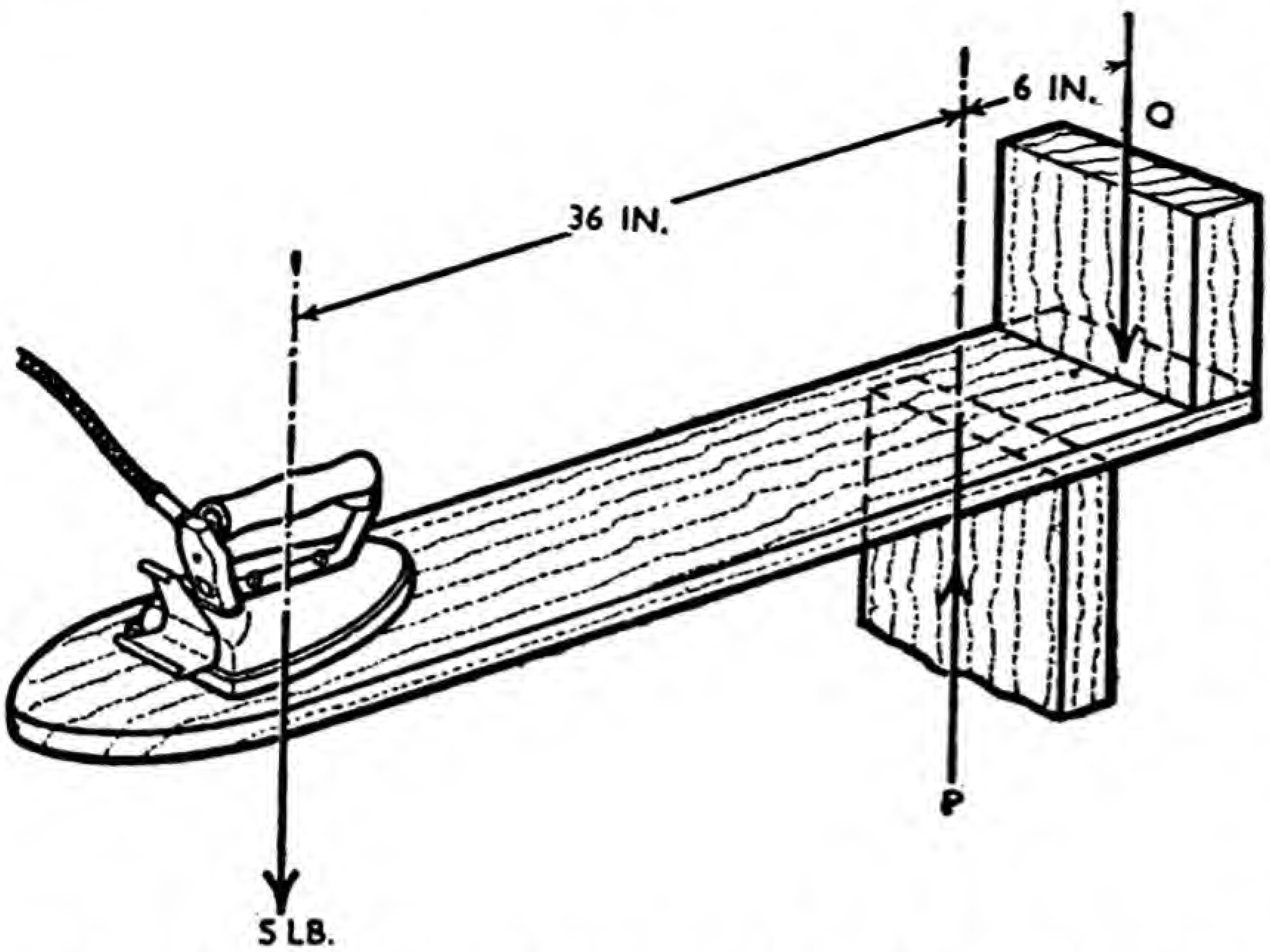


Fig. 36. An ironing-board is an example of a cantilevered beam. The supporting forces P and Q may be found by calculating moments.

up with the weights in the positions shown, it would be found that the balances would record the calculated values of 3 lb. and 7 lb., thus verifying the theory experimentally. It is not quite true to say that the calculated values would be recorded, because they would be increased by the weight of the beam itself. Provided the beam is light, this difference is small. Alternatively, it is possible to calculate the additions to a and b, when we know how to use centres of gravity which are considered later, or, more simply, to note the scale readings before the weights are applied to the beam.

More Practical Problem

Now consider a more practical problem. The front and rear axle loads of a steam roller are 2 tons and 6 tons respectively, and the vehicle has advanced across a bridge to the position shown in Fig. 35. Neglecting the weight of the bridge itself, what are the supporting forces A and B? By taking moments, the reader should, without difficulty, find that A is 3 tons, and B 5 tons.

Cantilever

It should also be noted that not all beams are supported at, or near, each end. Thus, Fig. 36 shows the familiar built-in ironing-board, which is designed to fit vertically into its recess when not in use.

The load, which is the weight of the iron together with any pressure applied by the hand, is outside the supports, and this type of beam is called a cantilever. Assuming the load and dimensions shown in the figure, and taking moments about P:—

5×36 (anticlockwise) $= Q \times 6$ (clockwise)

so that $Q = 30$ lb.

Similarly, taking moments about Q:—

$5\,(36+6)$ (anticlockwise) $= P \times 6$ (clockwise)

and thus $P = 35$ lb.

Note that the forces are in equilibrium for translation, since $(5 + 30)$

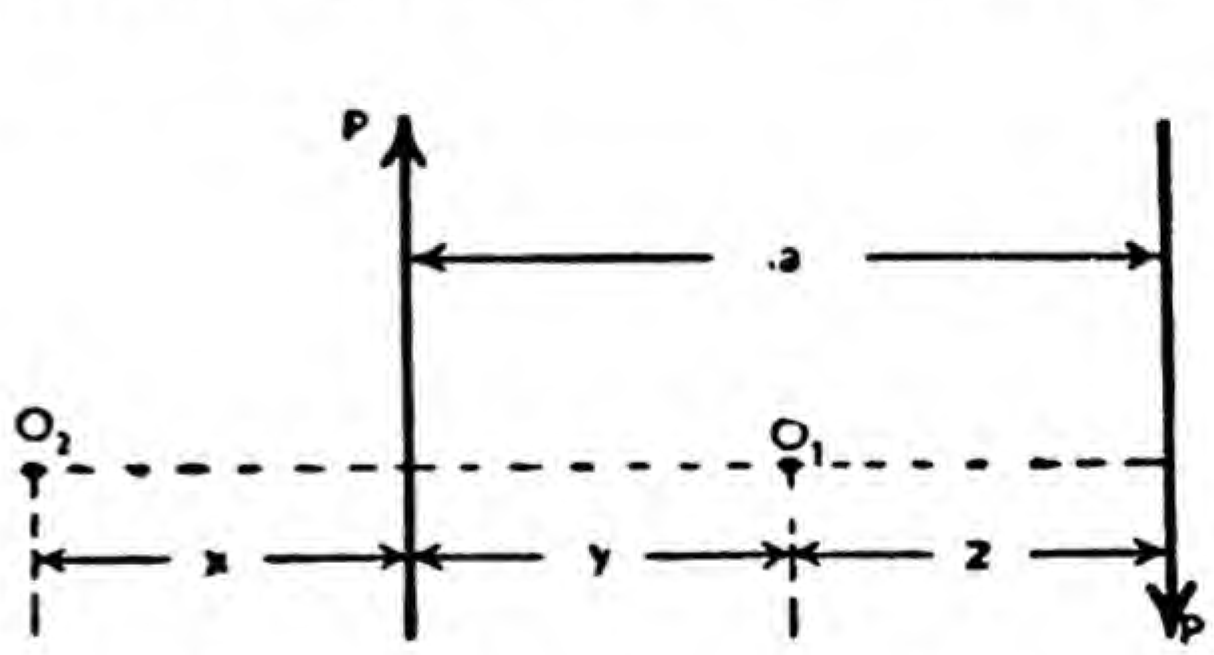


Fig. 37. These two equal and opposite parallel forces form a couple; a is the arm of the couple. System has no resultant force but has a resultant moment which equals Pa, irrespective of the point about which moments are taken.

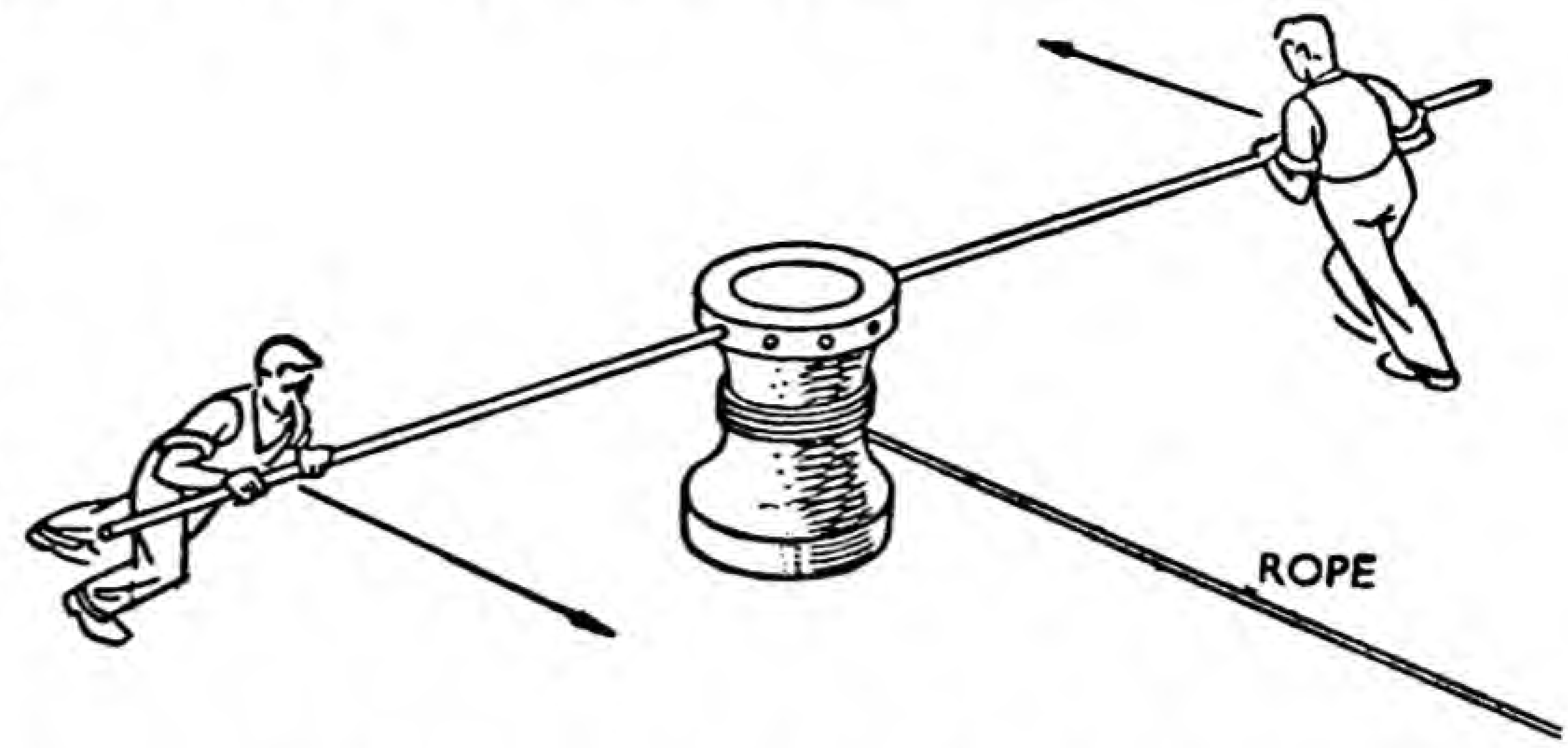


Fig. 38. If two men apply equal parallel and unlike forces to the arms of a capstan, the moment of the couple so produced rotates the capstan, and thus provides a large force in the rope.

lb. downward is balanced by 35 lb. upward.

This type of beam is commonly used in structural practice. For example, the galleries in a modern theatre are usually supported on cantilevers, because columns are undesirable in the auditorium.

Meaning of 'Couple'

A system of two equal parallel forces of opposite sense is termed a couple, and the perpendicular distance between them is the arm of the couple.

If we try to find a resultant force by applying the construction shown in Fig. 31, we cannot proceed because the two parallelograms are similar, and the diagonals are parallel, so that an intersection point is impossible to obtain. No doubt the reader has already realized that a couple can have no resultant force.

Considering Fig. 37, the force P acting upward is balanced by the force P acting downward and so there can be no translation and no resultant force. There is, however, a resultant moment, and it is interesting to note that it does not matter about which point we take moments. Thus, taking moments about O_1, we have :—

Moment of couple $= Py + Pz$
$= P(y + z) = Pa$ (clockwise).

This shows that, irrespective of the position of O_1, the moment does not vary.

The same is true if we consider the moment of the couple about any external point, say, O_2.

Moment of couple $= P(a + x)$ (clockwise) $-$ Px (anticlockwise) $= Pa$ (clockwise).

Therefore, the algebraic sum of the moments of two forces forming a couple about any point in their plane, is constant and equals the product of one of the forces and the arm of the couple.

A practical example of a couple is seen in the case of a capstan, (Fig. 38) which may be regarded as a form of lever. The tension in the rope has a moment about the centre of the capstan, and this is overcome by the moment of the couple produced by the two men. Since the lever arm of the rope about the centre is small, and the arm of the couple is great, a large tension in the rope can be readily produced by comparatively small forces applied at the ends of the radial bars.

Weight as Numerous Parallel Forces

We noted earlier that the earth exerts an attractive force on all bodies. Now, any body is made up of a very large number of small pieces or particles, and each of these is attracted towards the earth. Thus, we can consider the weight

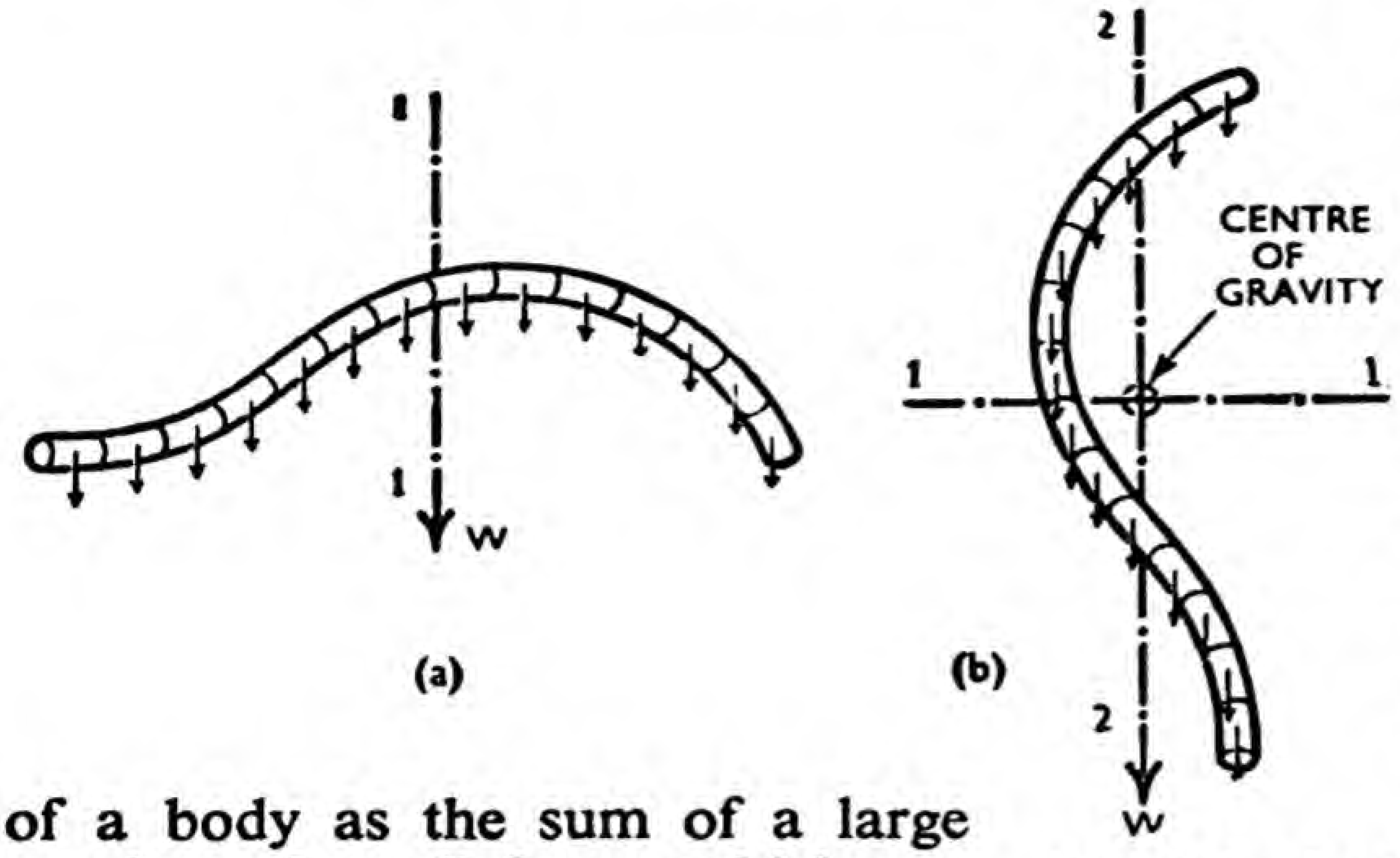


Fig. 39. Length of wire is considered to consist of a number of small pieces, each of which is attracted vertically towards the earth by gravity. The weight of the wire, which is the resultant of all these forces, is found to act through a point called the centre of gravity, no matter how the body is placed.

of a body as the sum of a large number of small forces which are all acting downward vertically, and are, therefore, parallel.

Centre of Gravity

If we find the position of the resultant of all these forces, its line of action will pass through a point called the centre of gravity. If the body is rotated to a new position, each particle is still attracted to the earth, but of course the direction of all these forces, relative to the body, is then altered. The resultant force, however, will still pass through the centre of gravity.

Consider the curved piece of wire shown in Fig. 39(a). We cannot represent a division into very small particles, but we can imagine that the wire is divided into short equal lengths, each having the same weight. Thus, we have a system of equal, parallel forces of which the resultant is the whole weight W and, by the methods already discussed, we can find its position 1—1.

Now, imagine that the wire is turned through a right angle as in Fig. 39(b). Again, each short length of wire is acted upon by a vertical force, and again the resultant is W, but acting now in the position 2—2. The intersection of 1—1 and 2—2 is the centre of gravity of the body, and it is found that whatever the angle or position of the body, the weight always appears to act through this point. We can now define the centre of gravity of a body as that point through which the line of action of the weight always passes, whatever the position of the body.

Frequently, it is necessary to find the centre of gravity of a lamina, or thin sheet of material, say, cardboard, of uniform thickness. As defined, the centre of gravity is the point through which the weight may be assumed to act. If we imagine that the thickness of the lamina is diminished, the position of the centre of gravity will obviously remain unchanged, although the total weight is reduced. If the lamina is assumed now to be infinitely thin, it becomes a figure having an area but no weight, and we then use the term centroid, instead of centre of gravity.

In many cases, because of symmetry, the position of the centre of gravity is obvious. Considering a straight, uniform rod, we can say at once that the centre of gravity must be at its midpoint. This follows because, if we divide the rod into short equal lengths, as the wire shown in Fig. 39 is divided, the

resultant of all the small forces must pass through the centre of the rod.

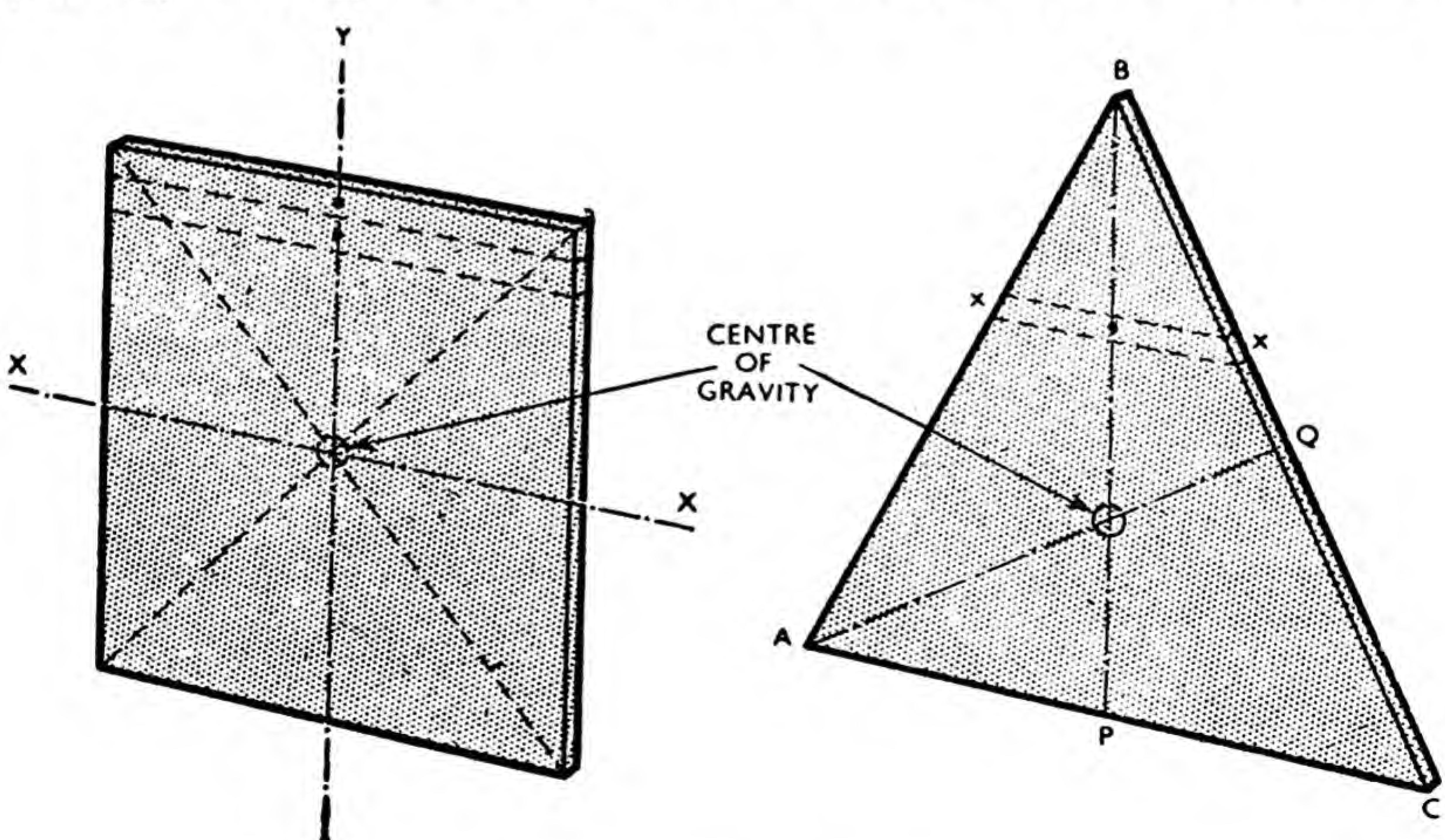
Now consider a square lamina (Fig. 40). We can divide this into a large number of narrow strips, each of which may be treated as a rod, with its centre of gravity at the centre of the rod. Thus, the axis *YY* contains the centres of gravity of all the component strips, and, therefore, must contain the centre of gravity of the whole. If, now, the lamina is turned through 90 degrees so that the axis *XX* is vertical, it is clear that the centre of gravity must also lie in *XX*. The intersection of *XX* and *YY* is, therefore, the centre of gravity ; this is also the point of intersection of the diagonals.

Triangular Lamina

Similar reasoning may be applied to the triangular lamina. The centres of gravity of all narrow strips such as *XX* must lie in the line *BP*, *P* being the midpoint of the side *AC*. The centre of gravity of the whole lamina must lie in *BP*, and similarly it must also lie in *AQ*, *Q* being the midpoint of the side *BC*. *PB* and *AQ* are called medians, and so we say that the centre of gravity of a triangular lamina lies at the intersection of the medians.

If we are considering a square and a triangle, geometrical figures having no weight, the centroids occupy the same positions as the centres of gravity of the laminæ. Now, as shown on page 54, we can tabulate these results, along with several others, which are obvious or can very simply be deduced.

The centre of gravity of a lamina of any shape can be found by experiment, as shown in Fig. 41. Several small holes are bored in the lamina, so that the body may be suspended freely from a pin passing through a hole. A vertical line is obtained by suspending



FINDING CENTRE OF GRAVITY

Fig. 40. By imagining a subdivision into narrow strips, we find that the centre of gravity of a square lamina lies at the intersection of the diagonals, and of a triangular lamina at the intersection of the medians.

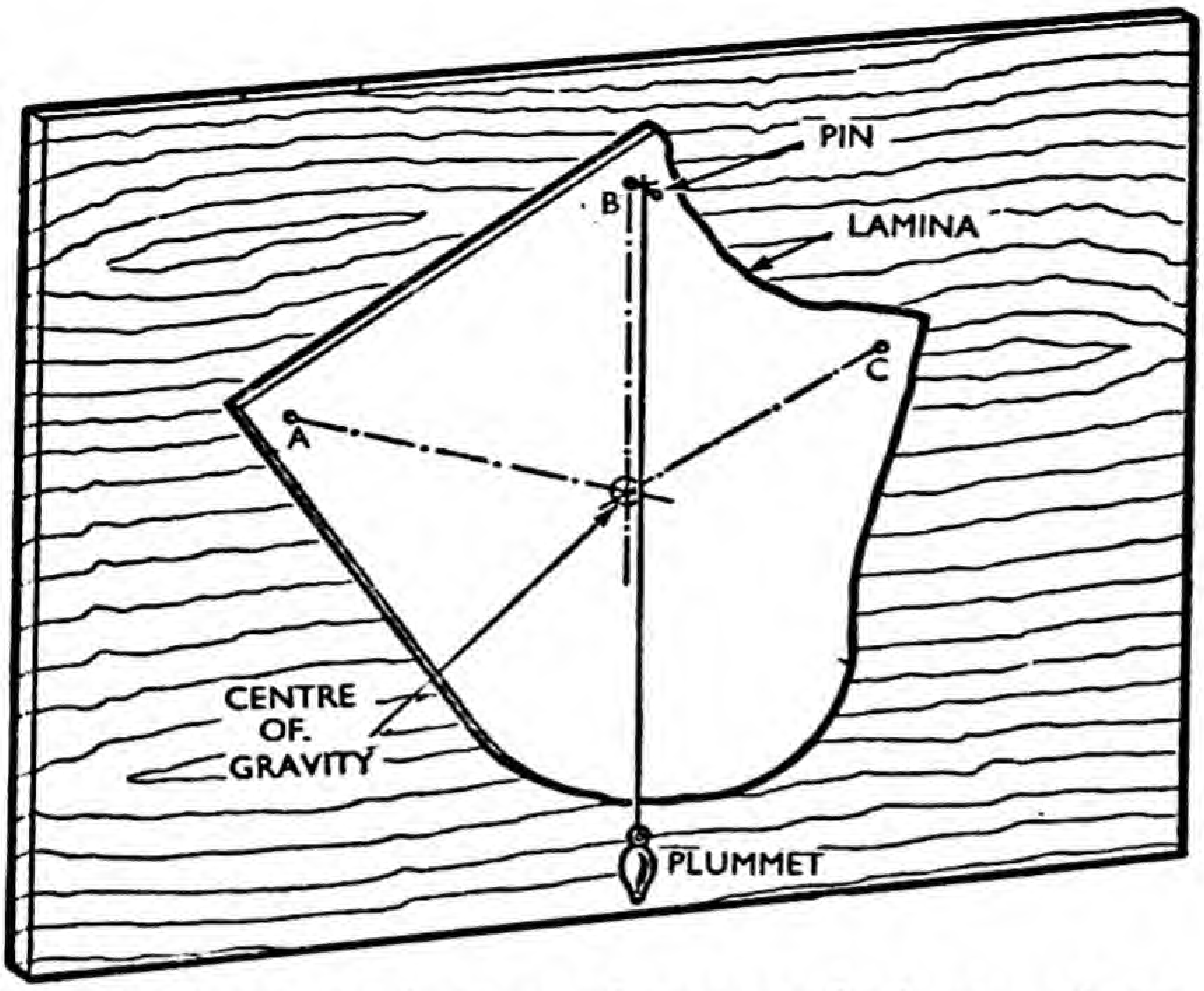
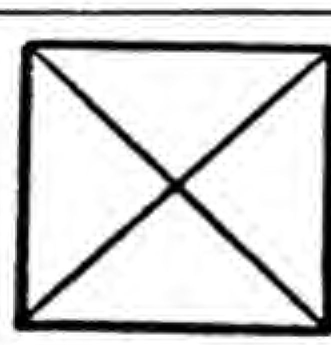
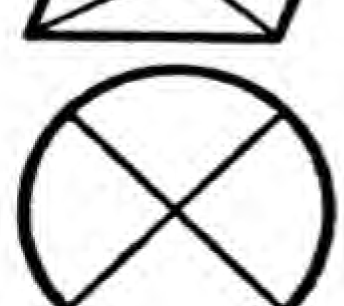
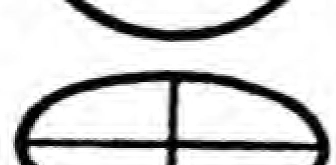
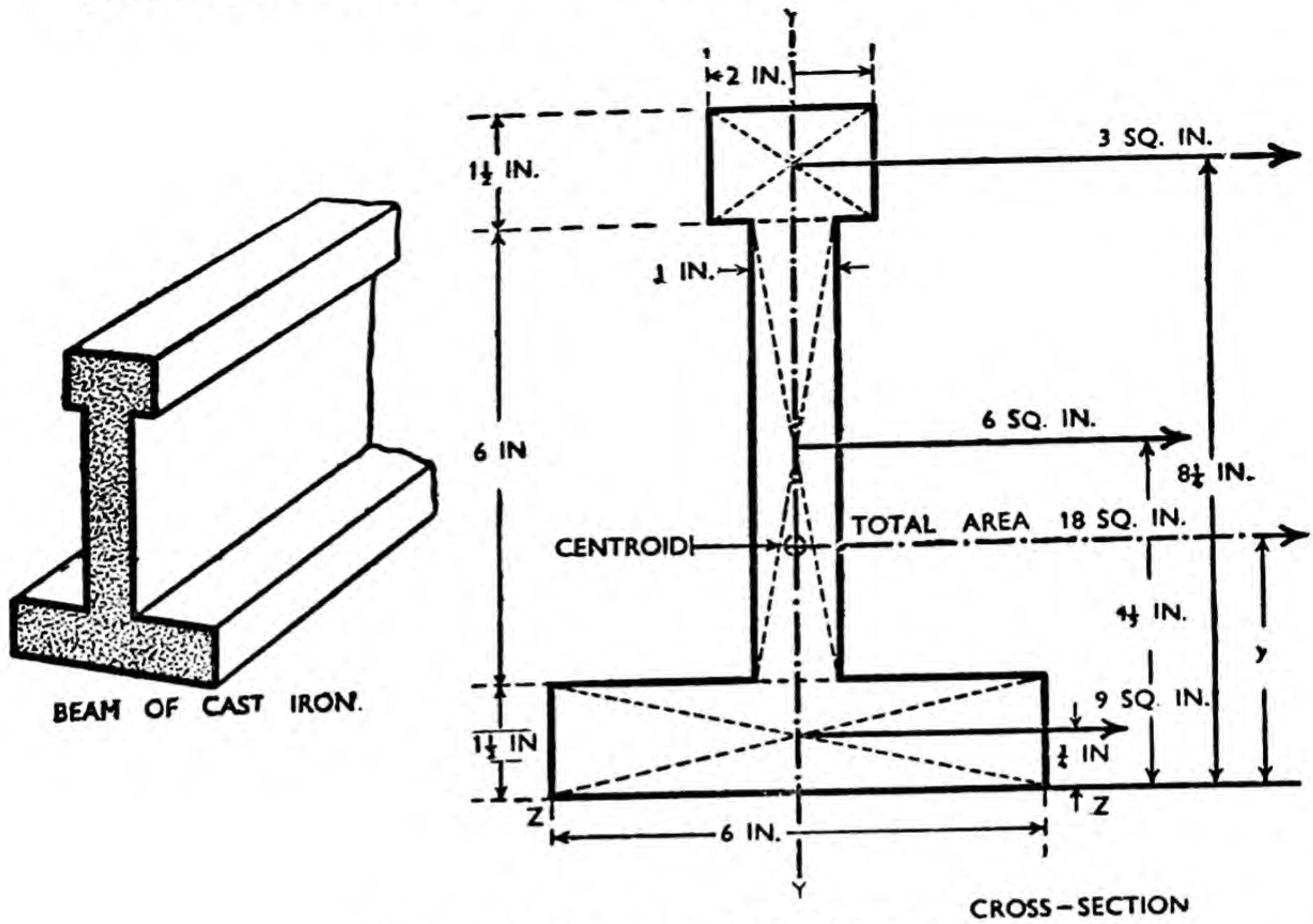


Fig. 41. Position of centre of gravity of a lamina of any shape can be found by experiment. Suspending lamina from various holes *A*, *B*, *C*, etc., we draw on it vertical lines by means of a plummet, and the intersection of the lines gives the position of the centre of gravity.

a plummet from the same pin. This line is carefully marked on the lamina with a pencil. Alternatively, the cord is chalked and then gently plucked so that it strikes against the surface and leaves a fine chalk line. This procedure is repeated with the lamina suspended in turn from the other holes, and it will be found that all the lines intersect at a point which is the centre of gravity.

With the lamina suspended as shown in Fig. 41, there are two forces acting on it, (1) the weight, acting vertically downward through the centre of gravity, and (2) the vertically upward reaction at the pin. But the body is in equilibrium, so these two forces must be in the

Shape		*Position of Centroid or Centre of Gravity of Lamina*
Square		Intersection of diagonals.
Triangle		Intersection of medians.
Rectangle		Intersection of diagonals.
Parallelogram		Intersection of diagonals.
Circle		Intersection of any two diameters.
Ellipse		Intersection of major and minor axes.



POSITION OF THE CENTROID

Fig. 42. Position of the centroid of the cross-section of a cast-iron beam is found by treating the areas of the individual rectangles as forces. By taking moments, the position of the resultant is found and this locates the centroid.

same line, and equal and opposite. The centre of gravity thus lies vertically below the pin, and so the intersection of the plummet-lines locates it exactly. If the experiment is performed with care, and a pin is later passed through a small hole bored at the centre of gravity, it will be found that the lamina will remain at rest in whatever position it is placed.

Finding the Centroid

In structural calculations, it is frequently necessary to find the centroid of a figure, compounded of several geometrical elements. For example, Fig. 42 shows the cross-section of a cast-iron beam. Because of the physical properties of cast-iron, the beam is not symmetrical about a horizontal axis, as is the case with steel beams, and before we can calculate the safe load that may be carried by the beam, we must find the position of the centroid of the cross-section.

If a piece of paper is cut to this shape and folded along the vertical line *YY*, it is obvious that each part of the left-hand side would cover exactly the corresponding part of the right-hand side. Therefore, we say that the figure is symmetrical about *YY*, so that we know at once the centroid must lie somewhere on *YY*. We then note that the figure consists of three rectangles, with the centroid of each lying at the intersection of its diagonals.

Now the centre of gravity of a body is the point through which the resultant weight acts, and the position of this resultant can be found by taking moments. In

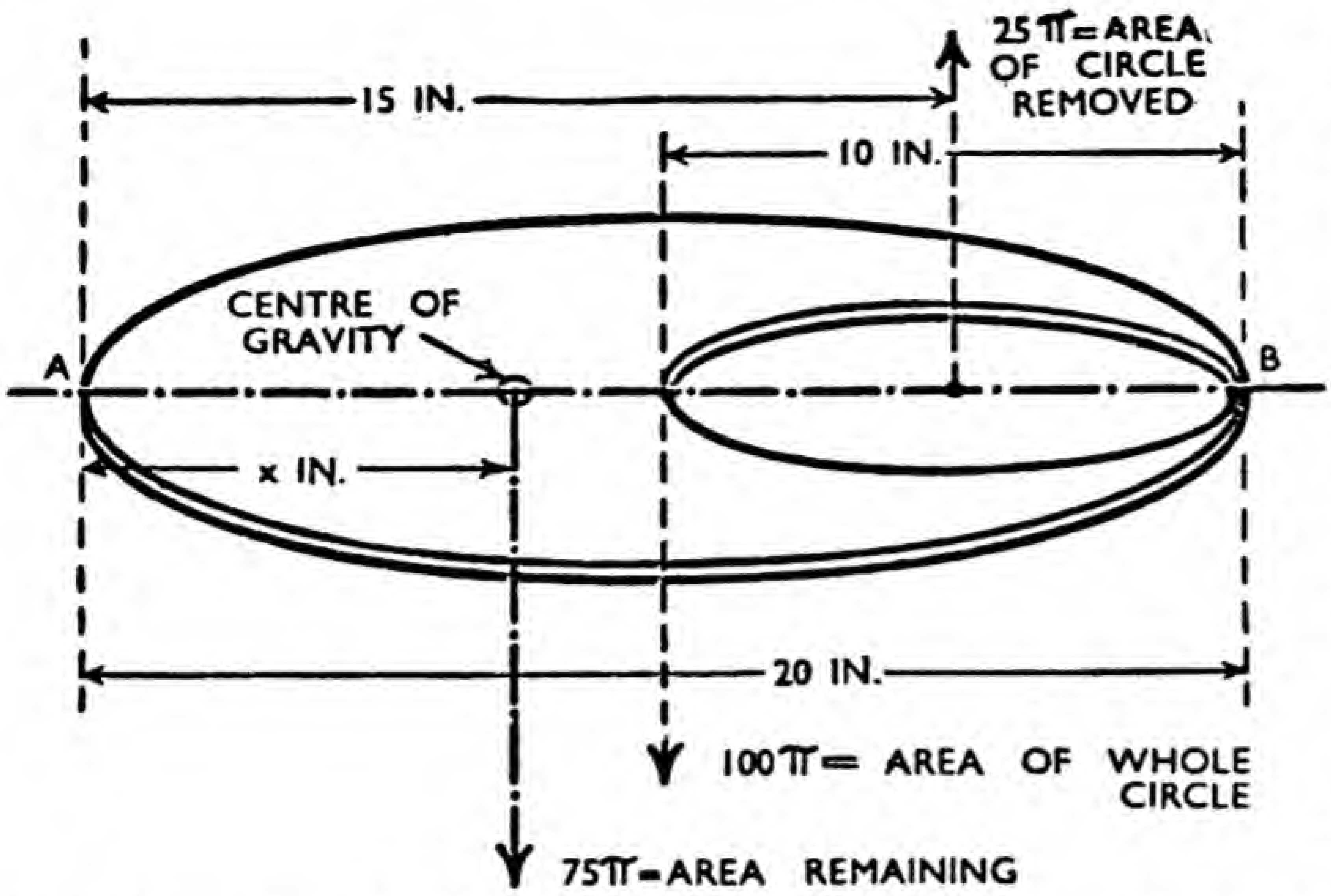


Fig. 43. When finding the position of the centre of gravity of this disk, the force representing the area of the smaller circle is given a contrary sign signifying that this area has been cut away.

this case we are considering areas which have no weight, but we can, in just the same way, take moments of areas. The areas of the three rectangles are 3 sq. in., 6 sq. in. and 9 sq. in., a total of 18 sq. in. Treating these areas as forces, and taking moments about ZZ, we have :—

Sum of moments of component areas = Moment of total area.

$$\therefore 3 \times 8\tfrac{1}{4} + 6 \times 4\tfrac{1}{2} + 9 \times \tfrac{3}{4} = 18 \times y.$$

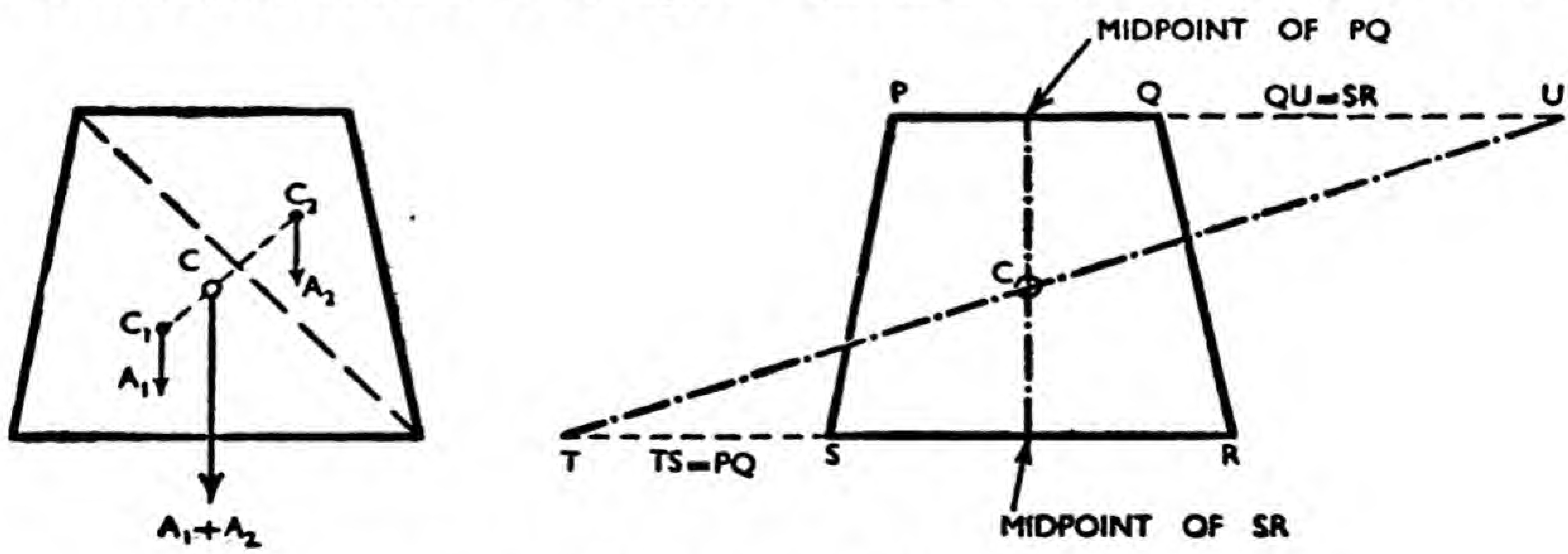
$$\therefore 58\tfrac{1}{2} = 18y, \text{ and } y = 3\tfrac{1}{4} \text{ in.}$$

Therefore, the centroid of the figure lies in the central axis YY, and is $3\tfrac{1}{4}$ in. above the base ZZ. Notice particularly that each individual area is treated as if it were a force, and the total area, 18 sq. in., as if it were the resultant of these forces.

Area Subtracted

We may now consider another example. From a uniform circular lamina 20 in. in diameter, a disk 10 in. in diameter is cut out, as shown in Fig. 43, and it is required to find the position of the centre of gravity of the remainder. First of all, notice that again there is an axis of symmetry AB, and that the centre of gravity will, therefore, lie somewhere in AB. Since the lamina is uniform in thickness and of the same material throughout, the weight of any portion is proportional to the area of that portion.

Thus, we take into consideration areas instead of weights, which is another way of saying that we



TWO METHODS OF CALCULATION

Fig. 44. Centroid of a trapezium may be found by dividing the figure into two triangles. Alternatively, the graphical construction shown may be employed.

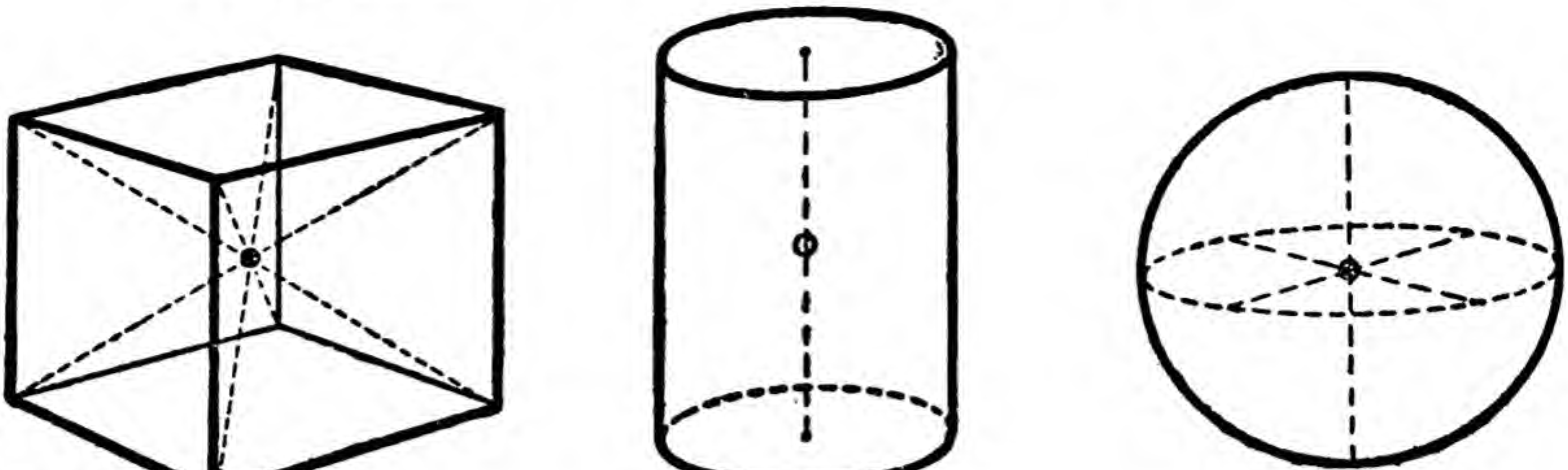


Fig. 45. Cube, cylinder and sphere are bodies having three mutually perpendicular axes of symmetry. The position of the centre of gravity in these cases is obvious without proof.

will find the centroid of the geometrical figure, and this centroid, of course, coincides with the centre of gravity of the disk. The area of the original circle is $10^2\pi = 100\pi$ sq. in., and this is considered as a force acting downward from the centre. (π is always used to represent a certain number which cannot be accurately expressed as a fraction, and is the result of dividing the circumference of a circle by its diameter. Its value is approximately 3.14 or $\frac{22}{7}$.) The area of the smaller circle is $5^2\pi = 25\pi$ sq. in., and, since it has been removed, it is treated as a force acting upward.

The area remaining is 75π sq. in. Now we take moments about any convenient point in the axis of symmetry, let us say, for example, A. Thus :—

$100\pi \times 10 - 25\pi \times 15 = 75\pi \times x$

Cancelling π throughout, we have :

$1{,}000 - 375 = 75x$

$\therefore\ x = 8\frac{1}{3}$ in.

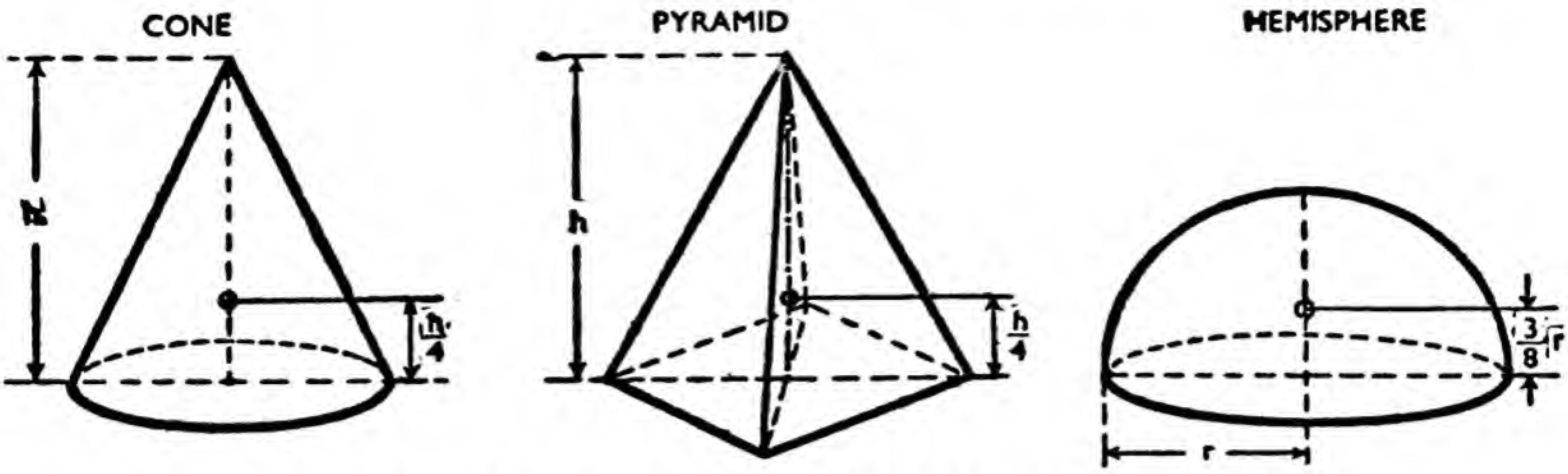
The centroid of the figure, or the centre of gravity of the lamina, is found to be $8\frac{1}{3}$ in. from the point A, or $1\frac{2}{3}$ in. from the centre. Note carefully that the parallel forces (areas) are in this case unlike, and so the position of their resultant, which locates the centroid or centre of gravity, lies outside them.

The Trapezium

Another figure of great practical importance is the trapezium, a four-sided figure having two sides parallel. When considering the stability of dams and retaining walls (Chapter 3), it is necessary to be able to find the position of the centroid of such a figure. A trapezium (Fig. 44) can be divided into two triangles of areas A_1 and A_2, having centroids C_1 and C_2, and the centroid C of the whole figure must lie on the line C_1C_2. As in the previous examples, by taking moments we can find the position of C.

There is, however, a convenient graphical construction. Produce the parallel sides PQ and RS as shown, making $QU = SR$, and $ST = PQ$. The intersection of TU and the line joining the mid-points of PQ and SR is the centroid.

The reader should verify this



CENTRE OF GRAVITY ON CENTRAL VERTICAL AXIS

Fig. 46. Each of these bodies has a central vertical axis in which the centre of gravity lies. It is found that for the cone and the pyramid, the height above the base is $h/4$, and for the hemisphere, $3r/8$.

construction by drawing an actual trapezium, and finding the position of the centroid by both methods.

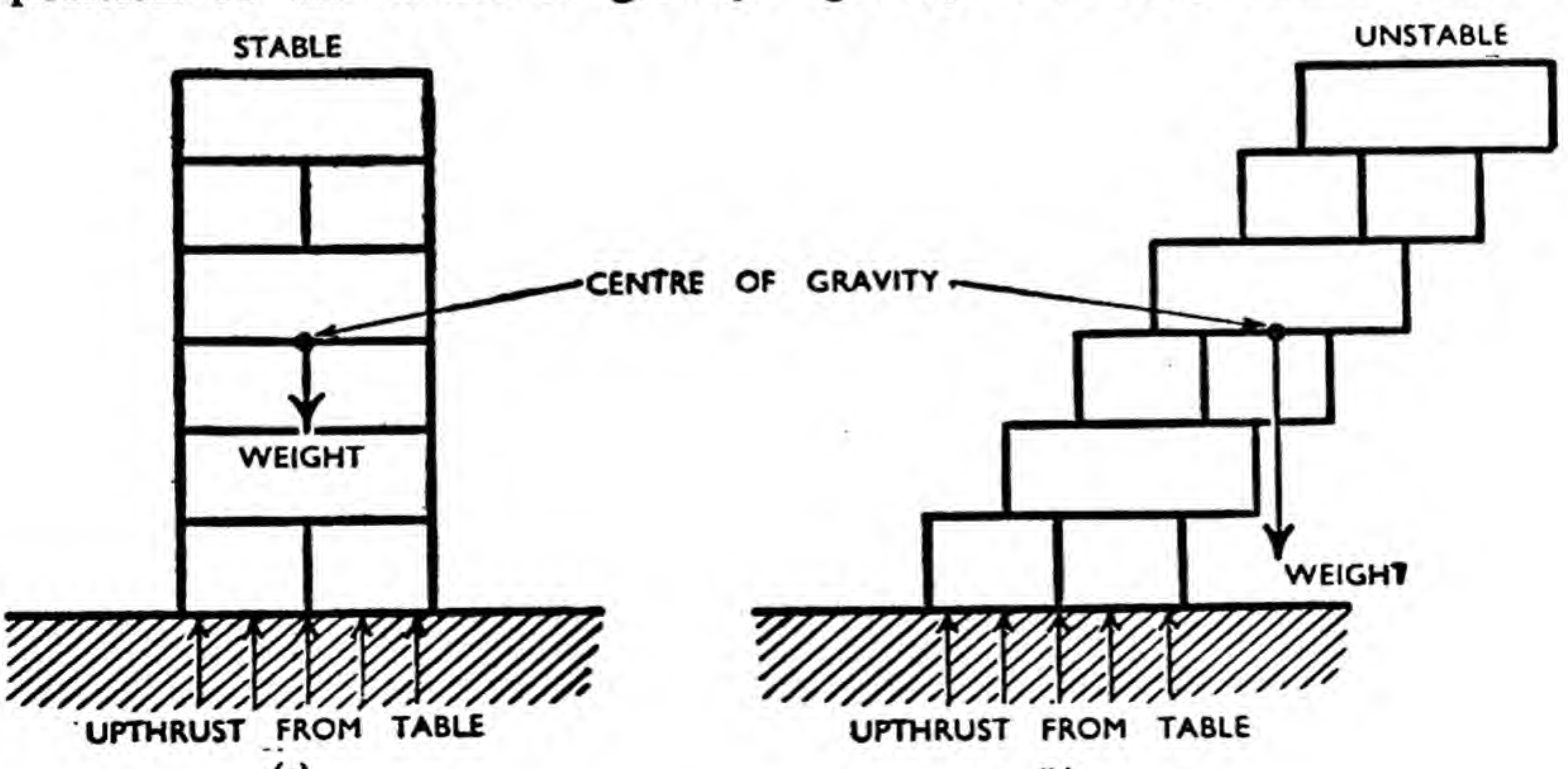
Solid Bodies

Let us now consider solid bodies. Our definition of the centre of gravity as the point through which the line of action of the weight passes, whatever the position of the body, still applies.

In certain cases (Fig. 45) there are three axes of symmetry perpendicular to each other, and the position of the centre of gravity is evident without any formal proof.

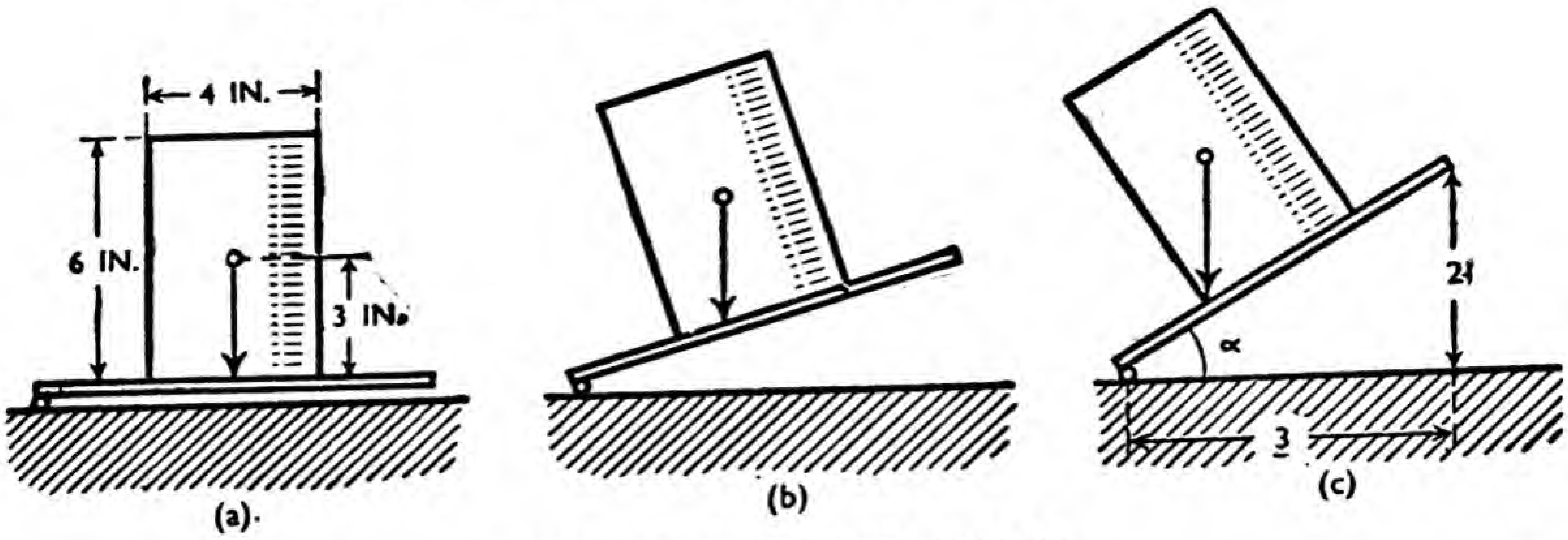
We cannot, however, by mere inspection, find where the centre of gravity lies in the case of a cone, pyramid or hemisphere. These bodies do not have three axes of symmetry, and the centre of gravity is obtained by mathematical processes which we cannot discuss here. The results, however, are important, and they are shown, for reference, in Fig. 46.

The position of the centre of gravity of a body with respect to



STABLE AND UNSTABLE CONSTRUCTIONS

Fig. 47. (*a*) Wall or pier of model bricks constructed as shown is stable, but if the bricks are cantilevered out (*b*) so that the centre of gravity falls outside the base, the structure cannot be in equilibrium and must fall over.



CRITERION FOR STABILITY

Fig. 48. If the inclination of the plane on which the cylinder stands is steadily increased, the body remains in equilibrium until the vertical, through the centre of gravity, falls outside the base. (*a*) Stable, (*b*) Stable, and (*c*) Unstable.

its base is very important. When the determining, by experiment, the centre of gravity of a lamina, we noted that the weight of the body acted in the same vertical line as the supporting force at the pin. Now, if a body rests on a horizontal plane, the supporting force must act through the base of the body in contact with the plane. If the body is in equilibrium, then the vertical through the centre of gravity, must pass within the base. By a simple example this can be made quite clear.

Stability of Walls

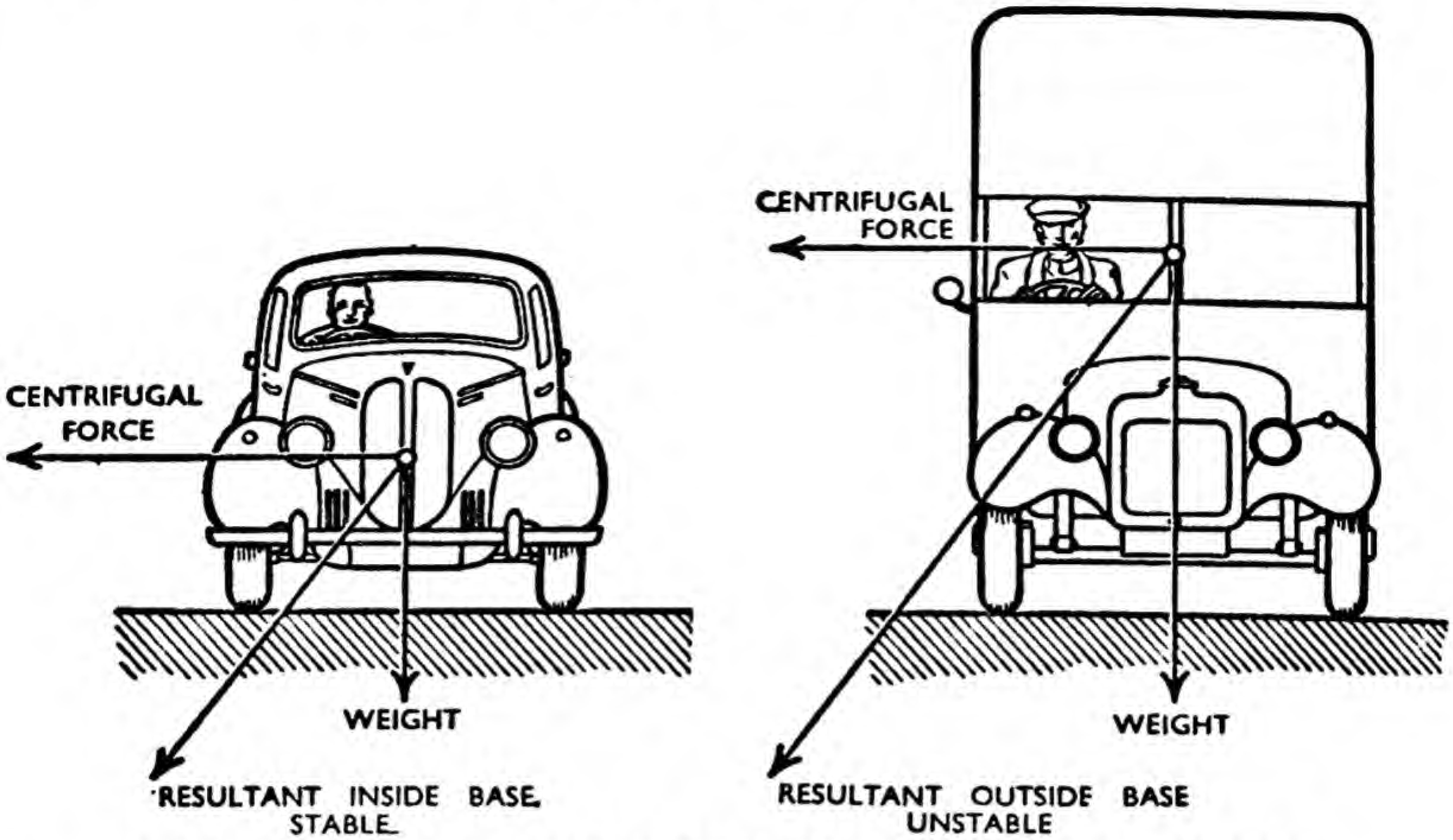
We have all, at some time or other, used model bricks to construct fanciful buildings which could be demolished by a stroke of the hand. Sometimes, however, our best efforts have collapsed, when ambition attempted to erect a structure which did not comply with the laws of equilibrium.

In Fig. 47 we find the first structure of model bricks is obviously stable. In the second case, it is noticed that the line of action of the weight through the centre of gravity falls outside the base. The other force acting is the upward pressure from the table, applied to the base. These forces, not being in the same line, cannot balance each other, and so the body cannot be in equilibrium, but must fall over.

As another example let us imagine that a cylinder 6 in. high and 4 in. in diameter, is placed on a plane, the inclination of which can be altered at will (Fig. 48). Assume also that the cylinder is prevented from sliding down the plane, although it may topple over.

Starting from the horizontal position as in Fig. 48(a), we imagine that the inclination of the plane is increased. In Fig. 48(b), the vertical through the centre of gravity still falls within the base, and the cylinder remains in stable equilibrium. When, however, the body is just about to overturn, the conditions are as illustrated in Fig. 48(c), and by simple geometry we deduce that the slope of the plane is 2 vertical in 3 horizontal, or that $\tan \alpha = \frac{2}{3}$, so that $\alpha = 33{\cdot}7$ deg. We can now say that if the inclination of the plane to the horizontal exceeds 33·7 deg., the cylinder is unstable and will topple over.

We have all experienced a sense of insecurity when riding round a curve at speed in a car or bus, even



MOVING VEHICLES SUBJECTED TO CENTRIFUGAL FORCE

Fig. 49. Vehicle rounding a curve is subjected to a lateral (centrifugal) force in addition to its weight. Provided the resultant of these two forces passes within the base, the vehicle is stable. Note that it is desirable to have the centre of gravity as near to the ground as possible. The laden furniture van is said to be top-heavy because the centre of gravity is high, and, for safety, the centrifugal force must be reduced by reducing the speed.

when the road was dry and skidding, therefore, unlikely. Instinctively, too, we feel that some vehicles are liable to overturn more readily than others under these conditions. This is undoubtedly true. When a body moves in a curved path, an outward, or centrifugal force (Chapter 4) acts on it through its centre of gravity. Also acting through the centre of gravity is the weight of the vehicle, and the point at which the resultant of these two forces cuts the base, determines the stability. We find then that the centrifugal force varies with the speed, but, as long as the resultant passes inside the base of the vehicle, it will not overturn.

Reference to Fig. 49 will show how important it is to keep the centre of gravity as low as possible.

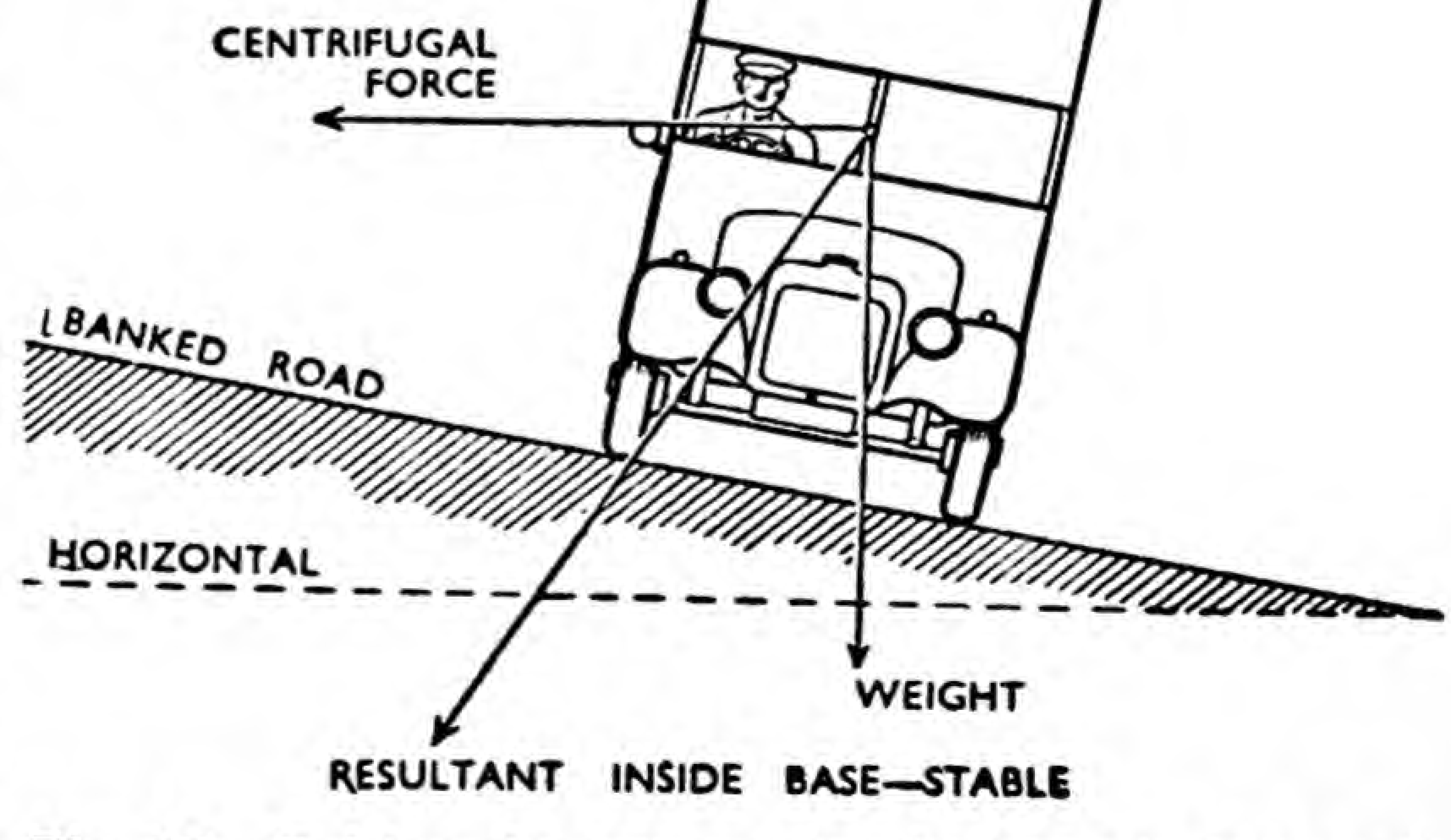


Fig. 50. Raising the outer side of a bend in the road assists in keeping the resultant within the base. A curve may then be negotiated safely by the vehicle at an increased speed.

A motor car can safely round a curve at a speed which would overturn a top-heavy furniture van. For this reason, a racing car is designed to have a very low centre of gravity. It should be clear, now, why omnibus companies insist upon the regulation of allowing no passengers to stand on the upper deck of a bus. By so doing, they ensure that the centre of gravity of the whole vehicle is not raised to a height above the ground which would be dangerous.

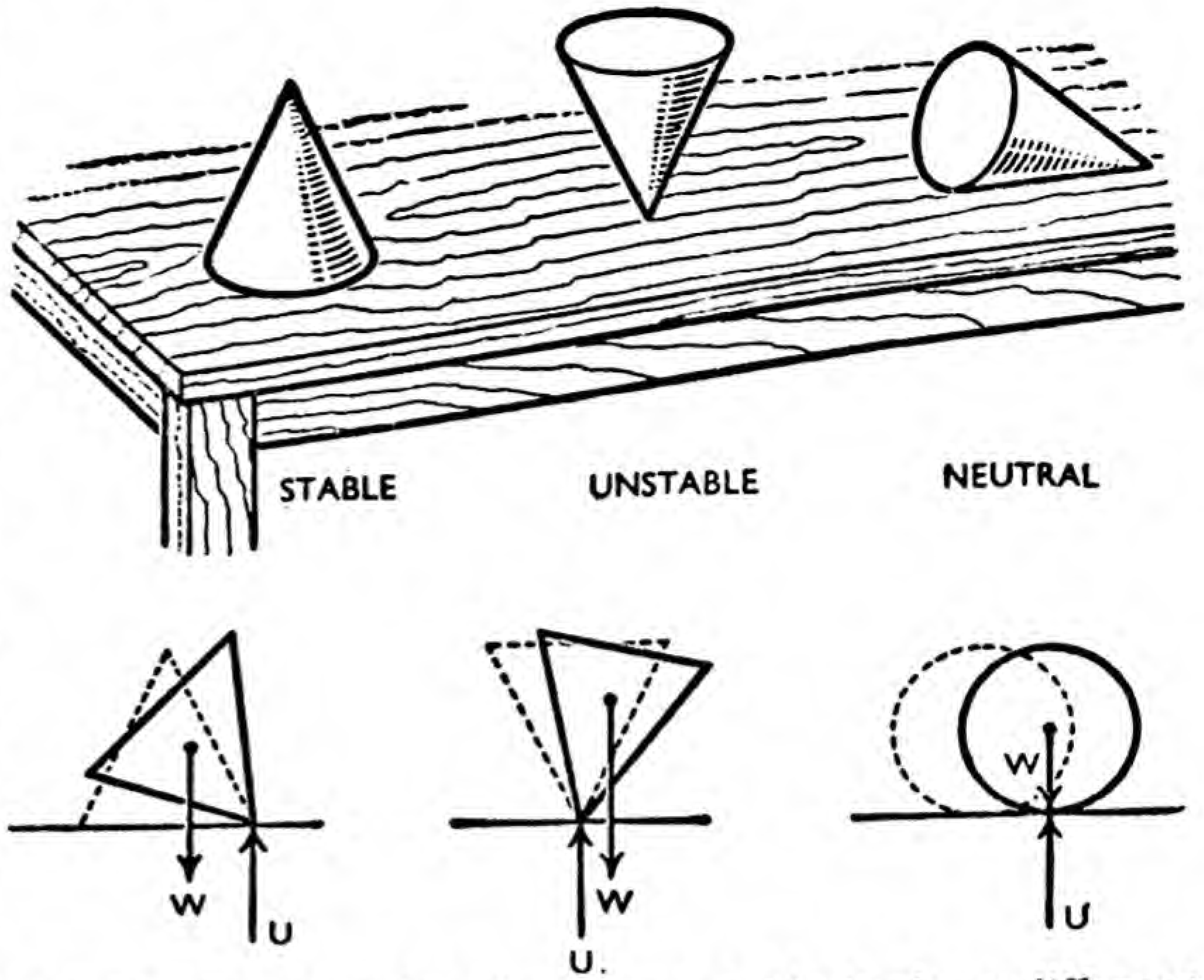


Fig. 51. Cone may be placed on a table in three different positions. When the body is slightly displaced, the relative positions of the weight *W* acting through the centre of gravity, and the upthrust *U* which is equal to *W*, determine the conditions of equilibrium, whether they are stable, unstable, or neutral.

Road and railway engineers are well aware of these facts, and they assist the movement round the curve by superelevation, or banking. The outer kerb, or rail, is raised above the inner, and this has the effect of bringing the resultant of the centrifugal force, and the weight, well inside the base (Fig. 50).

From the foregoing paragraphs, we realize that superelevation on a curve greatly increases the stability of a vehicle against overturning, and, it may be added, reduces the tendency to skid.

Neutral Equilibrium

We have used the terms stable and unstable with regard to equilibrium, but we require to amplify these, and to introduce a third condition, viz., neutral equilibrium. This is best described by a reference to a simple example. Assume that a cone (Fig. 51) with its base on a table, is tilted slightly. The moment of the couple formed by the weight and the upthrust from the table tends to restore the cone to its original position, and it is, therefore, said to be in stable equilibrium.

Now imagine that it is possible to balance the cone on its vertex. It is obvious that the slightest displacement will cause the body to move farther from its initial position, and so, in this second case, we say that the cone is in unstable equilibrium.

Lastly, imagine the cone laid with its curved surface in contact with the table. If we displace it slightly, it merely rolls to a new position and becomes stationary, because *W* and *U* remain equal and opposite ; this is known as neutral equilibrium.

The additional example shown in Fig. 52 should make these ideas

STABLE, UNSTABLE, AND NEUTRAL EQUILIBRIUM

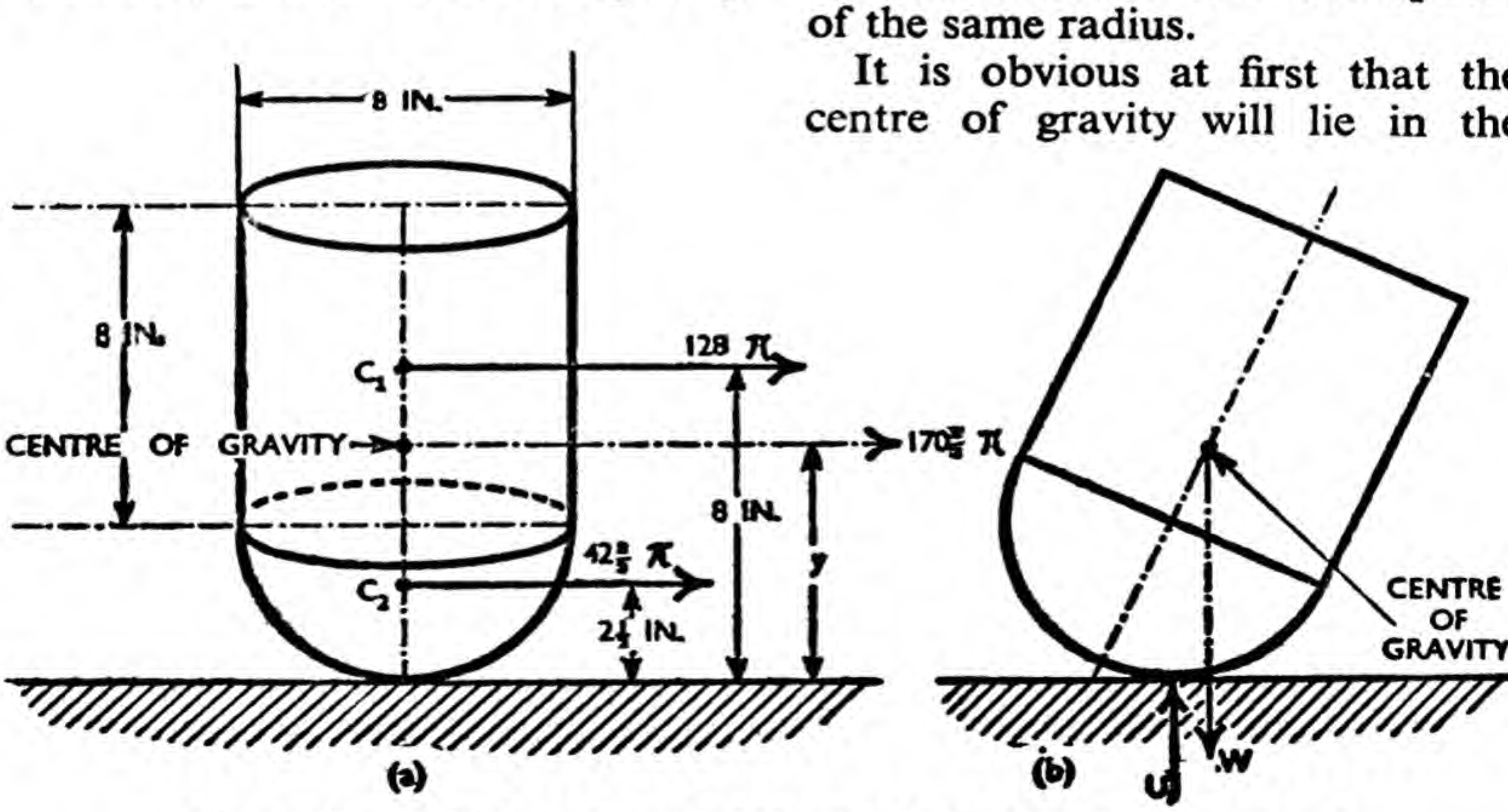
Fig. 52. Ball resting in a concave bowl is in stable equilibrium because, if displaced, it will return to its original position. Balanced on the convex surface of the inverted bowl it is in unstable equilibrium, and resting on the smooth horizontal surface of a table the ball is in neutral equilibrium.

quite clear, and the reader is advised to sketch the positions of the weight and the upthrust for the displaced position of the ball, in order that the mechanics may be perfectly understood.

More Complex Shapes

The solid bodies of everyday life are seldom perfect cubes, pyramids or spheres, but are made up of a variety of shapes, more or less complicated, and the centre of gravity may be difficult to find. If, however, a body consists of several components, of which we know the individual volumes and centres of gravity, we can apply the same method which we employed in the case of the cast-iron beam, for which the centroid of the cross-sectional area was required. To take a simple example, let us find the position of the centre of gravity of the body shown in Fig. 53(a), a cylinder attached to a hemisphere of the same radius.

It is obvious at first that the centre of gravity will lie in the



FINDING CENTRE OF GRAVITY BY TAKING MOMENTS

Fig. 53. By treating the volumes of the cylinder and hemisphere as forces and taking moments about the base, it is possible to find the position of the centre of gravity of the whole body. If the body is slightly displaced, it will move further from its initial position and is, therefore, in unstable equilibrium.

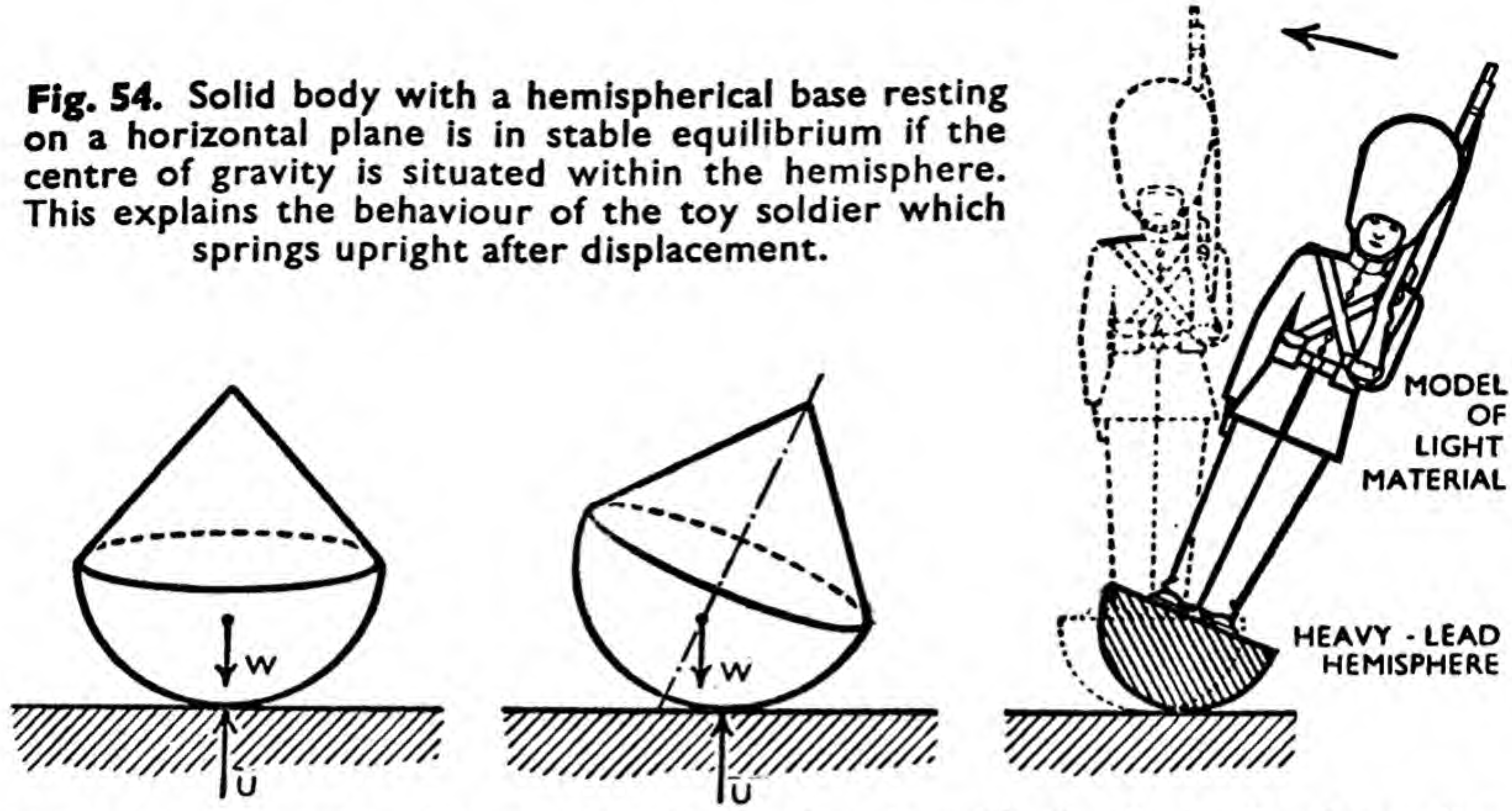


Fig. 54. Solid body with a hemispherical base resting on a horizontal plane is in stable equilibrium if the centre of gravity is situated within the hemisphere. This explains the behaviour of the toy soldier which springs upright after displacement.

central axis. We then mark the positions C_1 and C_2 of the centres of gravity of the cylinder and hemisphere. Now we find the respective weights of the two parts. Since, however, both are of the same material, it is only necessary to consider their volumes. The volume of the cylinder is $\pi \times 4^2 \times 8 = 128\pi$ cu. in., the volume of the hemisphere is $\frac{2}{3} \times \pi \times 4^3 = 42\frac{2}{3}\pi$ cu. in., and the volume of the whole body is $170\frac{2}{3}\pi$ cu. in. We then take moments about any convenient point in the central axis, say, the point of contact at the base.

$$128\pi \times 8 \text{ in.} + 42\tfrac{2}{3}\pi \times 2\tfrac{1}{2} \text{ in.} = 170\tfrac{2}{3}\pi \times y \text{ in.}$$

Cancelling π throughout, we have :

$$1{,}024 + 106\tfrac{2}{3} = 170\tfrac{2}{3}y$$

Whence $y = 6\frac{5}{8}$ in.

Unstable Equilibrium

In this example we find that the centre of gravity is $6\frac{5}{8}$ in. above the base of the hemisphere. Is this body in stable equilibrium? Answer this question by assuming the body to be displaced, as shown in Fig. 53(b). Clearly, the couple formed by the weight and the up-thrust will further displace the body from the initial position. The body will thus fall over, and is in unstable equilibrium.

Let us now suppose that a cone of small height, or, indeed, any body having a small weight because of its volume or density, is attached to a hemisphere. This will have the effect of lowering the position of the centre of gravity so that it lies within the hemisphere. What effect will this have upon the equilibrium?

If the body is displaced (Fig. 54), we see that the weight and the up-thrust form a couple which restores it to its initial position, and the body is, therefore, in stable equilibrium.

Now we can understand the action of the upstanding toy soldier. Although laid low, he springs upright again immediately on release, contrary to all the expectations of war, but not to the laws of mechanics. By making the hemisphere of lead, and the model figure of light material, we find that the centre of gravity is brought within the hemisphere, so ensuring stable equilibrium.

CHAPTER 3

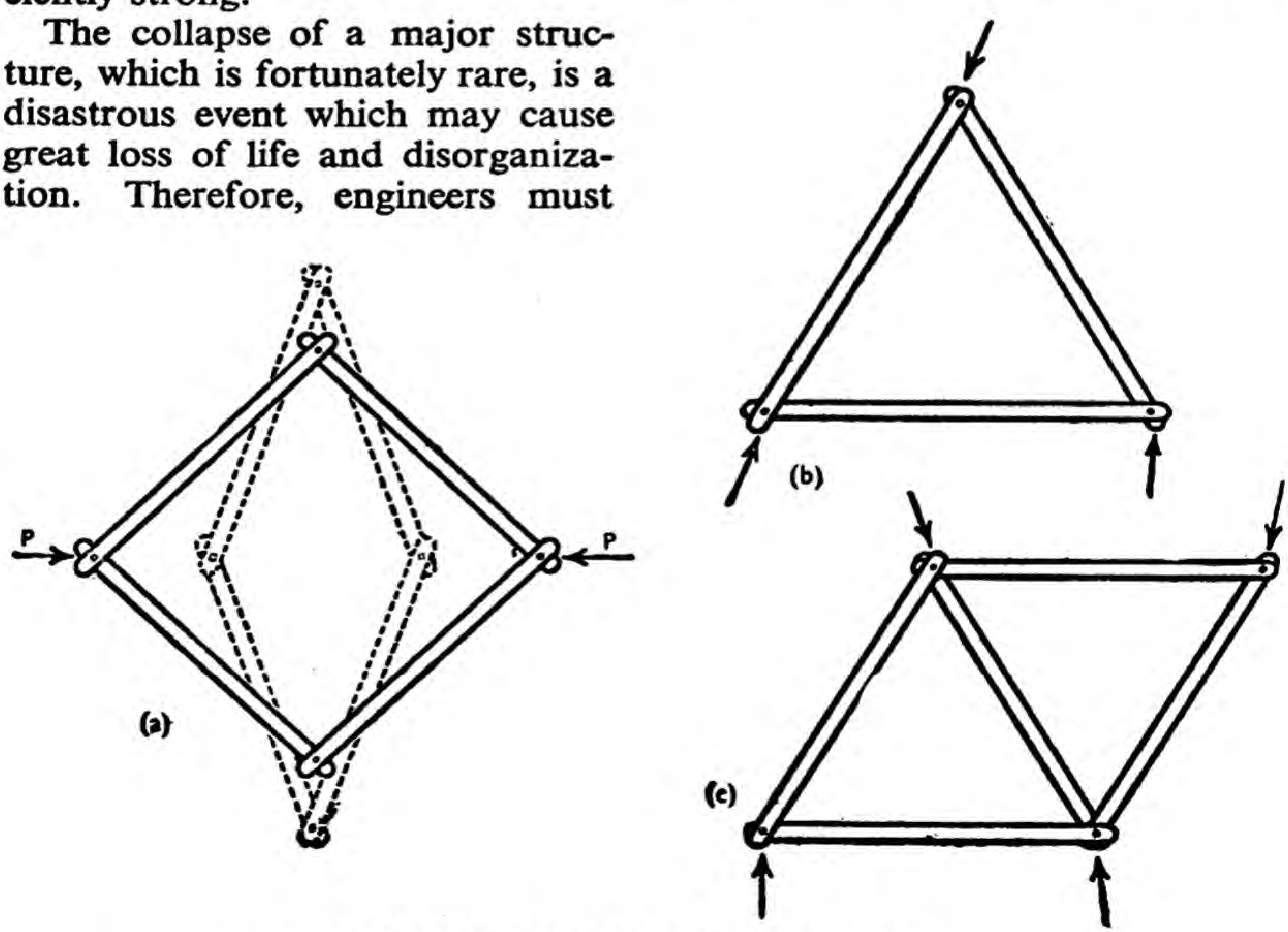
FORCES IN STRUCTURES

ECONOMY AND SAFETY IN STRUCTURAL DESIGN. THE PERFECT FRAME. PIN JOINTS AND GUSSET PLATES. THE DIFFERENCE BETWEEN DEAD AND LIVE LOADS. BOW'S NOTATION. HOW TO DRAW RECIPROCAL FIGURES. THE EFFECT OF ROLLER BEARINGS. WIND LOADS ON ROOFS. THE METHOD OF SECTIONS. COMMON TYPES OF TRUSS. WHEN STRUCTURES ARE REDUNDANT. THE DIFFERENCE BETWEEN TWO-PINNED AND THREE-PINNED ARCHES. DESIGNING DAMS AND RETAINING WALLS.

THE determination of the forces in structures is one of the most important applications of statics. It is necessary to know the forces which act in the various parts of a structure so that they may be made sufficiently strong.

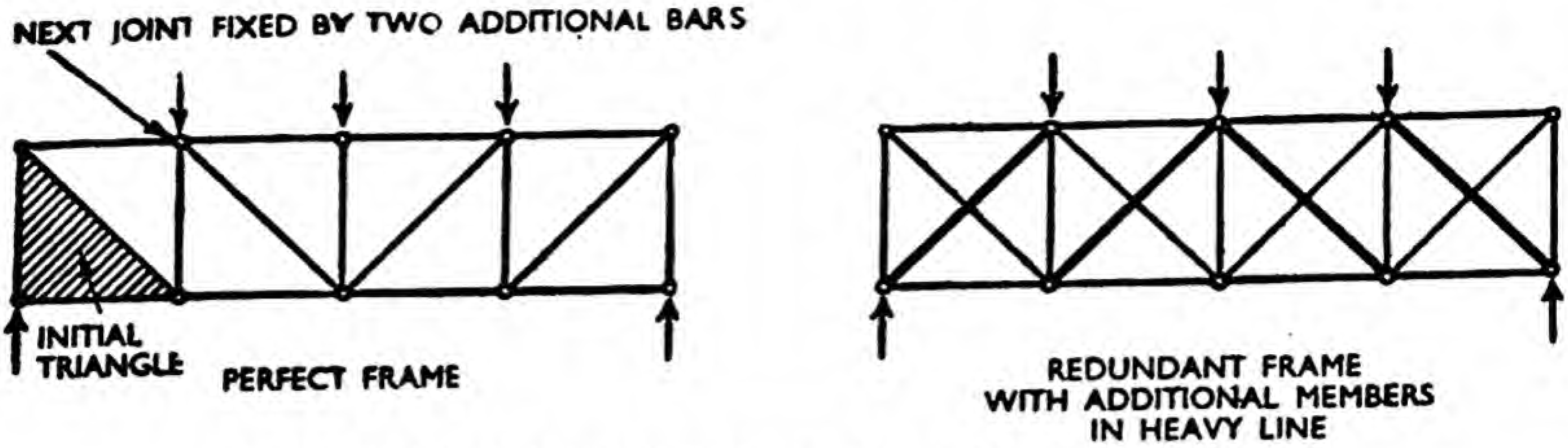
The collapse of a major structure, which is fortunately rare, is a disastrous event which may cause great loss of life and disorganization. Therefore, engineers must make sure that their structures are safe.

It is also true, however, that they endeavour to make them economical in material. Every structure is designed to withstand a definite maximum load with



TRIANGULAR FRAME IS STABLE

Fig. 1. Frame (*a*), consisting of four bars hinged at their ends, will collapse when loaded, but a triangular frame (*b*) is stable under any loading. By adding two additional bars, another joint is added to the frame (*c*) which is still stable.



PERFECT FRAME BUILT UP FROM INITIAL TRIANGLE

Fig. 2. This is accomplished by adding two bars for each additional joint. If more bars than the minimum number are used, the structure is redundant, and the forces in the members cannot then be obtained by simple methods.

safety, and it would, therefore, be wasteful to make it stronger than necessary.

It is possible for almost anyone to design, say, a bridge, but unless he fully understands the subject, it is almost certain that the finished construction would be either too weak and dangerous, or else too strong for the actual requirements and, therefore, uneconomical.

Determining Forces

Let us now consider some simple structures and the methods employed to determine the forces in the various parts. It will have been noted that bridges, roofs and other structures are frequently made up of a number of triangles. The reason for this will be easily understood.

Let us imagine a frame consisting of four bars connected by hinged joints, illustrated by Fig. 1(a). If forces *P* were applied, this frame would immediately collapse, as shown by the dotted lines. This is not the case with the triangular structure shown in Fig. 1(b) which is obviously stable for any system of loading. Now if we add two additional bars, as in Fig. 1(c), the frame is still stable, and it is possible to go on increasing the triangular cells at will. In this manner large structures of the required form can be constructed.

A triangulated frame of this kind is frequently called a truss, and the joints in it are called node points or simply nodes. Note that we start with three bars hinged together at the ends to form a triangle, and each new joint thereafter is fixed by the addition of two more bars. Such a truss is said to be a perfect frame because it has a sufficient number of bars, and no more, to keep it in equilibrium under any loading. If there are more bars than this, the frame is said to be redundant or statically indeterminate. The latter description suggests, quite correctly, that we cannot find the forces in a structure of this kind by the simple statics discussed in Chapter 2.

Fig. 2 illustrates a perfect frame of a type frequently used for bridges, and we note that, starting with an initial triangle, each new joint is fixed in position by the addition of two bars. If more bars than this are embodied, the frame becomes redundant.

It will have been noticed by this time that the members of our

Fig. 3. Bar held by hinges or pin joints at its ends may be pulled or compressed, but it cannot be bent.

frames are connected by hinges, or, as they are frequently termed, pin joints. Because of this, the four-bar frame in Fig. 1(a) collapses freely under load. Pin joints in a perfect frame have important effects upon the nature of the forces in the members, which can be explained by a simple example.

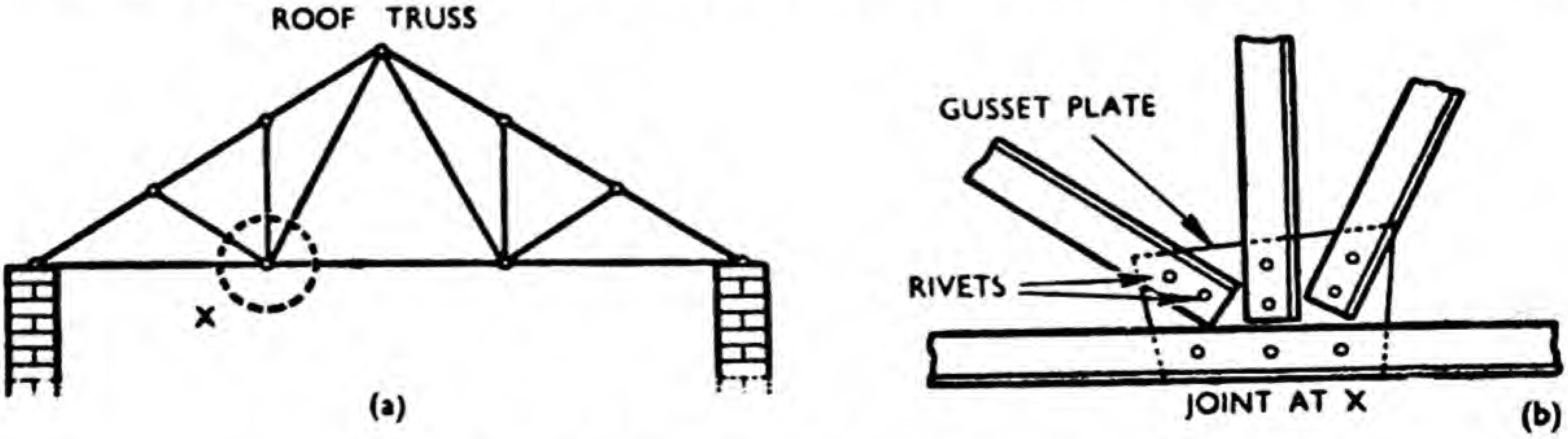
Suppose that a wooden rod (Fig. 3) has a handle at each end connected to the rod by hooks and eyes. Gripping the handles, it is possible to put the rod in tension or compression, but it is impossible to bend it as a beam. If the joints in a perfect frame are frictionless hinges, it follows that the forces in the members are either compressive or tensile, without any bending. Naturally, a bar of a frame will act as a beam if it is loaded between the nodes, but correct design does not allow this, and in all the examples which we shall consider, the frame will be loaded only at the nodes.

It will probably be said, from observation, that the joints in actual structures are not at all like hinges, and this is perfectly correct. For example, Fig. 4 shows the general outline of a roof truss together with a detail of one of the joints. Far from being a pin joint or hinge, it will be seen that the members are rigidly connected to a steel plate, called the gusset plate, by rivets or bolts. This means that the structure is not really like the pin-jointed frames which we have been considering. Despite this, however, it is justifiable to assume pin joints. Experiments have shown that the actual forces in such a structure closely approximate to those calculated on this assumption.

Putting this simply, it means that although it is known that the joints are not hinges it is convenient to assume that they are so, because, by making this assumption, the results obtained are found to be very nearly accurate.

Using Triangulated Frames

Triangulated frames can be used over longer spans than those bridged by a simple beam. This can be understood if the behaviour of a plank of wood used as a beam is considered. If it is laid flat over supports as in Fig. 5(a), we know by experience that, under a load it will deflect seriously, and may even



OUTLINE OF A ROOF TRUSS

Fig. 4. (*a*) Triangulated structure called a roof truss, and (*b*) a joint in it such as *X* is made by connecting the members to a steel gusset plate by rivets.

fracture if the load or the span is great.

In Fig. 5(b), the plank is placed on edge with the larger dimension as the depth, and it carries the load safely and with little deflection. If we imagine the internal triangles (which are shown dotted) to be cut out, a triangulated frame is formed which is almost as strong as, and is also much lighter than, the solid piece. Beams are, of course, very frequently employed (over moderate lengths), but when the span becomes great the weight of the structure itself becomes relatively more important. This weight can be reduced by using a triangulated truss.

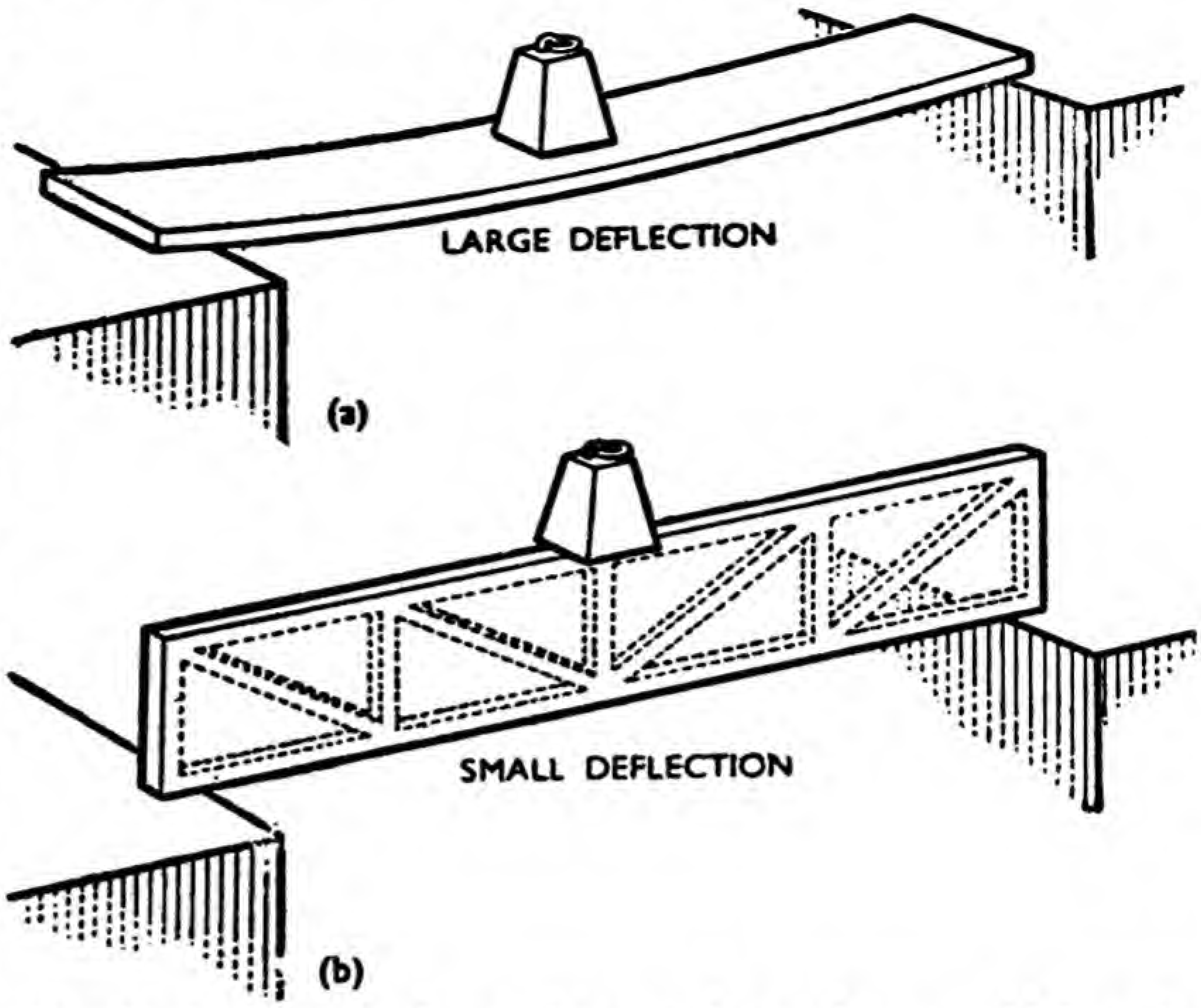


Fig. 5. (*a*) Plank of wood is used as a beam. When it is laid flat, it is weak and deflects seriously. (*b*) When it is placed on its edge, it is stronger and more rigid. The dotted triangles could be cut out with little detriment to the strength, and with a considerable saving in the weight of the beam itself.

Before discussing the methods of finding the forces in the members of a truss, let us go briefly into the manner in which structures are loaded. Fig. 6 shows a roof, partly cut away to show details, supported on trusses. The coverings—slates on boarding or corrugated iron or glazing—rest on longitudinal beams which are called purlins. These purlins, in turn, are supported by the trusses, which, it will be noticed, are loaded at the nodes only.

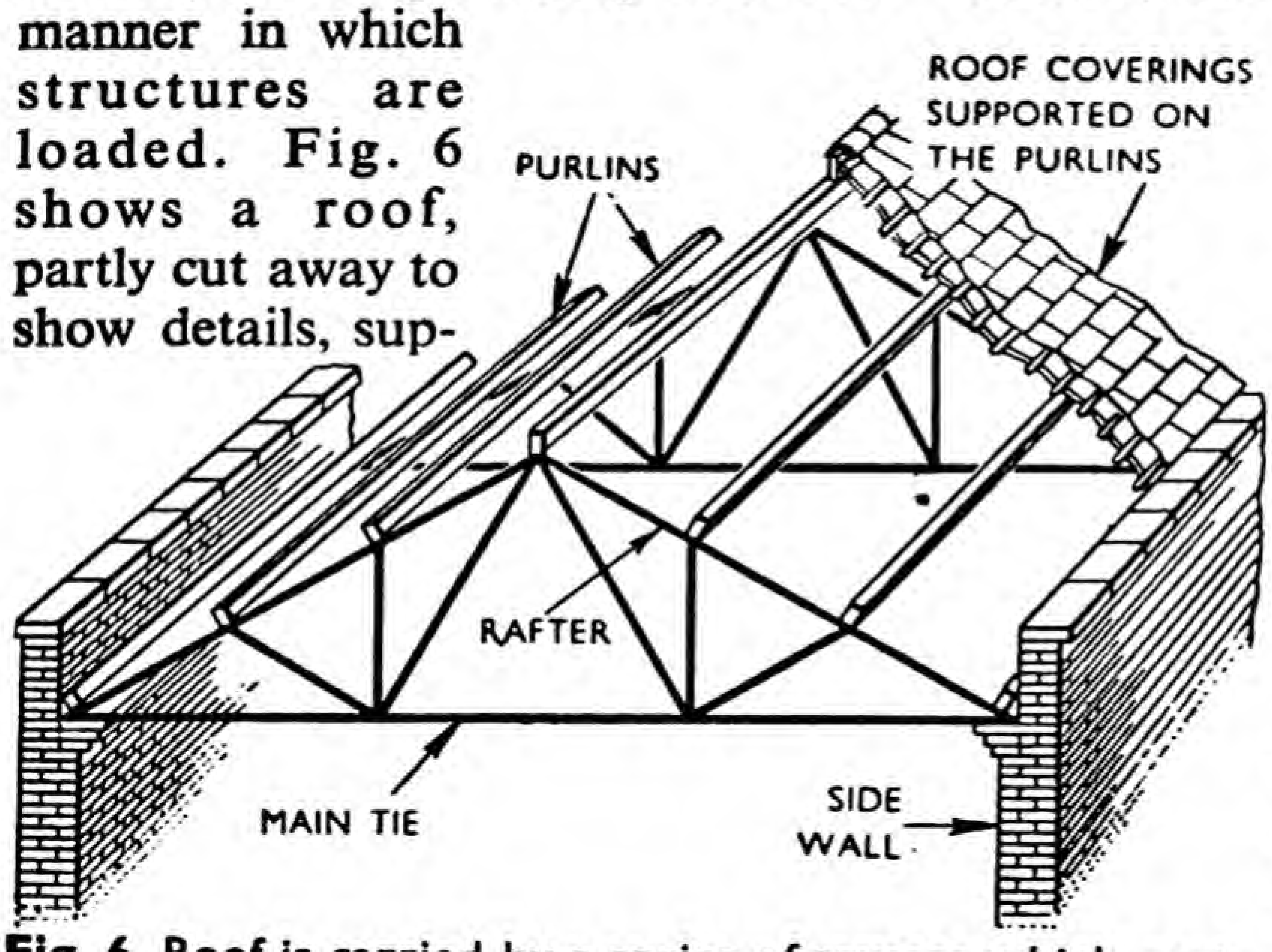
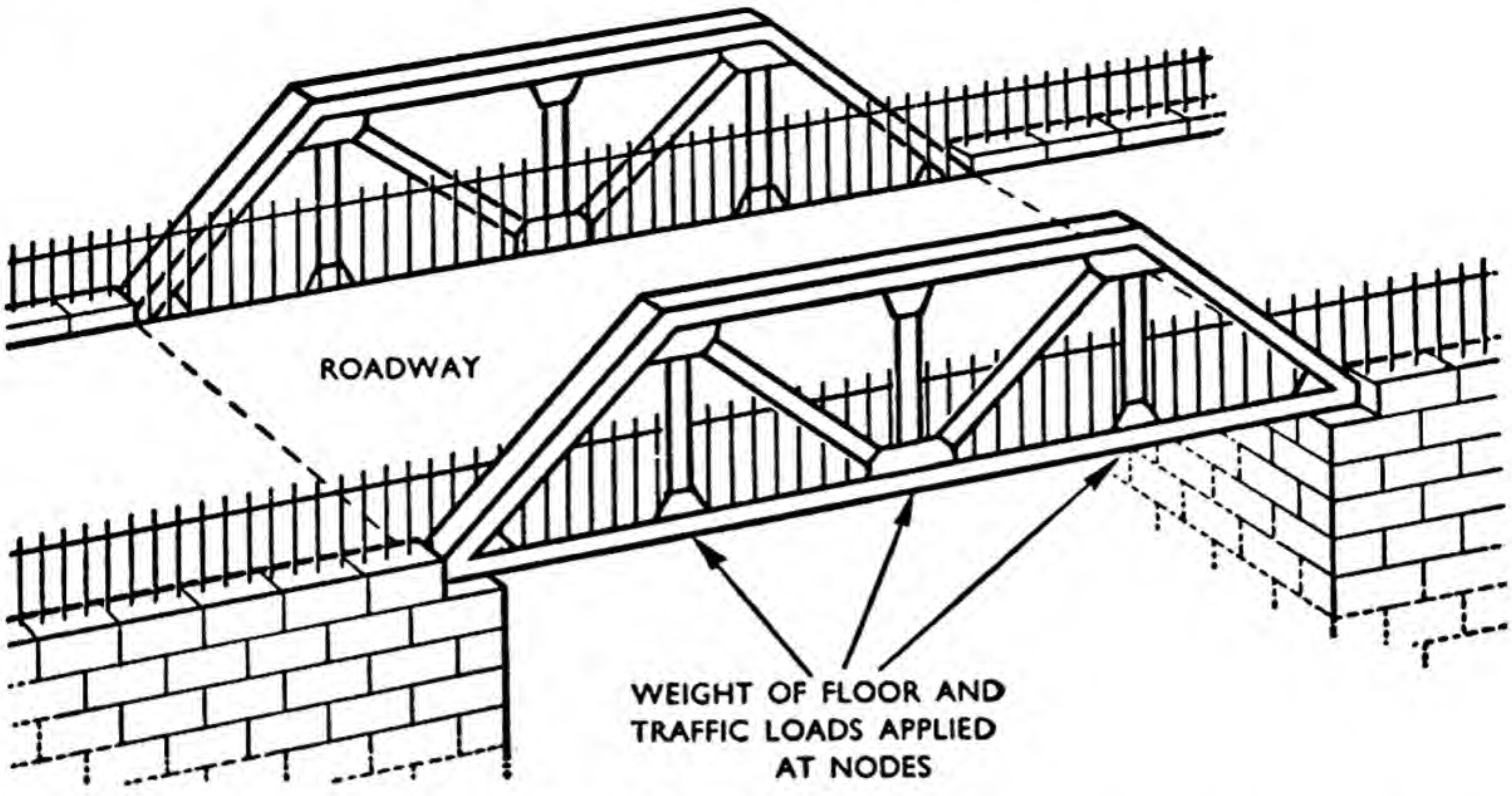


Fig. 6. Roof is carried by a series of trusses which rest on side walls. At the nodes, trusses support purlins which, in turn, carry roof coverings.

Finally, the trusses are carried by the walls, which transfer the load to the foundations.

The weight of such a roof may be considerable, especially when the span is great, and the designer must carefully estimate the vertical load applied by the purlin at each joint, and, in addition, make due allowance for the weight of the truss itself. These are the dead loads



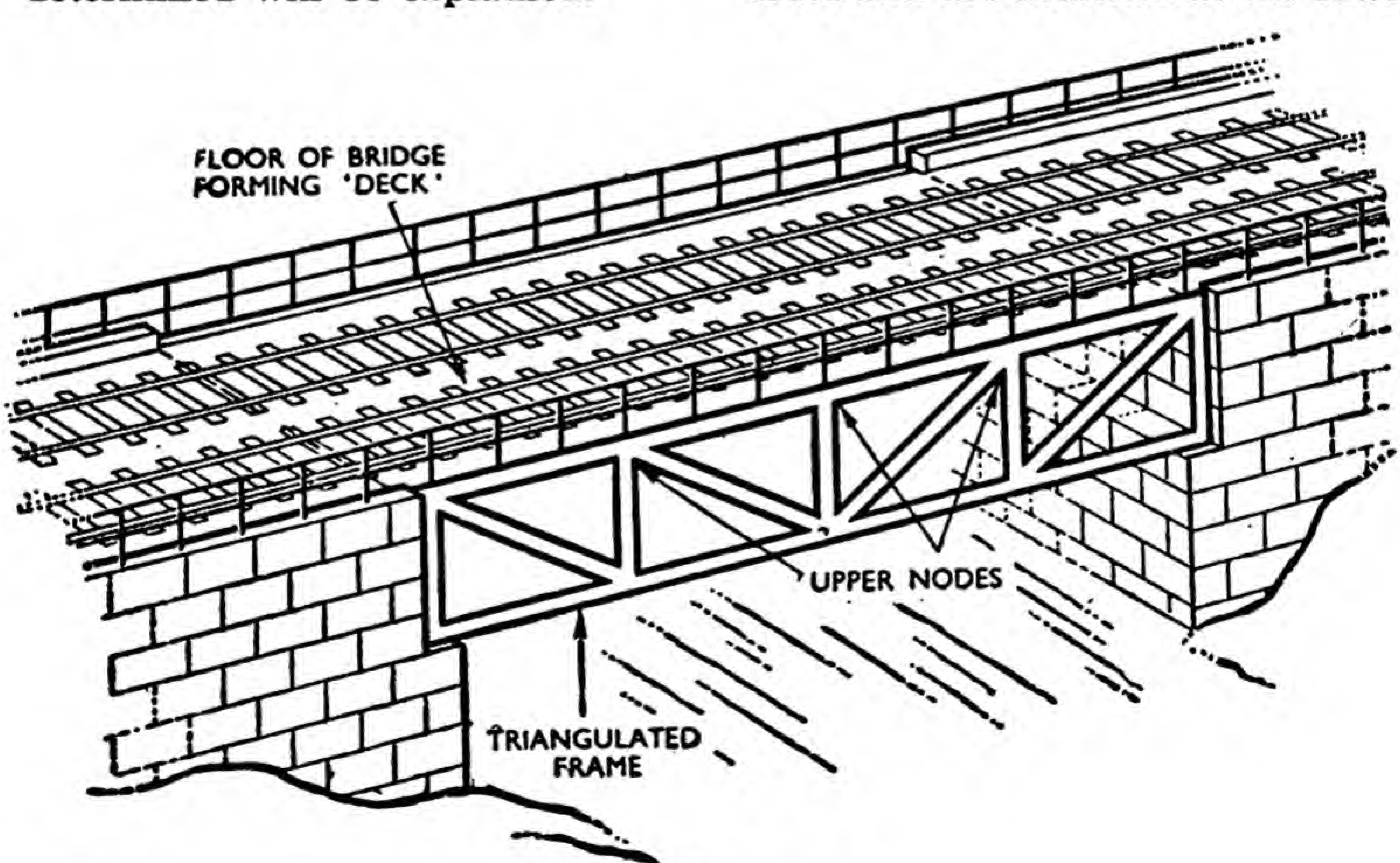
A THROUGH BRIDGE

Fig. 7. Two triangulated frames used to carry a roadway across a river. Floor of bridge rests on beams which are connected to main trusses at the nodes.

on the structure, so called because they are due to the dead weight and remain constant.

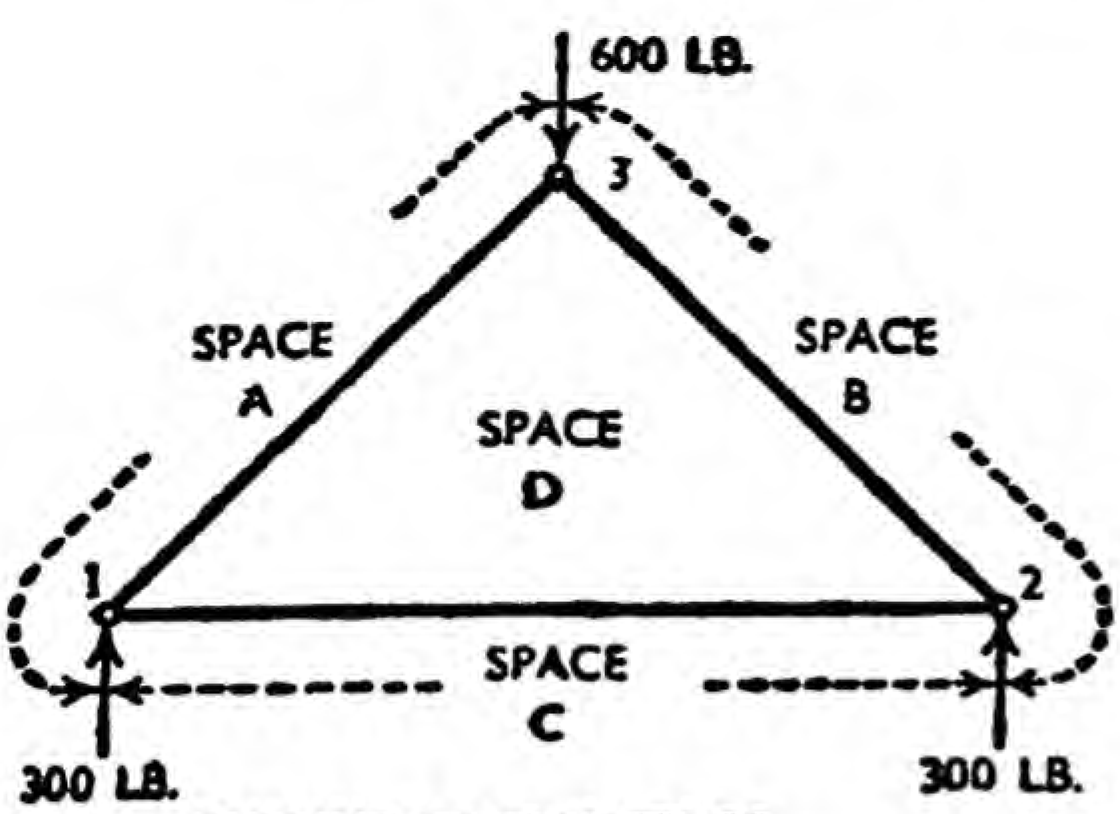
A wind pressure on this roof would be a live load, and, later in the chapter, the way its effects are determined will be explained.

Fig. 7 shows a simple road bridge. This type is termed a through bridge because the traffic passes through between the trusses. The floor of the bridge is carried on beams which span from truss to truss and are attached at the lower



A DECK BRIDGE

Fig. 8. This type of structure is termed a deck bridge because traffic is carried on a floor at the upper nodes, as in the case of the deck of a ship.



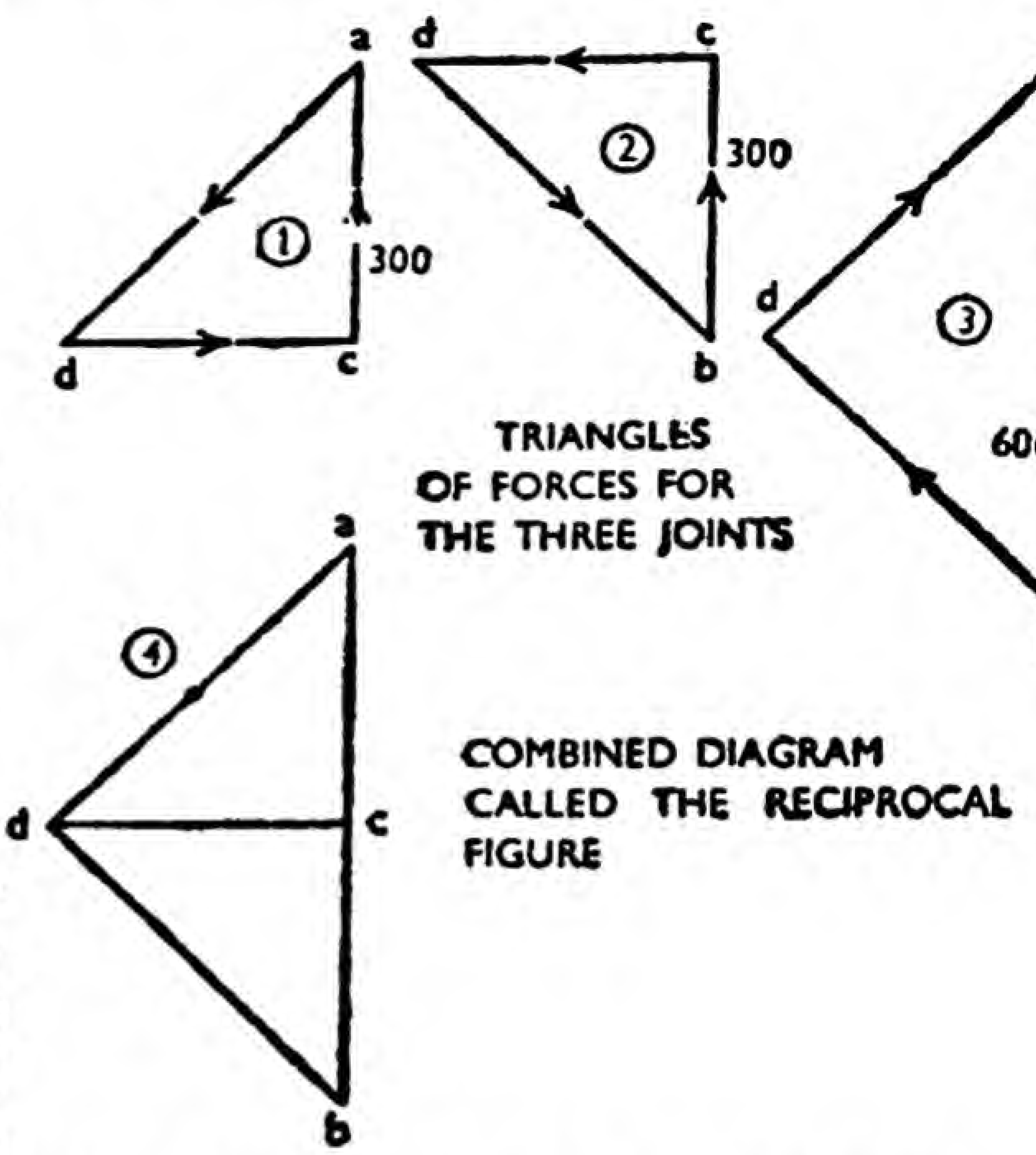


Fig. 9. Very simple roof truss is represented by single lines in the frame diagram. The forces in the members can be found by drawing triangles of forces (1), (2), and (3) for the three joints. It is more convenient, however, to draw for all the joints a combined vector diagram (4), which is called the reciprocal figure.

node points. Thus, loads are applied only at the joints and, therefore, there is no bending of the members of the trusses. The dead weight of the road surfacing, the floor and the truss itself is estimated, and the forces in the structure due to this dead loading are determined. The live load in this case is the vehicular traffic, and the designer must calculate the maximum force in each member when, say, a heavy traction-engine followed by laden trailers crosses the bridge.

The methods of calculating forces due to rolling live loads need not be pursued since they are more specialized, but it must be emphasized that it is the combination of the forces due to dead load and live load which each member of the frame must be able to resist with safety. In passing, it may be noted that deck bridges are also employed (Fig. 8), and are so called because the floor forms a deck resting on the upper nodes.

Adopting Suitable Notation

It has already been seen, in Chapter 2, how the forces in a wall-crane, lamp-standard, etc., may be found by the triangle or polygon of forces. The same procedure is applied in the case of triangulated frameworks, but first we must adopt a suitable notation, because the diagrams become difficult to draw and to interpret without a definite system. This method of lettering forces is known as Bow's notation, and is best explained by reference to the following example.

The most elementary form of roof truss, consisting of three bars, is shown in Fig. 9, and it is assumed that the apex is loaded with a vertical force of 600 lb. Since the frame is symmetrical, it is obvious that each supporting force is 300 lb., acting vertically upward.

Notice that the *spaces between the forces are lettered*, going first round the frame externally. Thus, starting with the space between the left-hand support and the 600-lb. load, this is called *A*; then proceeding to letter the spaces *B* and *C*,

we will have moved completely round the frame. Then allot a letter to each internal triangle. In the present case there is only one such triangle, *D*, but in larger frames as many additional letters are used as required.

Bow's Notation

This is Bow's notation, and, when using it, the 600-lb. force is referred to as the force *AB*, and when the vector for it is drawn, we letter the vector *ab*. Similarly, for example, the right-hand reaction of 300 lb. is the force *BC*, and the vector for it will be lettered *bc*. Remember that the problem is to find the forces in the three members of the frame. Therefore, according to this notation, these three forces will be *AD*, *BD* and *DC*, and the vectors representing them will be *ad*, *bd* and *dc* respectively.

Considering the left-hand joint 1, we can obviously draw a triangle of forces, since there are only two unknowns, viz., the magnitudes of the forces *AD* and *DC*, their directions being known. Thus, we draw vertically a vector *ca* representing the force *CA* of 300 lb.

Through *a* a line is drawn parallel to *AD*, and through *c* a line parallel to *DC* ; the intersection of these lines is lettered *d*, as shown in triangle (1). By scaling *ad* and *dc*, we know the magnitudes of the forces *AD* and *DC*. But the arrowheads must point in the same direction round the triangle as indicated, and it is deduced that the force *AD* is pushing towards the joint and, therefore, is a compressive force. Similarly, it is clear that force *DC*, pulling away from joint (1) is tensile. Notice particularly that the movement was in a clockwise direction round the joint from space *C* to *A* (known force), from *A* to *D* and from *D* to *C*. This should be done consistently for all joints.

In exactly the same way it is possible to draw triangles of forces (2) and (3) for joints 2 and 3, and the forces in the three bars of the frame are now known completely. But it will be noticed that the vectors *ad*, *bd* and *cd* each appear twice in the triangles. Therefore, to avoid unnecessary work, instead of drawing three separate triangles, construct at once a single combined diagram (4). This is called the reciprocal figure or reciprocal force diagram, and it gives the magnitudes of all the forces.

Arrowheads cannot be drawn on the reciprocal figure because a vector in it represents a force acting at two joints in the frame. For example, if we consider the force in the horizontal member *DC*, the sense with reference to joint 1 is *dc*, from left to right, and is *cd* from right to left, with reference to joint 2. This does not mean that it is impossible to obtain the nature of the forces from the reciprocal figure.

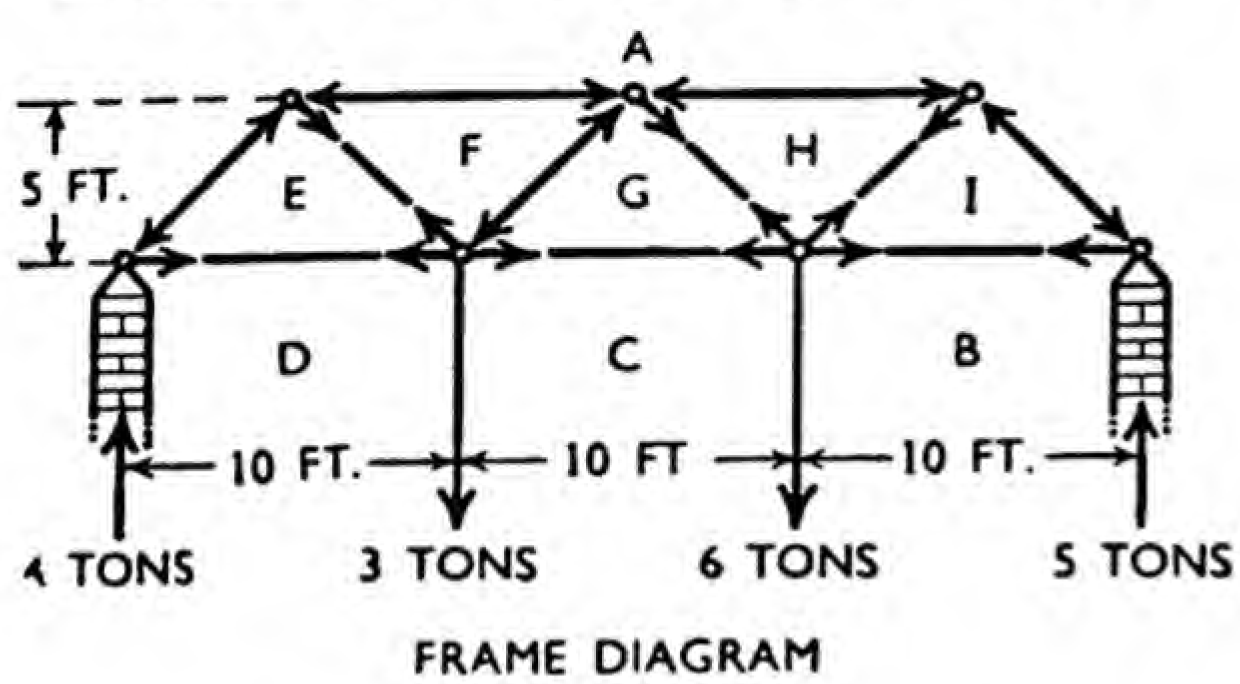
Concentrating attention on one node, and starting with a force of known sense, frequently one of the external forces, a path is traced round the triangle (or polygon) of forces for that joint. For example, considering node 3, it is known that the force *AB*, 600 lb., acts downward ; thus, in the reciprocal figure, we move from *a* to *b* downward.

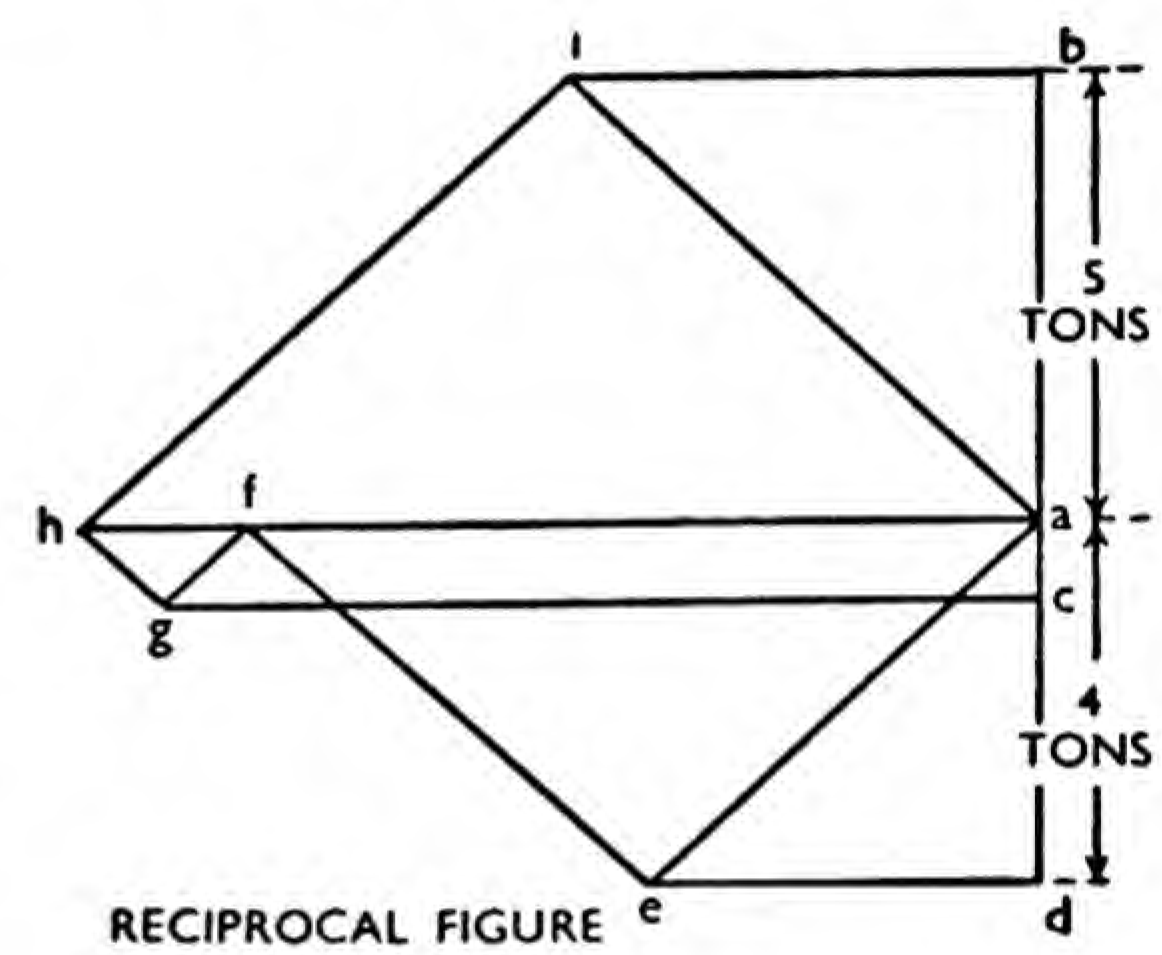
The next force in a clockwise direction at the joint is *BD*, and, moving from *b* to *d*, up and to the left, we deduce that *BD* is pushing up and to the left and, therefore, is a compressive force.

Lastly, the figure is closed by

moving from *d* to *a*, up and to the right, showing that *DA* is also compressive. This procedure can be followed for any joint. Notice that if there are more than three forces acting at a joint, the closed figure which is traced in the reciprocal diagram will be a polygon having a side for each force, but in exactly the same way the nature of the forces can be determined from the direction in which we move round the figure. Members which carry compressive forces are called struts and those which carry tensile forces are ties.

Another example, slightly more





FINDING THE FORCES IN THE PARTS OF A BRIDGE

Fig. 10. When a vehicle is on the bridge illustrated in this figure, it will cause forces to be applied at the lower nodes. The supporting reactions at the ends are found by taking moments as in the case of a beam. Then by drawing a reciprocal figure, the force in each member of the frame can be found.

difficult, should make the method quite clear. Suppose that when a traction-engine passes over the bridge shown in Fig. 10, it causes forces of 3 tons and 6 tons to be applied at a given instant to the lower joints. Before attempting to draw the reciprocal figure the supporting forces of 4 tons and 5 tons must be found. This is done quite easily by taking moments just as in the case of a beam.

Now letter the spaces *A*, *B*, *C* and *D* between the external forces; notice that the space *A* extends from the left-hand support round the top of the frame to the next external force, which is the right-hand support. Then letters *E*, *F*, *G*, etc., are allotted to the internal triangles. The reciprocal figure is commenced by drawing firstly, the vectors for the external forces. Thus, *AB*, equal to 5 tons upward, is drawn vertically to scale as the vector *ab*, then *BC* equal to 6 tons downward is represented as *bc*. Similarly, draw *cd* downward and *da* upward, thus arriving back at the starting point *a*. The polygon of the external forces in this case is a straight line. Then draw the triangle of forces for the joint *DAE* and so locate *e*.

Three Unknowns

If we now try to solve the joint *CDEFG* it is impossible to proceed, because there are three unknowns *EF*, *FG* and *GC*. Therefore, we are forced to consider next the joint *EAF*. Through *a* draw a line (horizontal) parallel to *AF*, and through *e* a line parallel to *EF*, whence the intersection *f* is obtained. Now the force *EF* is

known, leaving only two unknowns (*FG* and *GC*) at the joint *CDEFG*, which is solved by finding the point *g*. This procedure is continued for each joint, and in turn the intersection points *h* and *i* are found.

Observe that the direction of *IA* from the frame diagram is known, and that we already have the points *i* and *a* in the reciprocal figure. *IA* and *ia* should be found to be parallel, and this provides a check on the accuracy of the work. In this simple example the error of closure, if any, should be very small, but for certain frames very careful drawing of parallels is necessary to obtain a good closure. Note that the figure may be closed at any other point, say *g*, by starting from each end and working towards the centre of the frame. Generally this procedure is to be preferred because it tends to reduce cumulative errors in drawing.

Distinguishing Struts and Ties

Arrowheads are shown on the frame diagram indicating the sense of the forces in relation to each joint, and these are found from the reciprocal figure. For example, consider the joint *CDEFG* and trace the polygon for it; *c* to *d* is vertically downward (known sense), *d* to *e* is horizontal from right to left, and the arrowhead on *DE* is marked with this sense showing that *DE* is pulling away from the joint and, therefore, is tensile. Continuing round the polygon, proceed from *e* to *f*, up and to the left, *f* to *g* down and to the left, and *g* to the starting point *c* from left to right, thus closing the figure. From the directions traced, the corresponding arrowheads may be drawn showing that *EF* and *GC* are ties and that *FG* is a strut.

Instruments Required

To obtain a clear understanding of this method it is essential that actual reciprocal figures should be drawn. It is suggested that the reader should, as a first attempt, draw for himself the diagrams shown in Fig. 10. Set-squares, scale, paper and pencil are the only requirements. To a scale of 1 in. = 5 ft., draw the frame diagram by setting out a horizontal line 6 in. long; divide this into three parts each 2 in. long, draw lines at 45 deg., and so complete the outline of the frame.

Using a scale of 1 in. = 2 tons, commence the reciprocal figure by drawing the vectors for the external loads. Thus, *ab* will be a vertical line 2½ in. long, representing 5 tons, *bc* will be 3 in., *cd* 1½ in., and *da* 2 in.

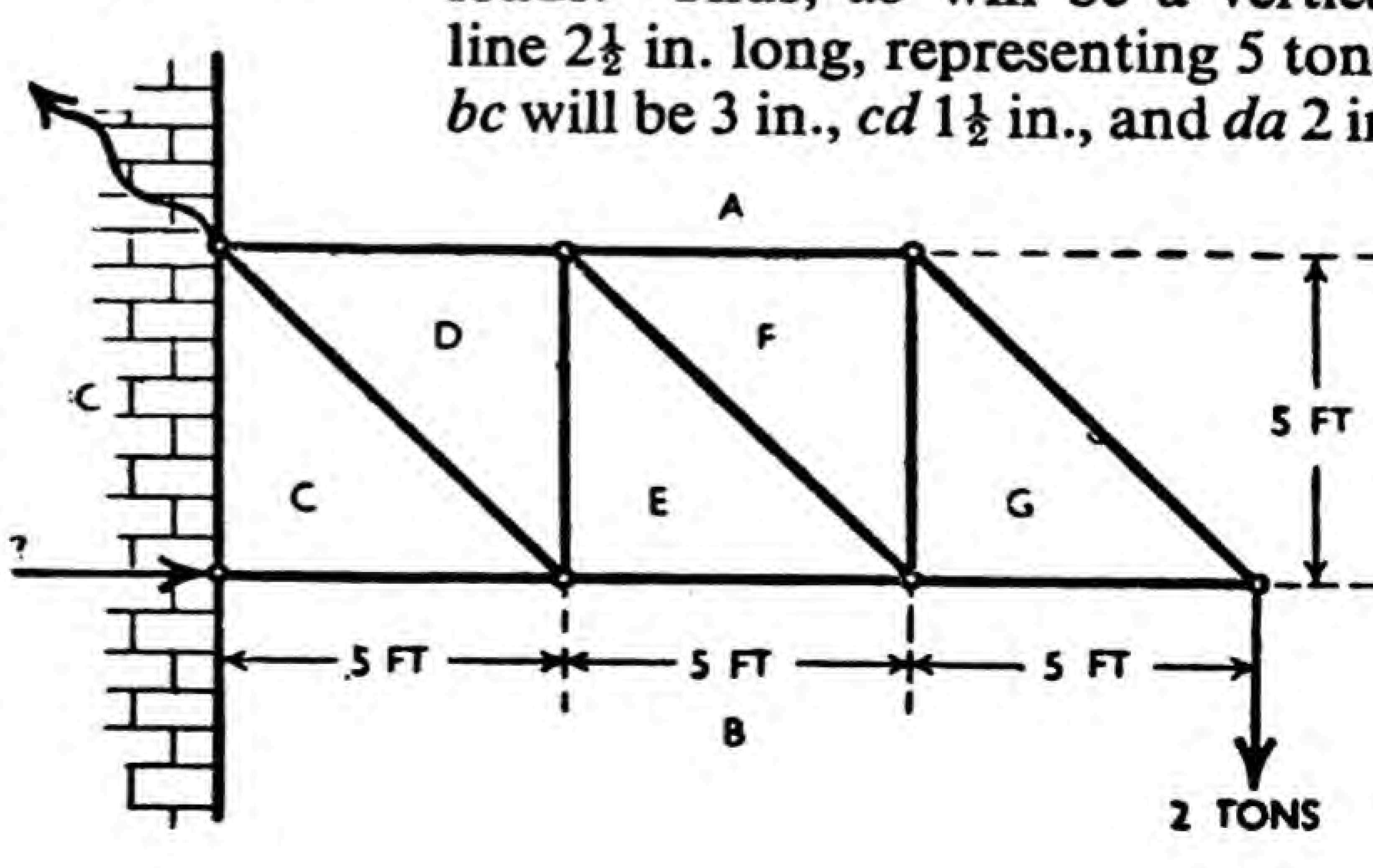


Fig. 11. It is possible to construct a reciprocal figure for this cantilevered frame without first determining the reactions at the wall. Vectors that are obtained directly from the reciprocal figure will represent the two supporting forces.

Draw then parallels for each joint in the manner described, and note with what accuracy the figure closes. The vectors represent to scale the forces in the members. For example, the line *gc* will be found to be $4\frac{1}{2}$ in. long; therefore, the force in the member *GC* is $4\frac{1}{2} \times 2 = 9$ tons, and this has already been proved to be tensile. In order that the reader may be sure that he has drawn the diagram correctly, he should check his results with the following :—

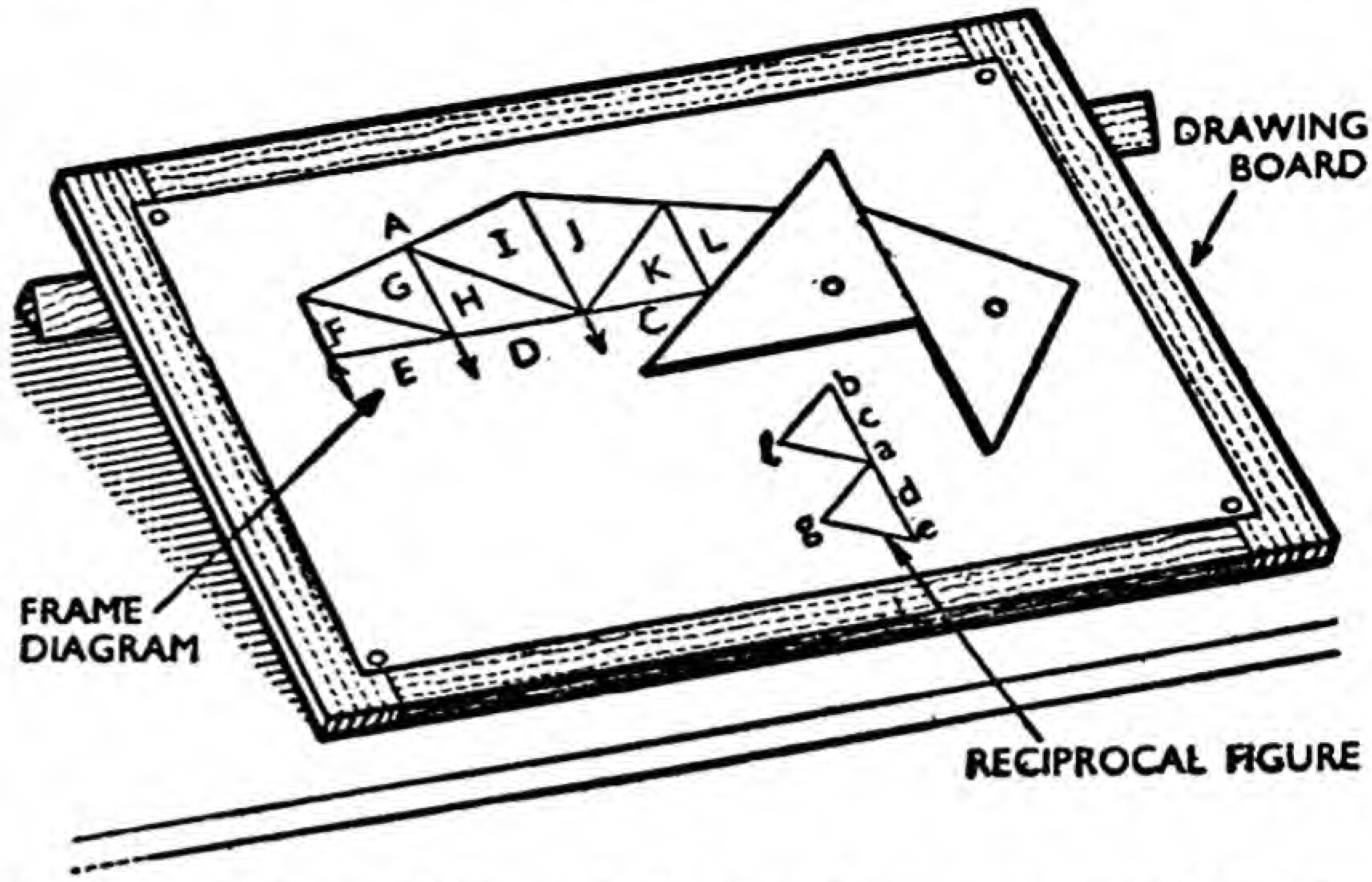


Fig. 12. When finding the forces in a structure, frame diagram should be made larger than the reciprocal figure in order to reduce inaccuracies. If the frame diagram is relatively small, a long line may have to be drawn parallel to a short line, and this leads to errors.

Member	Tie or Strut	Force (tons)
AE	Strut	5·7
AF	Strut	8·0
AH	Strut	10·0
AI	Strut	7·1
DE	Tie	4·0
CG	Tie	9·0
BI	Tie	5·0
EF	Tie	5·7
FG	Strut	1·4
GH	Tie	1·4
HI	Tie	7·1

It should now be possible, without assistance, to draw the reciprocal figure for the simple cantilever shown in Fig. 11. This brings out an interesting point. Clearly the joint *ABG* can be solved immediately, whereupon *AGF* becomes soluble. Then the polygon for the joint *BEFG* can be completed, and so on throughout the frame. Thus, without calculation or separate diagram, it is possible to obtain direct from the reciprocal figure the vectors *bc* and *ca*, which represent the lower and upper reactions respectively.

Notice that, since the wall is not a member of the frame, the letter *C* serves the space between the reactions as well as the internal triangle. Now the structure as a whole is in equilibrium under the action of three external forces, which, therefore, should meet at a point. Check the work by seeing if this is true for the forces that have been found. Further, it should not be difficult to calculate by moments the magnitude (6 tons) of the lower reaction, its direction being horizontal; this will provide the reader with an additional check.

Choosing a Scale

Beginners frequently choose a scale which makes the reciprocal figure very large in the hope that the diagram will give greater accuracy. But it must be remem-

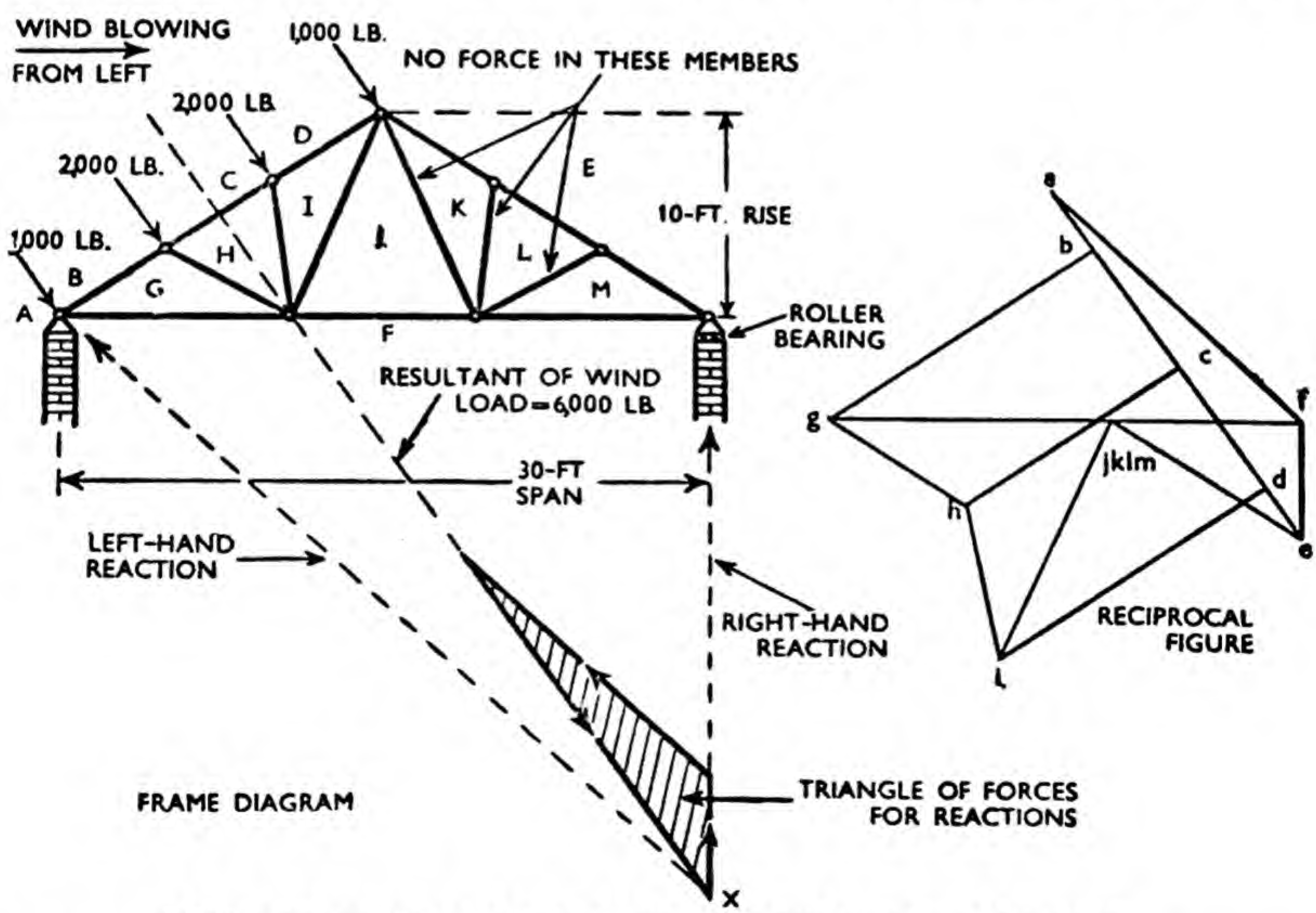
bered that each line in the reciprocal figure is drawn parallel to a member of the frame (Fig. 12). If the frame diagram is relatively small, a long line parallel to a short one may have to be drawn. In this way, any small angular error in drawing is magnified, and an inaccurate figure which will not close is the result.

Therefore, in general, it is better to make the frame diagram larger than the reciprocal figure, and it is helpful to sketch approximately the form of the figure before drawing it accurately. By this procedure a suitable scale may be selected and the reciprocal figure commenced in such a position that it fits conveniently into the space available.

Return now to the roof truss which has been considered earlier. The forces in the members of the frame due to dead load are easily obtained in the manner described, but the treatment of the live load (wind) requires some explanation.

Forces Caused by Wind

Any vertical surface, a wall, for example, is subjected to a pressure owing to wind which varies with the velocity of the air stream. When the wind impinges on a roof, it slides over it and produces pressure at right angles to the slope on the windward side. The steeper the slope, the greater is the pressure, and it is possible to calculate the magnitude of the pressure for a given slope and for a probable maximum wind velocity. This pressure on the roof coverings



ROOF CARRIED ON ROLLER BEARING AT ONE END

Fig. 13. Wind blowing on this roof from the left causes pressure at right angles to the rafter. This pressure on the roof coverings is transmitted to the purlins, which in turn apply forces at the nodes. The right-hand reaction is vertical because of the roller bearing, which allows free expansion of the truss. The reciprocal figure determines the forces in the members, and since the points *j, k, l* and *m* coincide, the members *JK, KL* and *LM* carry no force.

is transmitted to the purlins which, in turn, apply forces at the nodes of the truss, as shown in Fig. 13.

Our first problem is to find the supporting forces from the walls, but before we can do this it is necessary to consider how the truss is connected to the walls. It will be noted that at the right-hand end, there are shown small circles which represent diagrammatically a type of roller bearing frequently fitted to girders and trusses. Owing to daily and seasonal changes of weather, all structures are affected by considerable alterations in temperature and this causes variations in length, expansion with rise in temperature and contraction when the temperature falls.

If each end of the truss is firmly secured to the wall and the temperature rises, either the walls will be pushed slightly outward or the walls will remain in position and the truss will be compressed. These effects can be avoided if one end of the frame rests on rollers which allow free expansion. The action of such a roller bearing is well illustrated by placing, say, a match-box on several pencils laid on a smooth table (Fig. 14).

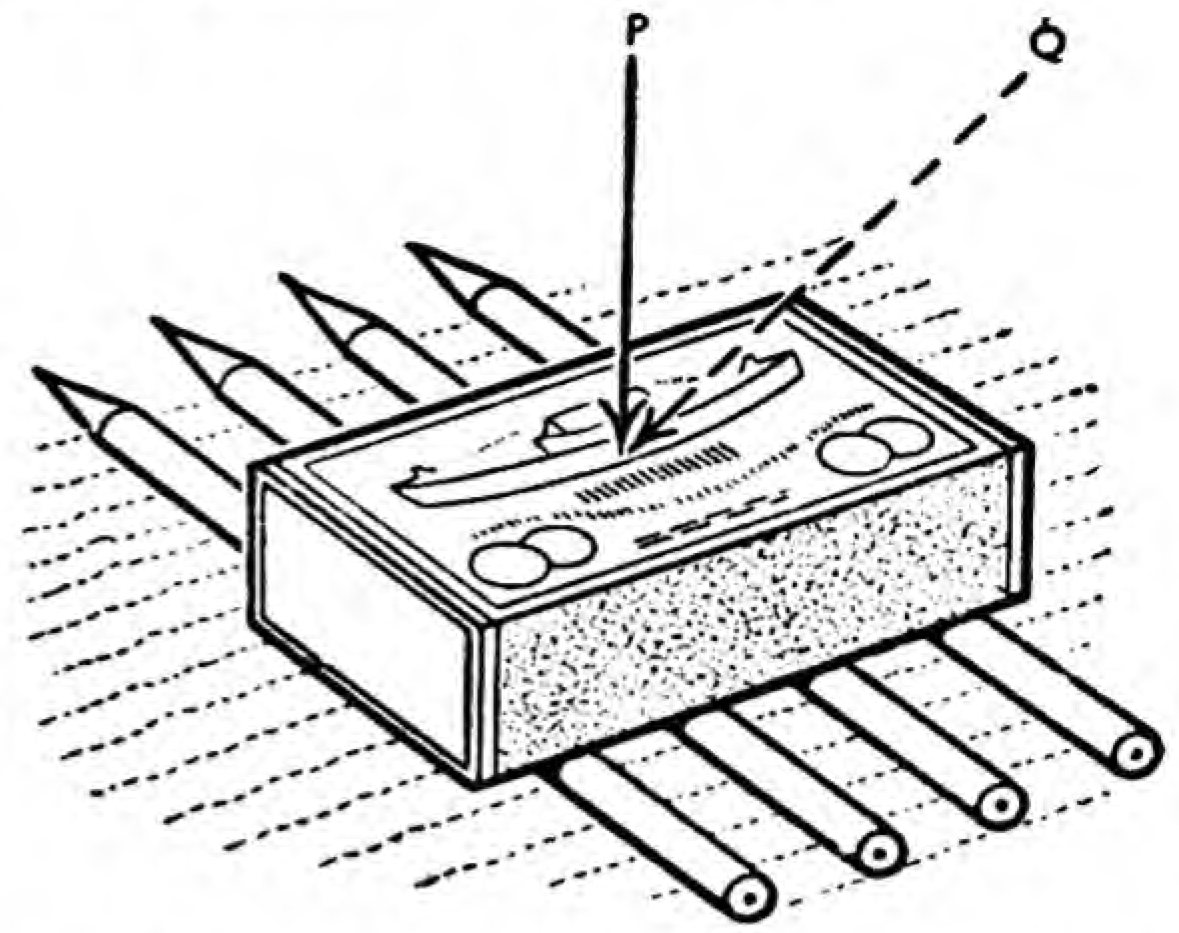


Fig. 14. Roller bearing can be likened to a match-box resting on several pencils, as illustrated in the above diagram. The box is in equilibrium under a vertical force P, but it will immediately move if an inclined force Q is applied. It will be found that reaction at a roller bearing must, therefore, be perpendicular to the base.

Effect of Roller Bearing

If we press vertically downward with a force P, an equal and opposite force is exerted by the table through the pencils, and the box remains in equilibrium. If an attempt is made to apply an inclined force Q, equilibrium is immediately destroyed and the box moves, because the pencils are unable to transmit any horizontal force. Therefore, it may be concluded that the supporting force at a roller bearing is always perpendicular to its base.

Returning to the roof truss, we assume, for the moment, that the four wind loads are replaced by their resultant of 6,000 lb. The structure is now a body in equilibrium under the action of three forces which, therefore, must meet at a point. Thus it is possible to draw the direction of the left-hand reaction to pass through the intersection X of the resultant wind load and the vertical reaction at the roller bearing. Then, by a triangle of forces, which is shown shaded, the two reactions are found completely.

The external and internal spaces are lettered in the usual manner, and the polygon of external forces, viz., the triangle *abcdefa*, is drawn to scale. Proceed then with the reciprocal figure by drawing parallels and finding intersection points g, h, i, etc. The reader is now invited to follow carefully the construction of the figure, or preferably, to draw it independently for himself without the aid of Fig. 13. This is suggested because

an interesting feature will be encountered.

When we try to draw parallels to *JK*, *KL* and *LM*, it will be found that the corresponding vectors have no length, and that the only possible solution makes *j*, *k*, *l* and *m* coincide in one point. The significance of this is that since *jk*, *kl* and *lm* have no length, the wind load causes no force in members *JK*, *KL* and *LM*. It does not follow that these bars could be omitted from the structure, because, of course, they carry forces caused by the dead loading. It is also obvious that the wind may blow from the right-hand side, which would alter the conditions and would, in fact, induce forces in these members.

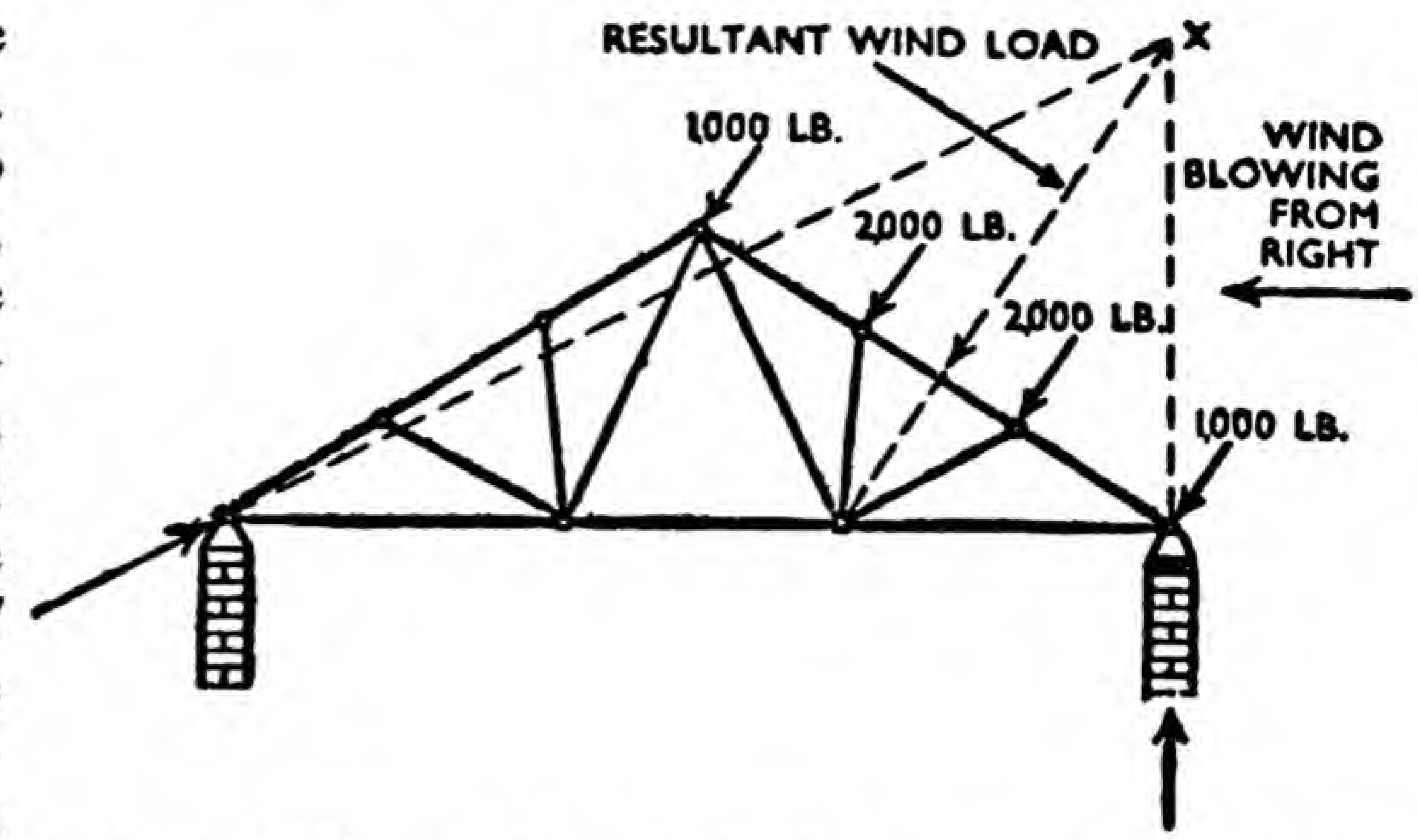
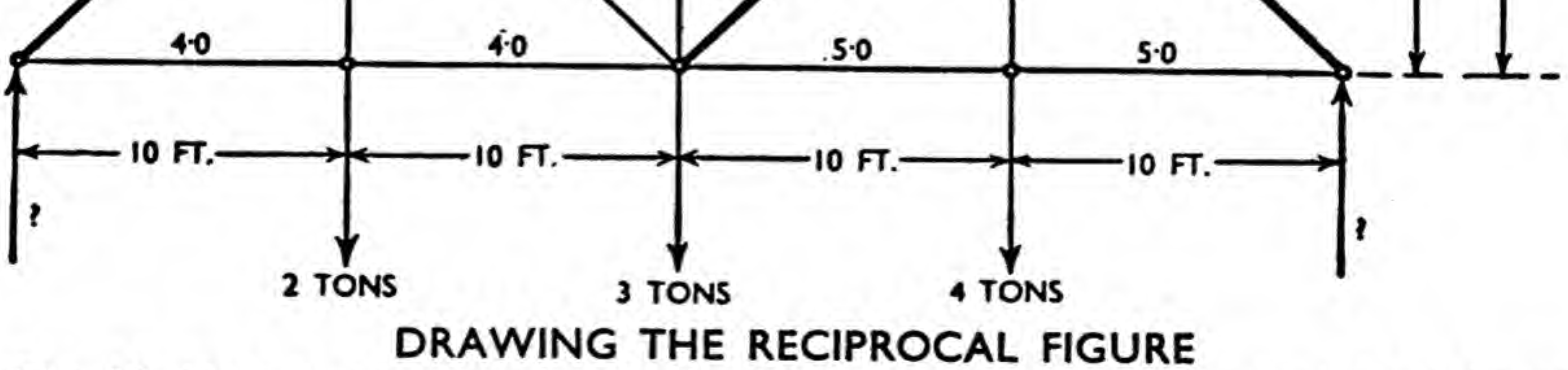


Fig. 15. When the wind blows on the right-hand side of the roof truss, the reactions can be found from the common intersection of the three external forces. The figure gives a clue to the solution of this.

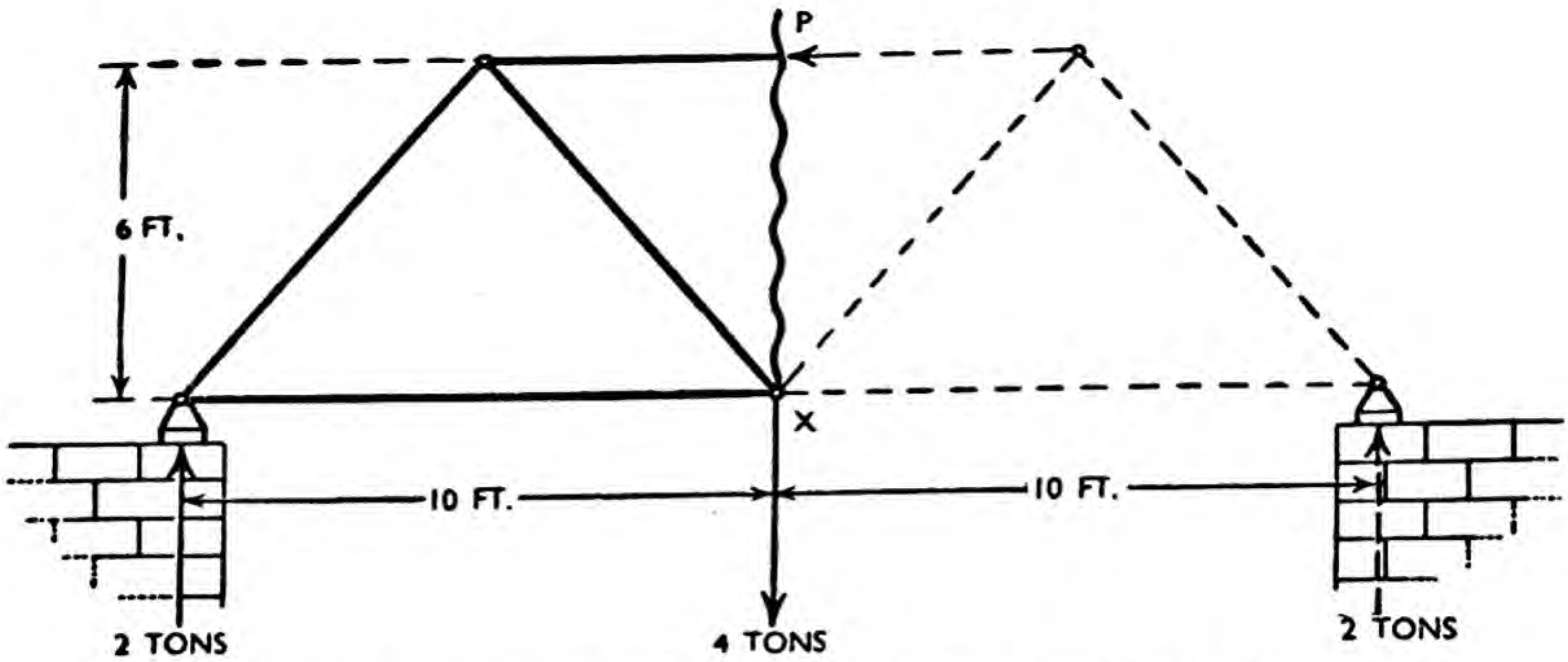
This suggests a suitable test by which an understanding of these methods may be judged. Consider the same truss and imagine the same forces of 1,000, 2,000, 2,000 and 1,000 lb. to act at right angles to the rafter on the right-hand side (Fig. 15). Using the same methods, draw the triangle of forces for the external loads (Fig. 15 gives a clue to the solution) and then find the forces in all the members by a reciprocal figure.

Having found the forces due to dead load, wind on the left, and wind on the right, the engineer is then able to determine the maxi-



DRAWING THE RECIPROCAL FIGURE

Fig. 16. Here is an example of a loaded frame for which the reciprocal figure is quite easy, first calculating by moments the vertical supporting forces at the end of the structure. So that work may be checked, forces in tons are shown beside each member, and struts are indicated by heavy lines.



CALCULATING FORCE BY TAKING MOMENTS

Fig. 17. Forces in a structure may be calculated by the method of sections. Imagine this frame to be cut along the irregular line, and consider the equilibrium of the left-hand side. By taking moments about *X*, force *P* in the upper horizontal can be calculated.

mum possible force in each member of the frame, and can proportion the structure so that it can safely resist these.

A point to notice is that the forces are not affected by the dimensions of the truss, but by its shape. Thus, for the structure shown in Fig. 13, the same forces will be found in the members whether the span is 30 ft. or 60 ft., provided the *proportions* of the frame are unaltered.

Fig. 16 is given as another example which is suitable for the reader to solve for himself. In the first instance, calculate by moments the vertical supporting forces at the ends of the structure. Draw the frame diagram to a scale of 1 in. = 5 ft., notate it, and then construct the reciprocal figure. The numerical figures adjacent to the members are the forces, in tons, which should be obtained from the reciprocal figure, and struts are differentiated from ties by heavy lines. This will enable the work to be checked.

Up to this stage graphical methods have been used to determine the forces in frames, but it is also possible to calculate the forces. Essentially, the method consists of considering the equilibrium of a portion of the structure, and of equating the moments of external forces about a suitable point to the moment of the unknown force in the member.

Simple Example

Take the simple example shown in Fig. 17, and assume that it is required to calculate the force *P* in the upper horizontal. Imagine that the frame is cut along the irregular line and the dotted part removed. Considering the equilibrium of the remaining part of the truss, the clockwise and anticlockwise moments about the point *X* are calculated. The clockwise moment is 2 tons × 10 ft. = 20 tons-ft. Notice that the 4-tons force has no moment since it passes through the point *X*.

There remains then only the moment of the unknown force *P*, and this must be anticlockwise to balance the clockwise moment of 20 tons-ft. Therefore *P* must be pushing from right to left with the

sense indicated by the arrowhead, viz., P is a compressive force. Equating moments, we have :—

20 tons-ft. (clockwise) $= P$ tons $\times$ 6 ft. (anticlockwise).

Therefore, $P = 3\frac{1}{3}$ tons.

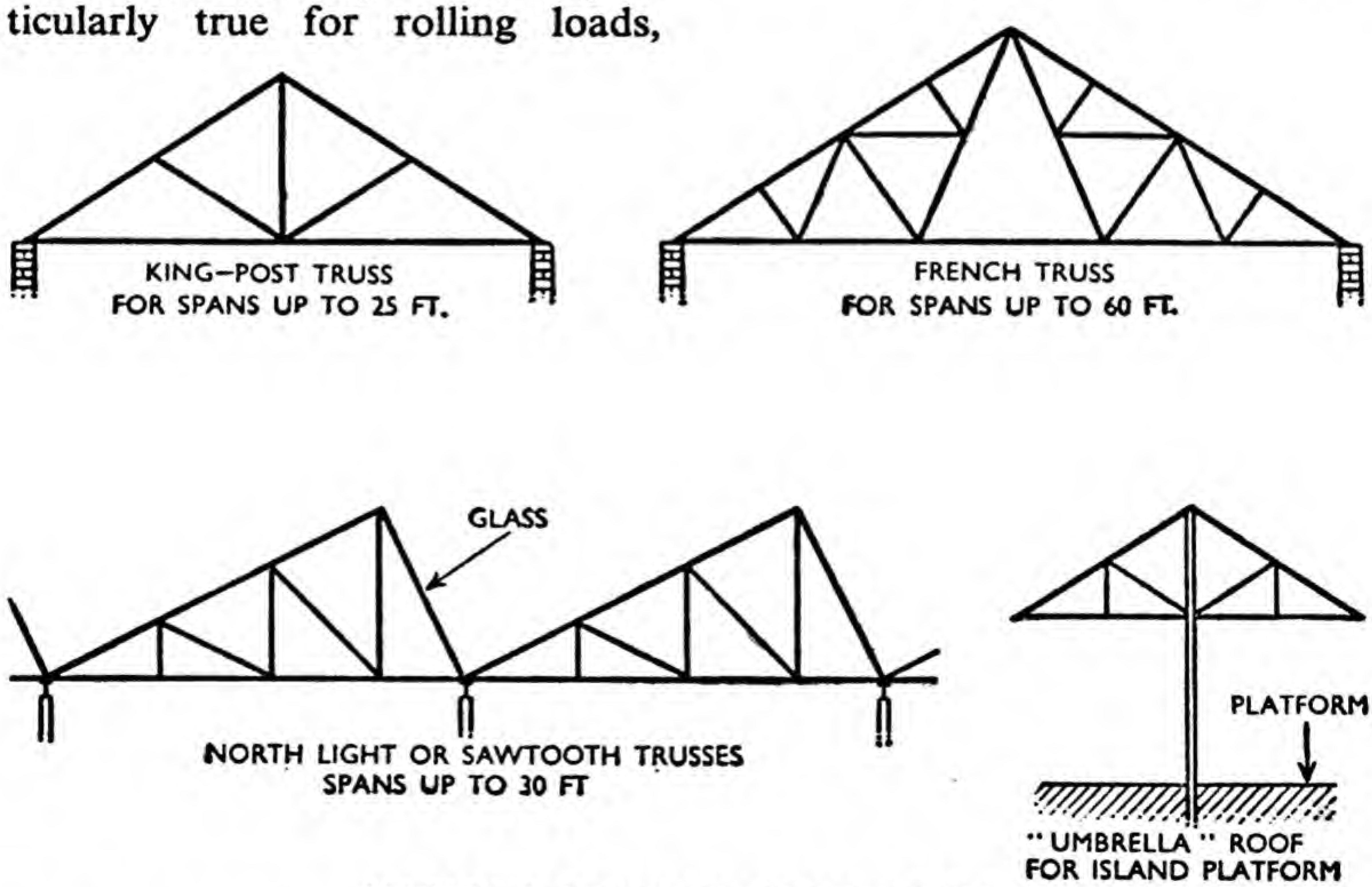
In the same way, by taking sections through the upper joints and cutting the lower horizontal members, it should be verified that the forces in these are each $1\frac{2}{3}$ tons. This is a very simple example, but it serves to indicate the usefulness of the method. Whenever it is possible, the accuracy of the results obtained graphically should be checked in this way.

Use of Method of Sections

This method of sections, as it is called, and several modifications of it, are used extensively by engineers. Indeed, when designing many structures, reciprocal figures are not drawn, and the forces are all calculated in this way. This is particularly true for rolling loads, which obviously cannot be treated by a single reciprocal figure.

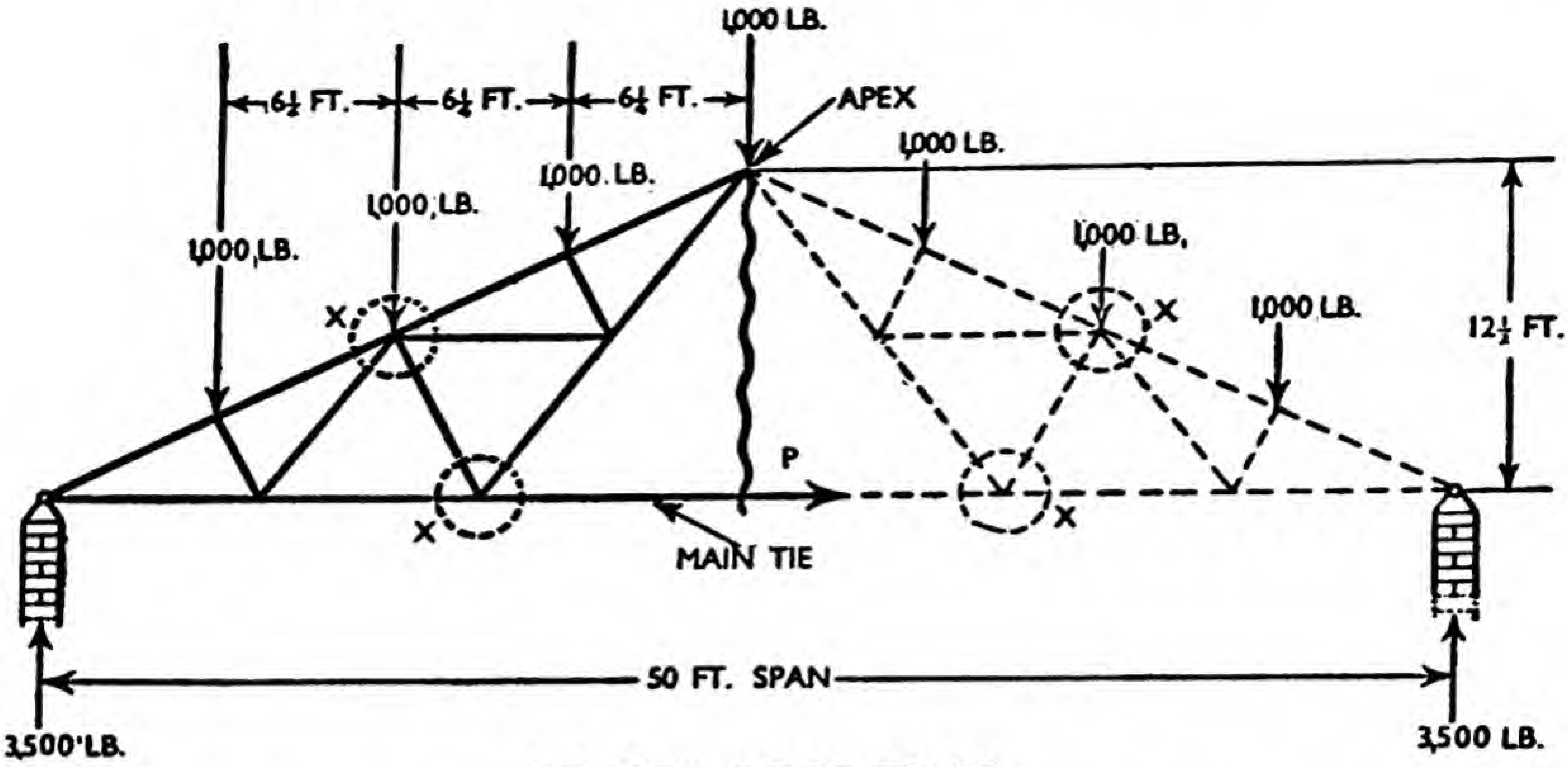
Having seen how the forces in framed structures are determined, let us now examine the types of truss which are used for various purposes. Fig. 18 shows a selection of roof trusses, and it will be noted that more triangles and members are used for larger spans. The reason for this can be readily understood if we take a long slender knitting-needle and load it in compression as a strut. It is known by experience that the needle tends to buckle laterally under even a moderate force, while a short length of the same needle will safely withstand a much greater load.

Some of the members in these trusses are struts carrying compressive loads, and, therefore, it is desirable to keep them relatively short. This is achieved by con-



VARIOUS TYPES OF ROOF TRUSS

Fig. 18. Various types of roof truss are employed for differing purposes and spans. In general, the larger the span the more numerous are the triangular cells in the structure. Umbrella roofs are often seen covering island platforms.



FRENCH ROOF TRUSS

Fig. 19. Reciprocal figure for this cannot be completed in the usual manner. Taking moments about the apex, calculate the force in the main tie. Corresponding vector is inserted in the reciprocal figure, which can then be completed.

structing the frame of numerous triangles, thus avoiding the use of long struts.

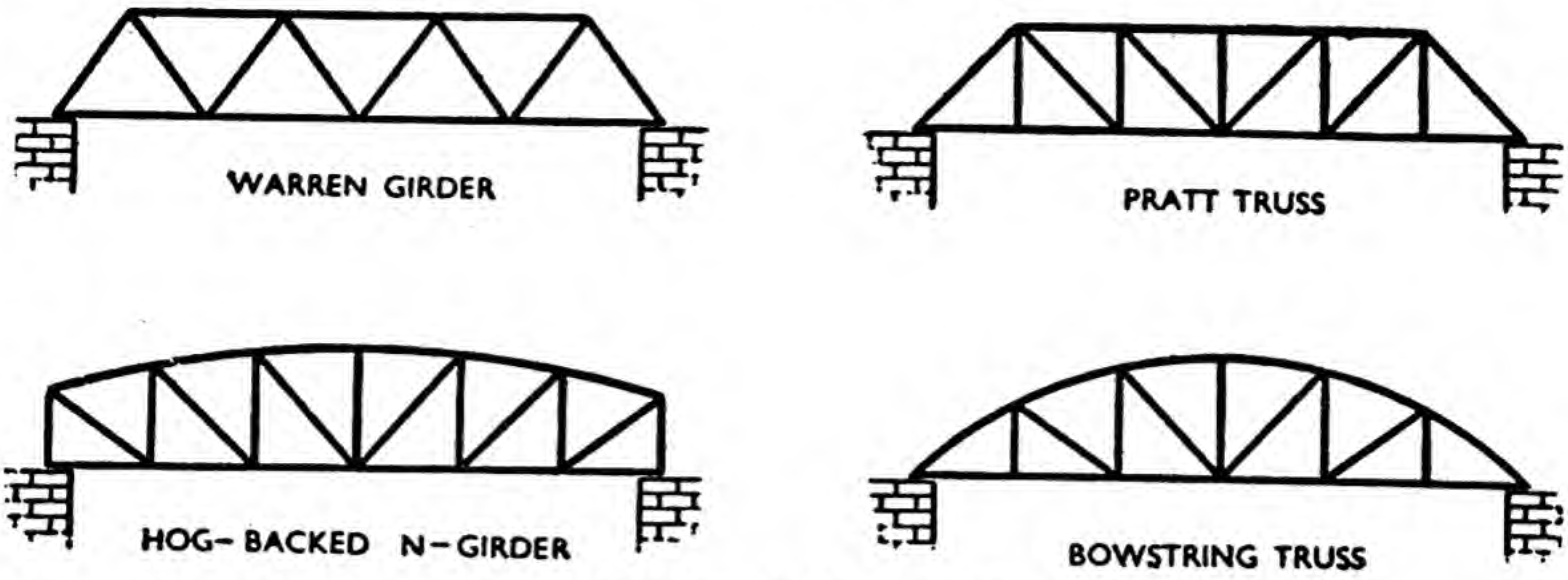
It will also be realized that it is impracticable to have the purlins spaced too far apart, and the positions of the nodes are also governed by this consideration. It may also have been noticed that the roofs of factories and workshops are frequently carried on a series of north-light or saw-tooth trusses of the type shown in Fig. 18. The roof covering on the steeper slope is of glass, and should, in northern latitudes, have a northern aspect. This ensures that the interior of the building is well illuminated without the disadvantage of direct sunlight.

The umbrella station roof is frequently seen covering island platforms. The French truss is used for larger spans, and is specially mentioned here because the reciprocal figure for it cannot be completed in the usual way. This can easily be verified.

Assume dead loads of, say, 1,000 lb. at the rafter nodes, as shown in Fig. 19, and draw the reciprocal figure as far as possible. It will be found that the figure can be drawn up to, but not including, any of the joints marked *X*, and that further progress cannot be made because at each of these joints there are three unknowns. The figure can, however, be completed with the aid of the method of sections.

Overcoming a Difficulty

Taking a central section cutting the main tie, it is possible to calculate the moments about the apex of all forces acting on the left-hand half of the truss. Thus: Clockwise $(3{,}500 \times 25)$ − anticlockwise $(1{,}000 \times 6\frac{1}{4} + 1{,}000 \times 12\frac{1}{2} + 1{,}000 \times 18\frac{3}{4})$ = a clockwise moment of 50,000 lb.-ft. due to the external forces. This is resisted by the anticlockwise moment of the force *P* in the main tie. Therefore, *P* is tensile and has a magnitude of $50{,}000 \div 12\frac{1}{2} = 4{,}000$ lb. This force is set out as a vector in the reciprocal figure, which can now be completed without difficulty,



COMMON TYPES OF TRUSS

Fig. 20. Illustrating some of the more common types of truss used for bridges, which may be recognized by the reader. The triangular cells are not limited to the numbers shown, which indicate only the arrangement of the members.

because at the joints X in the main tie there remain only two unknowns.

Fig. 20 illustrates some common types of bridge truss, examples of which may be recognized in actual road and railway bridges that are familiar. It will be understood of course that the triangular cells are not limited to the numbers shown in the figure, which indicate only the arrangement of the members. For those who desire to obtain facility, here are further examples providing useful practice in the drawing of reciprocal figures.

When curved members are included, as in the hog-backed and bowstring trusses, the reciprocal figure is drawn assuming straight chords between the nodes. When designing the bridge, the engineer makes allowance for the curvature so that the member is strong enough to resist the force so obtained.

In practice, a great variety of types of truss is employed, depending upon such factors as the span, loading, clearance, and local conditions for erection. It is hoped that sufficient interest has been stimulated for the examination with a more discerning eye of structures which were formerly passed by without notice. Here, it must be remarked that many structures, although apparently perfect frames, are in fact statically indeterminate.

Redundant Reaction

A redundant, or indeterminate, frame is one which has more members than are required to keep it stable under load. But a structure also becomes redundant if there are more supports than the minimum. Thus, the Warren girder shown in Fig. 21 is a redundant structure because of the additional support at the central pier. This should be readily understood because, for a given system of loading, it is impossible to obtain the reactions in

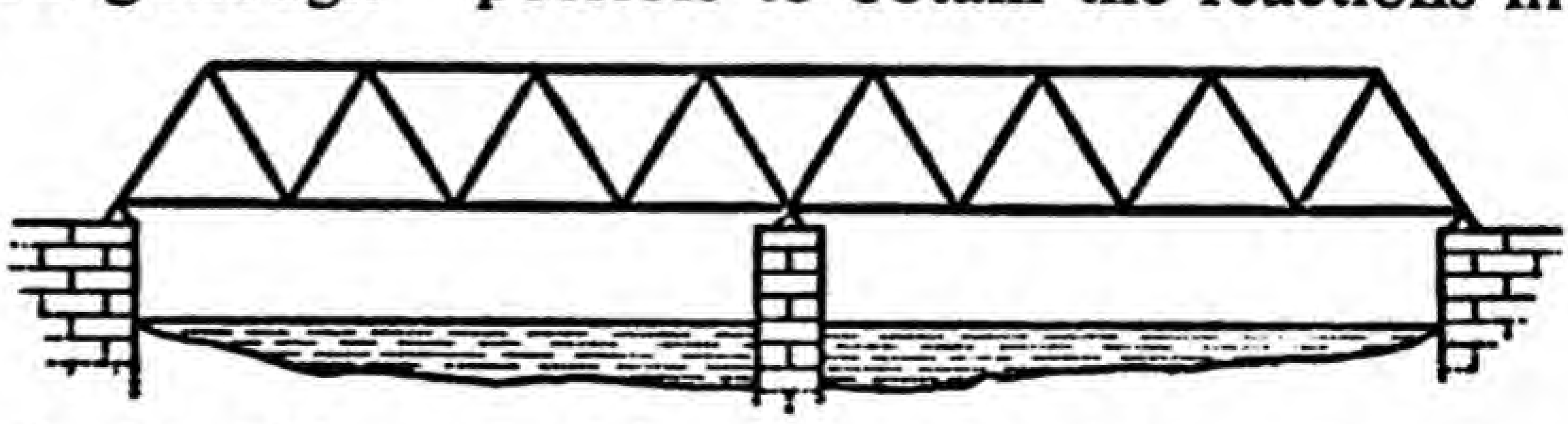
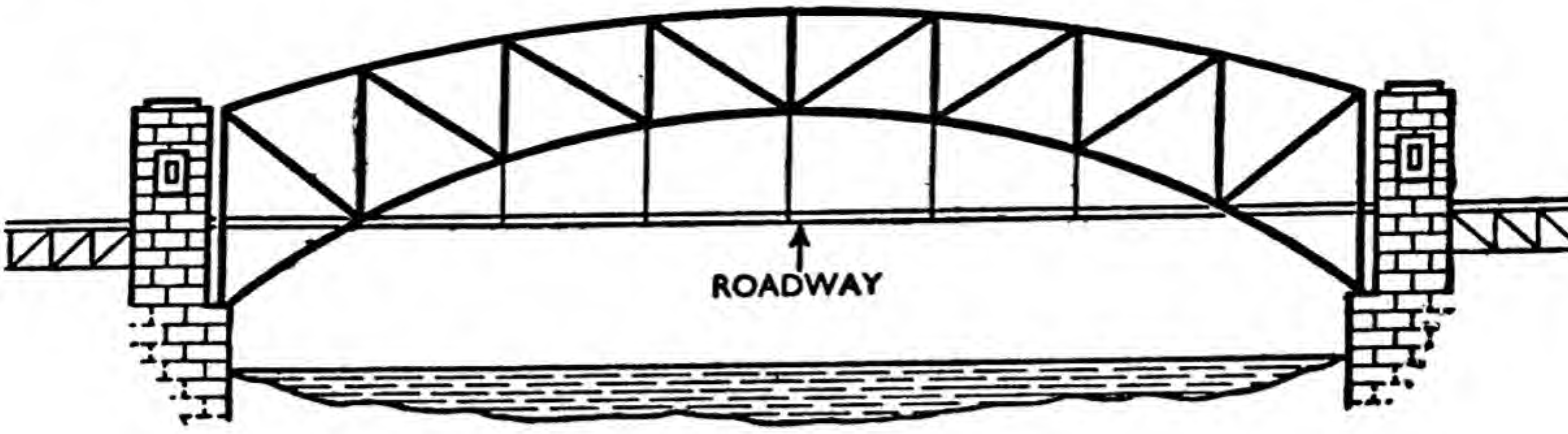


Fig. 21. This truss, although it appears to be a perfect frame, is statically indeterminate because there are three supports and, therefore, the reactions cannot be found by taking moments.



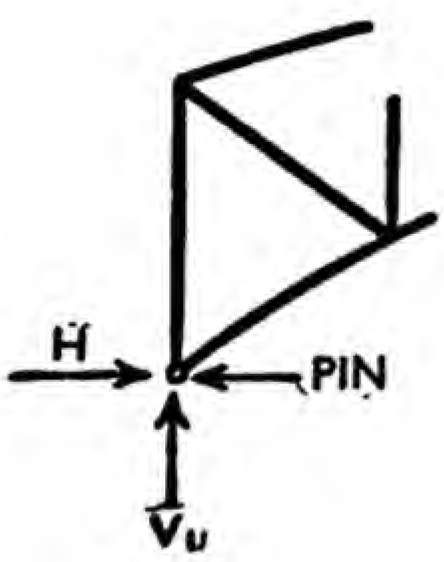
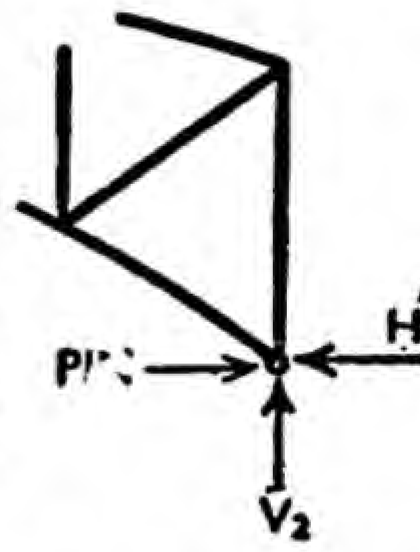


Fig. 22. Here is a two-pinned arch with an unknown horizontal thrust. The forces in it cannot be found by simple statics, because there is an unknown thrust H at the pins. The famous Sydney Harbour Bridge is an arch of this type.



the usual manner by taking moments, since there are three unknowns. Without the reactions, we cannot commence to draw a reciprocal figure.

Certain classes of arches are also statically indeterminate. For example, the structure shown in Fig. 22 is supported at each abutment by a hinge or pin, and this type, therefore, is called a two-pinned arch. At each of these pins, in addition to the vertical reactions V_1 and V_2, which are obtained in the same manner as for a beam, there is an unknown horizontal thrust H, the magnitude of which depends upon the properties of the members of the frame. The famous Sydney Harbour Bridge is an arch of this type, and the calculation of the forces in such a structure is an enormous and highly specialized task.

But not all arches are indeterminate. The simple frame shown in Fig. 23 is called a three-pinned arch because, in addition to the pins at the abutments, a third pin is provided at the centre. This is a determinate structure, a fact which the reader can easily verify for himself. Assuming that a load of 10 tons is placed at the centre, as shown, the vertical supporting forces will each be 5 tons. Considering the equilibrium of the left-hand half of the arch, and taking moments about the central pin :—

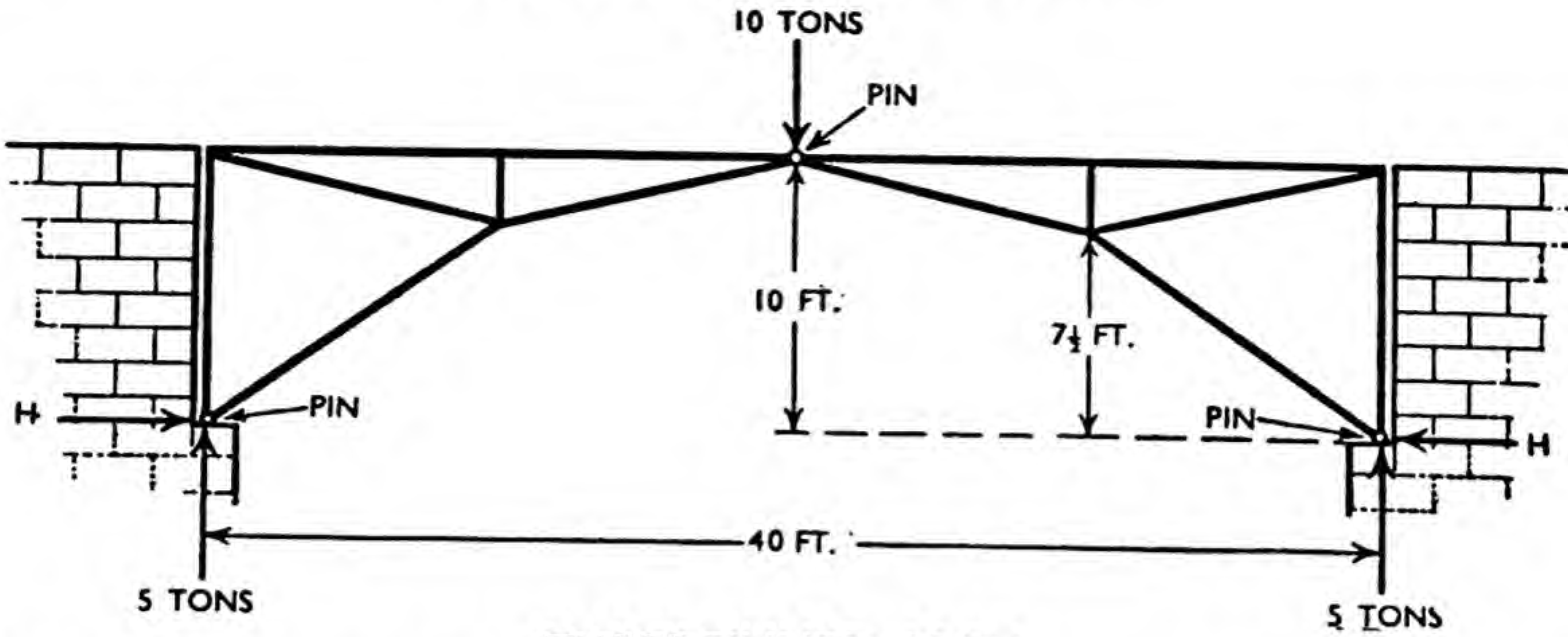
5 tons $\times$ 20 ft. (clockwise) $=$ H tons $\times$ 10 ft. (anticlockwise)

Therefore, $H = 10$ tons.

Instructive Example

Now that all the external forces are known, a reciprocal figure can be drawn, and it is suggested that since it will provide another instructive example, this should be done for the given loading. Of course, all the joints are assumed to be hinged, but this arch is three-pinned, and therefore determinate, only when the two halves are connected by a single pin at the centre ; compare carefully Figs. 22 and 23. Even if the arch in Fig. 23 had many more triangles and carried loading not symmetrically placed, it could still be solved by these simple methods.

The forces acting on a totally

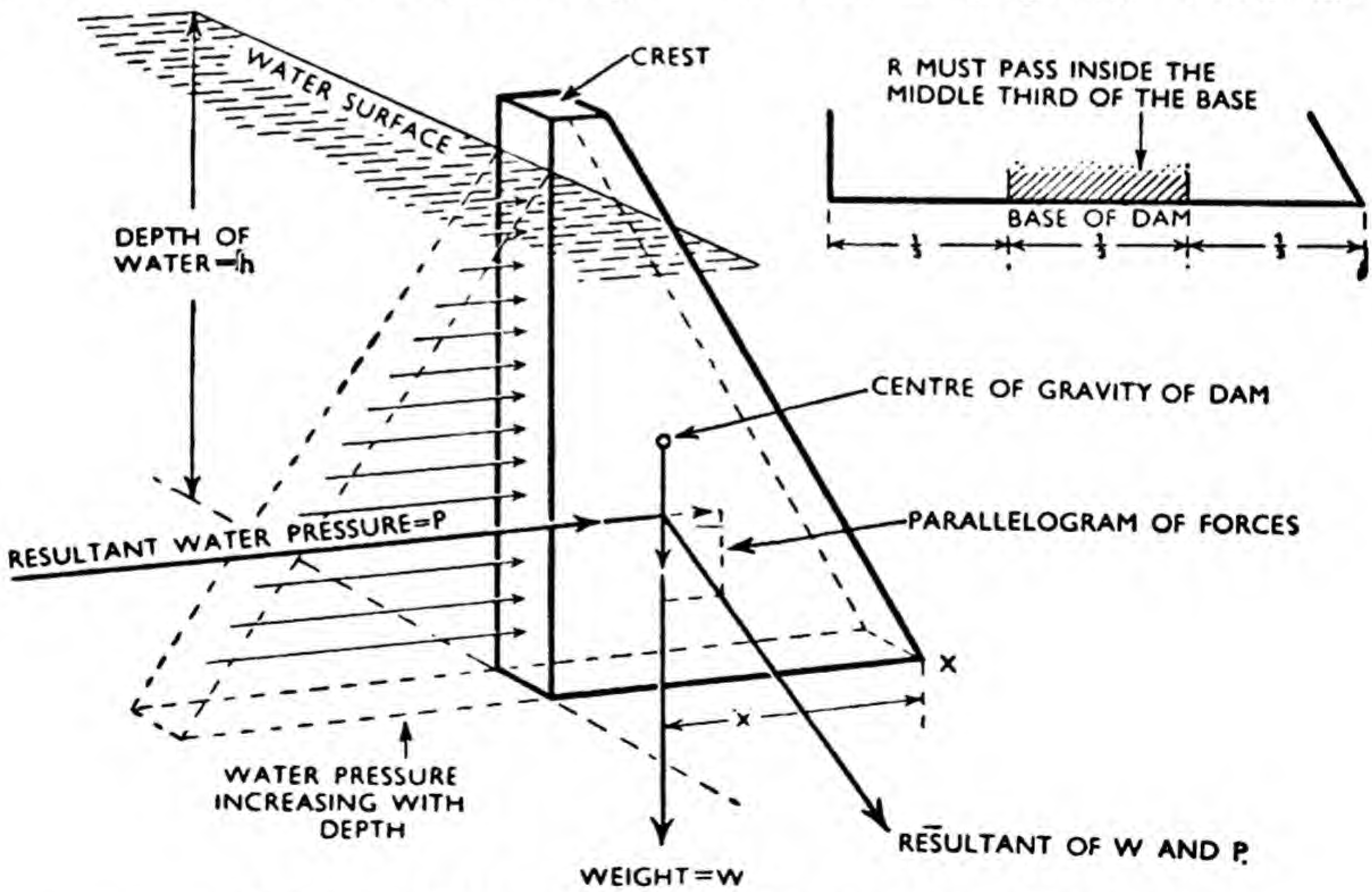


THREE-PINNED ARCH

Fig. 23. This is not indeterminate because the horizontal thrust *H* can be calculated by simple statics. Having found *H*, all the external forces are known, and a reciprocal figure will give the forces in all the members.

different type of structure may now be considered. Dams are employed to impound water in reservoirs for supply to cities and for hydro-electric power, and thus play a vital part in our civilization. The general principles of design equally apply to a dam as to, say, a bridge. The forces acting must be accurately determined, and the structure proportioned to resist them safely and economically.

Imagine that a narrow slice (say, 1 ft. wide) of a masonry dam is isolated from the remainder of the structure (Fig. 24). When the



SLICE OF A DAM ISOLATED FROM REMAINDER

Fig. 24. A narrow slice of a dam is under the action of its weight W passing through the centre of gravity, and the resultant water pressure *P* acting at *h*/3 above the base. In order that there shall be no tension in the dam, the resultant of these forces must cut the base within the middle third.

reservoir is full, the water level will approach the crest and a very large force is exerted on the face of the dam. The pressure is proportional to the depth, as shown, and the resultant force P, due to the water pressure, acts at $\frac{1}{3}$ of the depth, viz., $\frac{h}{3}$ above the base. Readers who are not quite clear about this should consult Chapter 11. The other force which must be considered is the weight W of the slice, acting vertically through its centre of gravity.

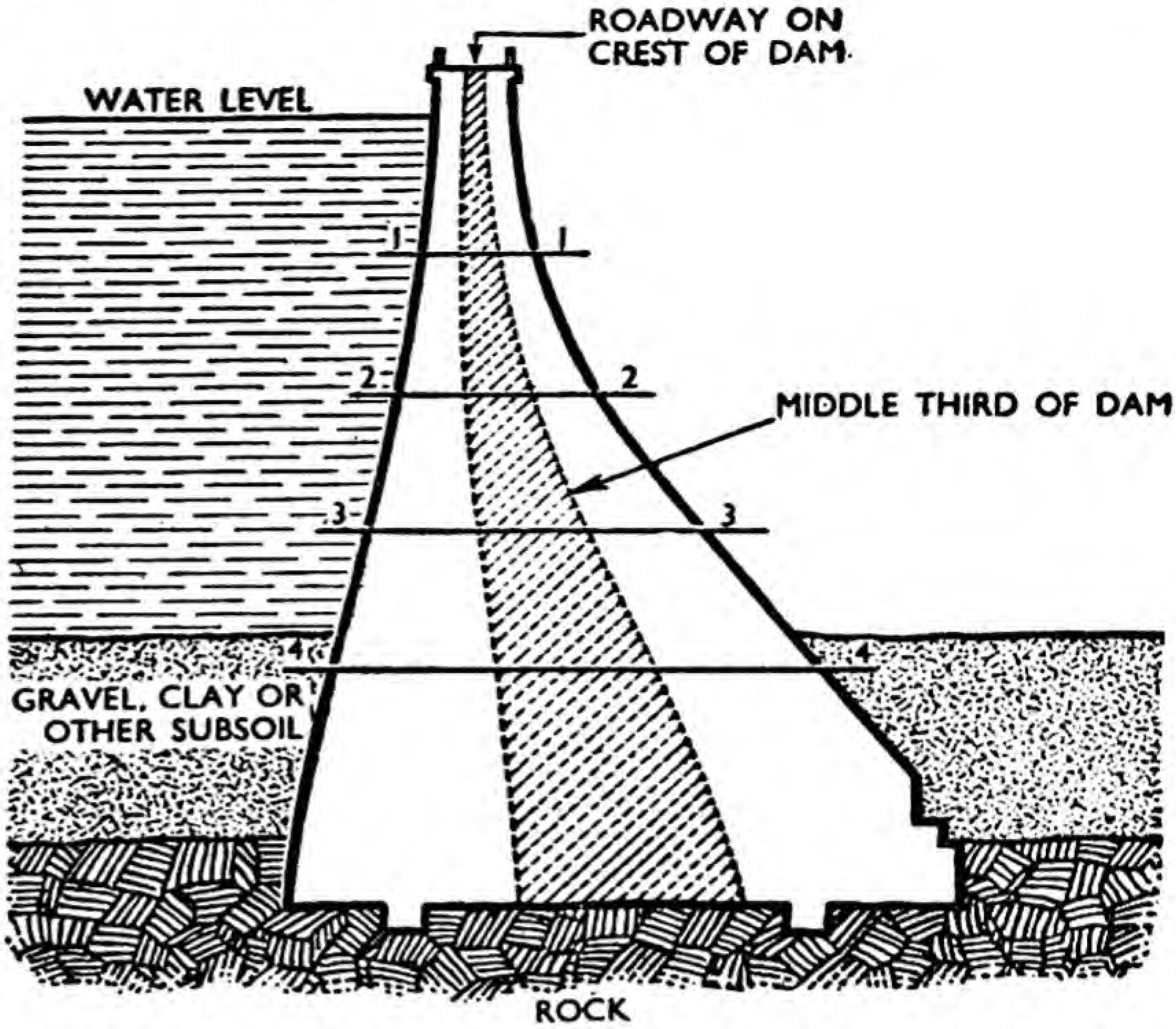


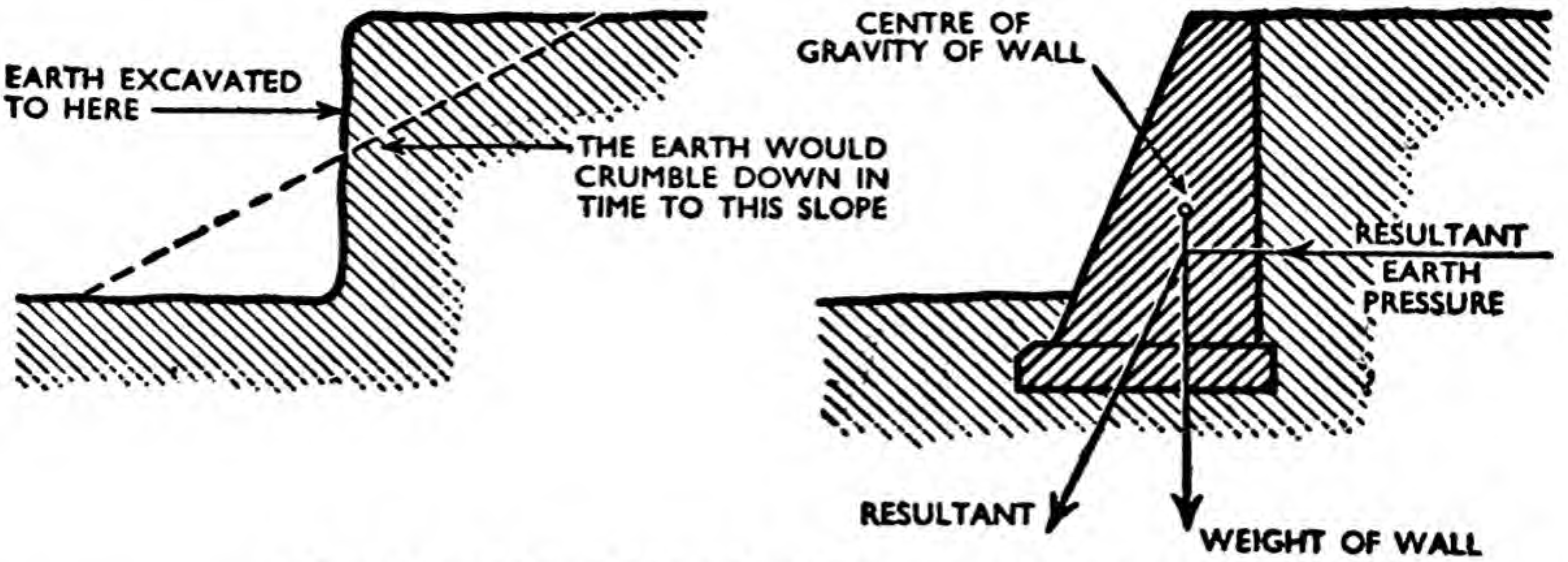
Fig. 25. Actual dams constructed of masonry may have a shape like this. Notice that the foundation of the structure is carried deep down through subsoil to solid rock. At all horizontal sections the resultant thrust must lie within the middle third.

If the water pressure were to push the dam over, it would tilt bodily about X. Taking moments about X, it can be said that, if the structure were about to be overturned, $P \times \frac{h}{3}$ would equal $W \times x$. Naturally, the dam is proportioned so that this could never happen, and in fact a great reserve of stability is provided. When considering such a structure the resultant R of P and W, as shown in the figure, is found by a triangle or parallelogram of forces, and the position of the point where R cuts the base is very important.

If the forces were such that the dam was on the point of overturning, it is known, from Chapter 2, that R would pass through X; this corresponds to the condition when $P \times \frac{h}{3} = W \times x$. As previously remarked, however, the structure is made much more stable than this by increasing the weight, and this has the effect of bringing R well within the base. For reasons that will be understood after reading Chapter 8, the resultant is made to cut the base at a point within the middle third part of the base. By proportioning the dam so that this condition is obtained, the engineer ensures that there is no tension in the structure. This is insisted upon because the masonry or concrete of which the dam is constructed, while strong in compression, has little or no strength in tension.

Essential Condition

It is also important to ensure that the line of action of the weight passes through the middle third of the base when the reservoir is empty, viz., when there is no water pressure. Clearly, this condition is



PREVENTING COLLAPSE OF AN EARTH FACE

Fig. 26. A retaining wall is employed to prevent the collapse of an earth face. If the characteristics of the earth are known it is possible to find the lateral pressure on the wall. The resultant of the weight of the wall, and the total pressure due to the earth should cut the base within the middle third. The wall is proportioned so as to secure this condition.

governed solely by the position of the centre of gravity with respect to the base, and the dam is proportioned accordingly.

Design of Dams

Actual dams do not usually have the simple trapezoidal shape shown in Fig. 24 but vary considerably in outline. For example, Fig. 25 shows a practical section. Although the shape is quite different, the general principles of design remain the same. It is necessary in practice, however, to consider not only the base but numerous horizontal sections, such as 1—1, 2—2, etc. At any section, say 2—2, the resultant of the weight and the water pressure above 2—2 must pass within the middle third part of 2—2. Similarly, if the reservoir is empty, the weight alone above any section must pass within the middle third. The major part of the design of such a dam consists of proportioning the outline so that both these conditions are realized for all horizontal sections.

Similar principles govern the design of retaining walls. When an excavation is made for, say, a road or railway, it is known by experience that the earth face would gradually crumble down under the action of the weather (Fig. 26). Therefore, a retaining wall is constructed to prevent this collapse. Knowing the characteristics of the soil, it is possible to estimate the pressure which it exerts on the back of the wall. Just as in the case of a dam, the resultant *R* of the weight of the wall and the total earth pressure is found by a parallelogram of forces, and *R* should cut the base within the middle third. The wall is proportioned so that this condition is obtained, and the designer then knows that the structure is stable against overturning, and is free from tension.

In this chapter the forces in some common structures have been considered in an elementary manner, and it is hoped that sufficient has been covered to stimulate interest in the structures under or over which we pass daily, and especially to show the applications of simple mechanics to this most important subject.

CHAPTER 4

MECHANICS OF MOVEMENT

MOTION. LINEAR AND ANGULAR VELOCITIES. RELATIVE VELOCITIES. LINEAR AND ANGULAR ACCELERATIONS. NEWTON'S LAWS OF MOTION. MOMENTUM. CONSERVATION OF MOMENTUM. BALLISTIC PENDULUM PRINCIPLE. CURVED BANKED TRACKS. WORK, POWER, HORSE-POWER. CENTRIFUGAL AND CENTRIPETAL FORCE. TORQUE AND ACCELERATION. SIMPLE HARMONIC MOTION. MECHANICS OF ROTATION. STATIC AND DYNAMICAL BALANCING. USE OF GEARS TO INCREASE TORQUE.

BODIES in motion are some of the most familiar sights in our everyday experience, on land, on or under water, and in the air. In this chapter we shall consider the motions of bodies, their causes and some of their effects.

If a body is moving in a certain direction and is not rotating, it is said to have motion of translation only. This applies to a ship, a motor car or an aeroplane moving in a straight path. If a body is rotating without moving forward as a whole, as in the case of the flywheel of a stationary engine, it is said to have motion of rotation only. It is, of course, possible for a body to have both motions at the same time ; for example, the wheel of a bicycle is rotating round the hub and moving forward simultaneously, and a rifle bullet is spinning at a high speed while it moves toward the target.

Motion of Translation

We shall confine ourselves to motion of translation in the first instance and deal with motion of rotation at a later stage.

Although we use the term velocity in many cases when we really mean speed, it is often important to distinguish between these two terms.

By speed we mean the rate of change of position, without reference to the direction in which the body is going. For instance, we may say that a car is moving at 30 m.p.h. without saying in what direction, and in that case we are referring to its speed. If, however, we say that it is moving at 30 m.p.h. in a north-westerly direction, we are specifying its velocity, which can now be represented by a straight line, the length of the line to a selected scale giving the speed, and the direction of the line showing the direction of motion. Since the same line could also show a south-easterly direction, an arrow is necessary to show completely the velocity of the body.

It is now necessary to have a clear idea of what is meant when we say that a car is moving at 30 m.p.h. Obviously, it cannot imply that in the next hour it will cover a distance of 30 miles, because its speed will almost certainly vary during that time. It will be more accurate to say that in the next second it will travel 44 ft., and still more accurate to say that in the next tenth of a second it will travel 4·4 ft. This would lead to

difficulties if we had to measure accurately the distance covered in a very small time, in the event of the speed not being uniform. Fortunately, this is not necessary, and a method of estimating the speed at any instant will be explained later.

If the speed is varying and a body travels 30 miles in an hour, we can say that its *average* speed is 30 m.p.h. or 44 ft. per sec.

Relative Velocity

It must be clearly understood that the velocity of a body is always *relative* to some other body which is regarded as fixed. In most of our problems we regard the surface of the earth as fixed ; when we say a car is travelling at 30 m.p.h., we mean that it is travelling at 30 m.p.h. relative to the earth's surface.

Owing to the rotation of the earth on its axis, however, all bodies on its surface are moving at speeds which increase from zero at the poles to a maximum at the equator (about 1,500 m.p.h.). The earth itself is moving round the sun, and the sun itself is moving, so that actually we are moving in space all the time at almost incredible speeds. The reason why we feel nothing of this is that the human body, in fact every moving body, is capable of appreciating only *changes* of speed which involve forces.

If the speed of a body is changing, it is said to have an acceleration. If the speed is increasing, the acceleration is positive ; if the speed is decreasing, the acceleration is negative, and is sometimes known as retardation.

Acceleration is measured by the rate of change of speed, usually the change of speed per second.

For example, if a vehicle accelerates from 30 m.p.h. (44 ft. per sec.) to 45 m.p.h. (66 ft. per sec.) in 5 sec., the acceleration would be found as follows :—

Change of speed $= 66 - 44 = 22$ ft. per sec.

Time taken $= 5$ sec.

Acceleration =

$$\frac{\text{Change of speed}}{\text{Time taken}} = \frac{22}{5} = 4{\cdot}4.$$

The acceleration is, thus, 4·4 ft. per sec. per sec. ; that is to say, a change of speed of 4·4 ft. per sec. every second. We could, of course, say that it is 3 m.p.h. per sec., but it is unusual to mix the time units in this way. (The expression ft. per sec. per sec. is usually abbreviated to ft. per sec.^{2}.)

Determining Relative Velocity

The determination of the velocity of one body A relative to another body B is of great importance in a number of cases. Let us begin with a simple case.

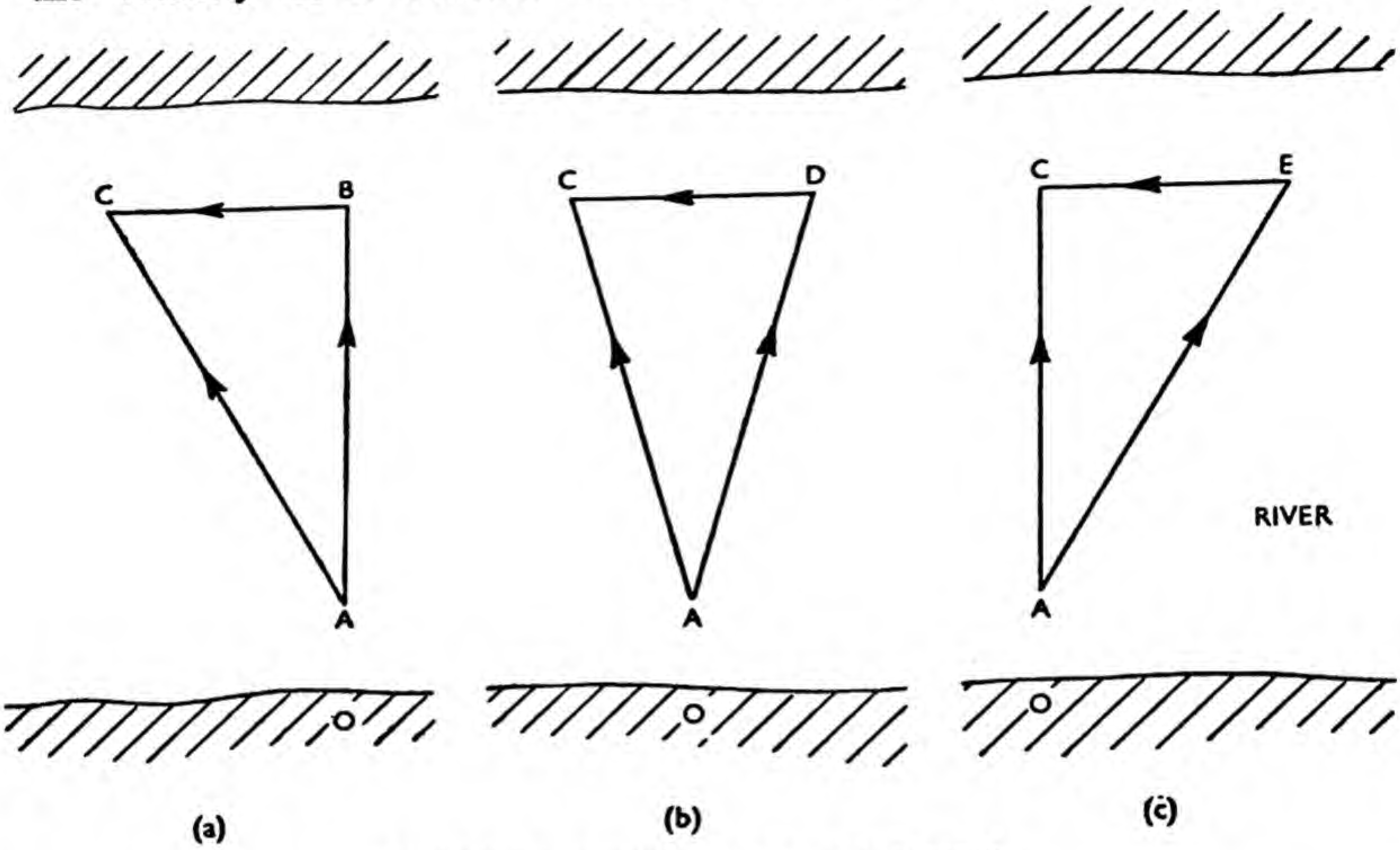
A car A, moving at 40 m.p.h. in a given direction, is being overtaken by another car B, moving at 45 m.p.h. in the same direction. If they are a mile apart, how long will it take car B to overtake car A ?

Now in any case, the velocity of B relative to A is the rate at which the distance between B and A is altering, and in this case it is $45 - 40 = 5$ m.p.h. in the forward direction, that is, it is the same as if A were stationary and B moving toward A at 5 m.p.h. Similarly, the velocity of A relative to B is the same as if B were stationary and A moving backward at a speed of 5 m.p.h.

If two trains are passing one another on parallel lines and both are travelling in the same direction at different speeds, a passenger in

the slower train will get the impression that he is travelling backward, as indeed he is, relative to the faster train.

To save unnecessary length of statement, we may use the expression ${}_Bv_A$ to indicate the velocity of A relative to B and ${}_Av_B$ to indicate the velocity of B relative to A.



FINDING RESULTANT VELOCITY

Fig. 1. (*a*) Swimmer crossing a river or stream, as seen by an observer *O* on the bank, crosses from *A* to *C* if he attempts to reach the other bank by swimming at right angles to that bank. (*b*) If he swims towards *D*, again as seen by the observer *O*, it looks as though he is swimming, again, from *A* to *C*. (*c*) Should the swimmer swim towards *E*, it still looks as though he is travelling from *A* to *C*, which is at right angles to the bank. The different directions achieved by the swimmer are due to the velocity of the river, and so the resultant velocity of the swimmer in each case will be *AC*.

Thus the above statement becomes :

$${}_Bv_A = -\,{}_Av_B$$

(Note that we are using the term velocity now, since we are considering direction as well as speed.)

Since the distance between A and B is decreasing at a rate of 5 m.p.h., the time taken to decrease this distance by one mile will evidently be 12 min. (one-fifth of an hour). Car A will then have travelled a distance of eight miles, and car B will have travelled nine miles. The advantage of a very high maximum speed in police cars, for instance, will be obvious. If the speed of car B had been increased to 50 m.p.h., both the distance and time required for overtaking would be halved. For a similar reason, one car should not attempt to overtake another on a congested road unless its speed *relative to the other car* is large.

If a vehicle 15 ft. long is attempting to pass another vehicle of the same length, the speeds being 42 m.p.h. and 40 m.p.h. respectively, the time taken for the overtaking vehicle to clear completely the overtaken vehicle will be 10 sec., and the distance travelled by the slower of the two vehicles

will be 600 ft. before this occurs. It is the *difference* of the speeds which determines the time taken to pass, but it is the actual speed which influences the distance travelled.

Resultant Velocity

Before we proceed to consider more complex cases of relative velocity, it is necessary to understand the meaning of the term resultant velocity.

Suppose a swimmer is crossing a river (Fig. 1(a)) and is swimming in a direction *AB* at right angles to the bank. Let *AB* represent his velocity through the water to some known scale. Then, if the river is flowing at a velocity represented to the same scale by *BC*, the actual direction of motion of the swimmer as seen by an observer *O* on the bank will be *AC*, and this also represents his resultant velocity. If he swims in some other direction *AD* (Fig. 1(b)), his resultant velocity is obtained in the same way. If he wishes to reach a point immediately opposite his starting point, he must swim in a direction *AE* (Fig. 1(c)) such that the resultant direction of motion is at right angles to the bank.

The necessary direction of *AE* can be found quite readily as follows: referring to Fig. 1(c), set out *EC* to represent the velocity of the stream to scale. Set a pair of compasses to a length *EA*, representing the velocity of the swimmer, to the same scale, and draw an arc of a circle cutting *CA* at *A*. Join *A* to *E*. This gives the direction in which he must swim, and the resultant velocity of crossing will be *AC*.

The same method is used in setting the course of an aeroplane in a cross wind. In this case *AC* (Fig. 1(c)) is the desired direction of flight relative to the earth, *AE* is the velocity of the plane in still air, *EC* is the velocity of the wind. The direction of *AE* gives the compass course on which the plane must be set to fly.

We may now return to the more general case of relative velocity.

Let us consider two passengers at some distance from each other on the deck of a ship. If one of them wishes to meet the other he will walk towards him at, let us say, 3 m.p.h. Now the time required for them to meet will be just the same whether the ship is moving or not, since this time depends only upon the velocity of each relative

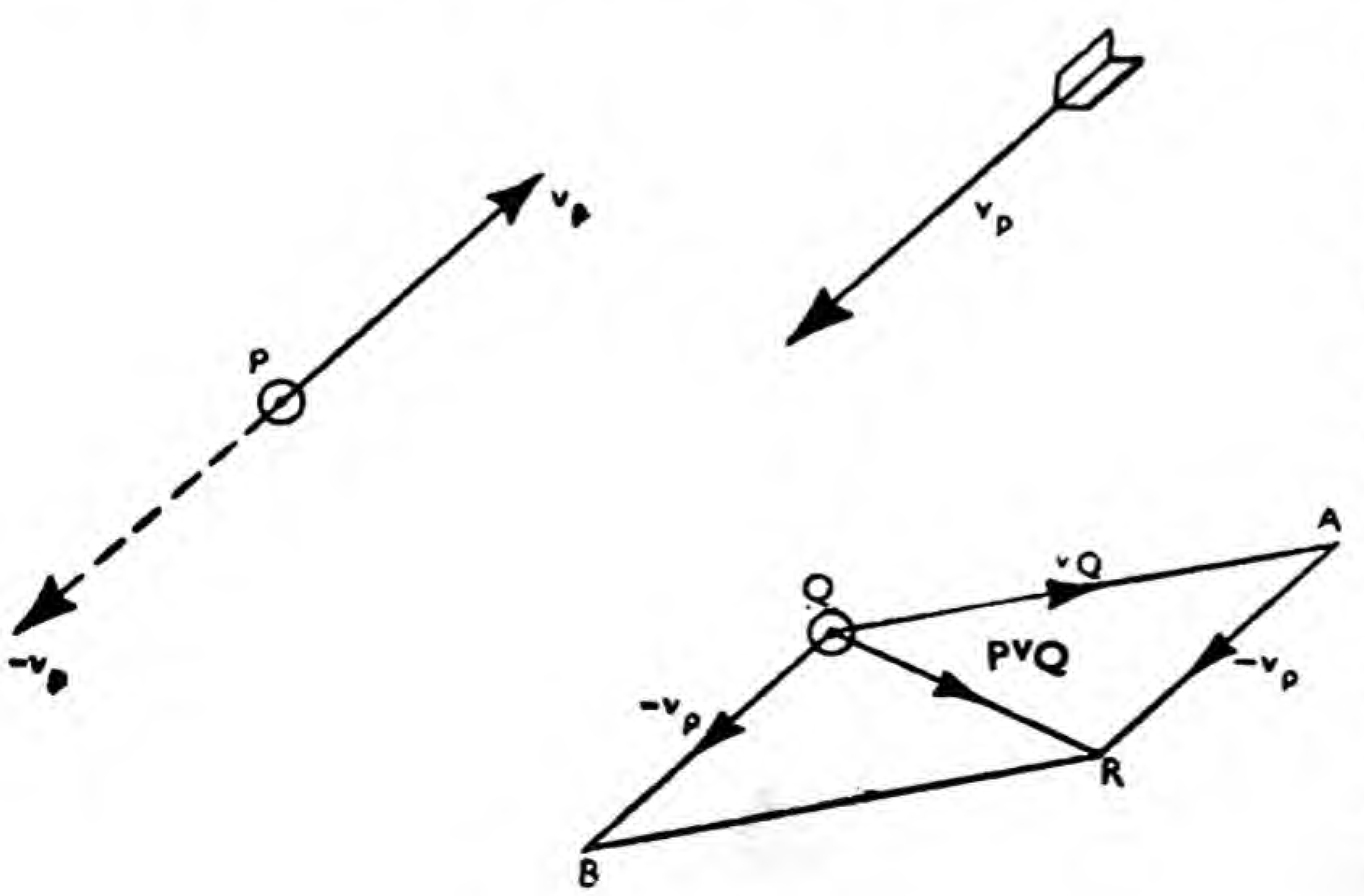
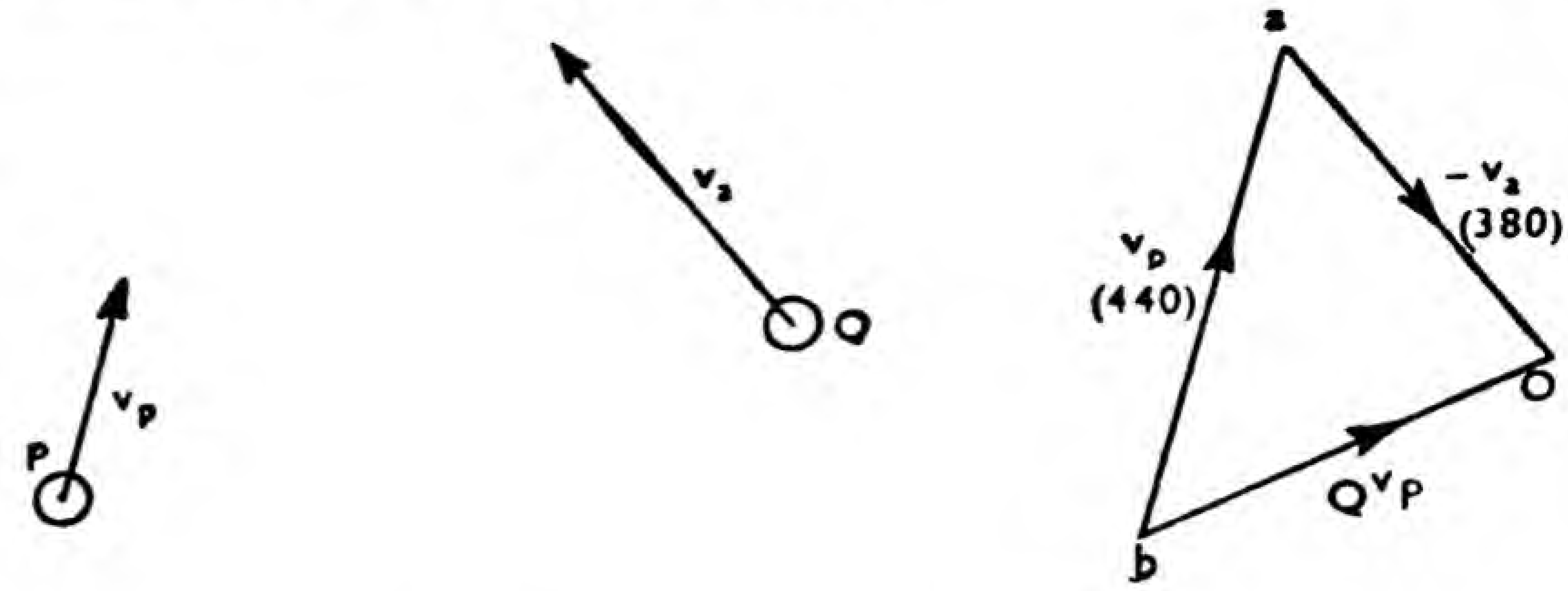


Fig. 2. Two bodies, *P* and *Q*, are moving with velocities V_P and V_Q. If each of them is subjected to an additional velocity, equal and opposite to V_P, then the relative velocity of *Q* to *P* will remain unaltered. *P* will be brought to rest, and the resultant velocity of *Q* will be the diagonal *QR* of the parallelogram *QARB*.

Fig. 3. *P* and *Q* represent positions of two aeroplanes at the same level, travelling in a horizontal plane. If *P* is going to overtake *Q*, then plane *P* must be set to fly in the direction of *ba*.



to the deck. The resultant velocity of each, regarding the earth as fixed, will, of course, depend upon the motion of the ship. If we subject each of the passengers to an additional velocity of the same magnitude and direction due, say, to the motion of the ship, we do not alter the velocity of either passenger relative to the other.

Additional Velocity

Let us now take the case of two bodies *P* and *Q* (Fig. 2) moving with velocities v_P and v_Q as shown. If we subject each of these to an additional velocity equal and opposite to v_P, the relative velocity of *Q* to *P* will be unaltered, as seen above. In this case, however, *P* is brought to rest and the resultant velocity of *Q* is the diagonal *QR* of the parallelogram *QARB*. *QR* is thus the velocity of *Q* relative to *P*, viz., $QR = {}_Pv_Q$, and *Q* is moving away from *P* in the direction *QR* with speed *QR*.

Instead of the parallelogram we can draw a triangle in which $QA = v_Q$, $AR = -\ v_P$, and the closing line $QR = {}_Pv_Q$.

Suppose *P* and *Q* (Fig. 3) are the positions of two aeroplanes at a given instant. *P* is flying at 440 ft. per sec., and *Q* is flying at 380 ft. per sec., both at the same level and flying in a horizontal plane. If *P* is to overtake *Q*, then ${}_Qv_P$ must be in the direction *PQ*. Applying the above principle, draw *ao* to represent the speed of *Q* reversed. Draw also a line *ob* of indefinite length parallel to *QP*. Set the compasses to a length representing v_P (440 ft. per sec.), and draw an arc with *a* as centre, cutting *ob* at *b*.

The plane *A* must then be set to fly in the direction *ba*, and it will be approaching plane *Q* at a speed represented by *bo*. The same principle is involved if a bullet fired from *P* is to hit *Q*, but the construction is not quite so simple in this case.

Obviously, there would not be time for calculations of this kind to be made while in action, and automatic sights are provided which perform the operation automatically.

A sportsman shooting at a flying bird or moving animal, a man trying to catch a train which is moving out of the station, or a person throwing a missile at an external object from a moving vehicle, all of these have to make mental estimates based on the above, and the degree of success they attain depends on the accuracy of these estimates.

Varying Speed

If the speed of a body is varying, we can observe the times taken to travel a number of known distances and these can be plotted as shown

in Fig. 4. If we wish to know the speed at a point P, a line PQ is drawn to touch the curve at P. This line is known as a tangent to the curve. Vertical and horizontal lines QR and PR are drawn as shown, from which we obtain :—

Velocity of P (ft. per sec.)

$$= \frac{QR \text{ (ft.)}}{PR \text{ (sec.)}}.$$

This is the velocity of the body when it has travelled a distance PX.

If a body is rotating about an axis regarded as fixed, it is said to have an angular velocity. This is measured by the angle turned through by the body in unit time, say deg. per sec., or r.p.m.

Measurement in Radians

It is found convenient in most problems to take the radian as the unit angle. If we step along the circumference of a circle a distance equal in length to the radius, the angle between the two radii at the beginning and end of this arc is

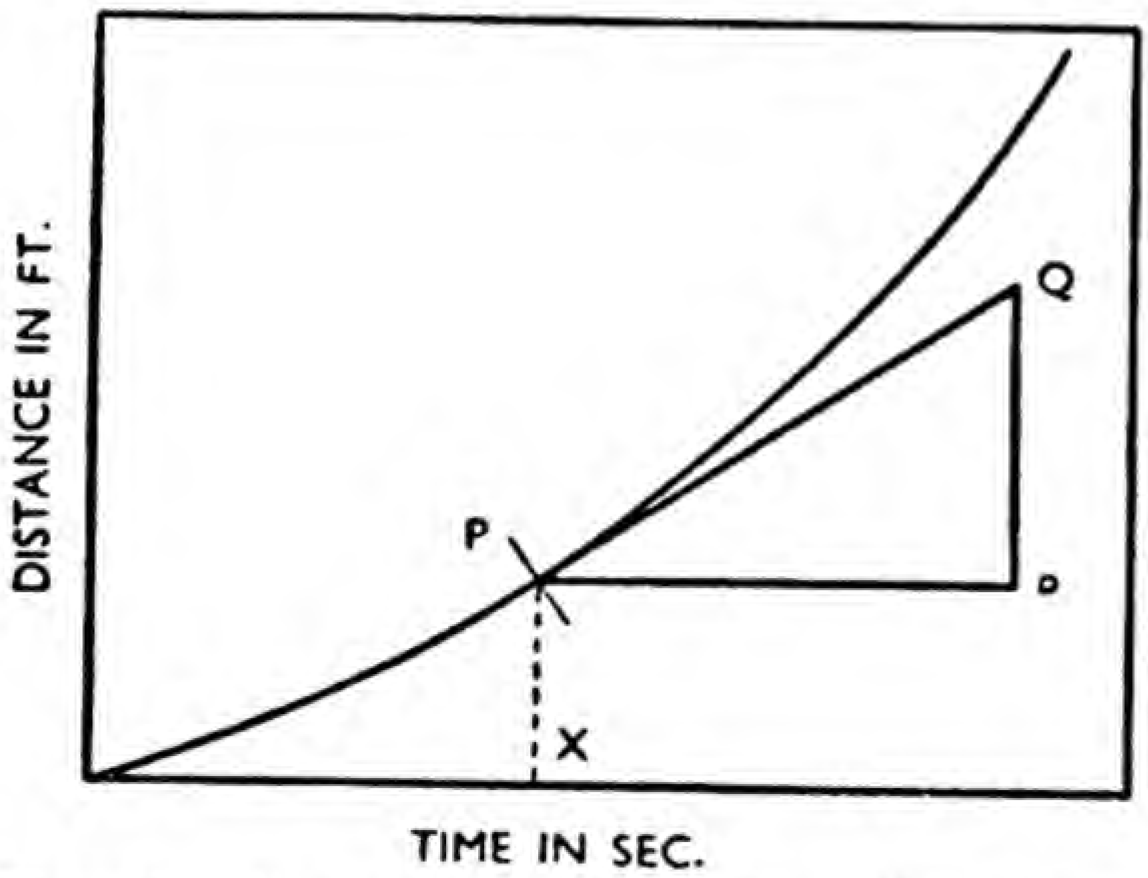


Fig. 4. Illustrated above is a distance-time diagram for a body whose speed is varying. To find the velocity of P at any given time X, draw the tangent PQ to the curve. PR and QR are then drawn to form a right-angled triangle. The velocity of P, therefore, will be found to be QR divided by PR, measured in the units to which the diagram is made. Above, these units are feet and seconds.

one radian. Since the length of the circumference is 2π multiplied by the radius ($\pi = 3{\cdot}142$), 360 deg. will be equal to 2π radians.

Therefore, 1 radian

$$= \frac{360}{6{\cdot}284} \text{ deg.} = 57{\cdot}3 \text{ deg.}$$

The reason for using this unit is that it makes it easier to calculate the linear velocity of a point on the rotating body from the angular velocity of the body (rad. per sec.).

Referring to Fig. 5, if P is a point on the body, distant OP from the axis of rotation, then if OP is 3 ft. and P is moving in its circular path at a speed of 18 ft. per sec., the number of radians traced out by OP will be $\frac{18}{3} = 6$ rad. per sec.

Thus, the angular velocity of OP is 6 rad. per sec.

Putting this in general terms, we have :—

Angular velocity (rad. per sec.)

$$= \frac{\text{linear velocity (ft. per sec.)}}{\text{radius (ft.)}}.$$

Angular velocities are frequently expressed in r.p.m., but these are easily converted to rad. per sec. if we remember that 1 rev. $= 2\pi$ radians, so that :—

Angular velocity (rad. per sec.)

$$= \frac{2\pi \times \text{r.p.m.}}{60}.$$

For example, if a flywheel is rotating at 120 r.p.m., its angular velocity is $\frac{6{\cdot}284 \times 120}{60} = 12{\cdot}57$ rad. per sec.

Angular Acceleration

Angular acceleration may be defined as change of angular velocity per sec.

Thus, if a wheel increases its speed at a uniform rate from 150

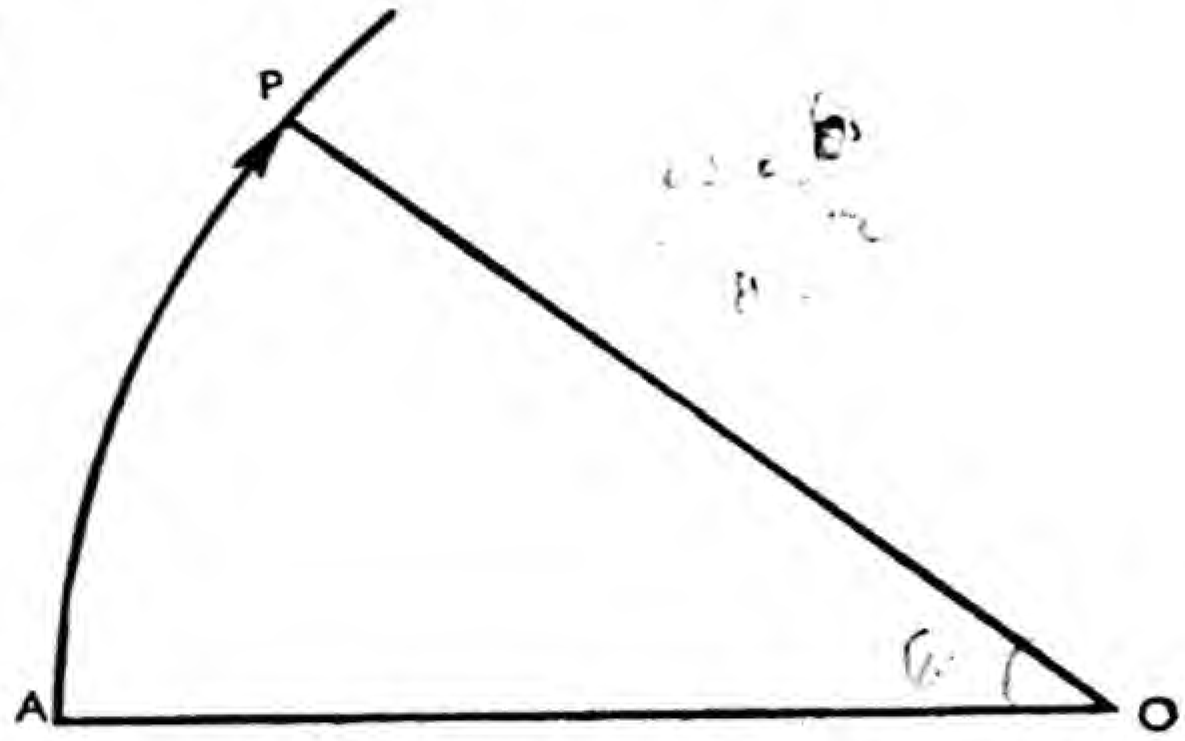


Fig. 5. *P* is a point on the circumference of a circle with centre *O*. The linear velocity of *P* is distance that it moves round the circumference per second. The angular velocity is the angle between the two radii at the beginning and end of the distance moved, and it is usually expressed in radians per second. To calculate the angular velocity of any body, find the linear velocity in feet per second, and divide it by the radius in feet.

r.p.m. to 200 r.p.m. in 12 sec., its angular acceleration will be found as follows :—

Increase of angular velocity

$$= \frac{6{\cdot}28\,(200\text{-}150)}{60} = 5{\cdot}23 \text{ rad. per sec.}$$

Time taken = 12 sec.

$$\therefore \text{Angular acceleration} = \frac{\text{change of angular velocity}}{\text{time}}$$

$$= 0{\cdot}436 \text{ rad. per sec. per sec. (or rad. per sec.}^2).$$

It will be useful to remember that 1 rad. per sec. is equivalent to approximately 9·6 r.p.m., so in this case the wheel is increasing its speed at the rate of 0·436 rad. per sec. every second, or 4·17 r.p.m. every second.

It is also necessary to bear in mind that, while we may speak of the linear velocity and acceleration of a point on a moving body, angular velocity and acceleration can refer only to a line on the body, and all lines on a moving body have the same angular velocity and acceleration, since, when the body performs one revolution, every line on the body also performs a revolution.

We may now proceed to find the relations between the motions of a body and the forces causing these motions. These are given by Newton's laws of motion, which may be stated as follows :—

1st Law. If a body is at rest, it will remain at rest unless some unbalanced or resultant force acts on it. If it is in motion it will move in a straight line with uniform speed, unless some unbalanced force is acting on it.

If a body is at rest or moving with uniform speed in a straight line it is said to be in equilibrium (equal balance). We sometimes find it convenient to say that the forces acting are in equilibrium, but actually we are referring to the body acted on by these forces.

Thus, the first law tells us that if a body remains at rest, all the forces acting on it are balanced, viz., there is no resultant force acting.

Resultant Force Acting

We can also say that if a body is not moving in a straight line, there must be some resultant force acting, even if it is travelling at uniform speed. For instance, a stone being swung round in a circular path at the end of a string needs the pull of the string to keep it in a circular path. If the string breaks, the force acting on the stone is removed, and the stone will move on in a straight line with whatever speed it had when the string broke.

It should be made quite clear that the stone *P* (Fig. 6) does not fly out radially along *OR*, as many suppose, but tangentially along *PQ*. The distance *OQ* from the

centre *O*, of course, increases, but in this case anyone standing in the direction *OR* would be quite safe if the string *OP* broke at this instant.

The first law also applies to motion of rotation, but in this case angular velocity must be substituted for linear velocity, and torque or moment substituted for force.

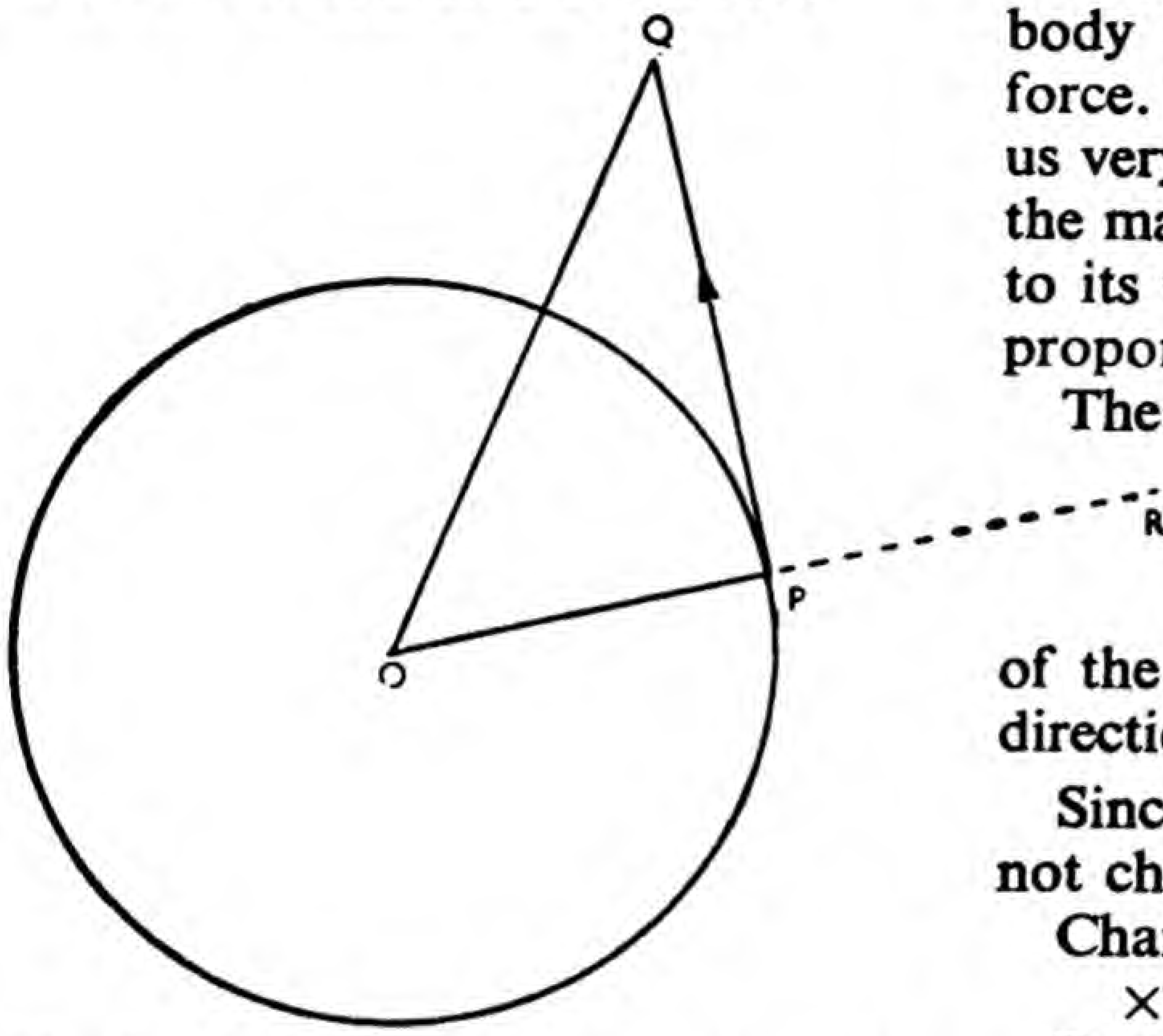


Fig. 6. *P* is a stone tied to a length of string and it is being swung round so that the path of *P* is the circumference of a circle with centre *O*. If the stone becomes detached from the string, it does not fly off on the line *PR*, as one might expect, but tangentially along *PQ*, so that any person standing in line with *OP* would be quite safe, and would not be hit.

Angular velocity is the rate of change of angular displacement of a body moving round a fixed axis, and is usually measured by the number of radians turned through per second (rad. per sec.). In order to change the angular velocity of a rotating body, a moment or torque is required, since the effect of a force in producing rotation depends upon the position of the line of action of the force from the axis, as well as the magnitude of the force.

We now come to the case where a body has an unbalanced or resultant force acting on it, and must introduce the additional terms mass and momentum.

Quantity of Matter

Mass is usually defined as the quantity of matter in a body and is a measure of the resistance of the body to being accelerated by a force. This definition does not carry us very far, but it may be said that the mass of a body is proportional to its volume and its density, viz., proportional to its weight.

The momentum of a body is obtained by multiplying its mass by its velocity. It is a measure of the resistance of the body to change of speed or direction.

Since the mass of a body does not change, it follows that :—

Change of momentum = mass × change of velocity.

Rate of change of momentum = mass × rate of change of velocity = mass × acceleration.

2nd Law. The rate of change of momentum of a body is proportional to the force acting on it, and takes place in the direction of that force.

Thus, from the above :—

Force acting is proportional to mass multiplied by acceleration.

If P is the force, m the mass of the body and f the acceleration, $P = k.m.f.$, where k is some constant.

The value of k depends upon the units we use for P, m and f. Suppose P is in lb. weight, $k = 1$ and f is in ft. per sec.^{2}.

Then Force (lb.) = Mass × acceleration (ft. per sec.^{2}).

How are we to measure the mass? Now the mass is quite independent

of how the body is moving, so that if we find the mass under any condition of movement, this will be the mass in all conditions.

Measurement of Mass

Let us, then, consider the body falling freely. The force acting is the attraction between the body and the earth, which we call its weight. The acceleration in this case, which is usually denoted by the letter g, is known to be 32·2 ft. per sec.^{2}.

Using the above equation, we now have :—

$$W \text{ (lb.)} = \text{Mass} \times g.$$

$$\therefore \text{Mass} = \frac{W}{g}.$$

Here we get finally :—

$$\text{Force (lb.)} = \frac{W}{g} \times \text{acceleration (ft. per sec.}^2\text{)}.$$

In scientific work generally, and particularly in electrical work in this country, the unit of mass is 1 gram (1 lb. = 454 grams), and the unit of force is 1 dyne, mass and weight being in the same units. Thus, 1 dyne is the force which will accelerate a mass of 1 gram by 1 cm. per sec. per sec. Thus :—

$$\text{Force (dynes)} = W \text{ (grams)} \times \text{acceleration (cm. per sec.}^2\text{)}.$$

Accelerating Force

It must be remembered that force in the above equations is what remains after all other resistances (such as friction, etc.) have been overcome.

A train weighing 200 tons, standing on a level track, is to attain a speed of 35 m.p.h. (51·2 ft. per sec.) from rest in 30 sec. If frictional resistances (assumed constant) amount to 40 lb. per ton weight, find the pull which the engine must exert on the train during this period.

$$\text{Mass of train} = \frac{200 \times 2{,}240}{32{\cdot}2}.$$

$$\text{Acceleration} = \frac{51{\cdot}2}{30} = 1{\cdot}707 \text{ ft. per sec.}^2.$$

Force required for acceleration

$$= \frac{200 \times 2{,}240 \times 1{\cdot}707}{32{\cdot}2} = 23{,}700 \text{ lb.}$$

Force required for overcoming friction $= 200 \times 40 = 8{,}000$ lb.

$\therefore$ Total pull required

$= 23{,}700 + 8{,}000 = 31{,}700$ lb.

When the train is running at a uniform speed of 30 m.p.h., the pull required will be only 8,000 lb., since the resultant pull on the train will then be zero, and the first law tells us that in this case the train will continue to move at constant speed. The excess pull of 23,700 lb. would then be available to draw the train up a gradient of 1 in 20, since the extra pull required would be $\frac{1}{20}$ of the weight.

Another way of stating the second law in the form of an equation is :—

Force = Momentum created or destroyed per sec.

This way of stating the law enables us to understand many of the things which we experience, either directly or indirectly, and to estimate their effects.

Air, for instance, is so light and so easily moved that it is difficult to imagine a strongly built fence being blown over in a gale of wind. In the first place, what is the source of pressure on the fence? We will imagine that the wind is blowing at 60 ft. per sec. (41 m.p.h.) at right angles to the fence. When it reaches the fence it is stopped and,

therefore, the whole of its momentum in this direction is destroyed.

Assuming that 12 cu. ft. of air at atmospheric pressure weigh 1 lb., we can now calculate the force on each square foot as follows :—

Volume of air reaching fence per sec. = 60 cu. ft.

Weight of air reaching fence per sec. $= \frac{60}{12} = 5$ lb.

$\therefore$ Momentum reaching fence per sec. $= \frac{5}{32{\cdot}2} \times 60 = 9{\cdot}32$.

Since this momentum is destroyed, the force on each square foot, therefore, will be 9·32 lb. weight, and if the fence is 8 ft. high and 100 ft. long, the total force on the fence due to wind pressure will be $9{\cdot}32 \times 800 = 7{,}460$ lb. weight (nearly $3\frac{1}{2}$ tons).

Effect of Speed

It will be noticed that the 60 ft. per sec. appears twice in the calculation, showing that the wind force is proportional to the square of the wind velocity, viz., doubling the wind velocity (to 82 m.p.h.) would increase the force four times (to about 14 tons).

The enormous force on a tall building in a gale, and the way in which strong bridges can be washed away by swiftly flowing rivers can be imagined, remembering that water is 750 times as heavy as air.

In the case of the fence, we must warn readers that the real action is not quite so simple, as there will probably be a suction effect on the other side (Fig. 7), making the actual force very appreciably greater. This is too complex to be dealt with by simple methods.

Let us now consider the case of a hammer striking a nail. We know that if we try to push the nail into the work a considerable force may be required. Suppose the hammer head weighs 1 lb. and is moving at 30 ft. per sec. when it strikes the nail, acting for one-fortieth (0·025) of a sec.

The momentum which is destroyed is $\frac{1}{32{\cdot}2} \times 30 = 0{\cdot}935$ units.

Momentum destroyed per sec. $= 0{\cdot}935 \div 0{\cdot}025 = 37{\cdot}4$ units.

Hence the average force exerted by the hammer on the nail in this case is 37·4 lb. weight.

Reducing Force of Blow

Note that by increasing the time of the blow we reduce the average force exerted. The object of spring buffers on railway carriages, and of buffer stops at the terminus, is to destroy the momentum of the train slowly and so reduce the shock involved if very large forces are required for a quicker destruction of momentum.

3rd Law. To every action on a body there is opposed an equal and opposite reaction.

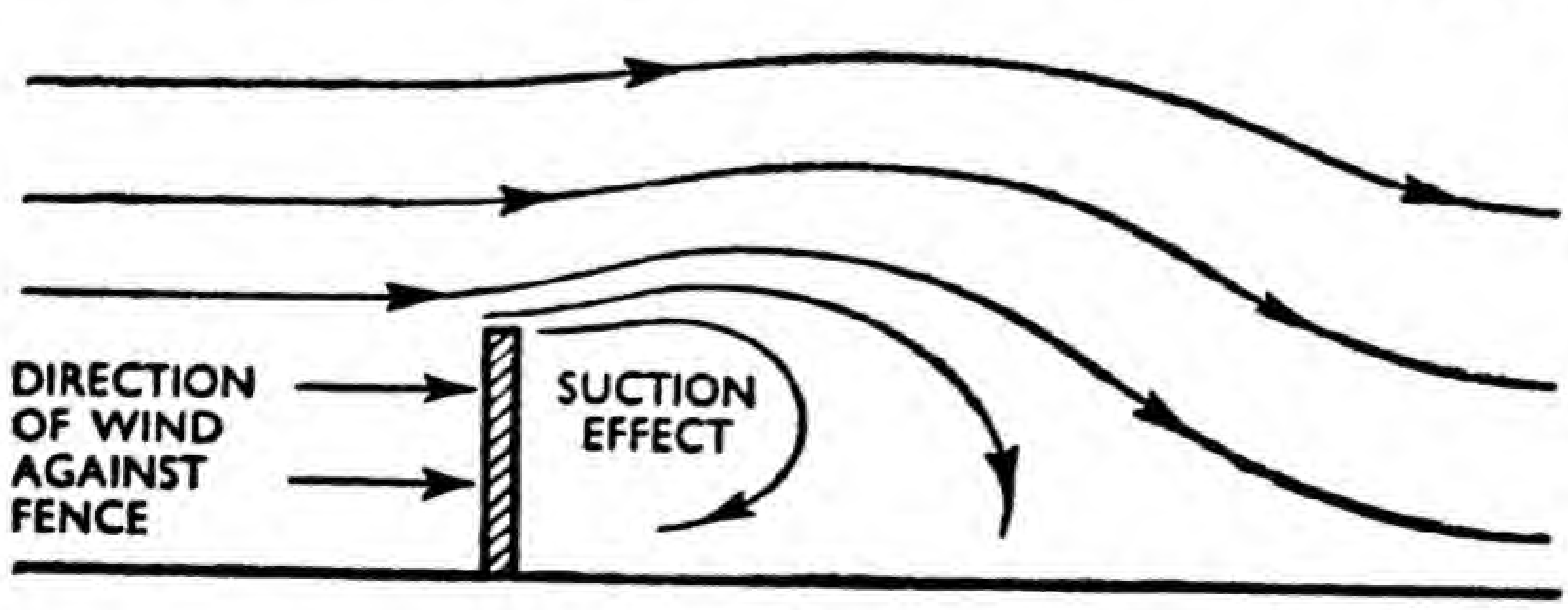


Fig. 7. Illustrating the action of the wind on a fence. The momentum is destroyed, and it will be found that a certain amount of negative pressure is also acting on the back of the fence.

Action and reaction in this case mean forces.

If we press downward on a table with a force of 10 lb. weight and the table remains stationary, it reacts with an upward force of 10 lb. weight.

If we press sideways on the table, friction between the table and the floor provides an equal and opposite force. If there is not sufficient frictional resistance to provide this force, the table will move in the direction of the applied force.

Exerting Downward Pressure

A man standing on the floor of a stationary lift exerts a downward pressure equal to his weight. The floor exerts an upward force of the same magnitude on the man, so that, according to the first law he remains stationary. This is also true if he is ascending or descending with constant speed.

If, however, the lift is accelerating upward, the man, of course, is also accelerating upward, and he can accelerate only if the upward force P (Fig. 8) is greater than W.

Let us suppose that the man weighs 180 lb. and that the lift is accelerating upward at 3 ft. per sec.^{2}. We wish to find P.

Using the equation on page 93:—

Accelerating force $= P - 180$.

$$\therefore P - 180 = \frac{180}{32{\cdot}2} \times 3 = 16{\cdot}8.$$

$\therefore P = 196{\cdot}8$ lb. weight.

Therefore, the man is exerting a pressure of 196·8 lb. weight downward on the floor of the lift and the floor is reacting upward with a force of 196·8 lb. weight.

By similar reasoning we find that if the lift is descending with an acceleration of 3 ft. per sec.^{2}, the pressure of the man on the floor is reduced to 163·2 lb. weight.

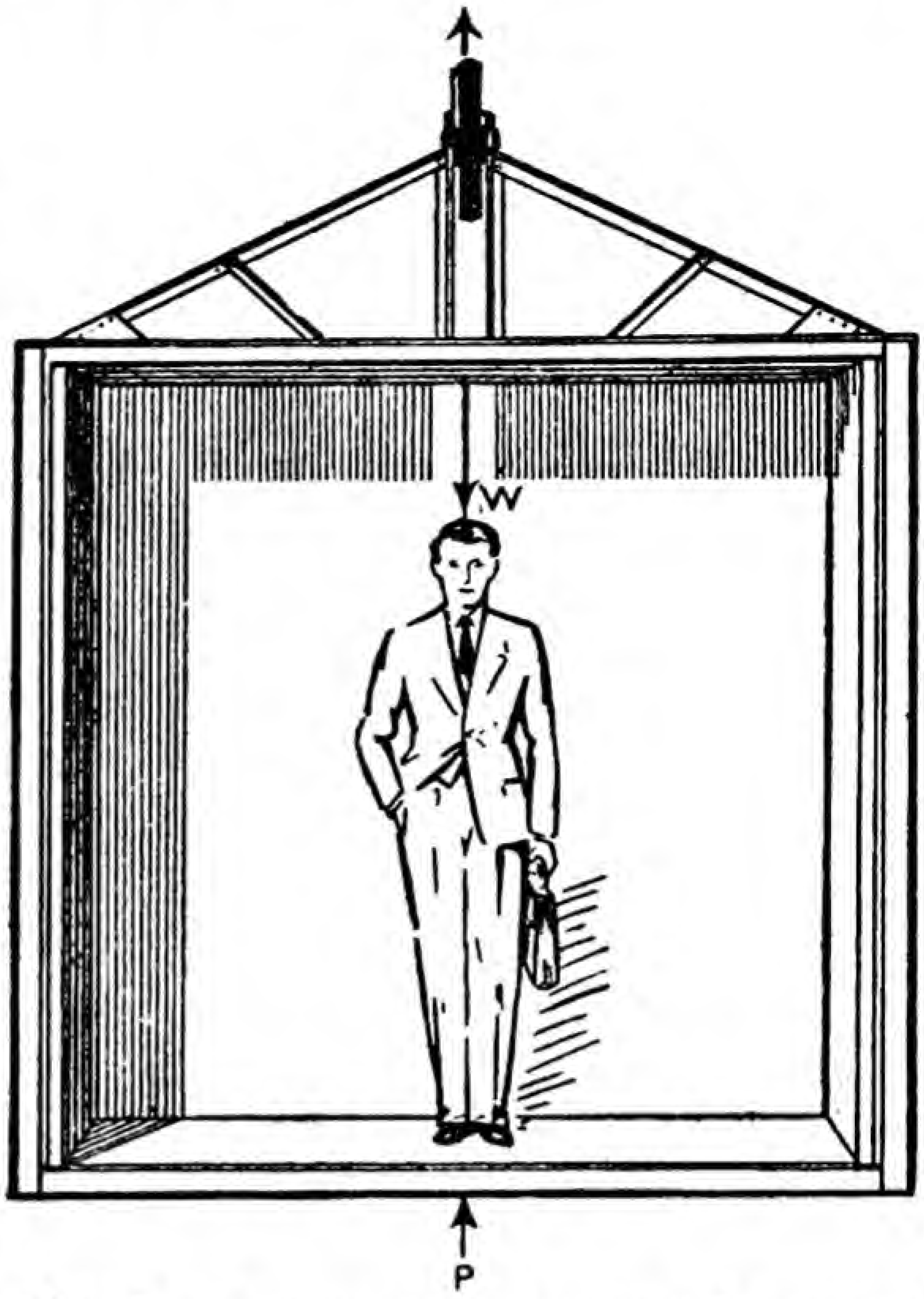


Fig. 8. Downward pressure P exerted by the man on the floor of the lift causes an equal pressure P in the opposite direction exerted by the floor on the man. If the lift is accelerated upward P must be greater than W.

It is often necessary to find the distance travelled in a given time by a body with a constant acceleration. This is not difficult if we remember the following rules.

Final velocity = initial velocity + acceleration × time.

Average velocity $= \dfrac{\text{Initial velocity} + \text{final velocity}}{2}$.

Distance travelled = average velocity × time.

Putting this into symbols; if u is the initial velocity, v the velocity after t sec., f the acceleration in ft. per sec.^{2}, and s the distance travelled, then :—

$$v = u + f.t.$$

Average velocity =

$$\frac{u + (u + f.t.)}{2} = u + \tfrac{1}{2}f.t.$$

$$s = u.t + \tfrac{1}{2}f.t^2.$$

If the body starts from rest ($u = 0$), then $v = f.t$ and $s = \frac{1}{2}f.t^2$. Looking back to the example of the train (page 94), we have $u = 0$, $f = 1{\cdot}707$ ft. per sec.^{2}, $t = 30$ sec.

$$\therefore\ s = \tfrac{1}{2}f.t^2 = \frac{1{\cdot}707 \times 900}{2} = 770 \text{ ft.}$$

Thus the train will travel a distance of 770 ft. before it reaches the speed of 30 m.p.h.

In the case of a body falling freely from rest, the acceleration is g (32·2 ft. per sec.^{2}), and the vertical distance fallen in t sec. is given by :—

$$s \text{ (ft.)} = \tfrac{1}{2}\,g.t^2 = 16{\cdot}1\ t^2.$$

Thus, in 1 sec. the distance fallen will be 16·1 ft., in 2 sec. it will increase to 64·4 ft., and in 3 sec. it will be 145 ft.

If we wish to find the time taken to fall 300 ft., then :—

$$300 = 16{\cdot}1\ t^2.$$

$$t^2 = \frac{300}{16{\cdot}1} = 18{\cdot}64.$$

$$t = \sqrt{18{\cdot}64} = 4{\cdot}33 \text{ sec.}$$

To find the final velocity v of a body under constant acceleration f in terms of the distance s travelled from rest, we have,

Final velocity $v = f.t.$

$$v^2 = f^2.t^2. = 2f \times \tfrac{1}{2}f.t^2.$$

Since $s = \frac{1}{2}f.t^2$.

$$\therefore\ v^2 = 2f.s.$$

Therefore, for a falling body, $v^2 = 2g\,h$, when h is the distance the body has fallen.

Effect of Air Resistance

If a bullet is fired in a horizontal direction (Fig. 9) at, say, 2,000 ft. per sec., and we neglect air resistance, its horizontal velocity will remain unchanged at 2,000 ft. per sec., but it will fall vertically in the same way as if it had been dropped from the firing point. Thus, at the end of 1 sec. it will have travelled 2,000 ft. horizontally and dropped 16·1 ft. vertically, and so on. If the target is 1,500 ft. away, $t = 0{\cdot}75$ sec., and the vertical distance dropped will be $16{\cdot}1 \times ({\cdot}75)^2$, or 9 ft. Thus, the rifle would need to be aimed at a point 9 ft. above the target it is desired to hit.

If a plane is flying horizontally at 200 m.p.h. (294 ft. per sec.) as in Fig. 10, at a height of 8,000 ft. above the target, at what horizontal

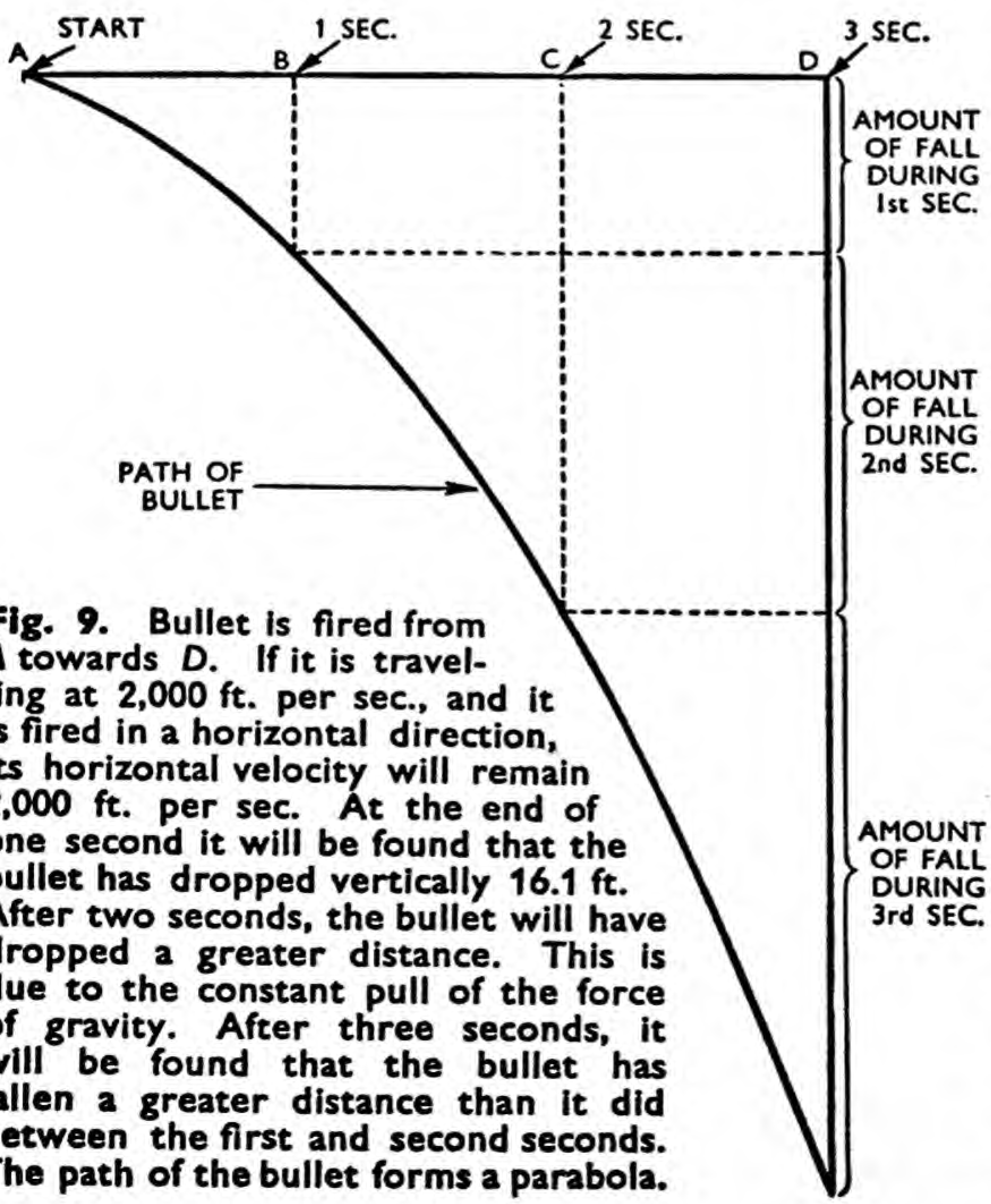


Fig. 9. Bullet is fired from *A* towards *D*. If it is travelling at 2,000 ft. per sec., and it is fired in a horizontal direction, its horizontal velocity will remain 2,000 ft. per sec. At the end of one second it will be found that the bullet has dropped vertically 16.1 ft. After two seconds, the bullet will have dropped a greater distance. This is due to the constant pull of the force of gravity. After three seconds, it will be found that the bullet has fallen a greater distance than it did between the first and second seconds. The path of the bullet forms a parabola.

distance from the target must the bomb-aimer release his bombs?

The time taken for a bomb to drop 8,000 ft. is given by :—

$$8{,}000 = 16{\cdot}1\ t^2.$$
$$t^2 = 496.$$
$$t = 22{\cdot}3 \text{ sec.}$$

In this time the bomb will have travelled horizontally a distance of $294 \times 22{\cdot}3$ ft. = 6,580 ft. ($1\frac{1}{4}$ miles). Therefore, the bomb must be released when the plane is $1\frac{1}{4}$ miles from the target.

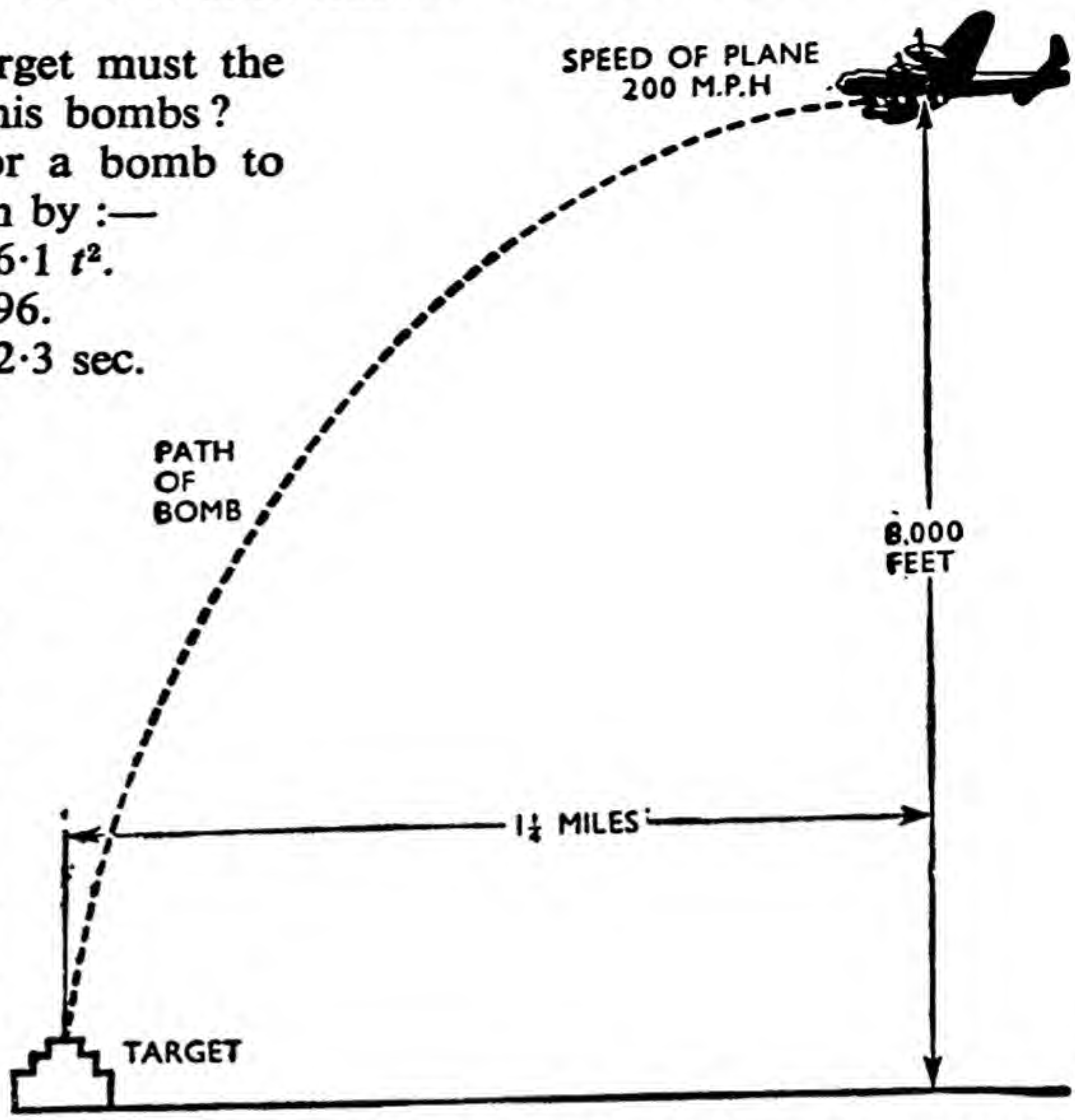


Fig. 10. Aeroplane has to release bomb a considerable distance from the target. This distance depends on two main items, the speed of the aircraft and the height at which it is travelling. In this example the distance is $1\frac{1}{4}$ miles.

Actually, the speed of falling is affected by air resistance, which becomes relatively large at high speeds. For instance, if the air resistance is 1 oz. at 10 ft. per sec., it becomes 900 oz., or 56 lb., at 300 ft. per sec., so that, if the falling body weighs 56 lb., it will reach a speed of 300 ft. per sec. and will continue to fall at this speed no matter how far it has yet to fall, since there is now no resultant force acting. The horizontal velocity of the bomb will be reduced gradually for a similar reason, and careful consideration must be given to the shape in order that the air resistance may be as small as possible.

Important Principle

Another important principle which follows from the laws of motion is known as the principle of conservation (or preservation) of momentum.

Before stating the principle formally, let us try to find out what happens when two bodies come into collision, either intentionally or unintentionally. Examples of this are a hammer striking a nail or a chisel, the monkey of a pile-driver striking the pile, a head-on or tail-on collision between two vehicles, in fact, all cases of so-called impact.

Let us take the simple case of two bodies A and B travelling in the same direction with different speeds, the speed of A being greater than the speed of B (Fig. 11). After a time A will meet B and exert a force F on it to the left for a short time. The third law tells us that the body B will react on A with an equal force F to the right, acting for the same time.

Since force $\times$ time = change of momentum, A will have its momen-

tum decreased by a certain amount; that it so say, its velocity will be decreased. *B* will have its momentum increased by the same amount, that is, its velocity will be increased. Thus the total change of momentum after impact is zero, or 'the sum of the momenta after impact is the same as the sum of the momenta before impact.' This is the principle of conservation of momentum.

This still holds if the bodies are moving in opposite directions when impact takes place, but in this case if we call momentum to the right positive, we must call momentum to the left negative, and it is the *algebraic* sum of the momenta which is unaffected by impact.

The actual speeds of the bodies after impact depend upon the degree of elasticity of the materials, which affect both the force *F* and the time of impact, and we shall deal here only with the case in which the bodies move on together after impact, a case which, in practice, frequently occurs.

Suppose *A* weighs 5 lb. (Fig. 11) and is moving at 10 ft. per sec., while *B* weighs 3 lb. and is moving at 6 ft. per sec., and that *A* and *B* move on together after impact has taken place.

Let v be their common velocity (ft. per sec.) after impact.

Total momentum before impact

$$= \frac{5}{32{\cdot}2} \times 10 + \frac{3}{32{\cdot}2} \times 6 = \frac{68}{32{\cdot}2}.$$

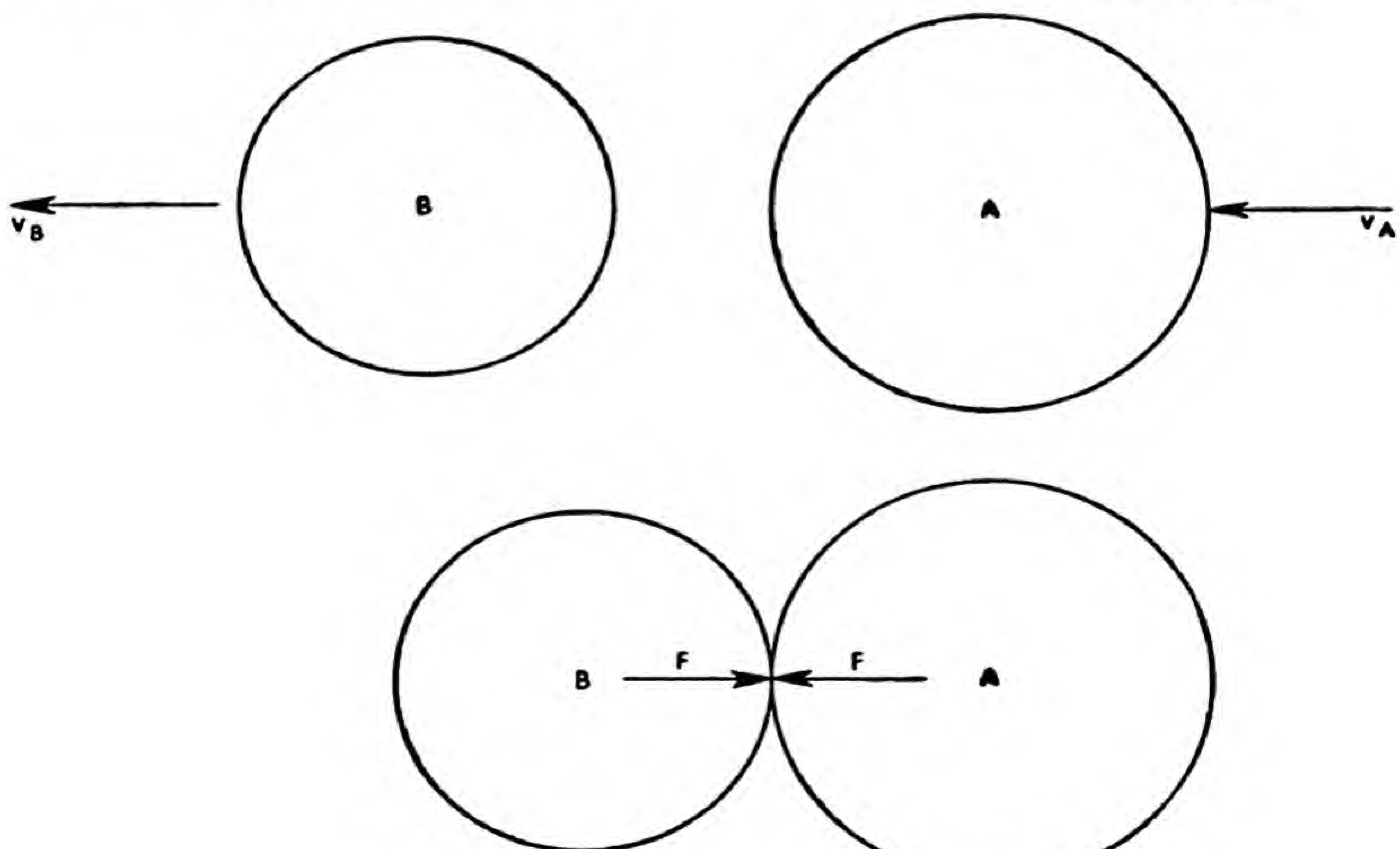
Total momentum after impact

$$= \frac{5+3}{32{\cdot}2} \times v = \frac{8v}{32{\cdot}2}.$$

Then, from the above, $\frac{8v}{32{\cdot}2} = \frac{68}{32{\cdot}2}$.

$$8v = 68.$$

$$v = 8{\cdot}5 \text{ ft. per sec.}$$



SPEED OF BODIES AFTER IMPACT

Fig. 11. *A* and *B* are two bodies moving with velocities V_A and V_B, the velocity of *A* being the greater. After impact they may move on together, the speed of *B* being increased and that of *A* being reduced, but the total momentum remains unaltered. Though there is no loss of momentum, there is a loss of energy.

Thus the two bodies will move on together at 8·5 ft. per sec.

It will be noticed that in this kind of problem we can use weight instead of mass, since the value of g cancels on both sides of the equation.

A bullet weighing 0·06 lb. is fired into a block of wood weighing 20 lb., which is at rest, but suspended so that it is free to move. If block and bullet move on together at a speed of 6·6 ft. per sec., what was the velocity of the bullet?

Reasoning as above, and taking weights instead of masses, we have:

$$0{\cdot}06 \times v = (20 + 0{\cdot}06) \times 6{\cdot}6.$$

$$v = \frac{20{\cdot}06 \times 6{\cdot}6}{0{\cdot}06} = 2{,}207 \text{ ft. per sec.}$$

This is the principle of the ballistic pendulum, an old method of measuring the velocity of a bullet. A velocity of 6·6 ft. per sec. is much easier to measure than a velocity of 2,200 ft. per sec. by direct methods, but modern electrical appliances have made a direct method of measurement of a high velocity quite practicable.

A weight of 4,000 lb. falls a distance of 8 ft. on to the head of a pile weighing 350 lb. With what speed will the pile begin to move immediately after impact if weight and pile move together?

Speed of weight just before striking the pile $= 8\sqrt{h} = 8\sqrt{8}$
$= 22{\cdot}6$ ft. per sec.

Total momentum before impact

$$= \frac{4{,}000}{g} \times 22{\cdot}6 = \frac{90{,}400}{g}.$$

Total momentum after impact

$$= \frac{4{,}350}{g} \times v.$$

$\therefore 4{,}350\, v = 90{,}400.$

$v = 20{\cdot}8$ ft. per sec.

How far the pile will move before its initial velocity of 20·8 ft. per sec. is destroyed depends, of course, upon the resistance to penetration.

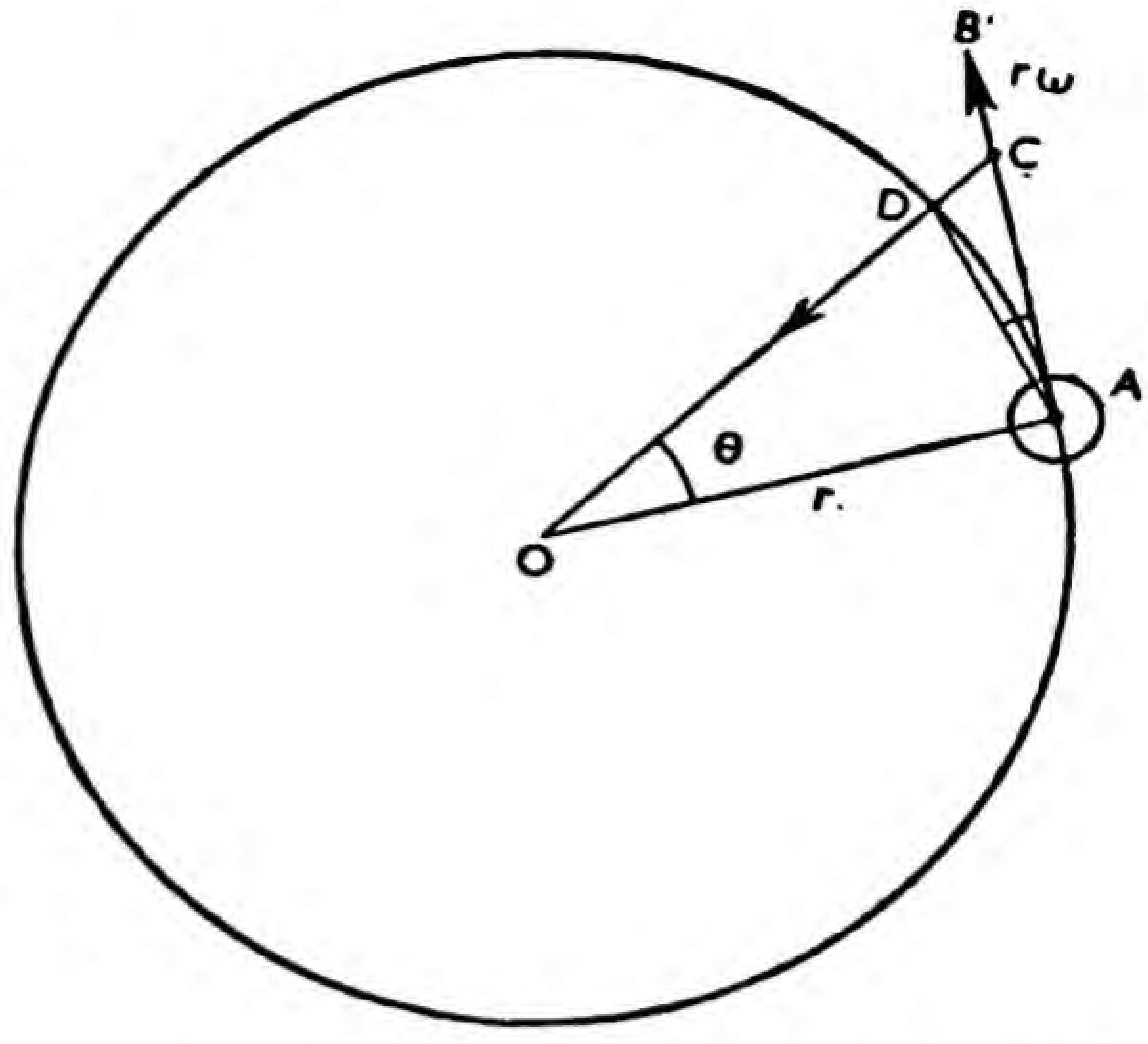


Fig. 12. If W is a weight at A moving in a circle about O, the centripetal force acting from A to O is $Wr\omega^2/g$. This will be the pull in OA. According to the third law, the pull at O will be equal and opposite to that at A.

If, for instance, the resistance to penetration is 15,000 lb., then since force × time = change of momentum (from Newton's second Law),

$$15{,}000 \times t = \frac{4{,}350}{32{\cdot}2} \times 20{\cdot}8.$$

$$t = \frac{4{,}350 \times 20{\cdot}8}{32{\cdot}2 \times 15{,}000} = 0{\cdot}19 \text{ sec.}$$

Since the average velocity =

$$\frac{20{\cdot}8 + 0}{2} = 10{\cdot}4 \text{ ft. per sec.,}$$

distance travelled by weight and pile before coming to rest =
$10{\cdot}4 \times 0{\cdot}19 = 1{\cdot}98$ ft. (nearly 2 ft.).
Obviously, by measuring the actual amount of penetration the average resistance to penetration can be estimated.

Estimating Recoil

The velocity of recoil of a gun can be estimated in a similar way. Suppose the projectile weighs 1 ton and leaves the muzzle at a speed of

2,400 ft. per sec. and that the weight of the part of the gun which recoils is 150 tons. Then, since the average force forward on the shot due to the explosion of the cordite charge equals the average force backward on the gun, according to the third law,

Momentum of gun backward = momentum of projectile forward.

$\therefore 150 \times v = 1 \times 2{,}400.$

$v = 16$ ft. per sec.

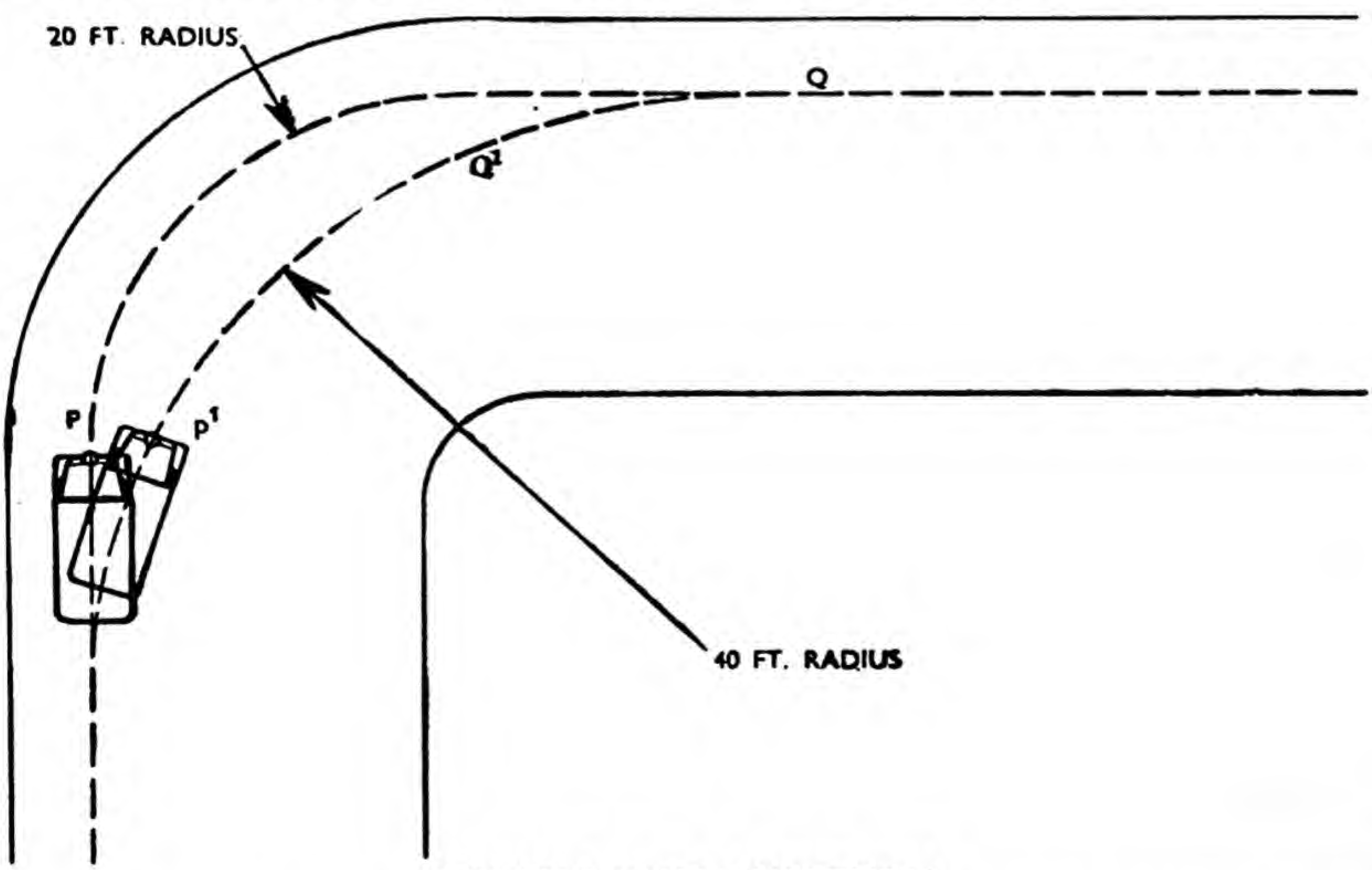
Thus the gun begins to move backward at 16 ft. per sec. and its momentum must be destroyed in a short distance by means of powerful springs and fluid resistance.

Radial Force

If a body is moving with constant speed in a circular path, the first law tells us that there must be a force acting on it. This force cannot be acting in the direction of motion, otherwise its speed would not remain constant. Therefore, we conclude that the force must be acting at right angles to the direction of motion, that is, along the radius of the circle.

Referring to Fig. 12, if the body is moving with speed v ft. per sec. in a circular path whose centre is at O and radius OA, which is equal to r ft., and a fraction of a second later it is at D, it will be seen that if no force had been acting the body would have travelled along AB from A to C.

Therefore, a force must be acting from A to O which would move the body from C to D in the time taken to travel from A to D. The radial acceleration due to this can be shown to be $\frac{v^2}{r}$ (ft. per sec.^{2}) so that the centripetal (toward the centre)



VEHICLE MAY OVERTURN

Fig. 13. Motor cars are liable to overturn, occasionally, due to incorrect movements made by the driver. Above is shown two possible ways of negotiating the corner, PQ and P^1Q^1. If the line PQ is followed, and the speed of the vehicle is too high, the result may be disastrous. It would not be so if line P^1Q^1 were followed.

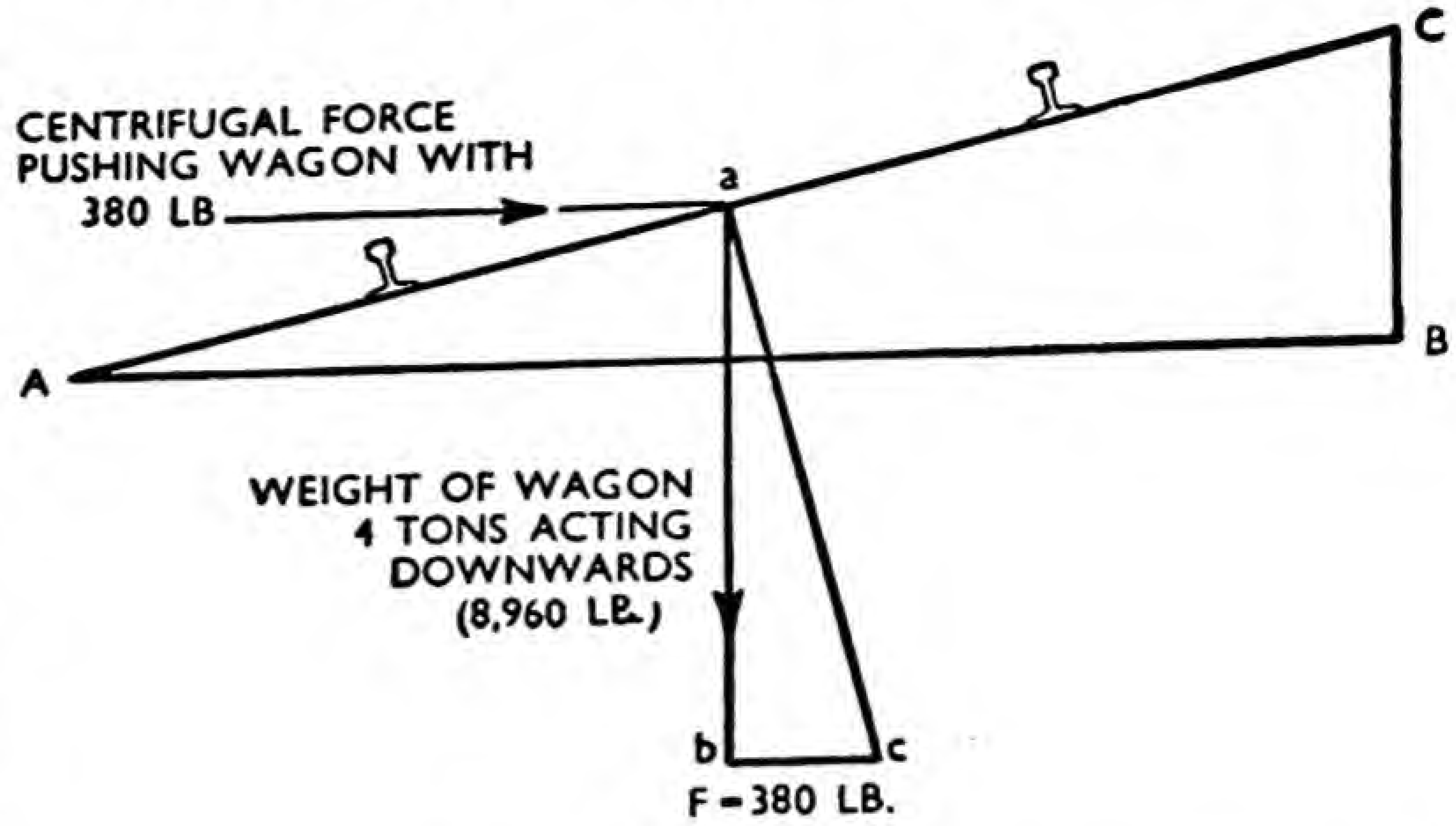


Fig. 14. A wagon weighing 4 tons is travelling round a curve at such a speed that the centrifugal force, acting horizontally, is 380 lb. The rails are set on an inclined plane *AC*. The angle of inclination *CAB* is so arranged that the resultant *ac* of the 4-tons load and the 380-lb. force is at right angles to *AC*. The angle *CAB* is equal to the angle *cab*

force acting is $\frac{W.v^2}{g.r}$ (by the second law).

This obviously is the pull in *OA*.

The equal and opposite pull at *O* is also $\frac{W.v^2}{g.r}$ (by the third law) and is known as the centrifugal (from the centre) force.

It must be remembered that an expression like the above is only a shorthand way of stating facts, and some useful conclusions may be drawn from it.

For instance, if the speed is doubled, radius remaining constant, the centrifugal force is increased four times. On the other hand, if the radius is doubled, the speed remaining constant, the centrifugal force is halved.

Avoiding Overturning

Thus, a car travelling round a right-hand bend of, say, 20 ft. radius at too high a speed (Fig. 13), may avoid overturning due to centrifugal force by cutting the corner in a circle of, say, 40 ft. radius, such as P^1Q^1. The expedient is obviously a dangerous one, as the car would be moving on the wrong side of the road at the corner, and reduction of v by braking would be safer, though less comfortable for the passengers and harder on the tyres.

Reduction of a rear wheel skid to the right by steering in the direction of the skid, which is by steering to the right, is explained by the centrifugal force produced tending to cause a skid to the left.

If a vehicle is moving in a circular path on a level road, the required force must be supplied by the friction between the tyres and the road. In the case of a train travelling round a curve this force is supplied by the side pressure of the outer rail on the wheels.

Insufficient Friction

At high speeds the friction between tyres and ground may be insufficient, and the road must be banked so that the force tending to cause the vehicle to slide down the inclined plane thus formed balances the centrifugal force. In the case of the train, the track is similarly banked by elevating the outer rail on the curve above the inner rail (Fig. 14).

If in Fig. 12 the link *OA* were replaced by a piece of elastic or, as in many mechanisms, by a spring, the distance *OA* would increase as the speed increased, as the extra force produced would increase the length of the spring. This is the

principle on which most governors for engines and other mechanisms work.

In many cases where complete revolutions are made, it is more convenient to use the angular velocity of OA.

Since angular velocity

$$= \frac{\text{linear velocity}}{\text{radius}},$$

the angular velocity is given by,

$$\omega \text{ (rad. per sec.)} = \frac{v \text{ (ft. per sec.)}}{r \text{ (ft.)}},$$

$$\text{or, } v = r.\omega.$$

The expression for centrifugal force now becomes,

$$\text{C.F.} = \frac{W}{g} r.\omega^2.$$

If the speed is given in r.p.m. (N), this becomes,

$$\text{C.F.} = \frac{W.r.N^2}{2{,}900}.$$

Thus, if a weight of 5 lb. is rotating in a circle of 4 ft. radius at 240 r.p.m.

$$\text{C.F.} = \frac{5 \times 4 \times 240 \times 240}{2{,}900}$$

$$= 398 \text{ lb.}$$

The tremendous force involved in the case of bodies revolving at high speed will be appreciated.

Work is said to be done by a force when it moves a body against a resistance. The work done is obtained by multiplying the resistance overcome by the distance moved.

Let us suppose that a packing case K is being dragged across a level floor at a steady speed. If the pull P is in the direction of motion (Fig. 15(a)), then the resistance is also P and the work done is P multiplied by distance moved.

If, however, the pull P is inclined to the direction of motion (Fig. 15(b)) the effective pull F is OB, when OA represents the magnitude of P, so that the work done is OB multiplied by distance moved.

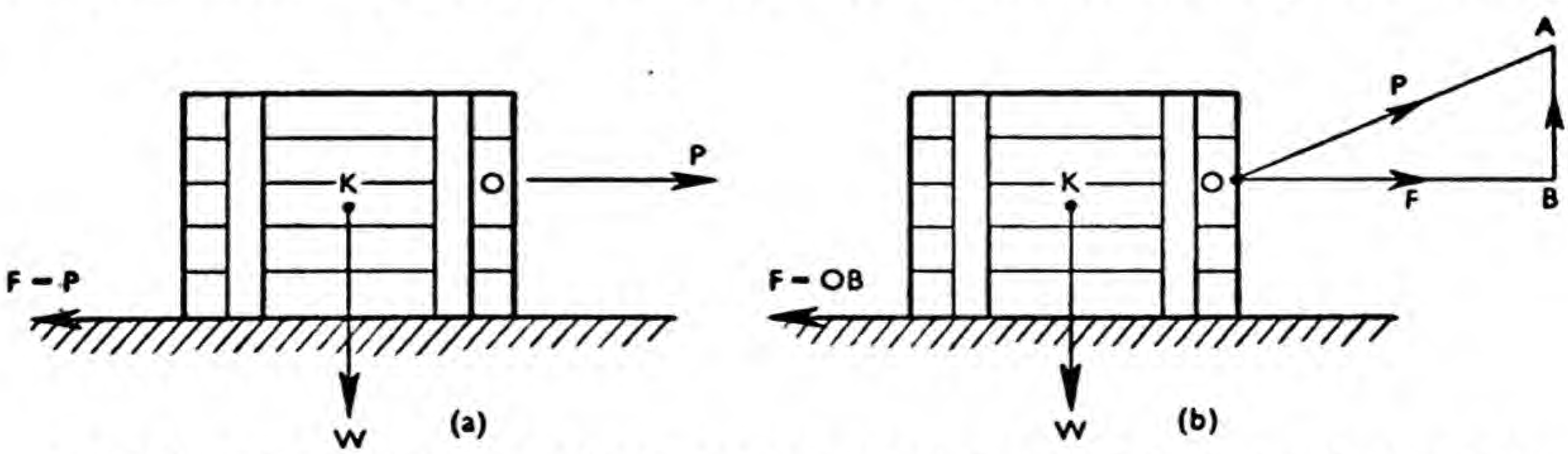
Therefore, we may say that the work done is the *effective* force in the direction of motion multiplied by the distance moved.

The vertical component BA of the force P is doing no work in this case, since the body K is not moving in a vertical direction.

No Work Done

A man carrying a heavy suitcase along a level path would not be doing any work in the above sense, since the force exerted by the case on his arm (viz., its weight) is vertical, while the movement is in a horizontal direction.

If a weight of, say, 1,200 lb. is being lifted vertically at a steady



WORK DONE IN DRAWING A PACKING CASE ACROSS THE FLOOR

Fig. 15. Amount of work done in overcoming a resistance is resistance overcome multiplied by the distance moved. (*a*) Packing case is dragged by horizontal force *P*, so amount of work done is *P* multiplied by distance moved. (*b*) Inclined pull applied, but effective force is *OB*, so work done is *OB* multiplied by distance moved.

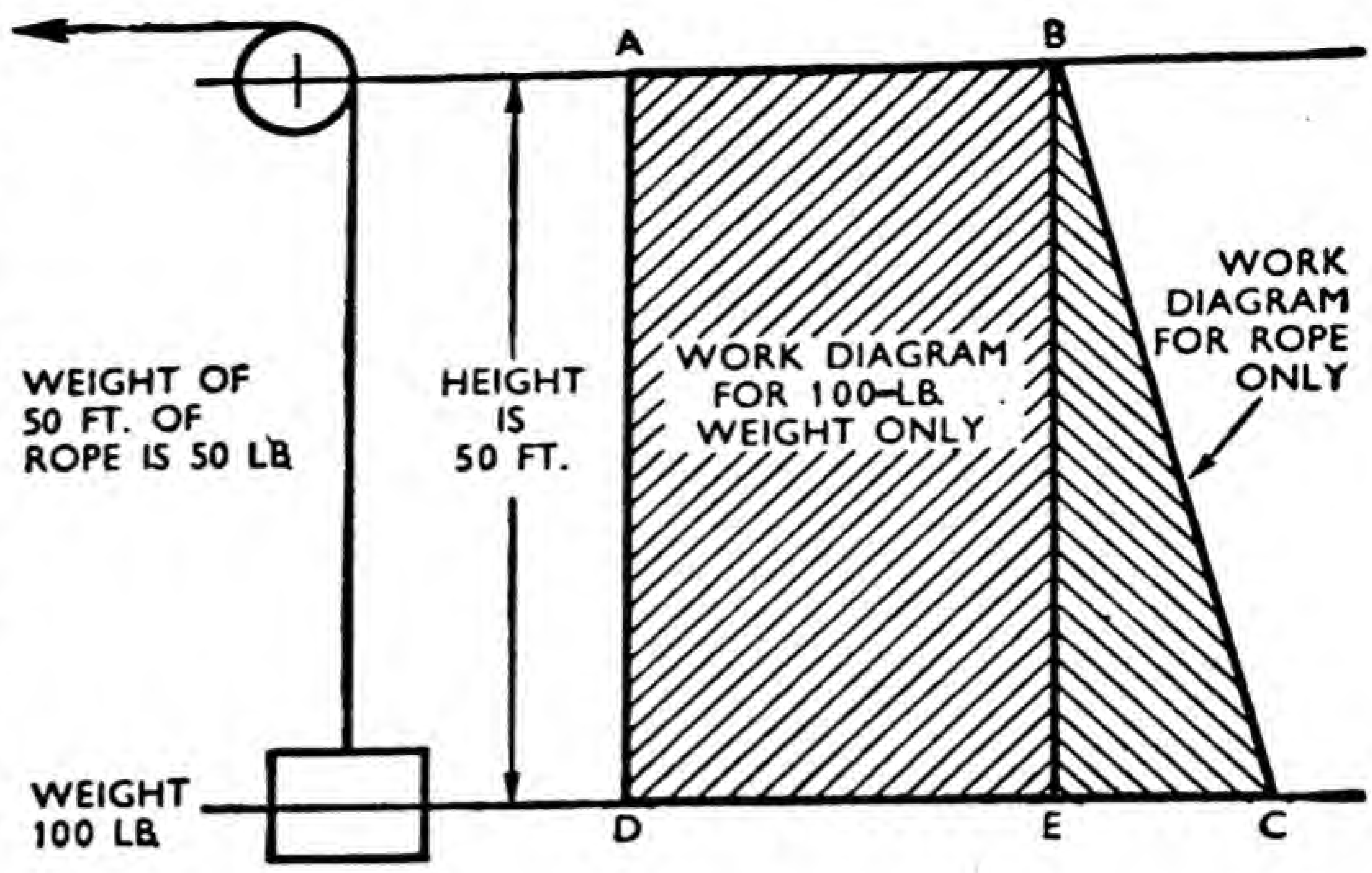


Fig. 16. At beginning of lift, pull required (neglecting accelerating force) is *DE* (100 lb.) plus *EC* (weight of 50 ft. of rope). At end of lift, pull required is *AB* (100 lb.). Since work done equals force times distance moved, the rectangle *ABED* represents the work done in lifting the 100 lb. weight, and the triangle *BEC* the work done in lifting the rope only. Thus the area *ABCED* represents the total work done.

speed by a crane through a distance of 14 ft., the resistance overcome is 1,200 lb., since this is the pull in the crane rope. The work done, therefore, will be 1,200 × 14 = 16,800 ft.-lb.

Measuring Work

In most problems, forces are measured in pounds (lb.) and distances are measured in feet (ft.). The work done is then in foot-pounds (ft.-lb.).

In the above case we have ignored the fact that the rope is being raised as well as the load and this may cause a little difficulty, since each foot of the rope is being raised through a different distance (Fig. 16). There is, however, a very simple rule which enables us to solve this and other similar problems. This is :—

Work done = total weight raised × vertical rise of centre of gravity.

With the data given on the diagram we have,

Work done in raising 100 lb.

$= 100 \times 50 = 5{,}000$ ft.-lb.

Work done in raising rope

$= 50 \times \frac{50}{2} = 1{,}250$ ft.-lb. (since the C.G. of the rope is 25 ft. below the pulley).

Total work done

$= 5{,}000 + 1{,}250 = 6{,}250$ ft.-lb.

In Fig. 17 a tapered pole is shown lying on the ground, its centre of gravity *G* being in the position shown. When it is in the vertical position it is shown by dotted lines, the vertical rise of the C.G. being *h* ft.

By the above rule, if *W* is 450 lb. and *h* is 4 ft., the work done in lifting the pole into a vertical position is 450 × 4 = 1,800 ft.-lb.

Power is the *rate* at which work is being done ; for example, ft.-lb. of work per sec. or per min.

Thus the power required to do a given amount of work depends upon the time taken to do it.

If, for instance, the time taken to lift the weight of 100 lb. (Fig. 16) is 12 sec., then the power used is 6,250 ÷ 12 = 521 ft.-lb. per sec., or 31,250 ft.-lb. per min.

Since, in this case, the total weight being lifted is becoming less as the weight ascends, the result given is the *average* power required.

The unit of power which is most generally in use is the horse-power, which is 33,000 ft.-lb. of work done per minute. It must be borne in

mind that this is not a quantity of work, but a *rate* of doing work. The unit dates from the time of James Watt, when horses were being displaced by the new steam pumping engine for pumping water from mines.

Experiment showed that a good horse could work at this rate for a short time only, while an engine could go on working at its rated horse-power indefinitely.

To find the h.p. required while a certain amount of work is being done we must know two things, the work done and the time taken to do it.

If it is desired to raise a weight of 2 tons through a height of 60 ft. in 30 sec., the h.p. required would be calculated as follows :—

Work done
$= 4{,}480 \times 60 = 268{,}800$ ft.-lb.

Time taken $= 0{\cdot}5$ min.

Work done per min.

$$= \frac{268{,}800}{0{\cdot}5} = 537{,}600 \text{ ft.-lb.}$$

$\therefore$ h.p. required for lifting

$$= \frac{\text{Work done per min.}}{33{,}000} = 16{\cdot}3.$$

Actually, a greater horse-power would be required, since there would be some friction to overcome, and we should probably specify an engine or electric motor of 18 h.p. for this particular operation.

The pull on the weight at starting would, of course, be greater than 4,480 lb., since it must be accelerated from zero velocity to 2 ft. per sec., and during this period a greater h.p. may be required.

The resistance to motion of a motor vehicle on a level track is 80 lb. at a speed of 40 m.p.h. What h.p. is required to drive the vehicle at this speed ?

40 m.p.h. $= 40 \times 88 = 3{,}520$ ft. per min.

Work done per min.
$= 80 \times 3{,}520$ ft.-lb.

$\therefore$ h.p. required

$$= \frac{80 \times 3{,}520}{33{,}000} = 8{\cdot}53.$$

Here, again, there is some loss due to friction in the transmission, and the engine would have to supply about $9\frac{1}{2}$ h.p.

If the vehicle weighs 2,000 lb.

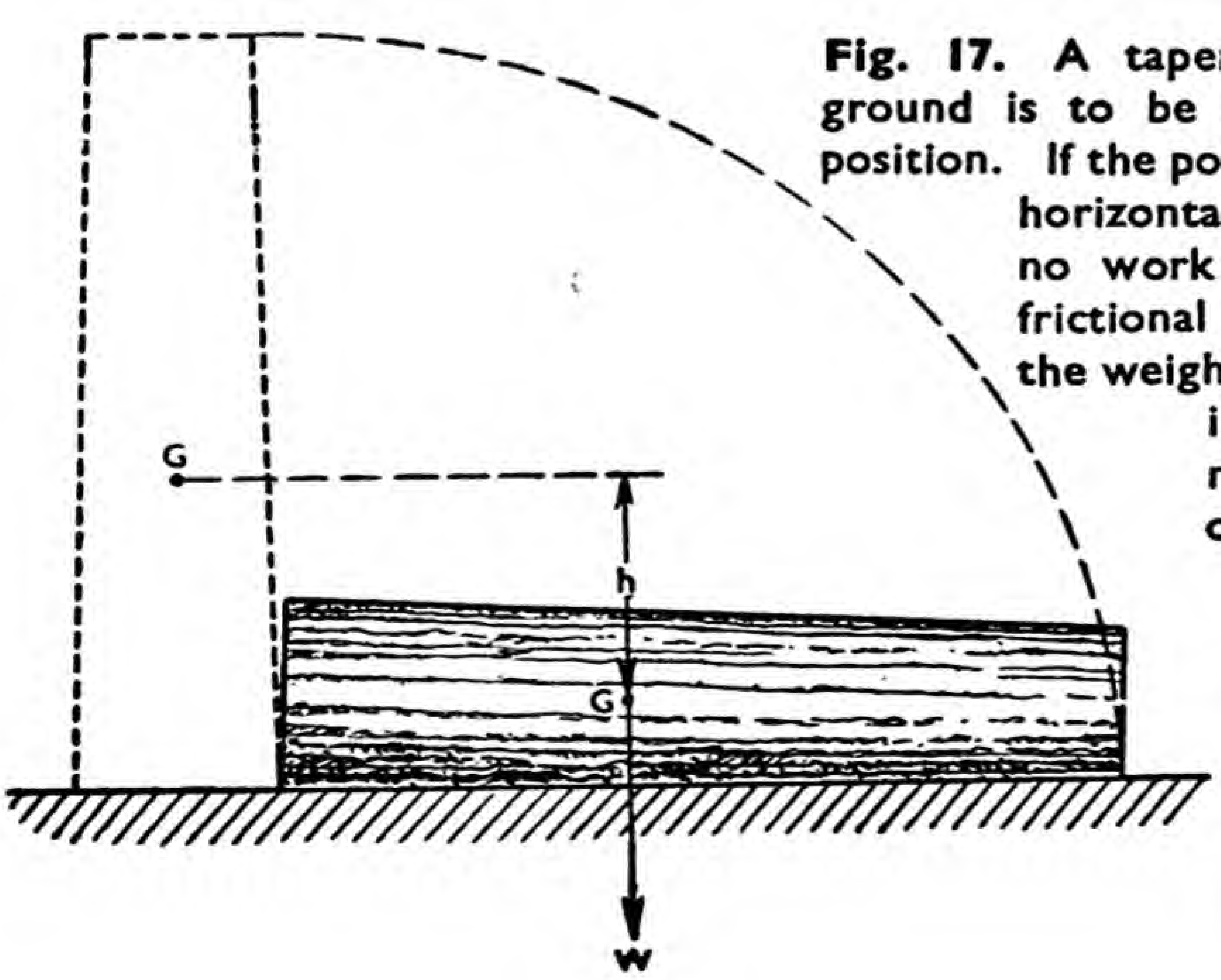


Fig. 17. A tapered pole lying on the ground is to be lifted into the vertical position. If the pole is not allowed to slide horizontally in the tilting process, no work will be done against frictional resistances, and since the weight acts vertically no work is done in horizontal movement of the centre of gravity *G*. We only need to know the vertical distance *h* between initial and final positions of *G*, the work done being the weight lifted multiplied by the distance *h*.

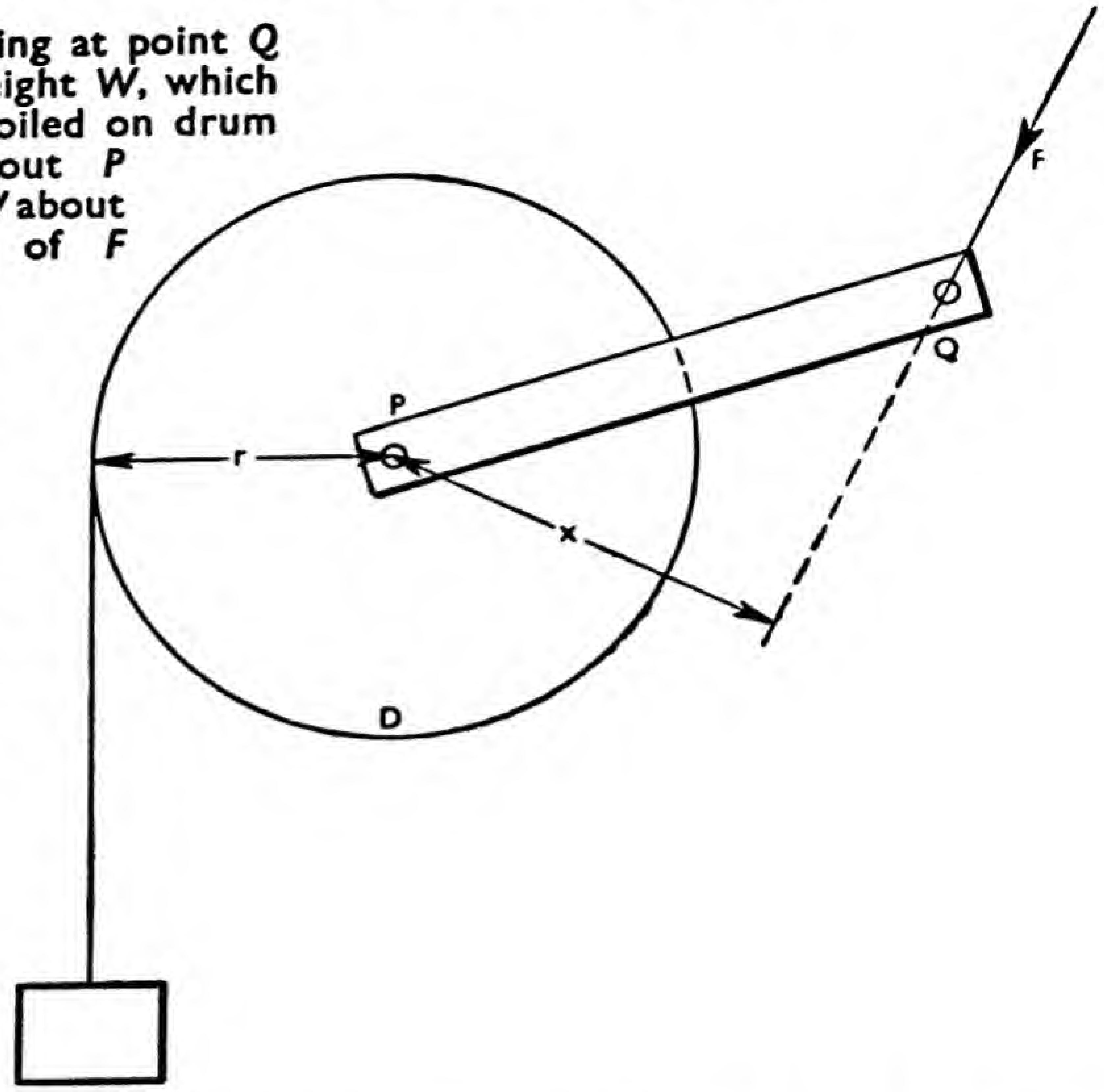


Fig. 18. If force F, acting at point Q on arm PQ, is to lift weight W, which is attached to a rope coiled on drum D, moment of F about P must equal moment of W about P. To find moment of F about P we need not know length PQ but must find length x of a perpendicular drawn from P to the line of action of F. Moment of F is then $F.x$, so that $F.x = W.r$. Thus, knowing the values of W, r and x, the necessary value of F is found. Evidently the greater x is, the smaller F can be, so that the best direction for F is at right angles to arm PQ, since the value of x is then the greatest possible.

and is climbing a gradient of 1 in 20 at 40 m.p.h., what additional h.p. would be required?

In one minute the weight of the vehicle has to be raised

$$\frac{3,520}{20} = 176 \text{ ft.}$$

$\therefore$ Additional work done per min. = 2,000 × 176 ft.-lb.

Additional h.p. required =

$$\frac{2,000 \times 176}{33,000} = 10{\cdot}7.$$

$\therefore$ Total h.p. required =

$$9{\cdot}5 + 10{\cdot}7 = 20{\cdot}2.$$

This shows quite clearly the effect of even a small gradient on the h.p. required.

Increased Speed

If the speed is to be increased to 60 m.p.h., the total h.p. required would be increased to

$$\frac{60}{40} \times 20{\cdot}2 = 30{\cdot}3.$$

What h.p. would be required to pump 10,000 gallons of water per hour to a height of 300 ft.? (1 gallon of water weighs 10 lb.).

Work done = 100,000 × 300 ft.-lb.

Time taken = 60 min.

Work done per min. =

$$\frac{100,000 \times 300}{60} = 500,000 \text{ ft.-lb.}$$

H.p. required for lifting water =

$$\frac{500,000}{33,000} = 15{\cdot}2.$$

Additional Horse-power

In practice, additional h.p. would be necessary to overcome friction in the piping, which might increase the effective head by 80 ft. or more.

When we are considering rotation of a body against a resistance, we have to consider the moment of the driving force about the axis of rotation. This is frequently referred to as a torque, and is usually expressed in lb.-ft.

Thus, if the force F (Fig. 18), acting on the arm PQ, raises the weight W, which is supported by a

rope round the drum D, then :—

Driving torque $= F \times x$.

Resisting torque $= W \times r$.

For constant speed, driving torque = resisting torque.

$\therefore F \times x = W \times r$.

Note that it is not the length of the arm PQ, but the perpendicular distance x which counts. Obviously, F would have its greatest effect if it were pushing at right angles to PQ, since x would then have its greatest value.

Suppose $W = 75$ lb. ; $r = 1{\cdot}3$ ft.; $x = 3{\cdot}6$ ft.

Then, $F \times 3{\cdot}6 = 75 \times 1{\cdot}3$.

$$\therefore F = \frac{75 \times 1{\cdot}3}{3{\cdot}6} = 27{\cdot}1 \text{ lb. weight.}$$

Work Done by Torque

We now wish to know how to find the work done by a torque. Consider a force F (Fig. 19) acting on a body rotating about an axis through O. Suppose that under the action of this force, acting

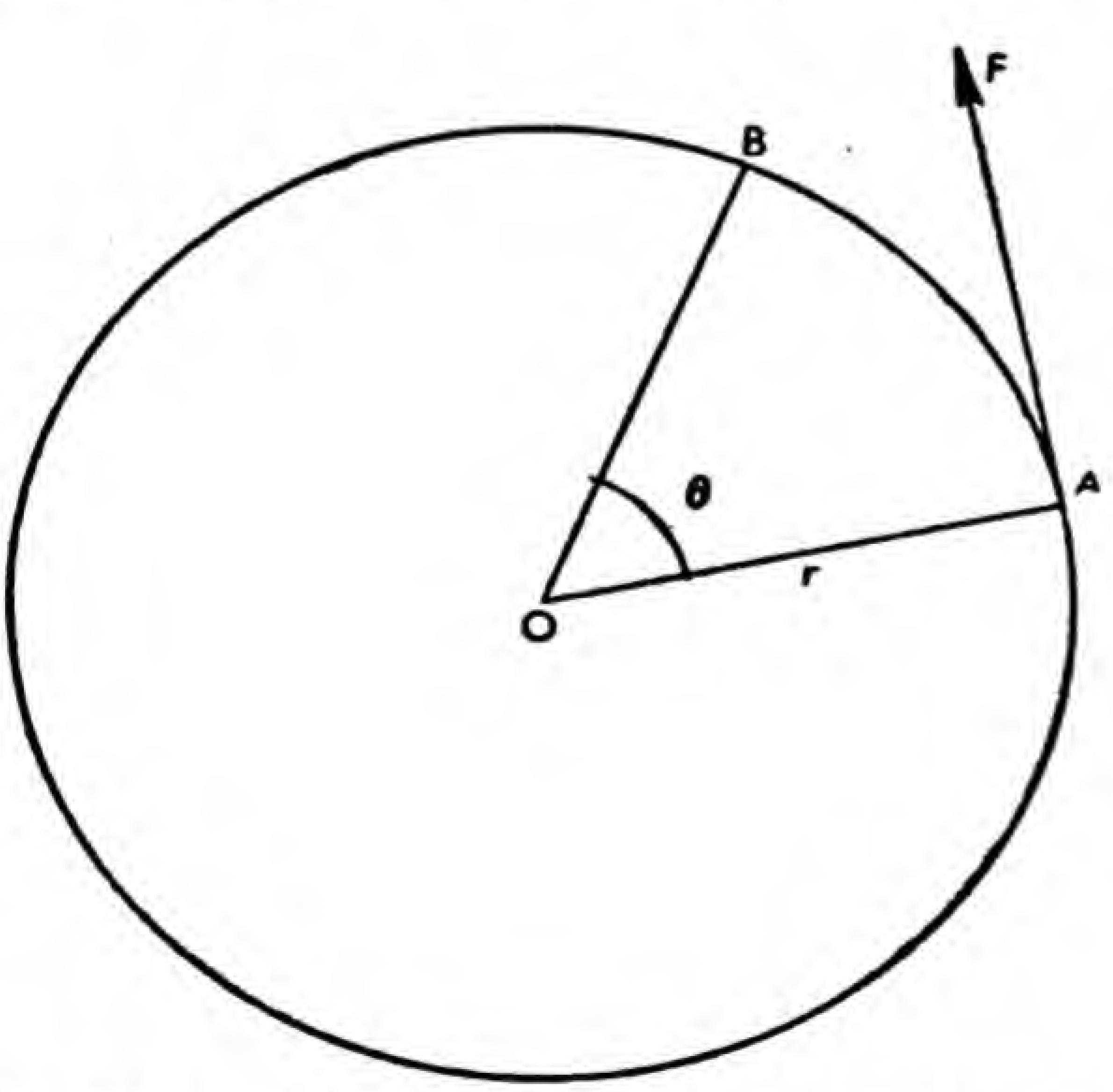


Fig. 19. The force F, acting at radius r, is causing a body to rotate steadily about an axis through O. When the body turns through an angle θ, OA moves to OB and the work done by F is $F \times$ arc AB. But, if θ is in radians, arc $AB = r.\theta$, so that the work done is $F.r.\theta$, or torque $\times\ \theta$.

always at right angles to the radius, the body moves through an angle AOB, which we will call θ. Now the torque acting is $F \times r$, and the work done is $F \times AB$ and, since the angle in radians is the arc divided by the radius, we have,

$$AB = r \times \theta \text{ (rad.)}.$$

Work done $= F \times r.\theta =$ Torque $\times\ \theta$.

Thus we have the rule :

Work done by a torque (ft.-lb.) = Torque (lb.-ft.) $\times$ angle turned through (radians).

If the body acted on by the torque is rotating at N r.p.m., and the torque is T (lb.-ft.), then, since one revolution $= 2\pi$ radians,

work done per min. $= T \times 2\pi N$.

$\therefore$ h.p. transmitted by a torque

$$T \text{ (lb. ft.)} = \frac{2\pi N.T}{33{,}000}.$$

If an electric motor is transmitting 3 h.p. at 1,800 r.p.m., the torque on the armature is given by,

$$\frac{6{\cdot}28 \times 1{,}800 \times T}{33{,}000} = 3.$$

$$T = 8{\cdot}76 \text{ lb.-ft.}$$

This is equivalent to a force of 8·76-lb. weight acting at a radius of 1 ft. By the third law of motion, the reverse torque on the frame of the motor will also be 8·76 lb.-ft.

If the driving torque is greater than the resisting torque, the speed of revolution of the body will increase, that is, it will have an angular acceleration, but each small portion of the body will have a different linear acceleration, depending on its distance from the axis of rotation.

Linear Acceleration

Let us consider (Fig. 20) a small portion w lb. of the body at a distance x ft. from the axis of rotation O, and let a represent the

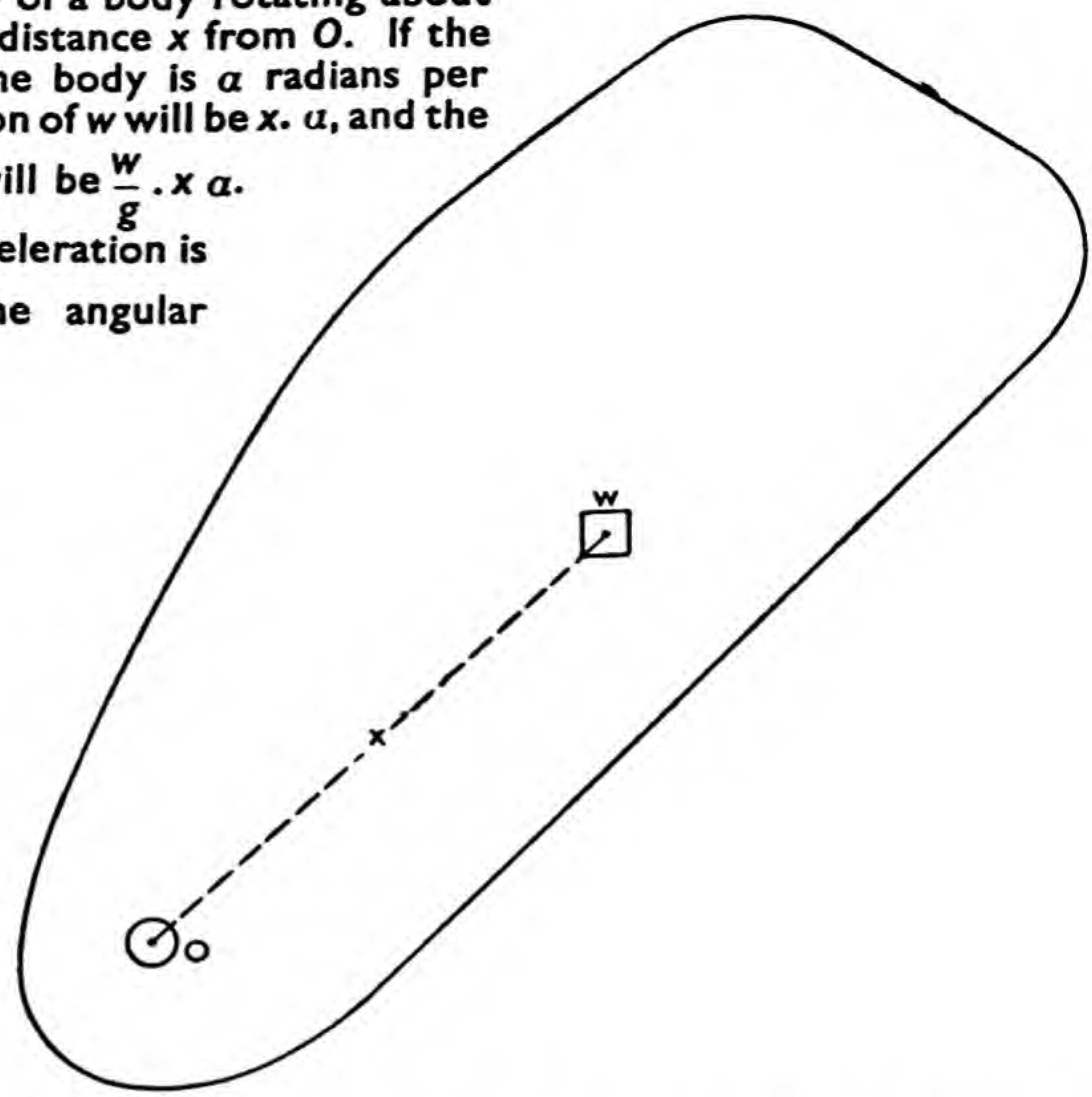


Fig. 20. A small piece w of a body rotating about an axis through O is at a distance x from O. If the angular acceleration of the body is α radians per sec.^{2}, the linear acceleration of w will be $x.\ \alpha$, and the accelerating force on w will be $\frac{w}{g}.x\ \alpha$. The torque due to this acceleration is $\frac{w}{g}.x.\alpha \times x = \frac{w}{g}.x^2.\alpha$. The angular acceleration is everywhere the same so that the total accelerating torque on the body is $\alpha \times$ (sum of values of $\frac{w}{g}.x^2$). If we take each little portion making up the whole body, multiply its mass by the square of its distance from the axis of rotation, and add all the results together, we obtain a quantity which is known as the moment of inertia.

angular acceleration of the body (radians per sec. per sec.).

Then the linear acceleration of w is $x \times \alpha$.

Force acting on $w = \frac{w}{g}.x.\alpha.$

Moment of force acting on $w =$ $\frac{w}{g}.x.\alpha \times x = \frac{w}{g}.x^2.\alpha.$

Thus the importance of w so far as this is concerned depends upon the *square* of its distance from the axis of rotation.

The total accelerating moment (or torque) acting on the body will be the sum of all these small moments and, since α is constant for the whole body, if we add up all the values of $\frac{w}{g}.x^2$ over the whole body and multiply by α, we shall obtain the accelerating torque.

The name given to the sum of the values of $\frac{w}{g}.x^2$ is moment of inertia, and the value of the moment of inertia (I) can be calculated for any body of known shape. We now have the rule,

Torque = Moment of inertia multiplied by angular acceleration.

We will take an example of a flywheel whose moment of inertia is 5 (lb. and ft. units), and find the torque required to increase its speed from 1,200 r.p.m. to 1,800 r.p.m. in 12 sec.

Change of speed = 600 r.p.m. = $\frac{6{\cdot}28 \times 600}{60} = 62{\cdot}8$ rad. per sec.

Time taken = 12 sec.

Angular acceleration $\alpha = \frac{62{\cdot}8}{12} =$ $5{\cdot}23$ rad. per sec. per sec.

$\therefore$ Torque required for acceleration $= 5 \times 5{\cdot}23 = 26{\cdot}15$ lb.-ft.

Notice that the smaller the time taken the greater is the torque required. This is, of course, in addition to the torque required for driving an external load.

We now come to another kind of

motion which is of very great importance in practice, since it is the motion of all vibrating bodies, such as a weight suspended from a spring, a pendulum, the strings of a violin, the air in an organ pipe, the waves of the sea, the vibration of the loud-speaker of a wireless set; in short, much of the phenomena which come within our everyday experience.

Simple Harmonic Motion

This kind of motion is known as simple harmonic motion, and we shall here deal only with some of its simpler applications.

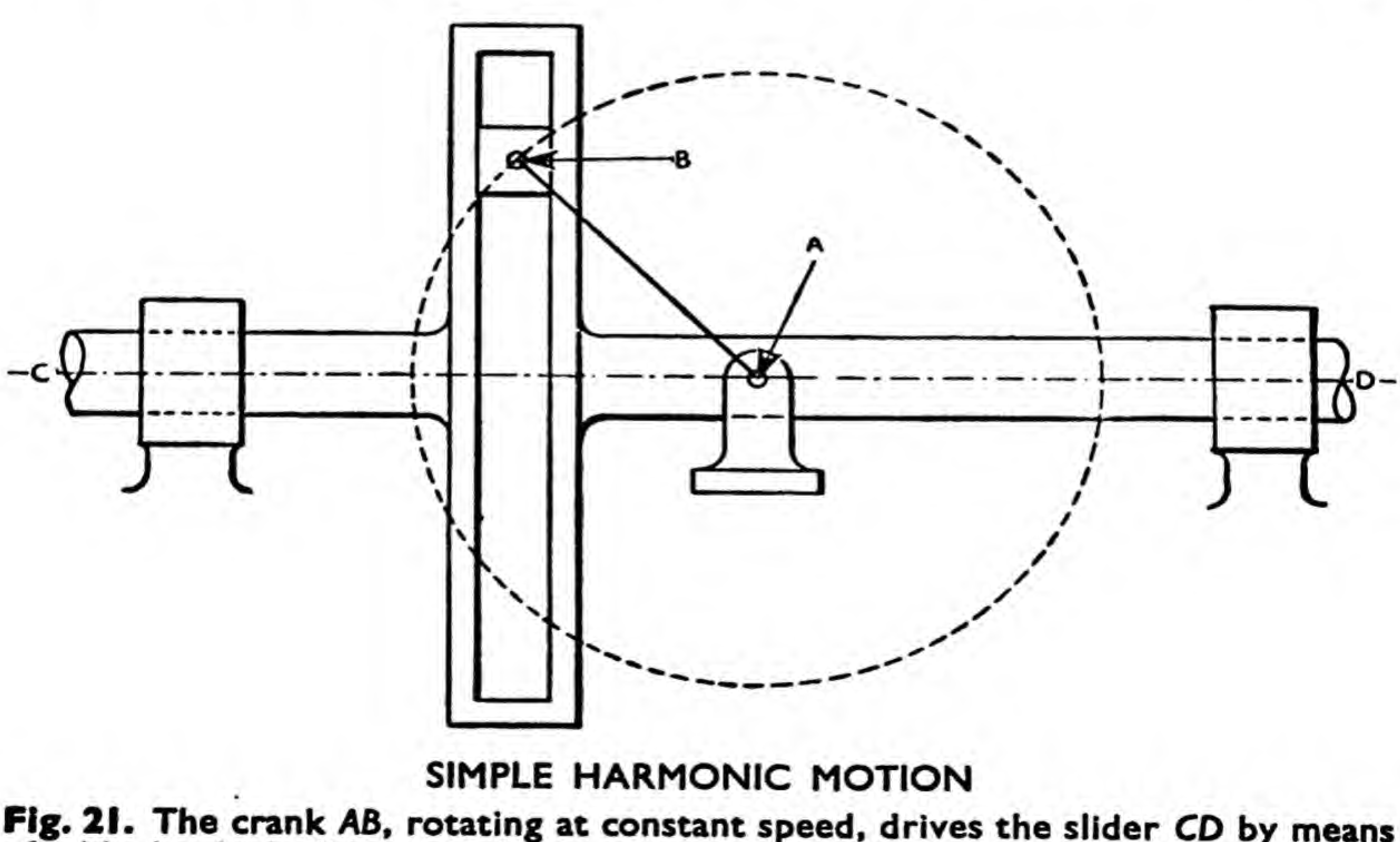
Let us imagine a crank *AB*, driving *CD* by means of a block sliding in a slotted link as shown (Fig. 21). If *AB* rotates with constant angular velocity, the motion of *CD* will be a simple harmonic motion.

Referring to Fig. 22, let ω be the velocity of *AB* (rad. per sec.), and r (ft.) the length of *AB*. Then, as shown in Fig. 22, *B* will have a linear velocity $r.\omega$ (ft. per sec.) and a centripetal acceleration $r.\omega^2$ (ft. per sec. per sec.). If *AB* has moved from *AD*, to the position shown, in t sec., the angle *DAB* will be $\omega.t$ (rad.).

The velocity of *N* along *DC* will be the horizontal component of $r.\omega$, and can be shown to be $\frac{BN}{BA}.r.\omega = B.N.\omega$. Therefore, it is zero at *D*, increases to $r.\omega$ at *A*, diminishing again to zero at *C*.

The acceleration of *N* along *DC* will be the horizontal component of $r.\omega^2$, and can be shown to be $\frac{AN}{AB}.r.\omega^2 = \omega^2.x$, and is always directed towards *A*.

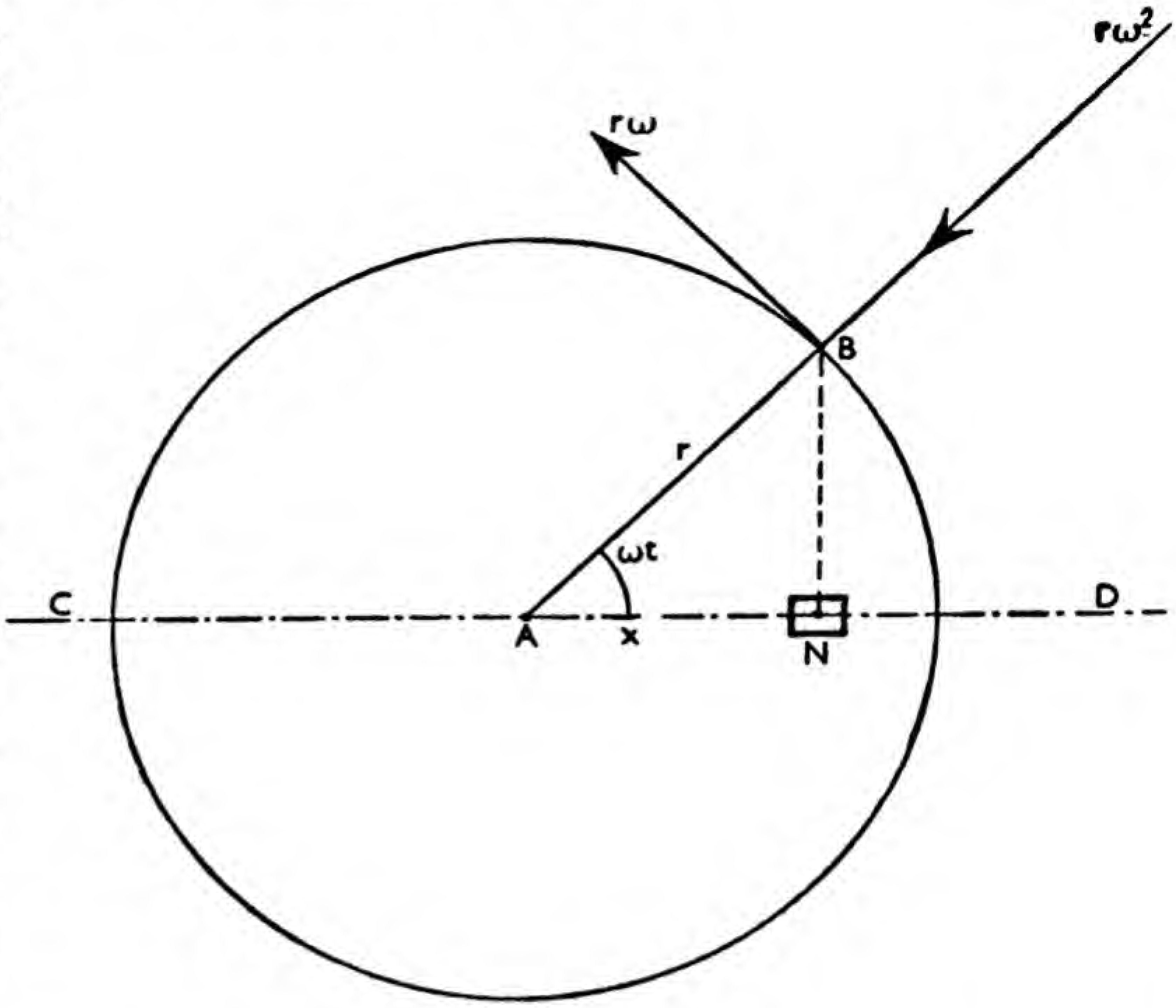
Thus, we can say that a body at *N* is moving with simple harmonic motion if its acceleration is always



SIMPLE HARMONIC MOTION

Fig. 21. The crank *AB*, rotating at constant speed, drives the slider *CD* by means of a block which slides in a slot as shown. *CD* will slide at a speed which varies from zero at the ends to a maximum at the centre of its stroke. The motion of *CD* is known as simple harmonic motion. Study of this motion is important for all kinds of vibration problems.

Fig. 22. This shows how velocity and acceleration of a body moving with S.H.M. can be found at any instant. Velocity of *B* is at right angles to *AB* and is $r.\omega$. Acceleration of *B* is radial from *B* to *A* and is $r\omega^2$. Velocity of *N* is $BN\omega$. and the acceleration of *N* is $AN.\omega^2$, so that the acceleration of *N* is from *N* to *A* and is proportional to its distance from *A*. We may thus conclude that if the acceleration of a vibrating body is proportional to its distance from its mean position, its motion is a S.H.M.



towards the centre *A* of its motion, and is proportional to its distance from *A*, viz., acceleration is proportional to displacement.

The time *T* of one complete oscillation of *N* (*D* to *C* and back again to *D*) is evidently the time of one complete revolution of the imaginary crank *AB*, so that :—

$$T \text{ sec.} = \frac{2\pi}{\omega}.$$

$$\text{Now, } \frac{\text{displacement}}{\text{acceleration}} = \frac{x}{\omega^2.x} = \frac{1}{\omega^2}.$$

$\therefore$ Periodic time $T =$

$$2\pi\sqrt{\frac{\text{displacement (ft.)}}{\text{acceleration (ft. per sec. per sec.)}}}.$$

Number of complete oscillations per sec. (frequency) is given by,

$$n = \frac{1}{2\pi}\sqrt{\frac{\text{acceleration}}{\text{displacement}}}.$$

We now have a means of finding out (1) whether a particular motion is simple harmonic, and (2) if it is shown to be simple harmonic, its periodic time and frequency.

In what follows we shall use the letters S.H.M. to indicate simple harmonic motion.

Fig. 23 shows a spring supported at its upper end, with a weight attached at its lower end.

Rate of Extension

We can show, by experiment, that the extension of the spring is proportional to the pull in the lower end of the spring. The rate of extension (lb. per ft.) of the spring is known as the stiffness of the spring, and we will use *s* to denote this.

Thus, if a force of 6 lb. extends the spring by 3 in., the stiffness is $\frac{6}{3}$ lb. per inch, or $\frac{6 \times 12}{3} = 24$ lb. per ft., so that for this spring $s = 24$.

Now suppose we attach a weight *W* to the spring and set it vibrating vertically.

When the weight is stationary, its level will be *XX* (Fig. 24), and the pull in the spring will be *W*. When the weight is vibrating, at some instant *W* will have moved downward a distance *x* (ft.) from

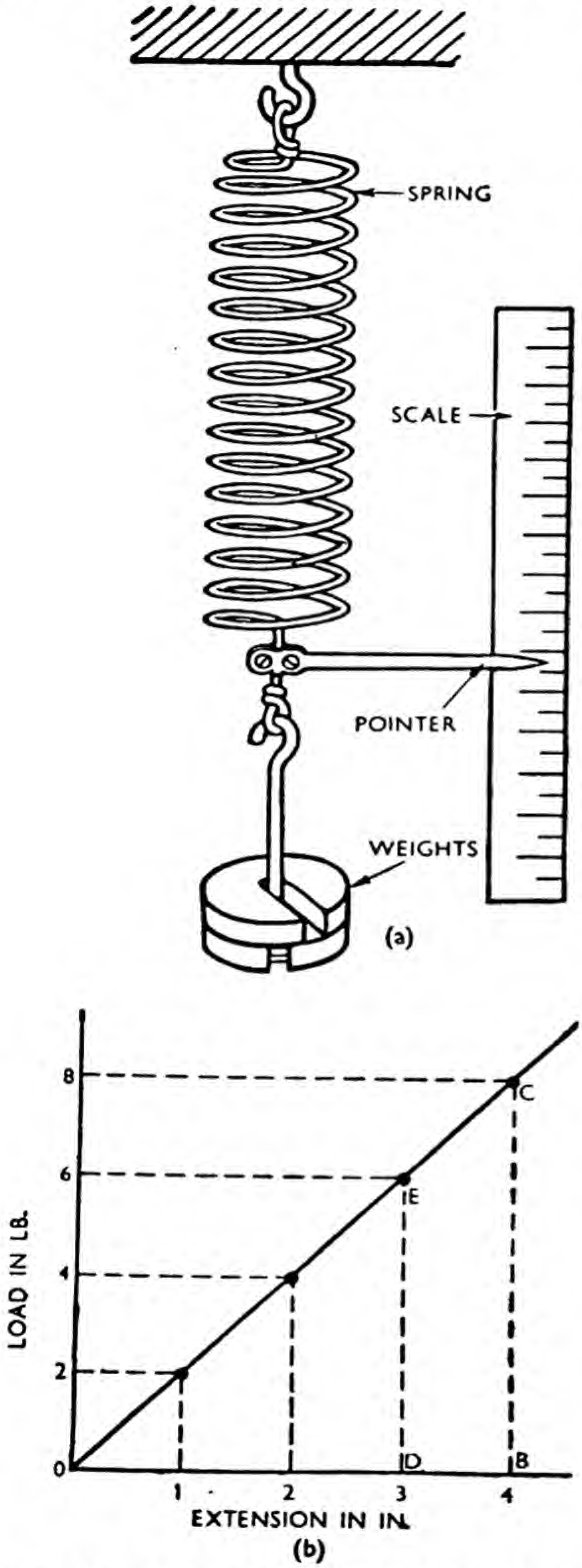


Fig. 23. If a spring is supported at one end and is loaded at the other with gradually increasing loads, it will be found that extension of spring is proportional to applied load, i.e., if loads are plotted against corresponding extensions, result is a straight line passing through zero. Other kinds of spring, such as coach springs, obey the same law.

its mean position XX, and at that instant the force acting upward on W due to the total extension of the spring will be $s.x + W$.

Thus, the accelerating force on W (toward XX) will be :—

$$(s.x + W) - W = s.x.$$

Since force = mass × acceleration, we have,

$$\frac{W}{g} \times \text{acceleration} = s.x,$$

$$\text{or } \frac{\text{acceleration}}{\text{displacement}} = \frac{g.s}{W}.$$

Now g, s and W are constants, so that, for any displacement of W, $\dfrac{\text{acceleration}}{\text{displacement}}$ is constant. Therefore, we conclude that the motion of W is a S.H.M.

We also know from the foregoing that,

$$\text{Periodic time } T = 2\pi\sqrt{\frac{W}{g.s}}.$$

$$\text{Frequency} \quad n = \frac{1}{2\pi}\sqrt{\frac{g.s}{W}}.$$

Thus, knowing the weight W and the stiffness s of the spring, the frequency of oscillation of W can be calculated.

Now $\dfrac{W}{s}$ is the static extension (or compression) in ft. of the spring when loaded with a weight W (lb.).

If we call this extension l (ft.), we get,

$$n = \frac{1}{2\pi}\sqrt{\frac{g}{l}},$$

and, as we shall show, this is the frequency of oscillation of a simple pendulum whose length is l ft.

This gives us a simple method of finding the natural frequency of oscillation of a loaded spring.

Find the static extension or compression of the spring due to the

load by lifting the load until the spring is just free. Now find the frequency of oscillation of a simple pendulum whose length is equal to the measured change of length as above.

If l is measured in inches and values of π and g are substituted, we get the number of double oscillations per minute to be,

$$n\,(\text{per min.}) = \frac{188}{\sqrt{l}\,(\text{in.})}$$

With the above method it is not necessary to have particulars of either the spring or the load so long as the static deflection can be measured.

The value of n in the above equation is frequently spoken of as the natural frequency of vibration of the weight which is attached to the spring. The term natural frequency of a spring is meaningless, since different weights attached to the same spring would have different natural frequencies.

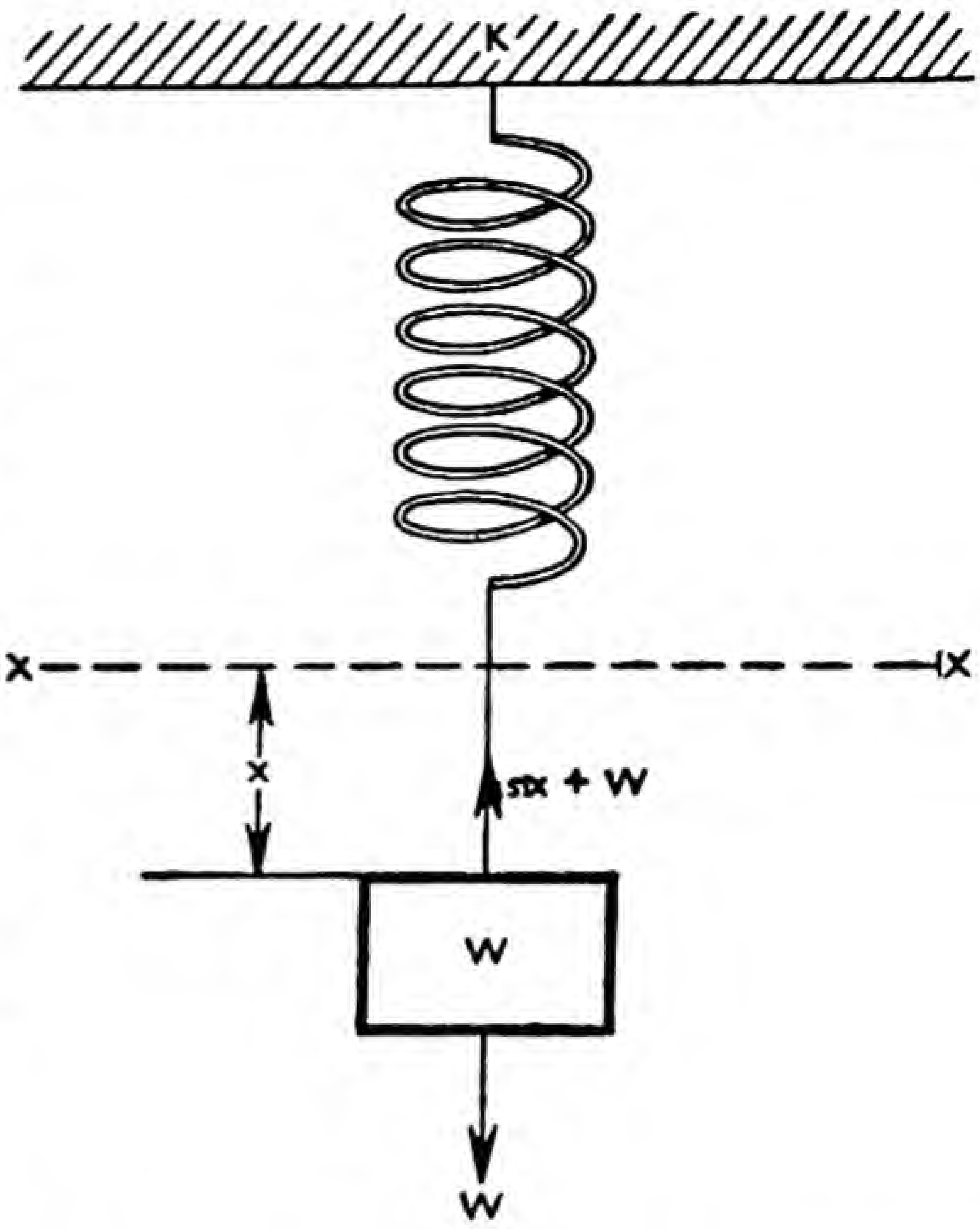


Fig. 24. If upper end of spring is made to oscillate through a small distance, weight at other end will oscillate with same frequency, but amplitude of its oscillation will depend upon how near the frequency of the oscillation of the upper part is to the natural frequency of the weight when upper end is stationary.

Forced Oscillation

Let us suppose that the upper end of the spring shown in Fig. 24 is held by the hand and given an up and down oscillation of, say, $\frac{1}{8}$ in. with some known frequency, a weight of 8 lb. being attached at the lower end.

Using the above equation, l will be 4 in., and the natural frequency

n will be $\dfrac{188}{\sqrt{4}} = 94$ per min.

Now as we increase the frequency of oscillation of the upper end, two things will be noticed; first, the weight will vibrate with the same frequency as the upper end of the spring; second, the amplitude of vibration of the weight will become greater as the frequency of the *forcing* oscillation increases, until at 94 oscillations per min., the vibration of the weight will be so great that the spring will break, or be seriously overstrained, if we continue to oscillate the upper end at this speed.

If we increase the speed of oscillation beyond 94 per min., the amplitude of oscillation of the weight will decrease, and at high speeds the weight will remain almost stationary.

Another thing that will be noticed is that at speeds above 94 per min., the weight will move in the opposite direction to that of the hand, producing a sort of concertina action.

Thus, if the support K (Fig. 24) is vibrating with a known amplitude (or half travel) and frequency,

the effect of this vibration on the weight W will depend upon how near the frequency of the forcing vibration of K is to the natural frequency of vibration of W as given above. In any case, the weight will eventually vibrate at the same frequency as that of K, but with a different amplitude.

Suppose the natural frequency of W, as given above, is n per min., and the frequency of vibration of K is p per min., its amplitude being a. The amplitude A of the forced vibration of W is given by,

$$A = \frac{a}{1 - \left(\frac{p}{n}\right)^2}.$$

For instance, if p is 720 per min.

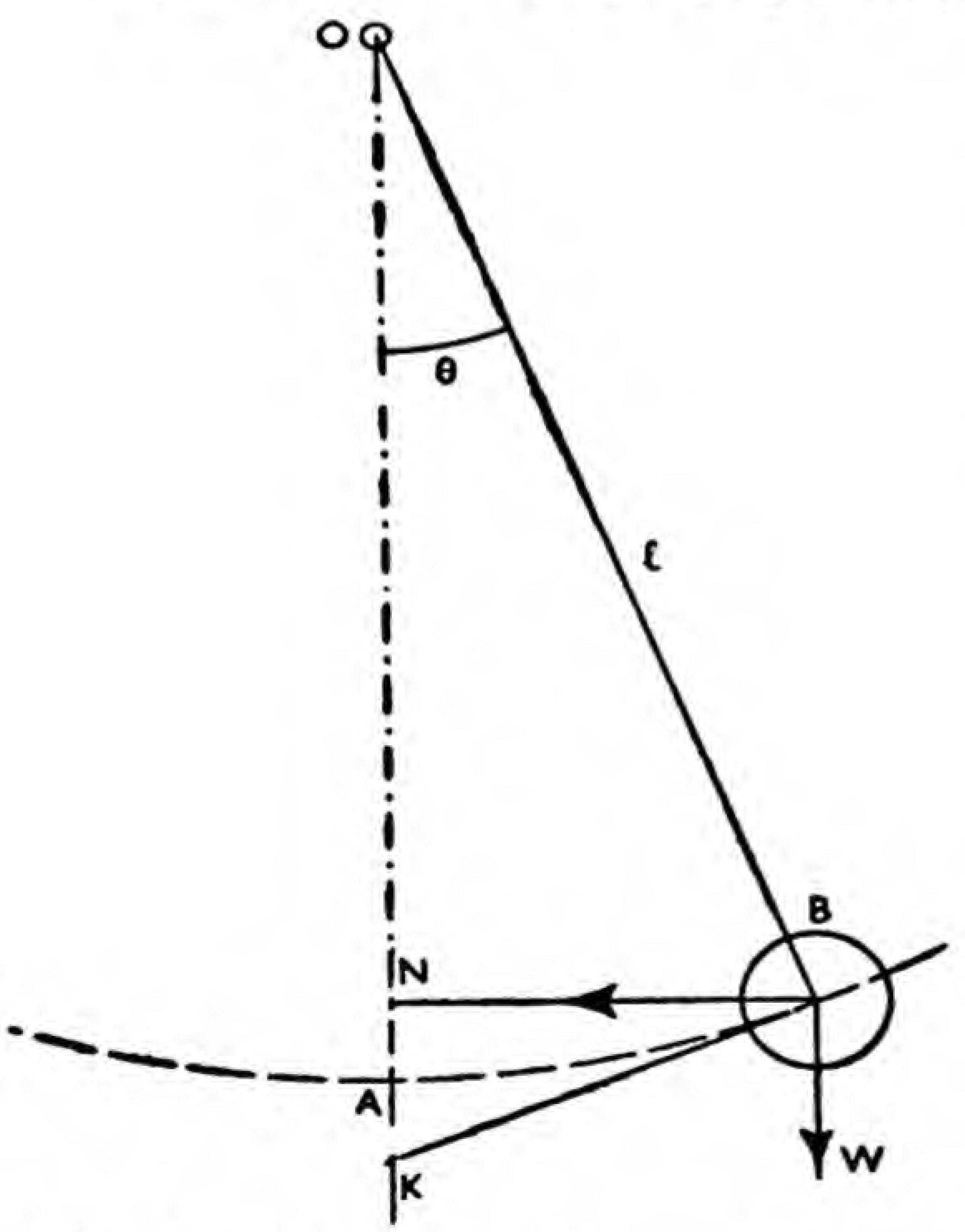


Fig. 25. The simple pendulum is actuated by gravity. The pendulum has swung to position OB and W is acting downward on the bob. The component of W in the direction BK is causing an acceleration $\frac{BNg}{l}$. If the angle θ is small, BN and BA may be taken as equal, so that the acceleration of B is $g.\theta$. Thus acceleration is proportional to displacement and hence the motion is simple harmonic.

and n is 900 per min., a being 0·05 in., then :—

$$\left(\frac{p}{n}\right)^2 = \left(\frac{720}{900}\right)^2 = 0{\cdot}64.$$

$$\therefore A = \frac{0{\cdot}05}{1 - {\cdot}64} = 0{\cdot}14 \text{ in.}$$

In this case the total distance through which W vibrates will be 0·28 in. at 720 per min.

If now p is increased to 850 per min., then,

$$\left(\frac{p}{n}\right)^2 = \left(\frac{850}{900}\right)^2 = 0{\cdot}892$$

$$\therefore A = \frac{0{\cdot}05}{1 - {\cdot}892} = 0{\cdot}464 \text{ in.}$$

The motion of K will thus be magnified 9¼ times at W.

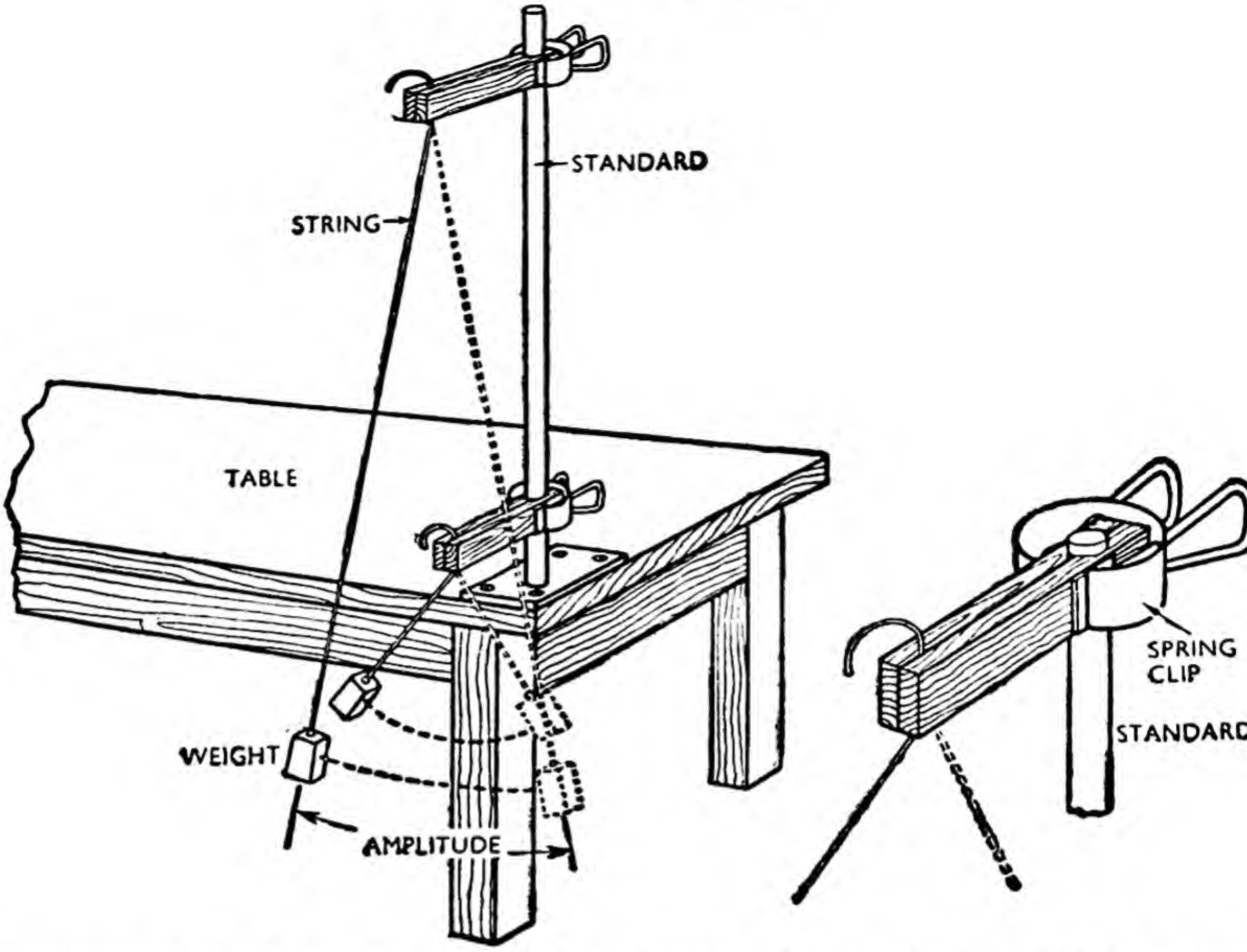
Similarly, if p is increased to 890 per min., A will be increased to 1½ in., so that when we are very near to synchronism ($p = n$), a very small amplitude of oscillation of K may produce excessively large amplitudes of oscillation of W.

Thus an almost imperceptible vibration in the vicinity of, say, an unbalanced armature of an electric motor attached to the floor of a steel-framed building, may produce uncomfortable vibrations in some other part of the building which is in synchronism with the speed of the motor.

Simple Pendulum

We have referred above to the simple pendulum, and it is now necessary to show that the motion of the bob of the pendulum is a S.H.M., and to find its periodic time T and frequency of oscillation.

Let W (lb.) be the weight of the bob and l (ft.) be the length of the pendulum OB (Fig. 25). At any instant let θ (rad.) be angle moved by OB from its central position OA.



CHECKING THE TIME OF OSCILLATION OF A SIMPLE PENDULUM

Fig. 26. Method is that the weight is set to swing through a small arc and by means of a stop-watch the time of, say, 50 oscillations is noted. The time of one oscillation is thus found accurately and can be checked by the formula.

Draw BN at right angles to OA. Then, if the angle θ is small, BN is approximately equal to BA (the error in the timing when the maximum value of θ is 10 deg. will be 0·15 per cent, and the corresponding error for 20 deg. is 0·6 per cent).

Thus $BN = BA = l.\theta$ approx.

The force tending to restore B to its central position is the component of W along BK, and this is given by $\frac{BN}{l} \times W$.

$\therefore$ Acceleration of B (B to N) =

$$\frac{\text{Force}}{\text{Mass}} = \frac{\frac{BN}{l} \times W}{\frac{W}{g}} = g \times \frac{BN}{l} = g.\theta.$$

Displacement of B from central position $= BA = l.\theta$.

$$\frac{\text{Displacement}}{\text{Acceleration}} = \frac{l.\theta}{g.\theta} = \frac{l}{g}.$$

Since l and g are constants, we conclude that the motion is a S.H.M., for which,

$$T = 2\pi\sqrt{\frac{l}{g}}.$$

Since the time of vibration of the pendulum is proportional to the square root of its length, if we had two pendulums, one 24 in. long and the other 6 in. long, the second pendulum would swing twice as fast as the first (Fig. 26).

Note that the weight of the bob does not appear in the final result, so that the time of swing for a light bob will be the same as for a heavy one (l the same for both), as long as the dimensions of the bob are

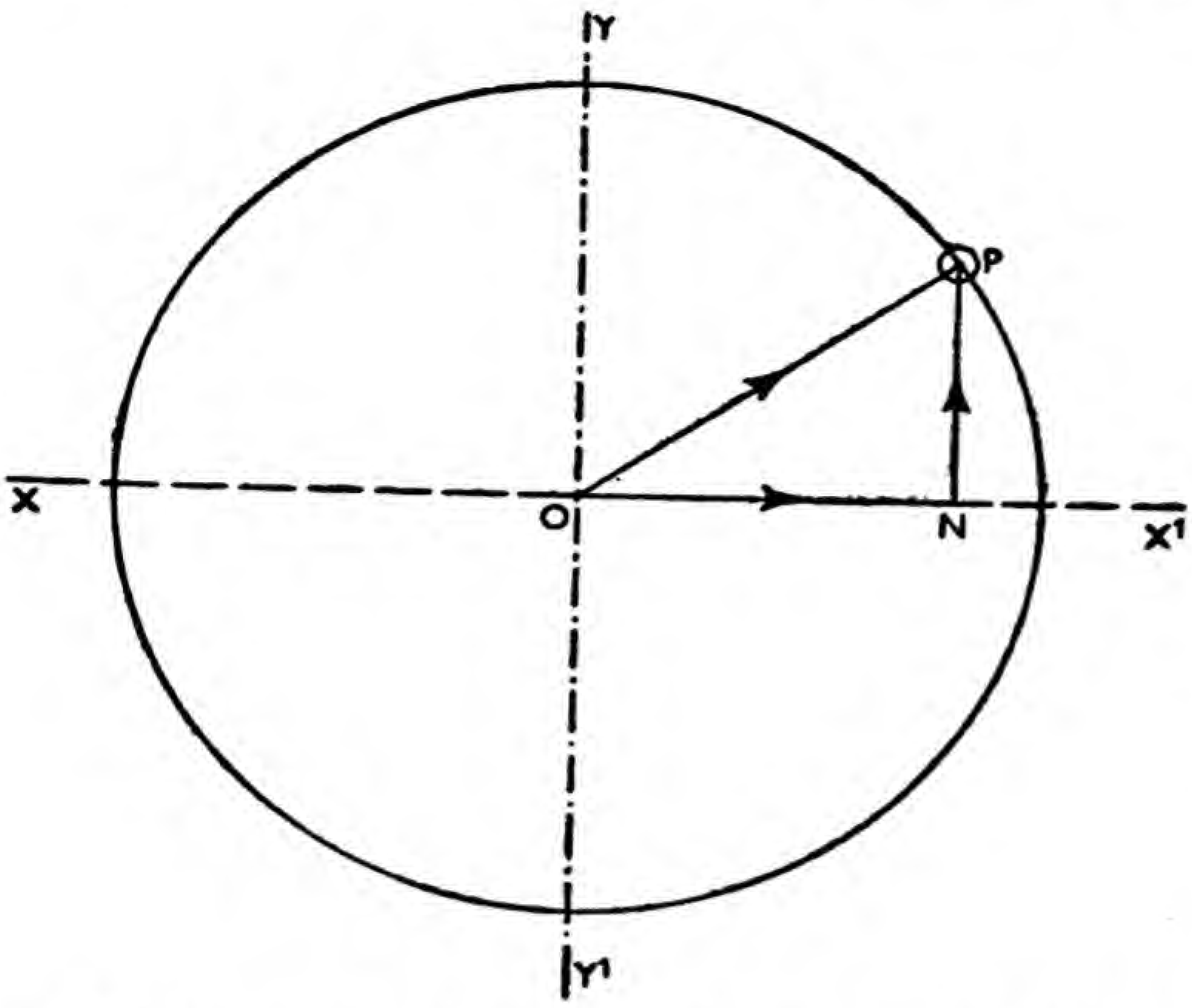


Fig. 27. *OP* is distance of centre of gravity of a rotating body from axis of rotation *O*. If body weighs 40 lb. and *OP* is 0·01 ft., then at 1,200 r.p.m. there will be a radial force of 196 lb. acting along *OP*. As the body rotates there will be a vertical force varying from 196 lb. upward to 196 lb. downward and a horizontal force varying from 196 lb. to the right to 196 lb. to the left. These forces will be acting on the bearings at the rate of 20 times per sec.

small in comparison with the length of the pendulum.

If the light string is replaced by a comparatively heavy rod, we shall have what is known as a *compound* pendulum, and in this case the simple formula given does not apply.

Rotary Balancing

We may now consider the effects of the forces due to unbalanced rotating parts and methods of reducing or eliminating these forces.

If a body is mounted on a rotating shaft and the centre of gravity of the body does not lie on the axis of the shaft, there will be a centrifugal force on the bearings which is constant in magnitude but varying in direction as the body rotates at constant speed.

If the weight of the body is W (lb.), distance of C.G. from the axis of rotation is r ft., and angular velocity of the body is ω (rad. per sec.), then the centrifugal force acting on the bearings is $\frac{W}{g}.r.\omega^2$.

Taking the case of a body weighing 40 lb., its C.G. being 0·01 ft. from the axis, if the speed of rotation is 1,200 r.p.m., then, $\omega = \frac{6{\cdot}28 \times 1{,}200}{60} = 125{\cdot}6$; $\omega^2 = 15{,}800$; C.F. $= \frac{40}{32{\cdot}2} \times {\cdot}01 \times 15{,}800 = 196$ lb. weight.

If *OP* (Fig. 27) represents the 196 lb. to a given scale, then for

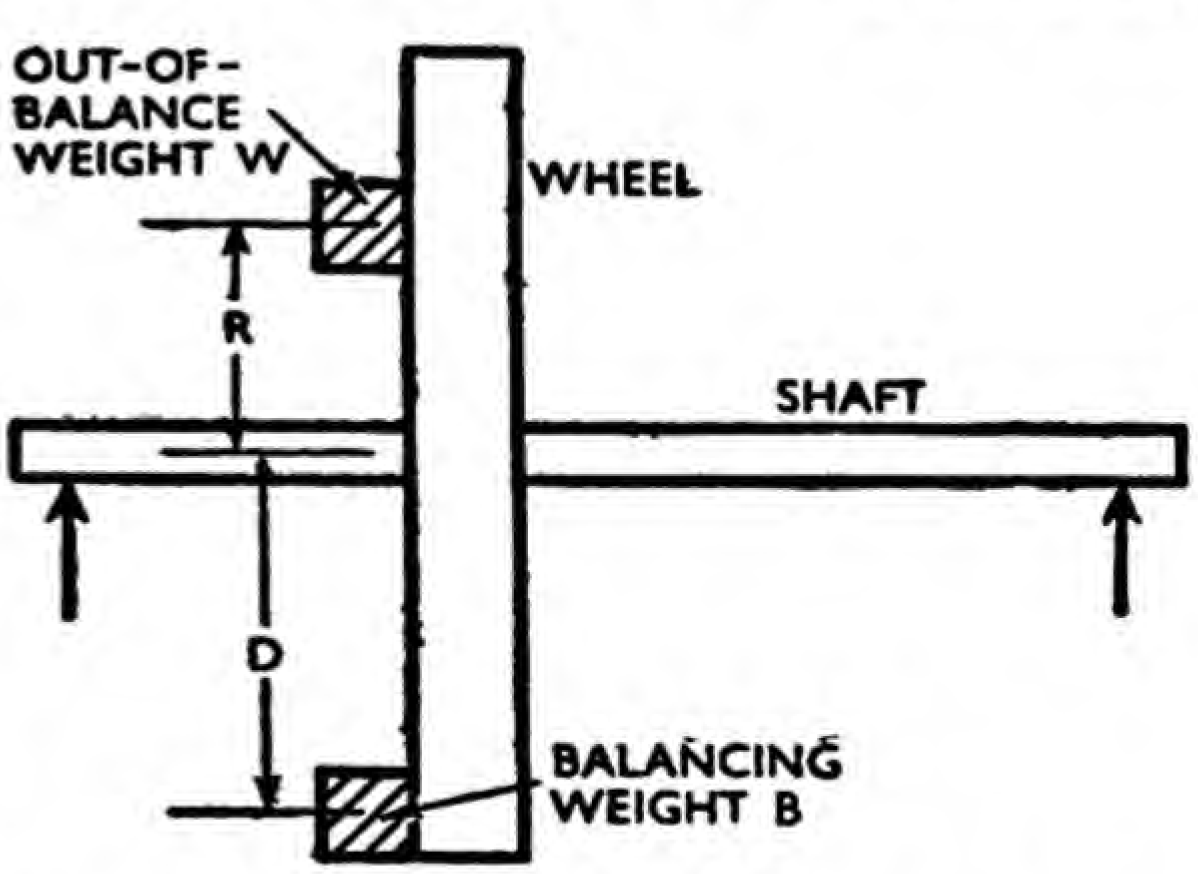


Fig. 28. Out-of-balance weight *W* at radius *R* can be balanced by a weight *B* at radius *D*, provided that $B \times D = W \times R$, the two weights are in a line passing through axis of shaft, and the two weights are in a plane perpendicular to the axis. If the two weights are not in the same plane we shall get static balance but there will be a rocking couple when wheel is rotating.

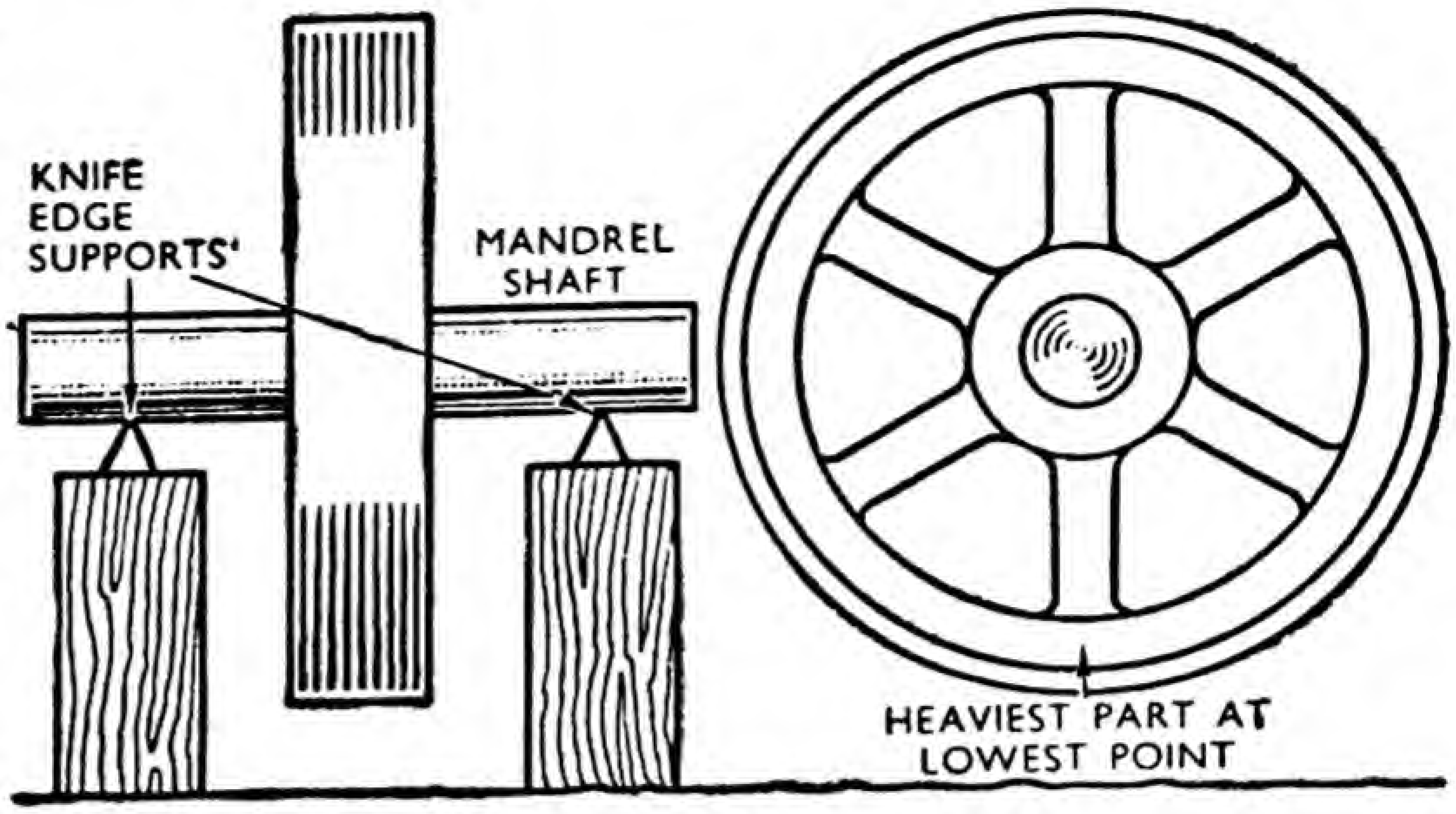


Fig. 29. Flywheel to be balanced is placed with shaft on levelled knife edges. It comes to rest with heaviest part at the lowest point. Weight is clamped to wheel in a line with its lowest point and adjusted until it remains stationary in any position. Knowing the weight and its distance from axis, balancing product is found.

any angular position of OP there will be a horizontal force ON and a vertical force NP acting. Thus the frame supporting the bearing will be subjected to a vertical force varying from 196 lb. upward at Y to 196 lb. downward at Y^1, also to a horizontal force varying from 196 lb. to the right at X^1, to 196 lb. to the left at X. These forces may cause severe vibrations at a frequency of 20 per sec. both vertically and horizontally and, in such a case, we must try to *balance* the rotating weight; for example, by attaching an additional weight which will provide an equal and opposite force to the bearings of, in this case, 196 lb. weight.

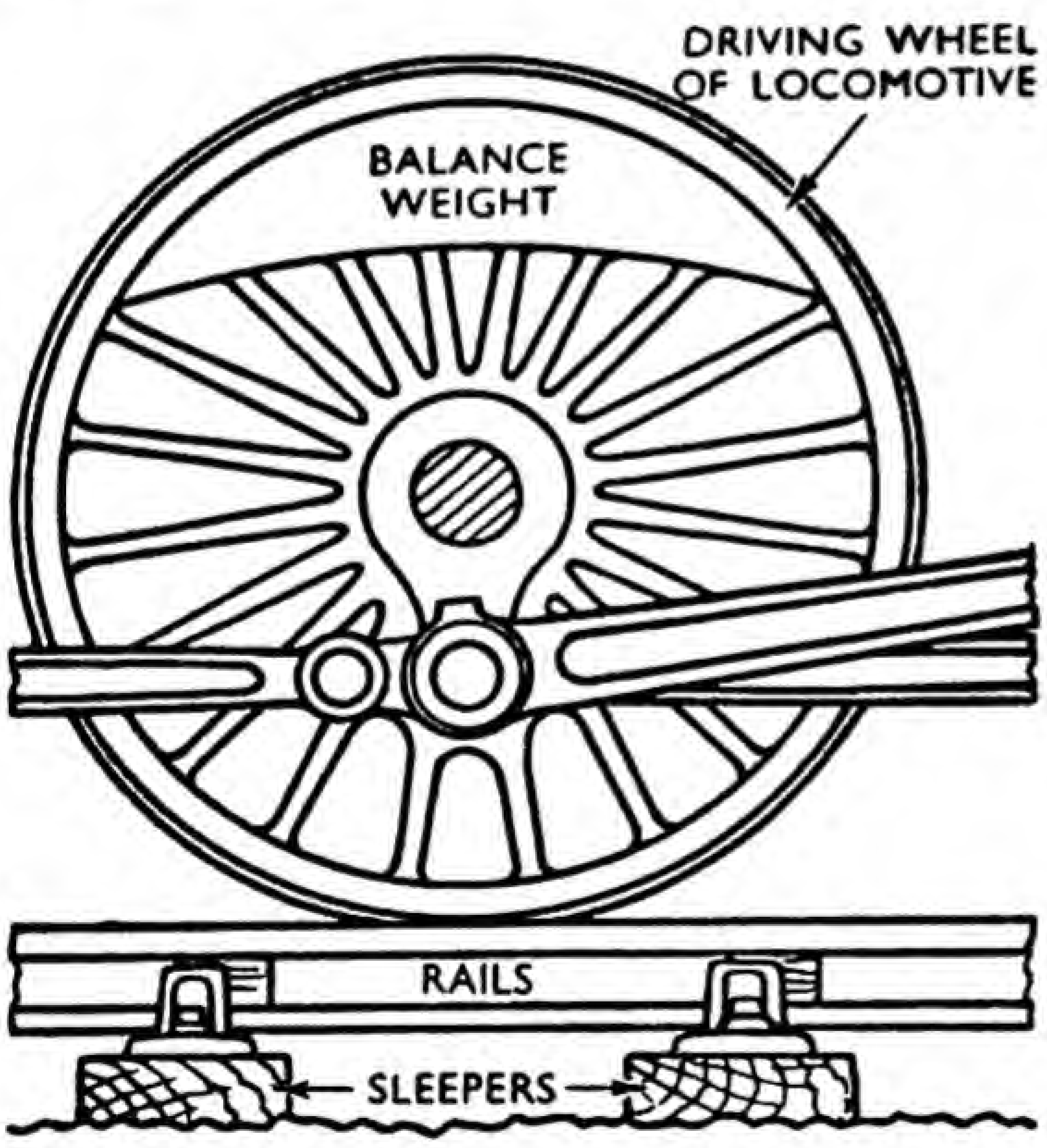


Fig. 30. Driving axle of locomotive has rotating cranks, eccentrics, and connecting rod ends. These must be balanced by rotating balance weights. Part of horizontal forces due to reciprocating parts is also balanced by adding to rotating balance weights. This introduces vertical forces which may cause the wheels to slip or even leave the rails momentarily at high speeds.

Obtaining a Balance

Since the angular velocity is the same for both weights, we shall get a balance if the value of $W.r$ is the same for both, provided the weights are opposite each other on a line passing through the axis of rotation.

Fig. 28 shows the application of the principle, the position of the balancing weight B being arranged so that $B \times D = W \times R$.

Since this is equivalent to saying that the moment of B must be the same as the moment of W for all angular positions, it suggests a method of balancing, say, a flywheel (Fig. 29).

Place the flywheel and its shaft on a pair of carefully levelled knife-edge supports. The wheel will come to rest with its heaviest part at the bottom. Now clamp a weight to a point in a vertical line above the centre. If the weight and distance are correct, the wheel will remain stationary in any position,

since there is no resultant moment. By a few trials, the required value of $W.r$ for balance can be found. Balance may be secured either by adding weight to the light side or by subtracting weight (e.g., by drilling holes) from the heavy side. This is known as the static method of balancing. It does not, however, give perfect balance unless the out-of-balance force and the balancing force are in the same plane, which is not always practicable.

Complex Calculations

Fig. 30 shows the arrangement of a balance weight in the driving wheel of a locomotive. The calculation of this weight is, however, a very complex business, since on the driving axle we have at least two cranks, coupling rods, etc., producing centrifugal forces in different planes, and also there are reciprocating parts, producing horizontal forces of varying magnitudes. We can balance all of the rotating parts by weights in the wheels (by methods which are beyond the scope of this book), but it is not possible to balance horizontal forces by rotating weights revolving with the cranks, since in this case we should be introducing vertical forces which are not balanced.

There is, however, a means of balancing horizontal forces which vary with S.H.M., by means of weights rotating in opposite directions (Fig. 31).

Using Two Weights

In this case two equal weights w at P and P^1 are geared together so that they rotate with the same speed in opposite directions.

The horizontal components ON and O^1N^1 of the centrifugal forces are in the same direction and add together, while the vertical components NP and N^1P^1 are in opposite directions and cancel one another.

Sometimes it is not practicable to place a single balance weight in the required position, and in this case two balance weights may be used as shown in Fig. 32.

An out-of-balance weight W, at radius r, is to be balanced by

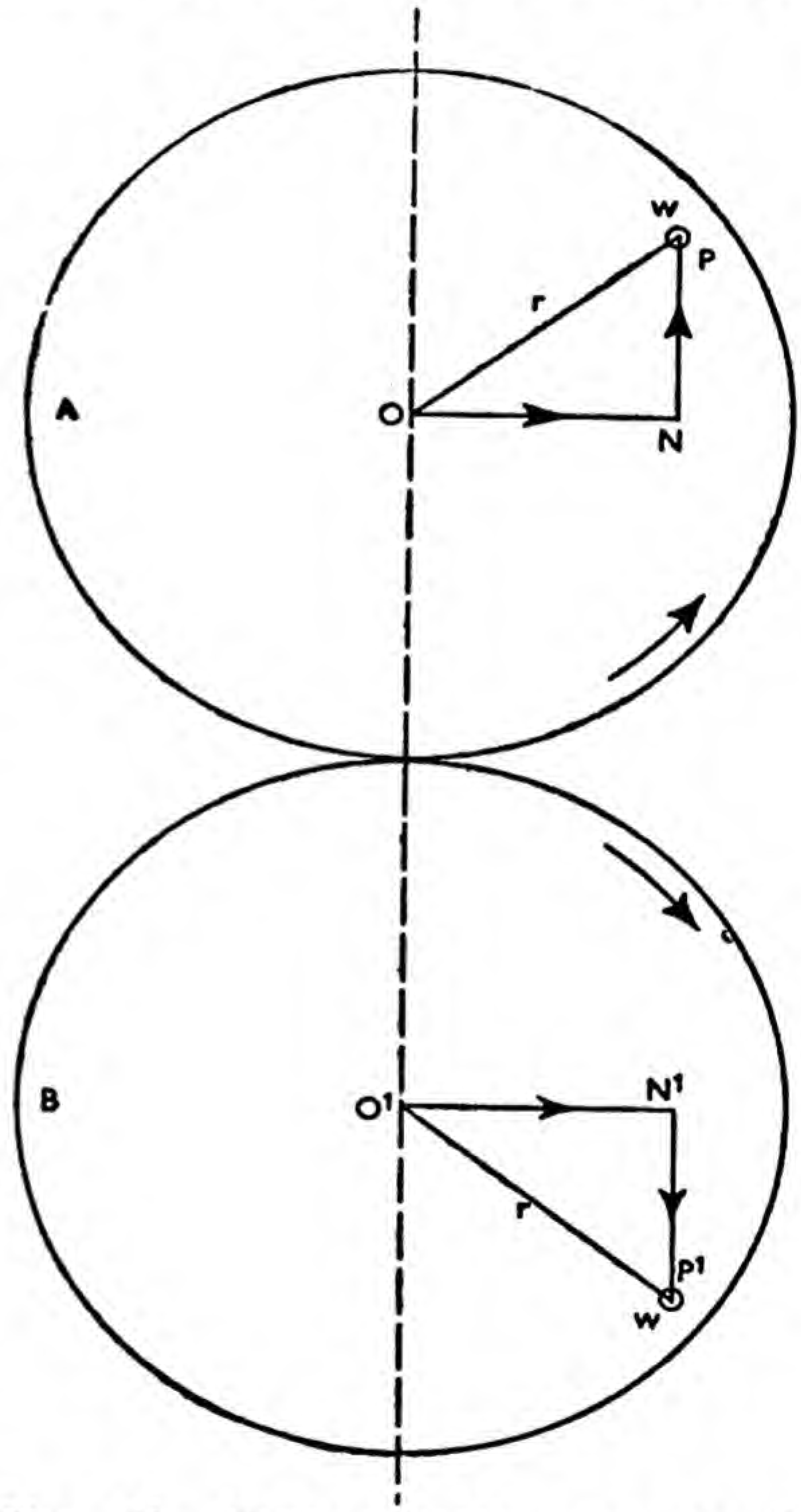
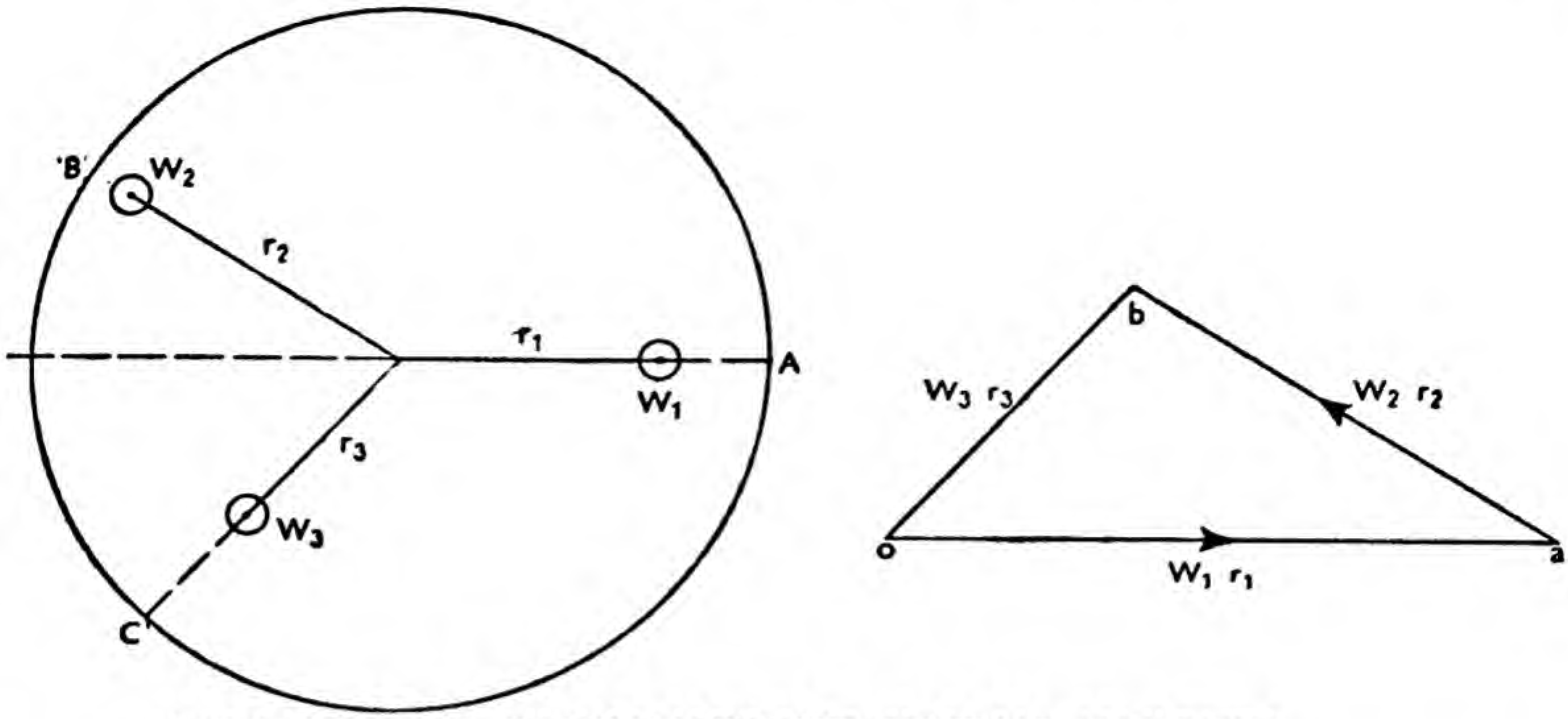


Fig. 31. Two wheels are geared together so that they rotate in opposite directions. Each of these is out of balance by the equivalent of a weight W at radius r. The centrifugal force on wheel A is equivalent to a vertical force NP (upward) and a horizontal force ON. Similarly the force on wheel B is equivalent to a vertical force N^1P^1 (downward) and a horizontal force O^1N^1.



BALANCING A SINGLE WEIGHT BY TWO WEIGHTS

Fig. 32. It is required to balance W_1 at radius r_1 by weights at radii r_2 and r_3 acting along OB and OC respectively. OB and OC can be in any convenient directions. Draw the vector oa representing the value of W_1r_1 to a chosen scale. Draw ab parallel to OB and draw ob parallel to OC, cutting ab in b. ab represents to the chosen scale the necessary value of W_2r_2 and ob similarly represents the necessary value of W_3r_3 for balance.

weights on the lines OB and OC, in the same plane as W_1.

Draw the line oa, representing to scale the value of W_1r_1. From o draw ob parallel to OC and from a draw ab parallel to OB, thus completing the triangle oab. ab is then the required value of W_2r_2, and ob is the required value of W_3r_3. If convenient values of r_2 and r_3 are chosen, the necessary values of W_2 and W_3 are thus found.

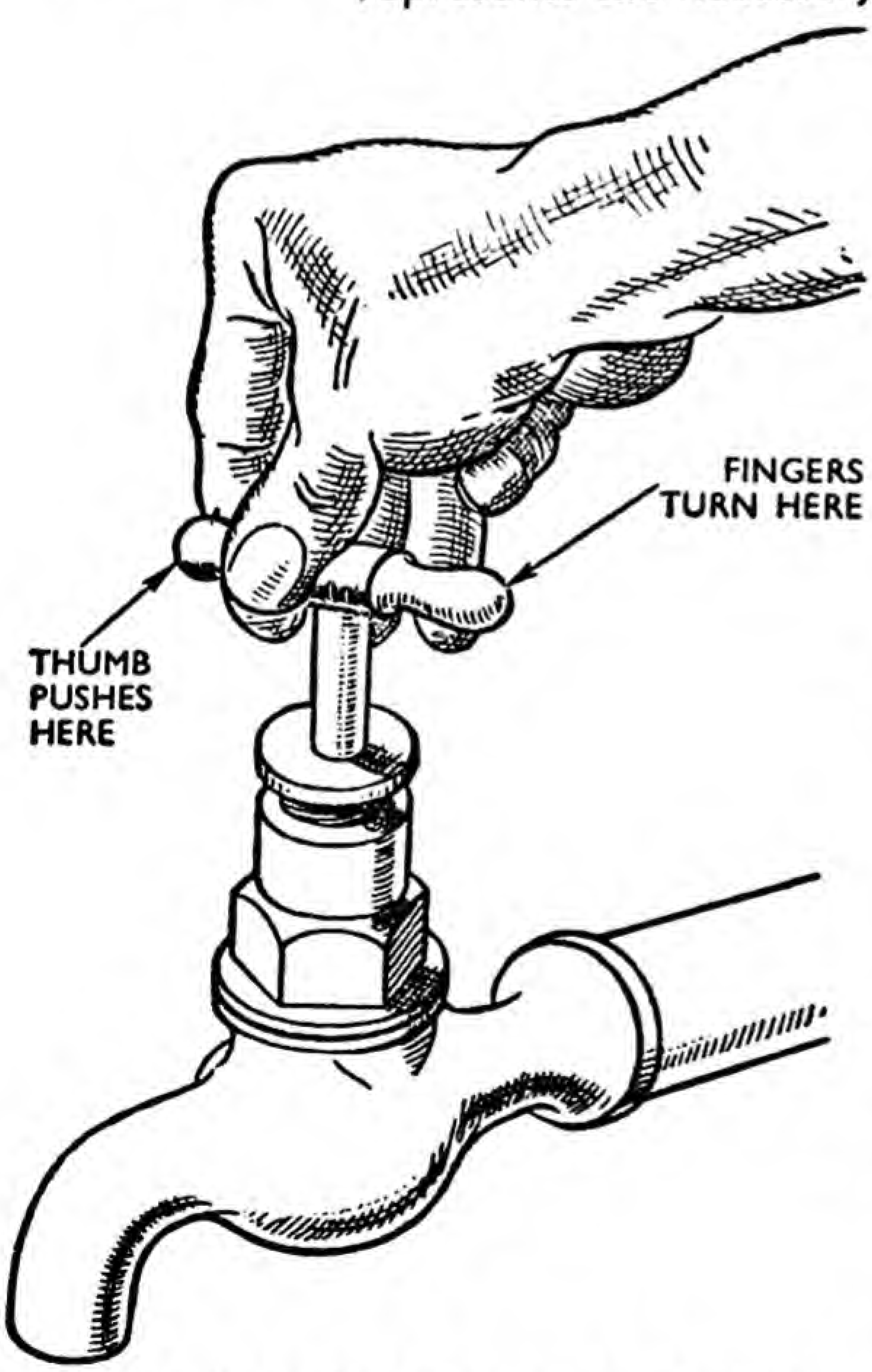


Fig. 33. In order to turn an ordinary water tap, a torque must be applied to the spindle. This is done by a pressure from the thumb on one side and an equal and opposite force from the fingers on the opposite side.

Applying a Torque

We have seen that in every case where we wish to produce or maintain rotation, a torque, or twisting moment, must be exerted. In turning an ordinary water tap, for instance, a torque must be applied by the thumb and fingers applying equal and opposite forces about $1\frac{1}{4}$ in. apart (Fig. 33). These forces must be equal and opposite, since we do not wish to produce any bending effect on the spindle of the tap.

If a large stop valve is to be

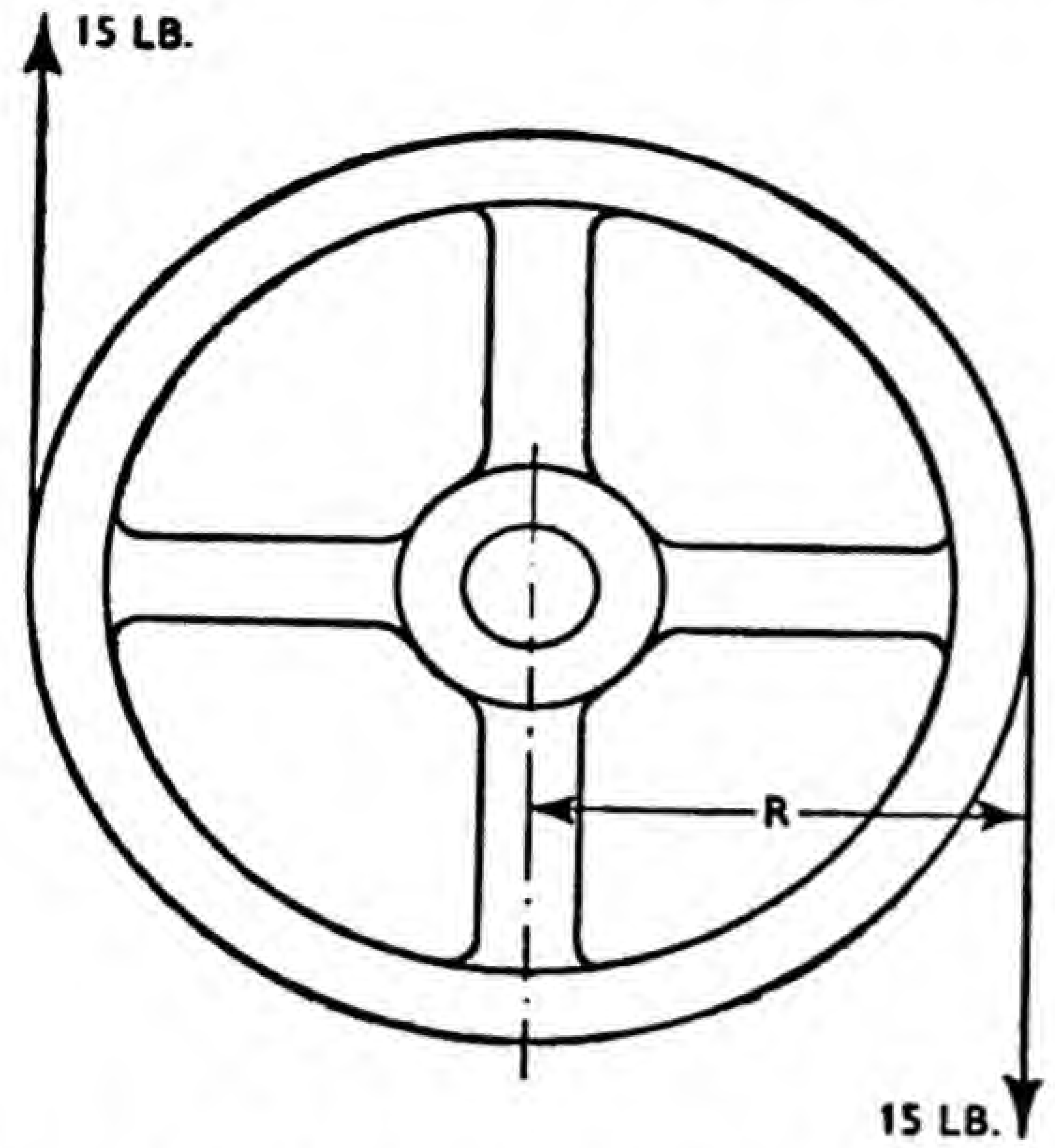


Fig. 34. The torque to be applied to the handwheel for closing a valve is 25 lb.-ft. and the forces to be applied on each side must not exceed 15 lb. Hence R must be chosen so that $2 \times 15R$ is 25 lb.-ft. or 300 lb.-in. R must, therefore, be at least 10 in. and a wheel 20 in. diameter is necessary.

closed, we must choose the diameter of the hand-wheel so that the force we are able to apply will give the required torque.

If the force applied to the rim of the wheel by each hand is 15 lb. weight, and the required torque for closing the valve is 25 lb.-ft. (Fig 34), then,

$$2 \times 15 \times R\,(\text{ft.}) = 25$$

so that R must be at least $\frac{25}{30}$ or $\frac{5}{6}$ ft. (10 in.). In this case the diameter of the handwheel must be at least 20 in.

Overcoming Resistance

If, by means of a driving agent providing a small torque at a high speed (e.g., an electric motor or a petrol engine), we wish to overcome a much larger resisting torque on the driven shaft, we have to bear in mind that the rate of doing work (say, ft.-lb. per min.) must be the same for the driving and the driven shaft if we include friction as part of the resisting torque. This means that,

Driving torque $\times$ r.p.m. of driver $=$ Resisting torque $\times$ r.p.m. of driven shaft.

Thus, if the resisting torque is 3 times the driving torque, the speed of the driven shaft must be one-third of the speed of the driving shaft. One method of obtaining this speed ratio is by gear wheels (Fig. 35).

If the radius of the driving wheel A is 4 in., the radius of the driven wheel B must be 12 in. in this case. If the tangential load on each

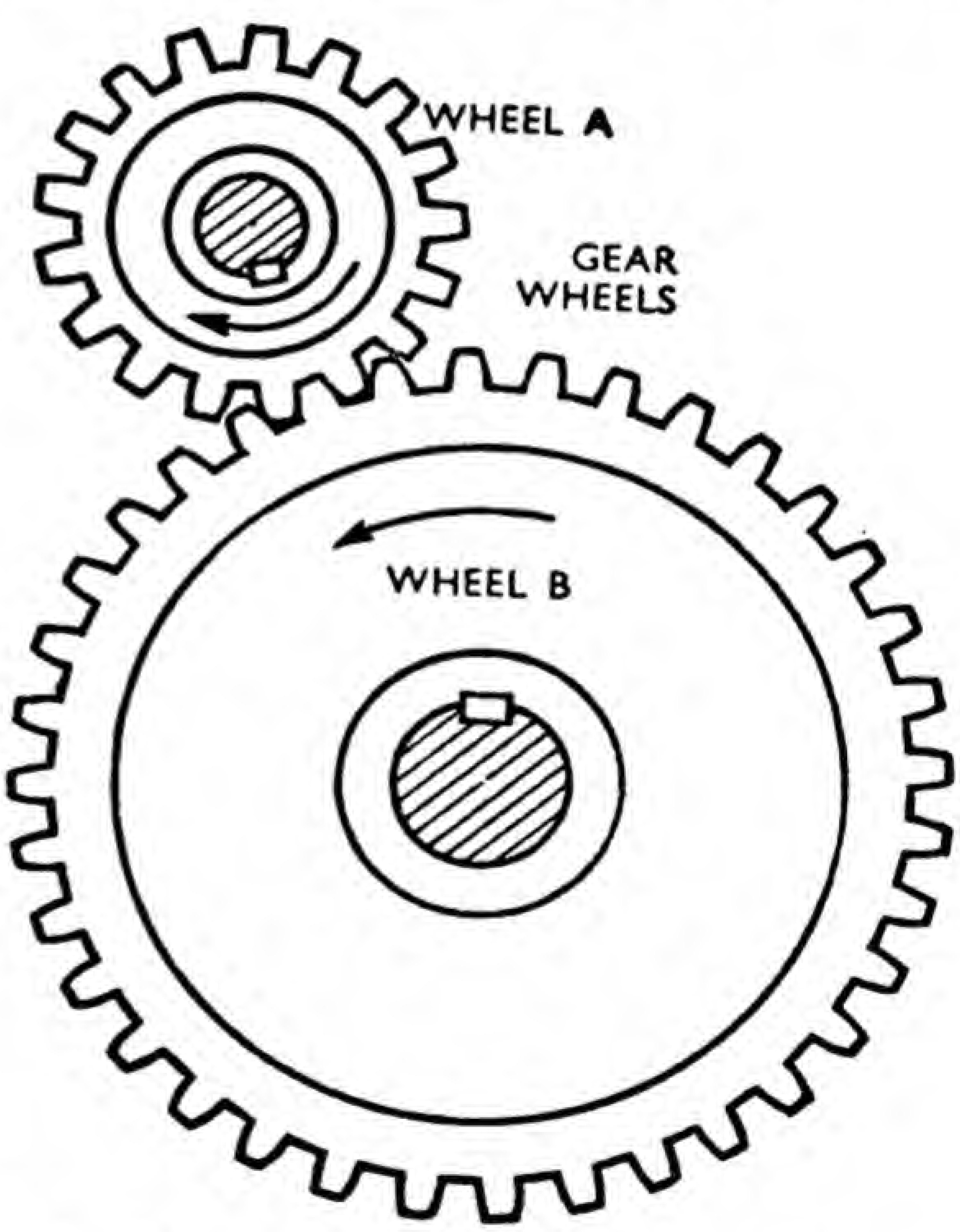
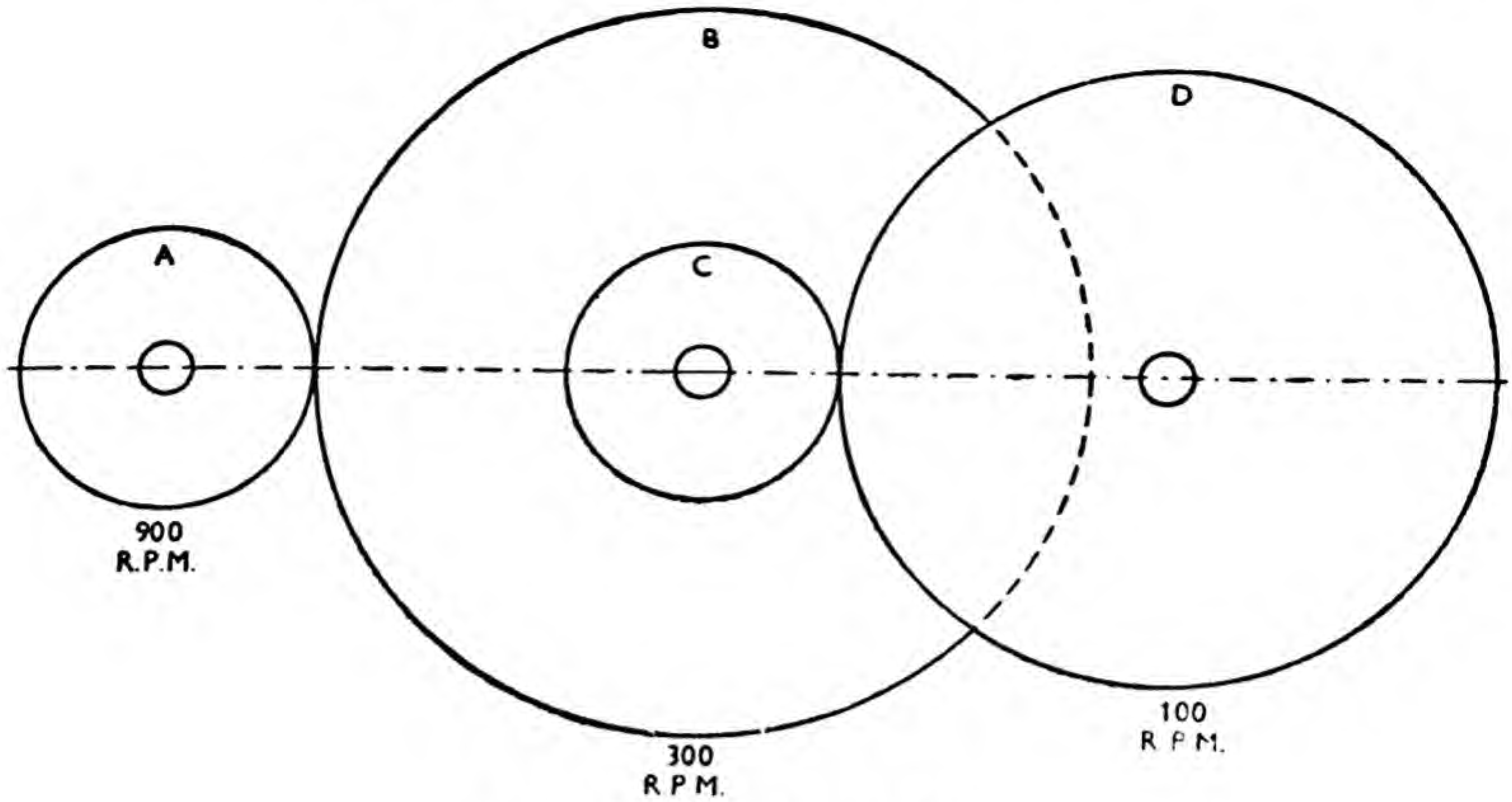


Fig. 35. This is a means of overcoming a large resisting torque on a driven shaft B by means of a smaller torque on the driving shaft A. If the torque on the driven shaft is to be three times the torque on the driving shaft, then the speed of the driven shaft must be one-third of the speed of the driving shaft. Thus the driven wheel must have three times as many teeth as the driving wheel. The speed ratio is evidently the reciprocal of the torque ratio.



COMPOUND GEARING

Fig. 36. If a large torque ratio is required with one pair of wheels, either the driving wheel will be too small or the driven wheel will be much too large. In this case compound gearing may be used. Wheel *A* is geared to *B*, *B* and *C* rotate together, and *C* is geared to *D*. If *B* is three times as large as *A*, speed of *B* (also *C*) is one-third of speed of *A*. If *D* is three times as large as *C*, speed of *D* is one-third of speed of *C* and consequently one-ninth of speed of *A*.

wheel at the point where contact takes place is 300 lb.,

Torque on wheel *A*

$$= 300 \times \frac{4}{12} = 100 \text{ lb.-ft.}$$

Torque on wheel *B*

$$= 300 \times \frac{12}{12} = 300 \text{ lb.-ft.,}$$

which verifies the above rule.

If a torque ratio of 9 to 1 is required and the radius of wheel *A* remains at 4 in., the radius of wheel *B* would need to be increased to 36 in., that is, wheel *B* would have to be 6 ft. in diameter. Even larger gear ratios are required in practice, and we must find some means of keeping down the size of the driven wheel.

Compound gearing may be used (Fig. 36). In this case the wheel *A*, 8-in. diameter, drives the wheel *B*, 24-in. diameter. *B* and *C* are fixed to the same shaft and rotate at the same speed. Wheel *C*, say 9-in. diameter, drives the wheel *D*, 27-in. diameter.

$$\text{Speed of } B = \frac{8}{24} \times 900 = 300$$

r.p.m. = speed of *C*.

$$\text{Speed of } D = \frac{9}{27} \times 300 = 100$$

r.p.m.

By this arrangement we have succeeded in reducing the size of the driven wheel *D* from 6 ft. to 2 ft. 3 in.

It should also be noted that wheels *B* and *C* are now intermediate wheels.

Augmenting Torque

The object of fitting gear boxes to motor vehicles is to augment the torque of the engine to that required at the rear axle for driving the vehicle. If, for instance, the

torque provided by the engine at its working speed is 120 lb.-ft., and the torque required at the rear axle is 1,200 lb.-ft., then gears must be provided which will reduce the speed of the rear axle to $\frac{1}{10}$ of the engine speed. If the vehicle is climbing a gradient which increases the torque on the rear axle to 2,400 lb.-ft., the gear ratio must be altered so that the speed of the rear axle is now $\frac{1}{20}$ of the engine speed. We do not wish to run at the slower speed on the gradient, but we can magnify the engine torque only at the expense of the speed of the driven axle, that is, also, of the vehicle.

Let us illustrate this by a concrete example.

Given weight of vehicle = 2,200 lb. Resistance to motion on the level = 40 lb. (assumed constant). Diameter of wheels = 2·5 ft. Torque provided by engine = 30 lb.-ft. To find the gear ratio required for climbing a gradient of 1 in 10 at a steady speed.

Total resistance to motion

$= 40 + \frac{2,200}{10} = 260$ lb.

$\therefore$ Torque on rear axle

$= 260 \times \frac{2\cdot5}{2} = 325$ lb.-ft.

$\therefore$ Required gear ratio

$= \frac{325}{30} = 10\cdot8$ (say 11).

If the engine delivers the torque of 30 lb.-ft. at 3,000 r.p.m., then, B.h.p. of engine (see Chapter 10) $= \frac{2\pi NT}{33,000}$

$= \frac{6\cdot28 \times 3,000 \times 30}{33,000} = 17\cdot2.$

Speed of rear axle

$= \frac{3,000}{11} = 273$ r.p.m.

$\therefore$ Speed of vehicle $= 273 \times \pi \times 2\cdot5 = 2,140$ ft. per min.

$= \frac{2,140}{88} = 24\cdot3$ m.p.h.

since 1 m.p.h. = 88 ft. per min.

If the distance between the driving and driven shafts is too large for

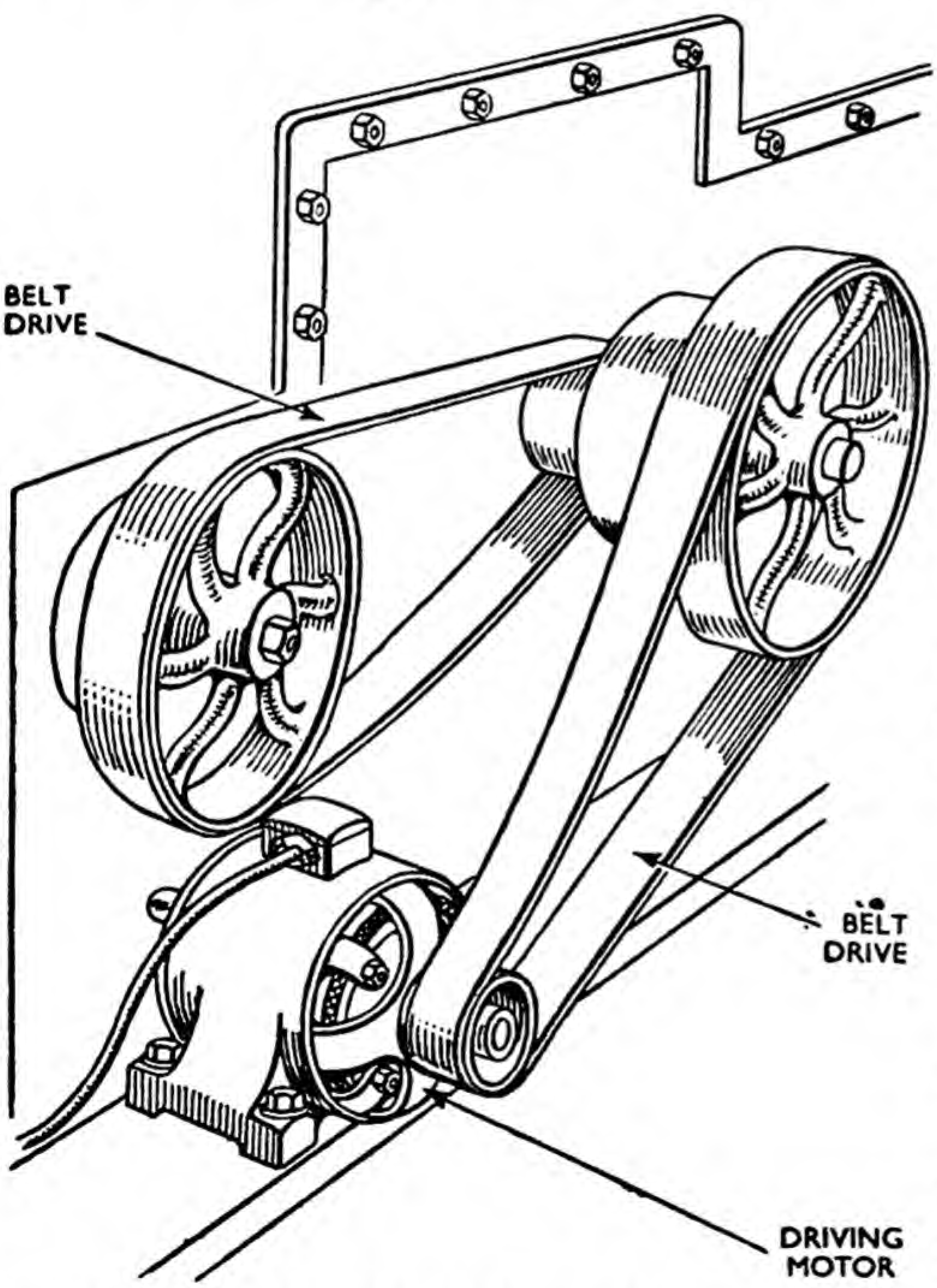


Fig. 37. If the driving and driven shafts are some distance apart or inconveniently situated, belt drives may be used. In this case a high-speed motor is driving a comparatively slow-speed shaft. The pulleys on the intermediate shaft are connected to the driving and driven shafts respectively by belts. The overall speed ratio is calculated from the diameters of the pulleys.

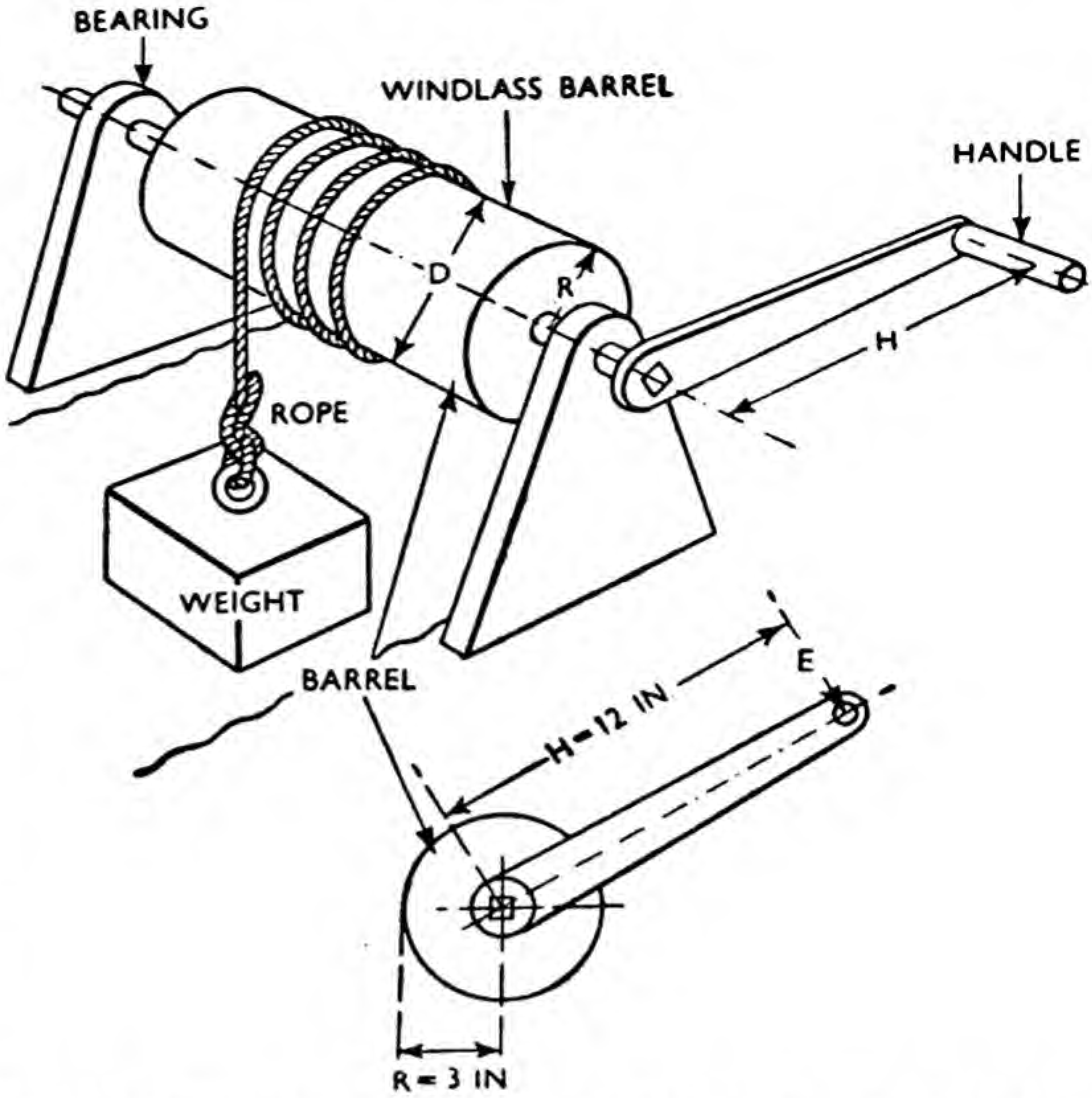


Fig. 38. This is one of the simpler applications of the law of work. In one turn of the handle the effort (assumed to be acting at right angles to the handle throughout) moves a distance $2\pi \times 12$ in. while the weight is lifted $2\pi \times 3$ in. If the weight is 70 lb., work done by effort $E = E \times 2\pi \times 12$; work done on weight $= 70 \times 2\pi \times 3$. By equating these we find that an effort of 17·5 lb. is required.

the use of gearing, chains or belts may be used (Fig. 37). The method of calculating the velocity ratio is the same.

All of the above are special instances of the general law known as the law of work (or principle of work). It may be stated as follows :—

If in any machine whatever (lever, screw jack, crane, hydraulic machinery, machine tool), an effort E, moving through a distance x, overcomes a total resistance R through a distance y, then the work done by the effort is equal to the work done on the resistance. (The total resistance will include frictional resistances.)

Taking the simple case of the wheel and axle (Fig. 38), consider one turn of the handle and let the weight be 70 lb.

Work done by effort at right-angles to handle =

$E \times 2\pi \times 12$ in.-lb.

Work done on weight =

$70 \times 6{\cdot}28 \times 3$ in.-lb.

By the above law,

$E \times 6{\cdot}28 \times 12 = 70 \times 6.28 \times 3.$

$\therefore E = 17{\cdot}5$ lb. weight.

It will be seen that it is not necessary to know the details of construction of a machine in order to find the ratio of the total resistance overcome to the effort. All we have to do is to move the point at which the effort is applied through a known distance x and measure the distance y through which the resistance is overcome.

$$\text{Then, } \frac{R}{E} = \frac{x}{y}.$$

The ratio of the external resistance (70 lb.) to the total resistance R, obtained as above, is known as the efficiency of the machine.

If, for instance, the efficiency of the machine is 90 per cent, the 17·5 lb. would only be 90 per cent of the actual effort required, which should, therefore, be 19·5 lb.

Efficiencies vary considerably in practice, according to the complexity of the machine, and the effectiveness of the lubricating arrangements.

CHAPTER 5

FRICTION AND LUBRICATION

FRICTION BETWEEN BODIES. LAWS OF DRY FRICTION. RESISTING MOTION. RESULTANT REACTION. INCLINED PLANE AND WEDGE. ANGLE OF SLOPE. WHY A SCREWTHREAD HOLDS. FRICTION BRAKES. KINDS OF CLUTCH. BELT AND ROPE TRANSMISSION. EFFECT OF LUBRICATION. REDUCING BEARING FRICTION. PRESSURE OF THE LUBRICANT. THE MICHEL THRUST BLOCK. REDUCTION OF WEAR BY LUBRICATION. EFFECTIVENESS OF OIL FILM.

WHEN one body slides over the surface of another, a resistance to sliding is felt. This resistance is called the friction between the bodies. The effects of friction whenever a match is struck against a box, or when a sack of potatoes is dragged across a floor are familiar. The small amount of friction experienced when one of the bodies is ice may be a source of inconvenience and danger on frosty days.

Many theories have been put forward to explain the friction between sliding bodies, but none is altogether satisfactory. We are concerned mainly with the effects of friction, and, in order to observe and classify these effects a few simple experiments will be given.

There are several kinds of friction, the three most important being called dry or solid friction, boundary or greasy friction, and fluid friction. Dry friction is encountered most often in everyday life, as when a book is pushed across a table top. The second, greasy friction, occurs when a lubricant such as oil or grease is present. Some of the effects of grease are observed when it is applied to the axle bearings of a wheelbarrow. Fluid friction is not so well known. It occurs at the bearings of high-speed shafts in machines and engines when a film

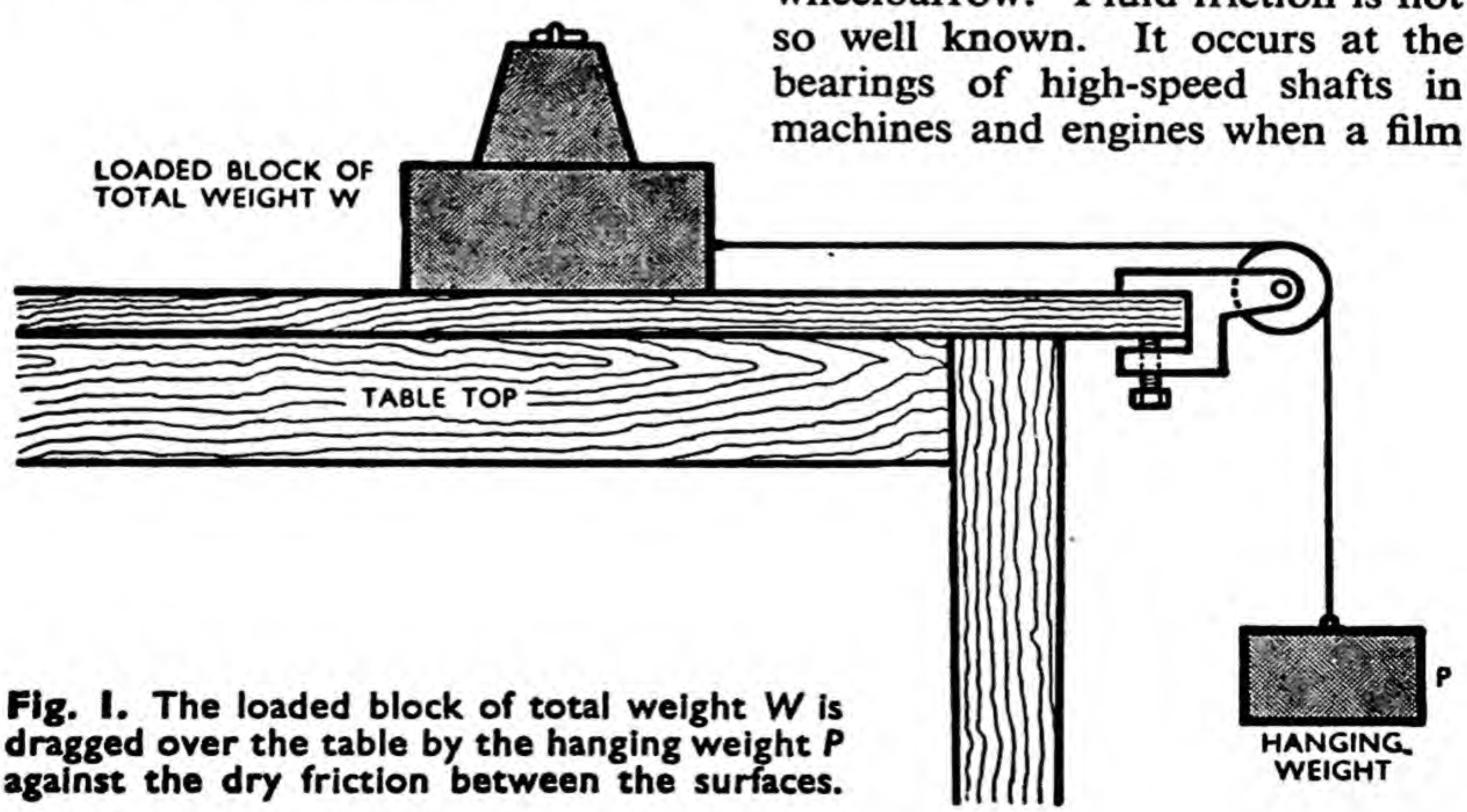


Fig. 1. The loaded block of total weight *W* is dragged over the table by the hanging weight *P* against the dry friction between the surfaces.

or layer of oil separates completely the sliding surfaces.

If a block of wood is placed on a level table top, it is found that a definite force is required to push it along. This force is equal to the friction force which always opposes the motion. The greater the weight of the block, the greater will be the force required. The force required to slide the block by means of a cord attached to it and passing over a pulley can be measured (Fig. 1). By hanging weights P on the other end of the cord, it is possible to make the block slide smoothly over the table, and to measure the force required, which will be P. By adding weights to the block we shall then be able to obtain the value of the friction force P for each of a series of weights. If W is the total pressure between the sliding surfaces, a set of readings is obtained similar to the following :—

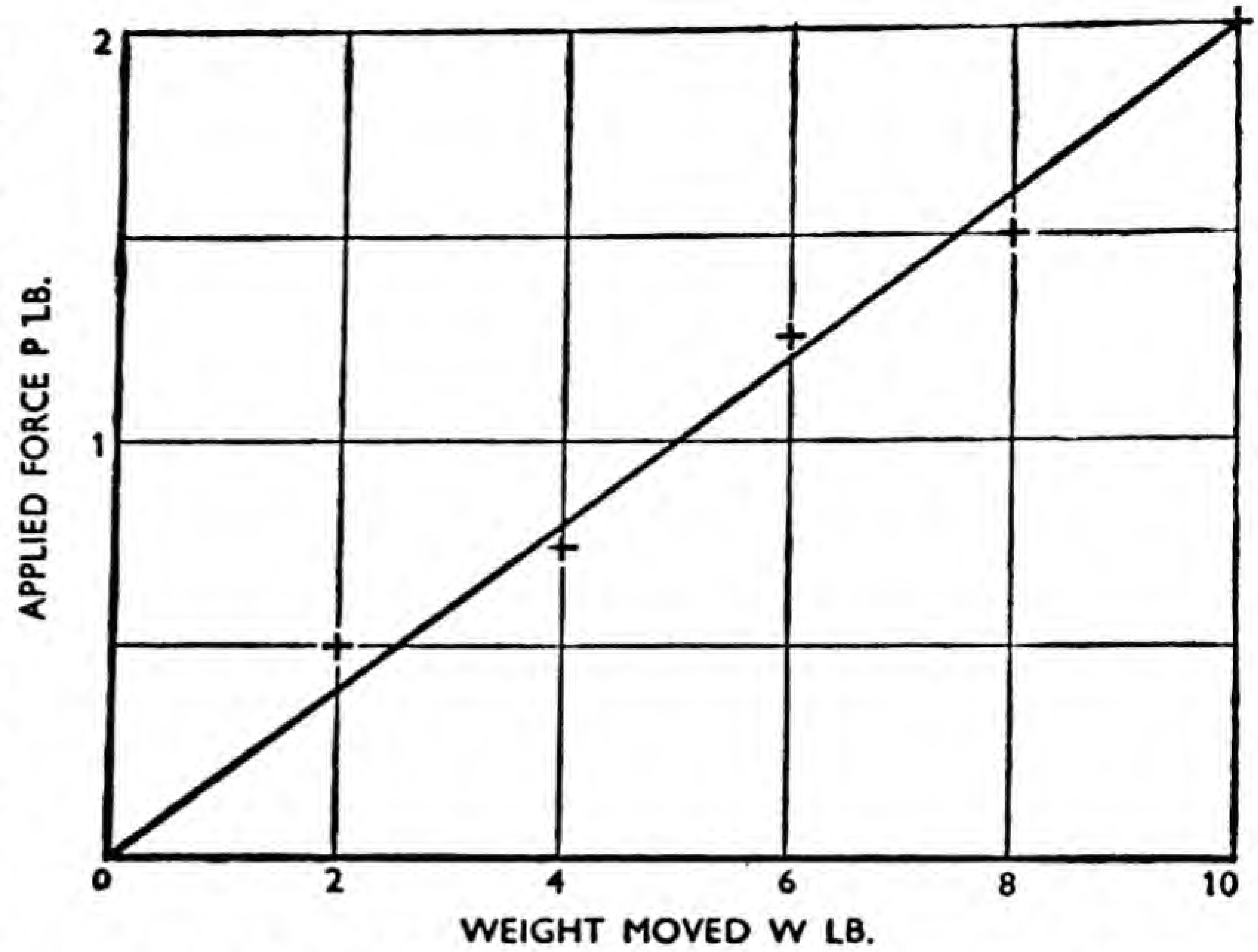


Fig. 2. The points marked + represent values obtained from an experiment carried out on apparatus illustrated in Fig. 1, and it is noticeable that they fall more or less on a straight line.

W lb.	2	4	6	8	10
P lb.	0·5	0·75	1·25	1·5	2·0

Depending on Pressure

Looking at these results, it can be seen at once that the friction depends upon the pressure between the sliding surfaces, and the force P is approximately one-fifth to one-quarter of W. We now draw a graph by plotting P against W (Fig. 2), and we find that the points lie on or near a straight line which passes through 0. If, to be more accurate, smaller weights had been used, it is probable that the points would have been closer to the straight line.

It is now very nearly certain that the friction, or resistance to movement, is directly proportional to the pressure between the sliding surfaces, or P is a certain fraction of W, in this case one-fifth or 0·2. This fraction is called the coefficient of friction and is usually denoted by the Greek letter μ. It can be found by dividing the friction force by the pressure between the sliding surfaces. If the coefficient of friction is known, then the friction force will be equal to the coefficient of friction multiplied by the pressure between the surfaces.

The same block is now placed on a smooth plank or board, one end of which rests on the table and the other end is supported on a pile of books or on another block. When the slope of the board is at a certain

angle (marked φ in Fig. 3), the block will slide gently down the board after it has been given a slight tap. It is possible to find that this angle is quite definite and remains the same whatever the weight on the block. If the angle is increased, the block will accelerate, and, if it is decreased, the block will not move.

The angle φ can be obtained by measuring the height h and the distance l. The ratio h/l is called the tangent of the angle φ. We shall find a list of tangents of angles in a set of Mathematical Tables.

Block in Equilibrium

When the block is just able to slide down the board, it is in equilibrium, and there are only three forces acting on it ; W, the weight of the block, R, the normal reaction between the board and the block and F, the friction force acting upward along the board in a direction opposing the motion. It is now possible to draw the triangle of forces *abc* for the forces W, F and R as shown in Fig. 3.

Now the coefficient of friction is the ratio of the friction force to the force between the surfaces, viz., $\mu = F/R$. But the angle between W and R is φ, and the tangent of φ is bc/ac, viz., F/R ; so that tan φ is equal to the coefficient of friction μ. The angle φ is called the angle of friction. This experiment forms a quick method of finding the coefficient of friction as it is necessary to measure only h and l.

The inclined board may be used to carry out further tests. If the base of the sliding block is roughened by rubbing it with a coarse sandpaper, it will be found that the slope of the board needs to be much greater to let the block move. The friction, therefore, must be greater. By trying blocks of different materials and different roughnesses it will be found that the angle φ depends upon both. For instance, a flat iron will do very nicely for a metal block, and it is noticed that the friction of metal on wood is less than for wood on wood, and that for metal on metal would be less again. A table may now be drawn up giving the angle of friction and the coefficient of friction for various pairs of surfaces.

It is noticed that the friction varies considerably for different pairs of materials. This is because

FRICTIONAL VALUES FOR VARIOUS PAIRS OF SURFACES

Surfaces in contact	Angle of friction φ (deg.)	Coefficient of friction μ
Wood on wood	11 to 26	0·2 to 0·5
Metal on wood	8 to 22	0·14 to 0·4
Metal on metal	8 to 17	0·14 to 0·3
Leather on metal	17 to 31	0·3 to 0·6
Stone on stone	22 to 33	0·4 to 0·65

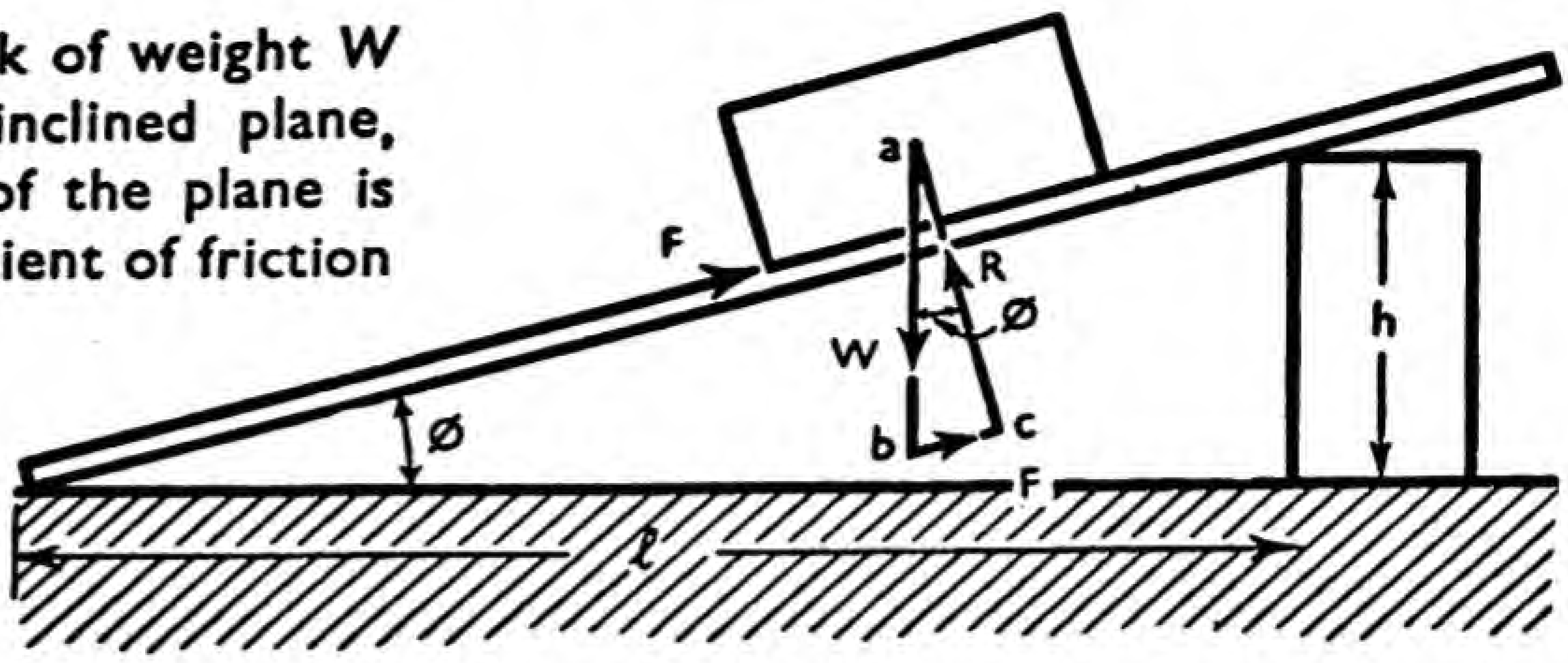


Fig. 3. When the block of weight W just slides down the inclined plane, the angle of slope ϕ of the plane is a measure of the coefficient of friction μ between the block and the plane, since μ is equal to tan ϕ, which is equal to $\frac{h}{l}$.

the nature as well as the smoothness of the surfaces has a very great effect upon the friction.

Summing up the above experiments, two of the laws of dry friction may be stated as follows :—

1. The friction is directly proportional to the normal pressure between the two surfaces.
2. The friction depends upon the nature and condition of the surfaces in contact.

Two other laws which may be obtained in the same way are :—

3. Friction is independent of the speed of sliding.
4. Friction is independent of the area of the surfaces in contact.

The last two laws may be applied only when the two sliding surfaces are perfectly dry and when the pressure between them is moderate. They are not correct for extreme conditions of pressure or speed of sliding.

Now let us see how these laws operate in practice. A chest when empty weighs 20 lb., and a horizontal force of 6 lb. applied to the handle will move it slowly over a smooth floor. What is the coefficient of friction, and what force will be required to drag the chest over the floor when it is filled with books weighing 80 lb. ? (Fig. 4).

Since the floor is horizontal, the pressure between the sliding surfaces will be equal to the total weight of the chest. When empty, the weight is 20 lb. and the friction force must be 6 lb. The coefficient of friction is equal to 6 lb. divided by 20 lb., viz., 0·3. When the chest contains 80 lb. of books, the total pressure on the floor will be 100 lb. The force required to pull it over the floor will be equal to the coefficient of friction multiplied by the total pressure, which is 0·3 × 100 lb., viz., 30 lb.

Limiting Friction

In the case considered above, it has been assumed that the body was on the point of sliding or was actually sliding. This kind of friction is known as limiting friction, and it is the greatest friction force that can be obtained between two given faces with a given pressure between them.

Returning to the block of wood on the table top. If the weight of the block were 10 lb. and the

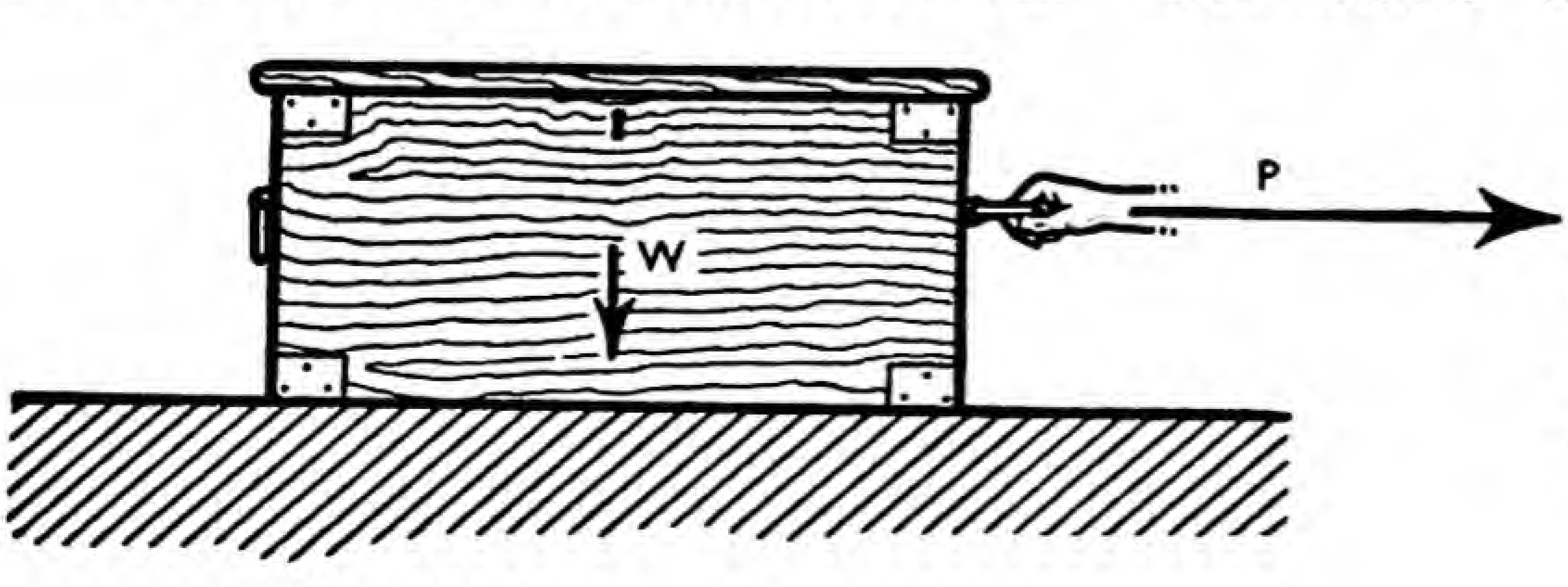


Fig. 4. Force P that is required to pull this chest full of books across the floor is equal to μW.

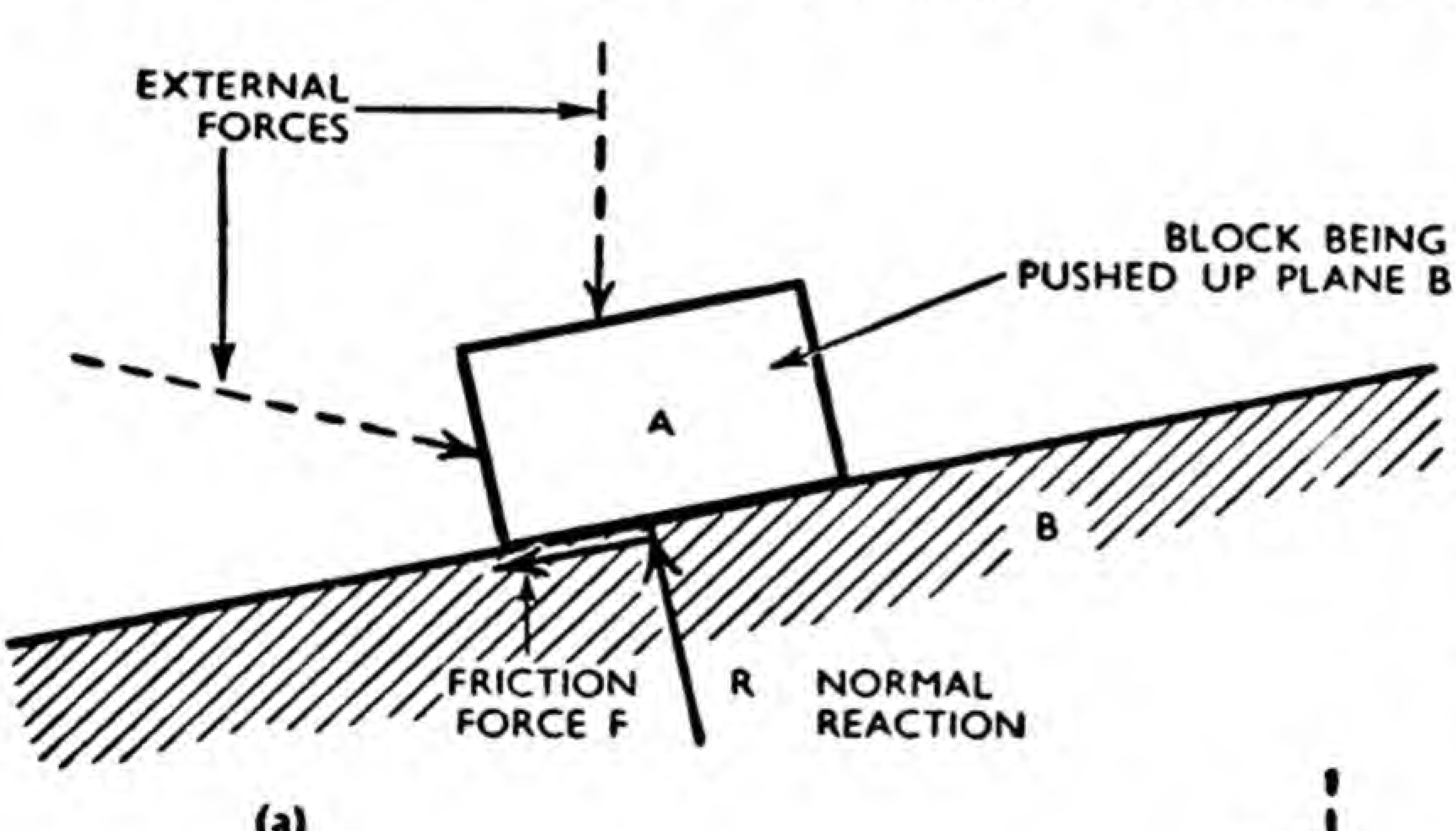
coefficient of friction were 0·2, a horizontal force of 2 lb. would just keep it sliding ; or if it were at rest, it would be on the point of sliding. This is the limiting friction force and is what is usually meant when speaking of friction.

Equal and Opposite Forces

Suppose that instead of applying the 2-lb. force, a force of 1 lb. only was applied. The block of wood would remain at rest, and would be in equilibrium under the action of the forces acting on it. The horizontal force of 1 lb. must be balanced by an equal and opposite force which can only be supplied by the friction. The friction force is now 1 lb. and will increase as the applied force is increased until it reaches the limiting value, namely μW. If the applied force is greater than the limiting friction, then there will be an unbalanced horizontal force and the block of wood will accelerate.

In all cases where forces acting on a body when friction is acting are considered, the friction force always acts in conjunction with a normal reaction at the surface where sliding may occur. In Fig. 5(a), we have a block A on which are acting a number of external forces, shown dotted, the normal reaction R and the friction force F.

It is assumed that A is going to slide up the surface of B. The resultant reaction of the plane B on A is the sum of the two forces R and F. This force, which is called Q, can be found by means of the



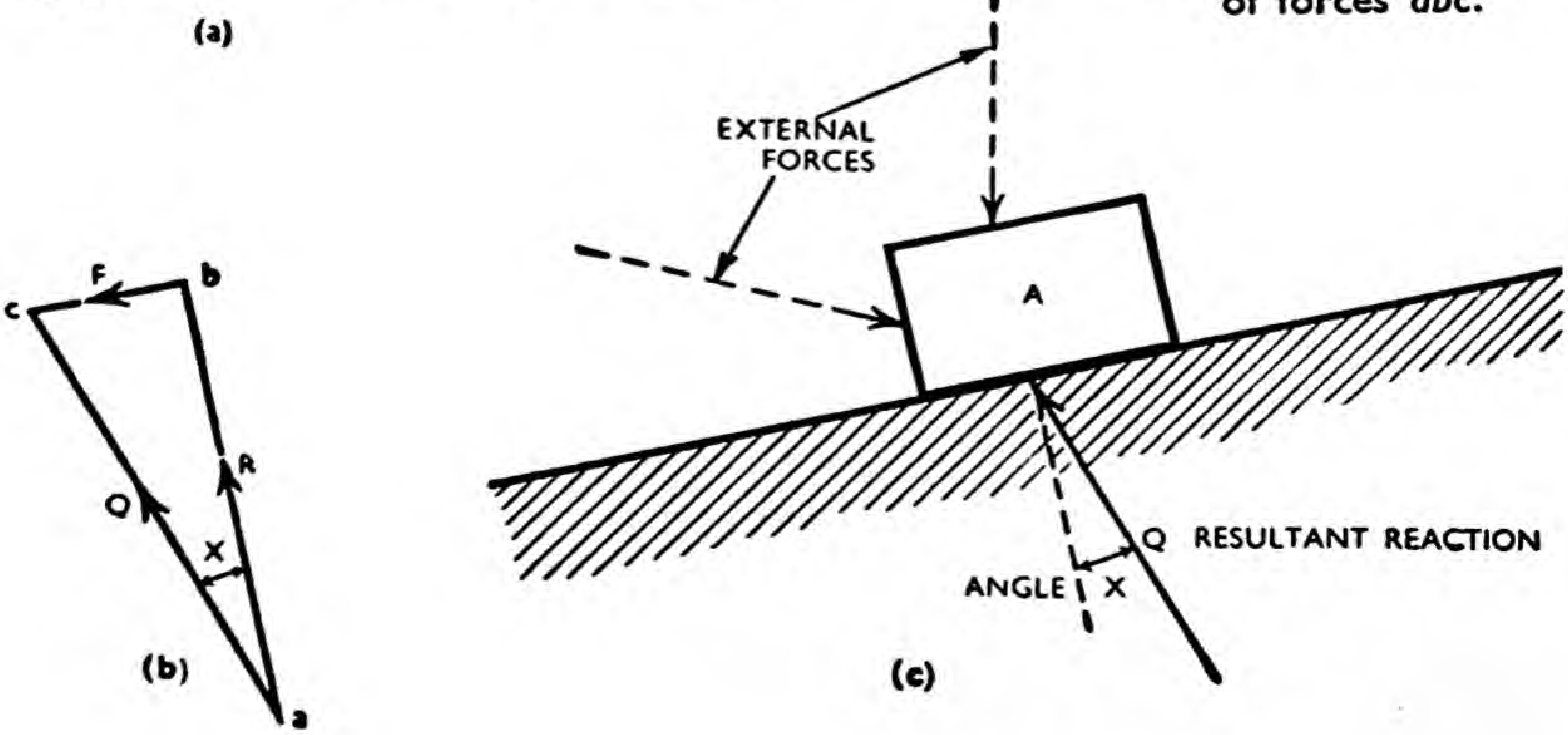


Fig. 5. The forces acting between the sliding block and the fixed plane are the normal reaction R and the frictional force F. These may be replaced by a single force Q, called the resultant reaction. This may be obtained from the third side ac of the triangle of forces abc.

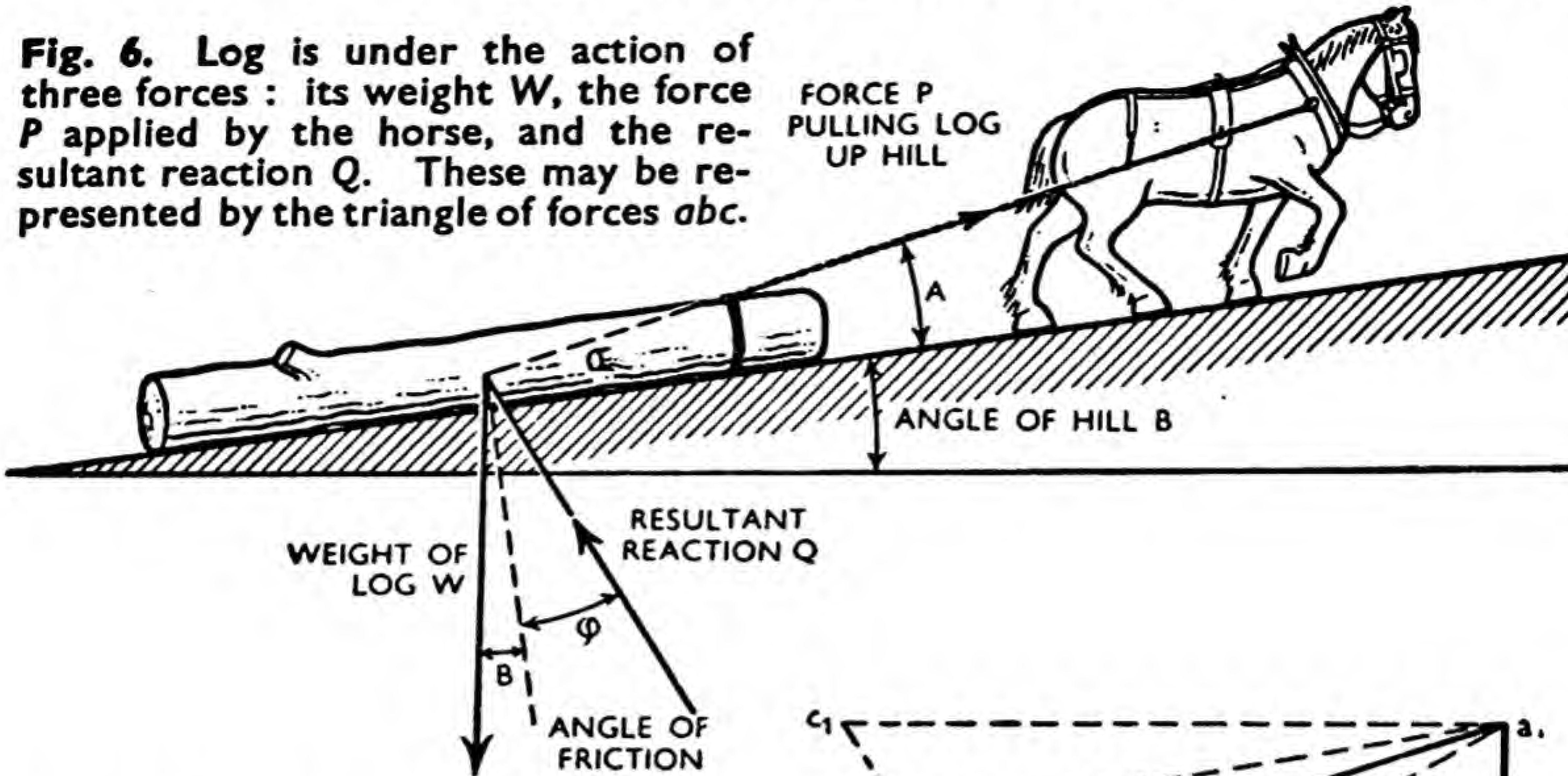


Fig. 6. Log is under the action of three forces: its weight *W*, the force *P* applied by the horse, and the resultant reaction *Q*. These may be represented by the triangle of forces *abc*.

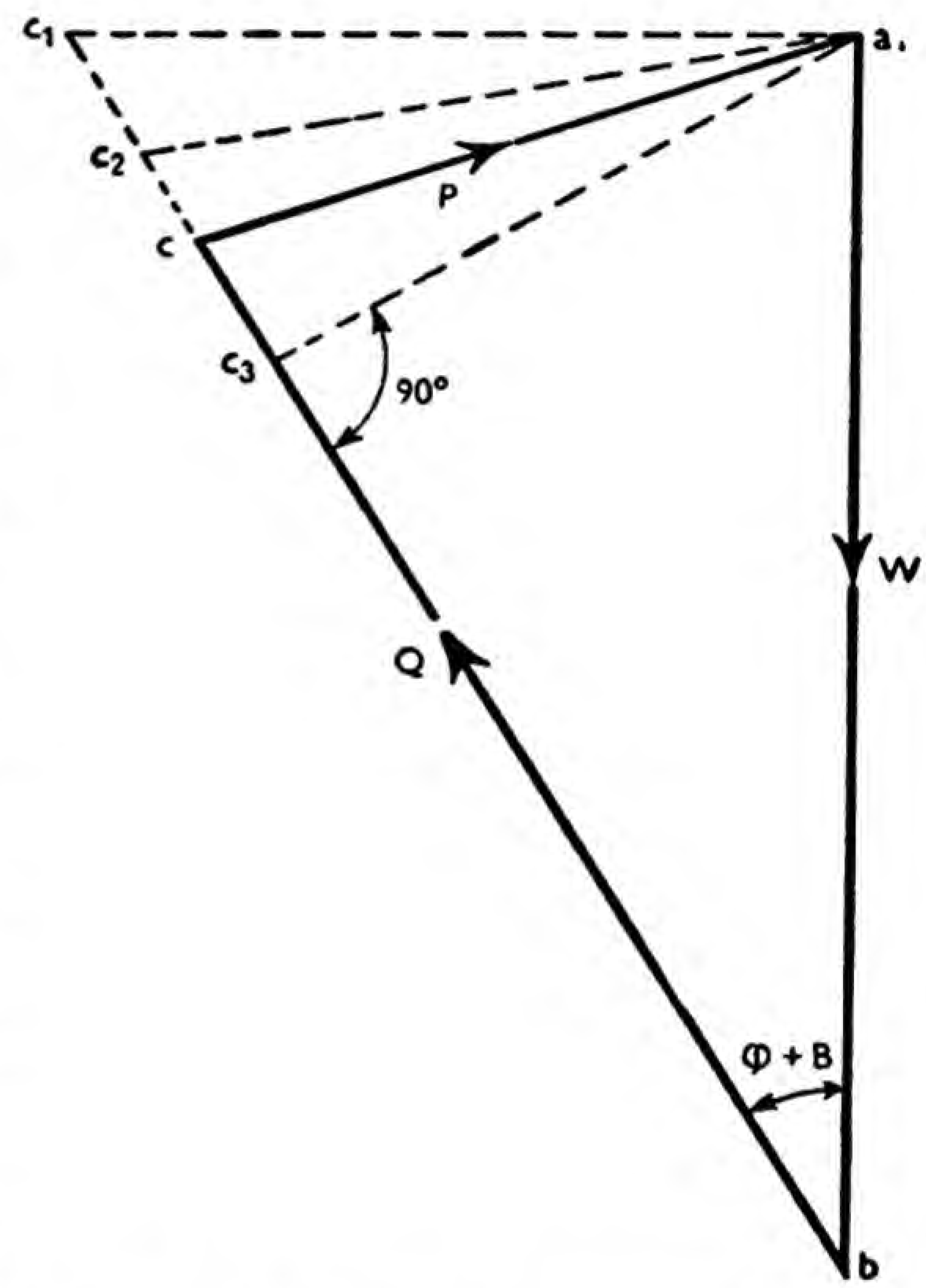
triangle *abc* (Fig. 5(b)). The forces *R* and *F* can now be replaced by the resultant reaction *Q* as in Fig. 5(c).

The angle *X* can be found from $\tan X = F/R$.

If *A* is on the point of sliding, the friction will be limiting, and $F = \mu R$, so that $\tan X = \mu R/R = \mu = \tan \varphi$, viz., $X = \varphi$, the angle of friction.

The case of a load being dragged up an inclined plane may now be considered. In Fig. 6, a log is being dragged up a hill. The log weighs 800 lb., the slope of the hill is 1 in 7, which is about 8 deg., the coefficient of friction is 0·4 and the drag-rope is at an angle *A* to the plane. The resultant reaction *Q* must lie in the direction shown in the figure, since we know that $\tan \varphi = 0{\cdot}4$, and the friction force is resisting the motion of the log.

Of the three forces acting on the log we know the magnitude and direction of one, namely the weight, and the direction of the other two, so that the triangle of forces *abc* can be drawn. The force *P* required to drag the log up the hill can be found by measuring the length of the line *ca*. The force *P* will depend upon the angle *A*, viz., the direction of the drag-rope. Commencing with



the drag-rope horizontal, the force will be c_1a; when it is parallel with the plane, the force will be c_2a. The force will go on decreasing as the angle *A* is increased until the rope is perpendicular to the resultant reaction *Q* when it is given by c_3a. This will be the minimum force which will be required to haul the log up the hill. If the figure has been carefully drawn to scale it is found that the minimum force *P* is 400 lb.

In Fig. 6, the angle between the

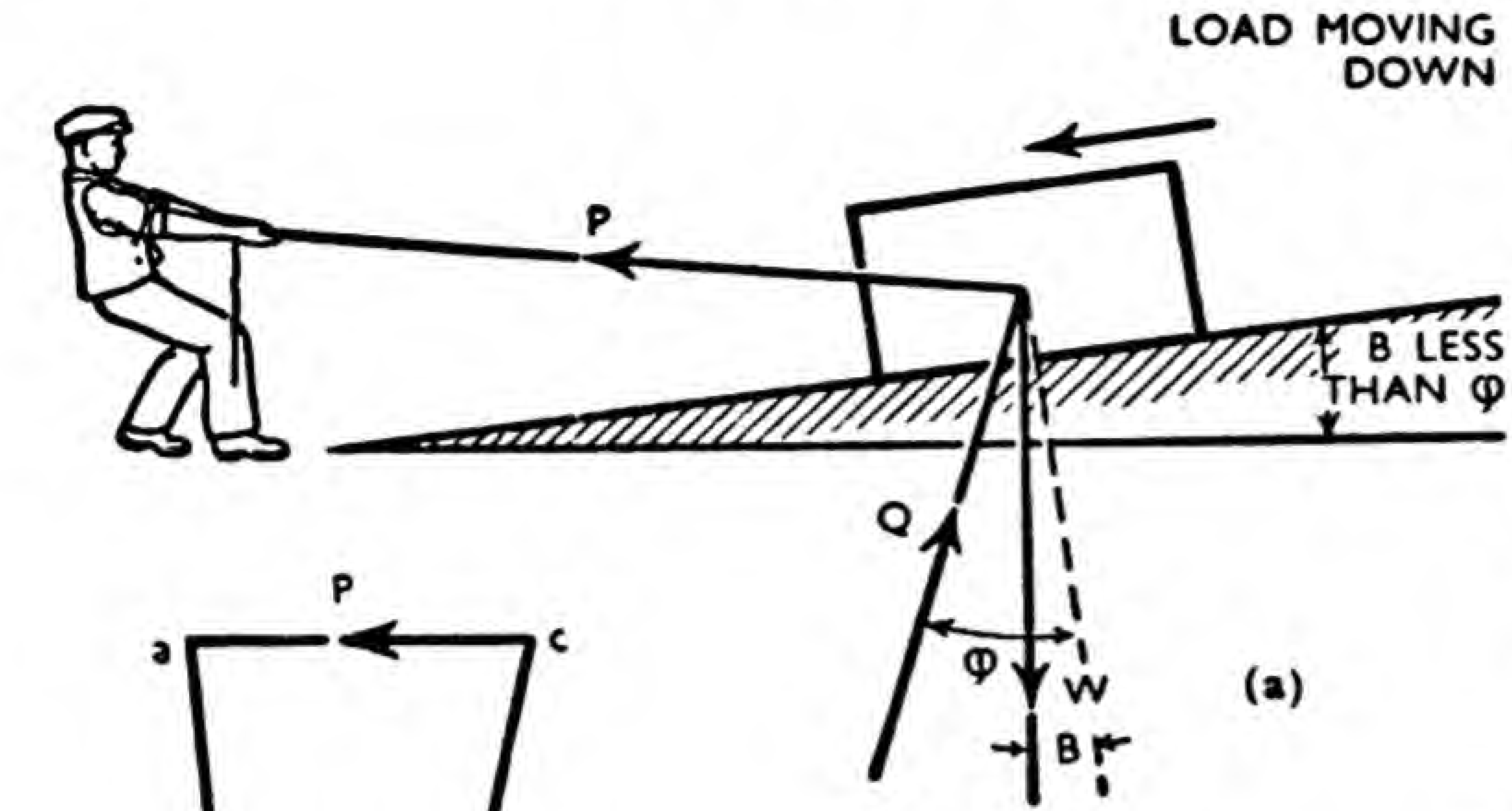
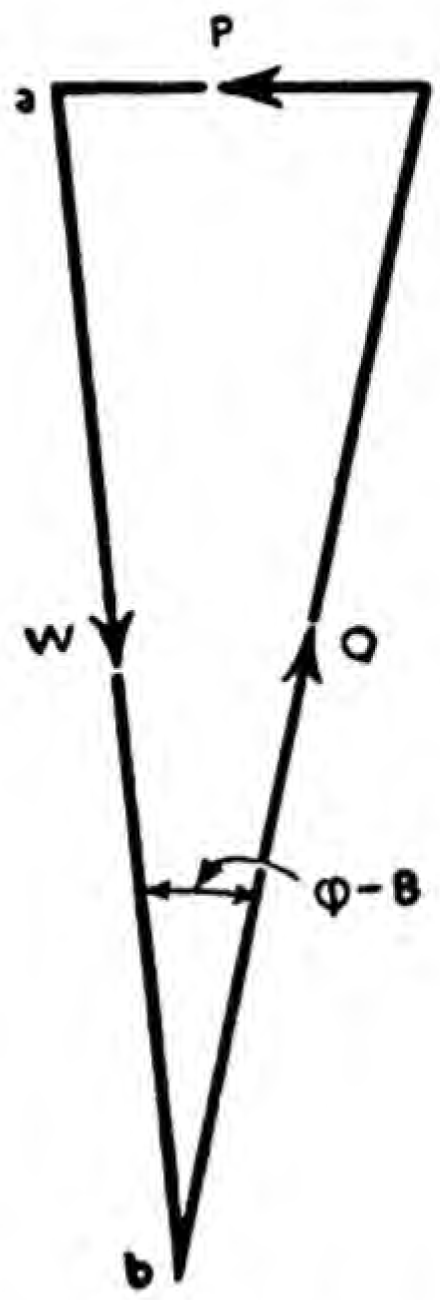
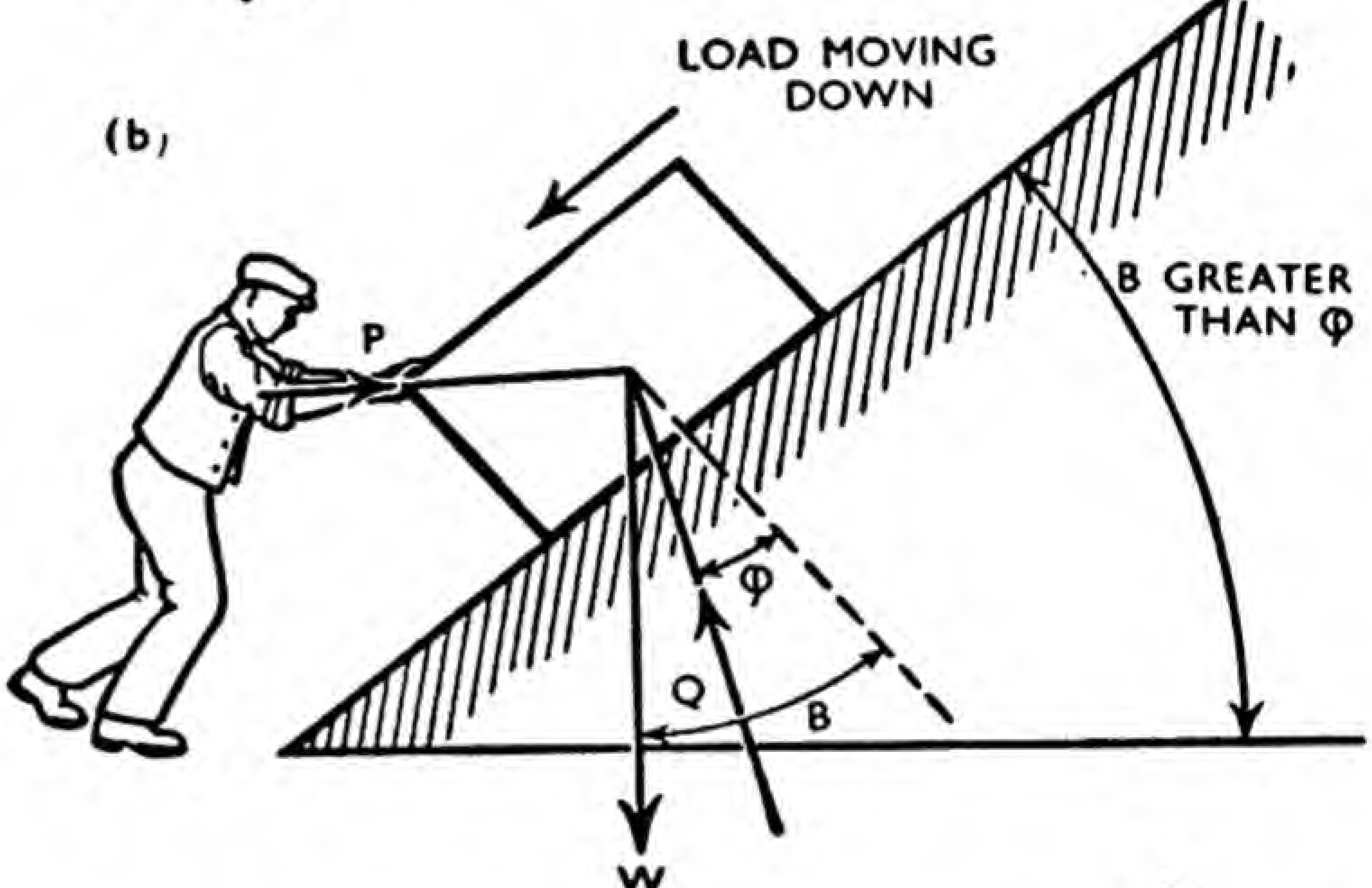


Fig. 7. (*a*) When the slope of the plane is less than the angle of friction, the load must be *pulled* down. (*b*) When the slope of the plane is greater than the angle of friction, the load, if it is to be lowered gently, must be pushed.

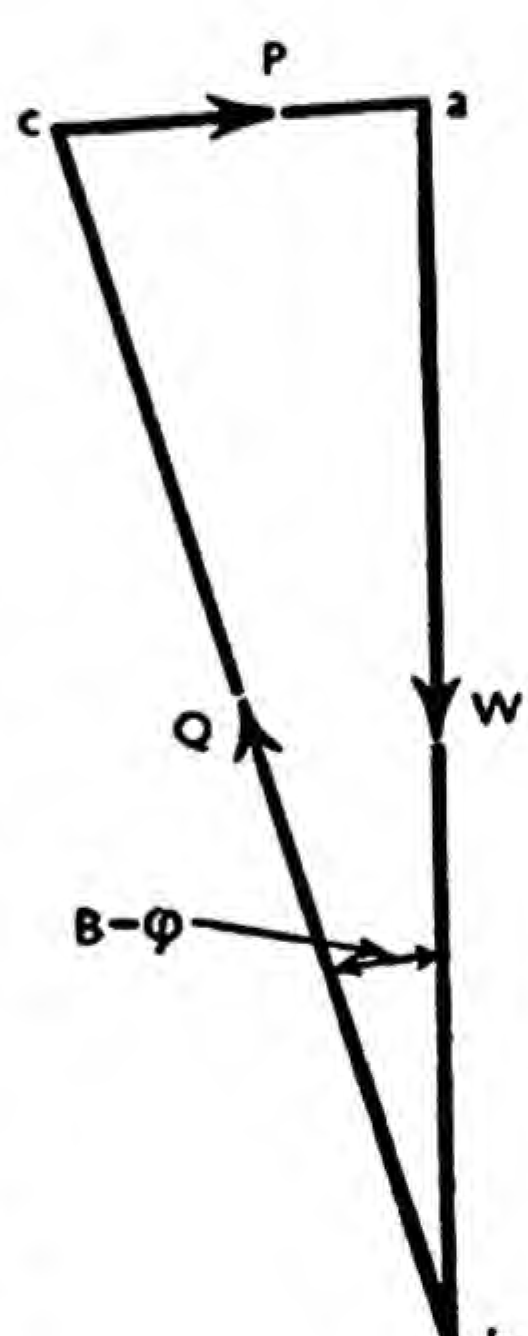




weight W and the resultant reaction Q, is $(\varphi+B)$. Now the tangent of the angle $(\varphi+B)$ is the length c_1a divided by the length ab. But c_1a represents the horizontal force required to pull the log up the hill, and ab represents the weight, so that it can be said that the horizontal force required is equal to the weight multiplied by the tangent of the angle $(\varphi+B)$. Putting it as an equation: $P = W \times \tan(\varphi + B)$, when P is horizontal.

Finding Force Required

To get a load down a slope, it may have to be pulled down or it may have to be held back. It depends upon the steepness of the slope. Fig. 7(a) shows a load W on a slope where the angle B is less than the angle of friction φ. By drawing the triangle of forces abc it is found that the force P must act down the slope to move the load. Care must be taken in this case to put the resultant reaction Q in so that it will oppose motion *down* the plane.



If the force P is horizontal, ca will be at right angles to ab. The angle between the weight W and Q is now $(\varphi-B)$, so that the horizontal force required to pull the load down the plane is the weight W multiplied by the tangent of the angle $(\varphi-B)$, or $P = W \times \tan(\varphi-B)$.

If the force is not applied to the load it will remain at rest on the plane, no matter how heavy it is.

In Fig. 7(b), the angle of the slope B is greater than the angle of friction φ, and again the load is to be lowered down the plane. In this case it is found that the force P must be applied so that it acts *up* the plane, and if it is horizontal, the equation is :— $P = W \times \tan(B - \varphi)$. The force is required to hold the load back, otherwise it would accelerate down the slope.

Small Coefficient

When a ship is launched, the slipways are greased so that the coefficient of friction is small. The angle of the slipway is then greater than the angle of friction and it is necessary to apply restraining forces to the ship in order that it may enter the water at a reasonable speed.

If a load is being pushed up a plane, it will be much easier when the angle of the slope B is less than φ, as a rest may be taken. When the slope is greater than φ the load must be pushed continuously until the top is reached.

Action of Wedge

A wedge is like an inclined plane in its action. It may be used to lift a load, or to force two things apart. In Fig. 8 a wedge being used to split a log is illustrated. If the head of the wedge is hammered down, the two portions of the log are forced apart, the split follows the grain of the wood, and by inserting another wedge in the split, the log can be divided. The force on the head of the wedge is P and the resultant reaction on each side of the wedge is Q. The wedge is sliding down through the log so that friction is acting upward on it, and the forces Q must be inclined as shown. The forces on the wedge can be represented by the triangle abc.

If we attempt to withdraw the wedge by an upward pull, the friction will now act in the reverse direction, since friction always acts so as to oppose motion or attempted motion. The resultant reactions will be inclined at the angle of

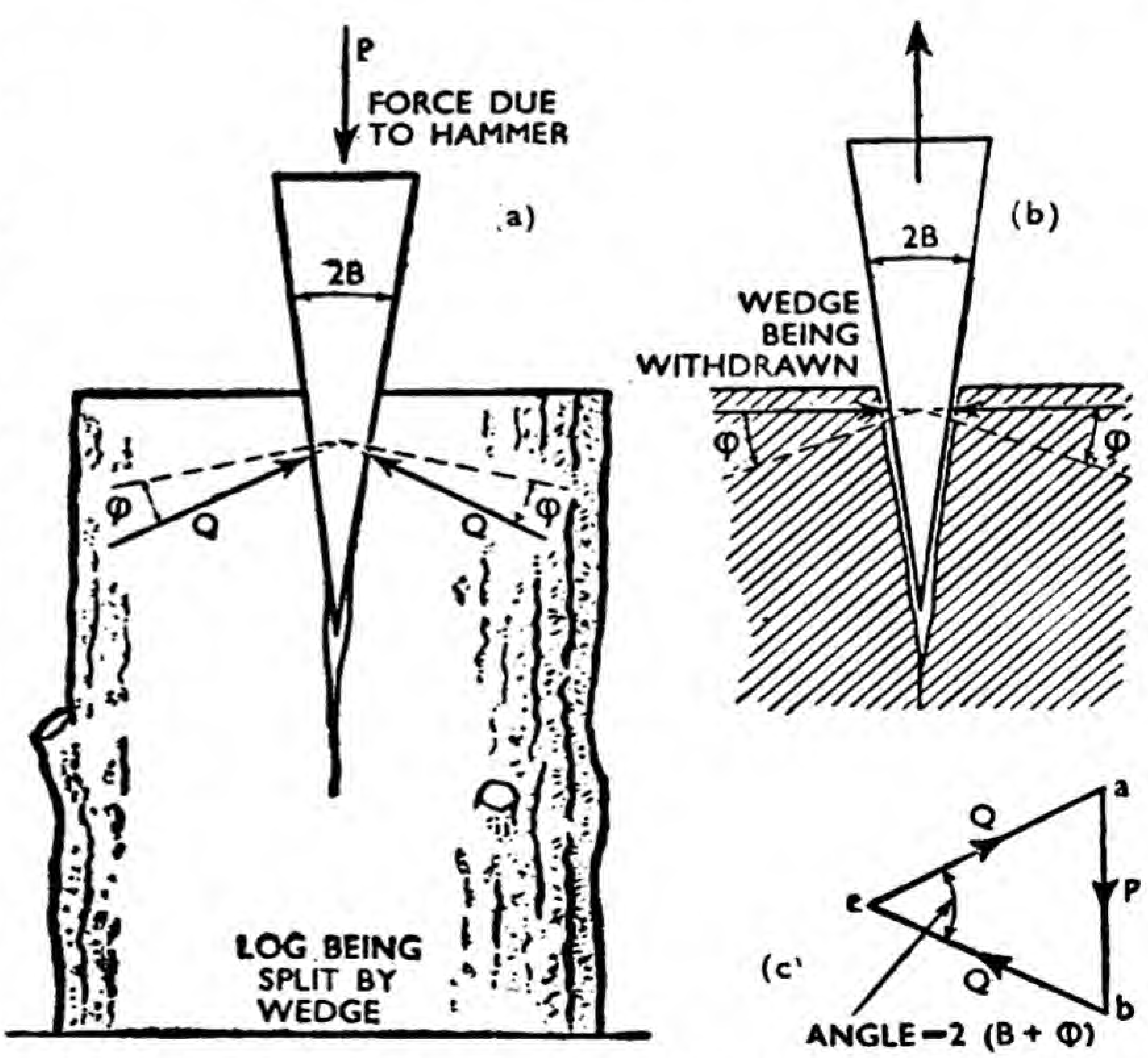


Fig. 8. (*a*) When a wedge is driven into a log, the frictional forces act upwards on the wedge. (*b*) When the wedge is withdrawn, the frictional forces oppose the withdrawal, and if B, the half angle of the wedge, is less than the angle of friction φ, the resultant reaction may be downward, so locking the wedge in the crevice

friction on the other side of the normals. The angle of the wedge is $2B$, and if B is greater than φ, a downward force will be necessary to prevent the wedge moving upward, and it will not stay in place.

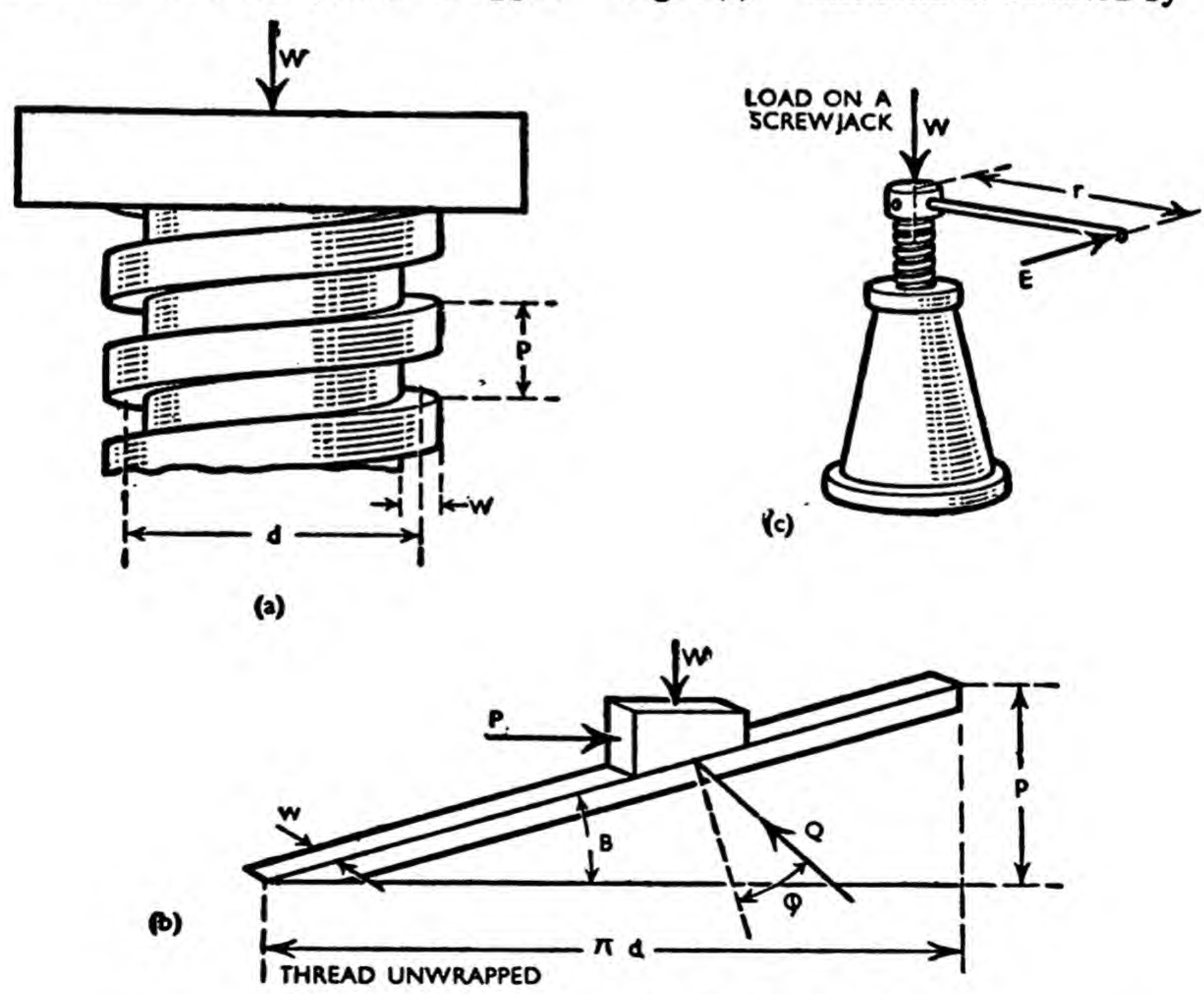
Small Angle of Wedge

We generally use wedges with a small angle of about ten or fifteen degrees, so that B is about five or eight degrees. This is less than the angle of friction; therefore, the wedge will remain in place. It very often requires a considerable force to withdraw a wedge from a log.

A screw thread is, in effect, a narrow inclined plane wrapped round a cylinder. In Fig. 9(a) we have a square-threaded screw. The pitch p is the distance moved when the screw is given one turn, and the mean diameter is d. When unwrapped, one turn of the thread will give an inclined plane of width w, of length πd and height p. The inclined plane formed by the unwrapped thread is shown in Fig. 9(b). The angle of the plane B can be found from $\tan B = p/\pi d$.

Use of Square Thread

This type of thread is used in machine tools, and for purposes such as the screwjack, shown in Fig. 9(c). The load to be lifted by



EXPLAINING A SCREW THREAD

Fig. 9. (*a*) Here is a square-threaded screw. (*b*) If one complete turn of the thread were unwrapped, it would look like this; in other words, it would form an inclined plane. (*c*) A familiar application of the screw thread is the screw-jack, which is used for raising or lowering heavy loads.

the screwjack is W, and the effort E is applied at the end of a bar at a radius r from the axis of the screw.

The leverage of the bar will depend upon the radius r. If P is the effective force at the mean diameter d, then the turning moment of the force P is $\frac{1}{2}Pd$, and it will be equal to the turning moment of E, which is Er. Therefore, $E \times r$ must equal $\frac{1}{2}P \times d$. The action of the screw when it is lifting the load W is exactly the same as the action of the sliding block on the inclined plane in Fig. 9(b). We have already dealt with this case and have found that the force P is equal to the load W multiplied by the tangent of the angle $(\varphi + B)$.

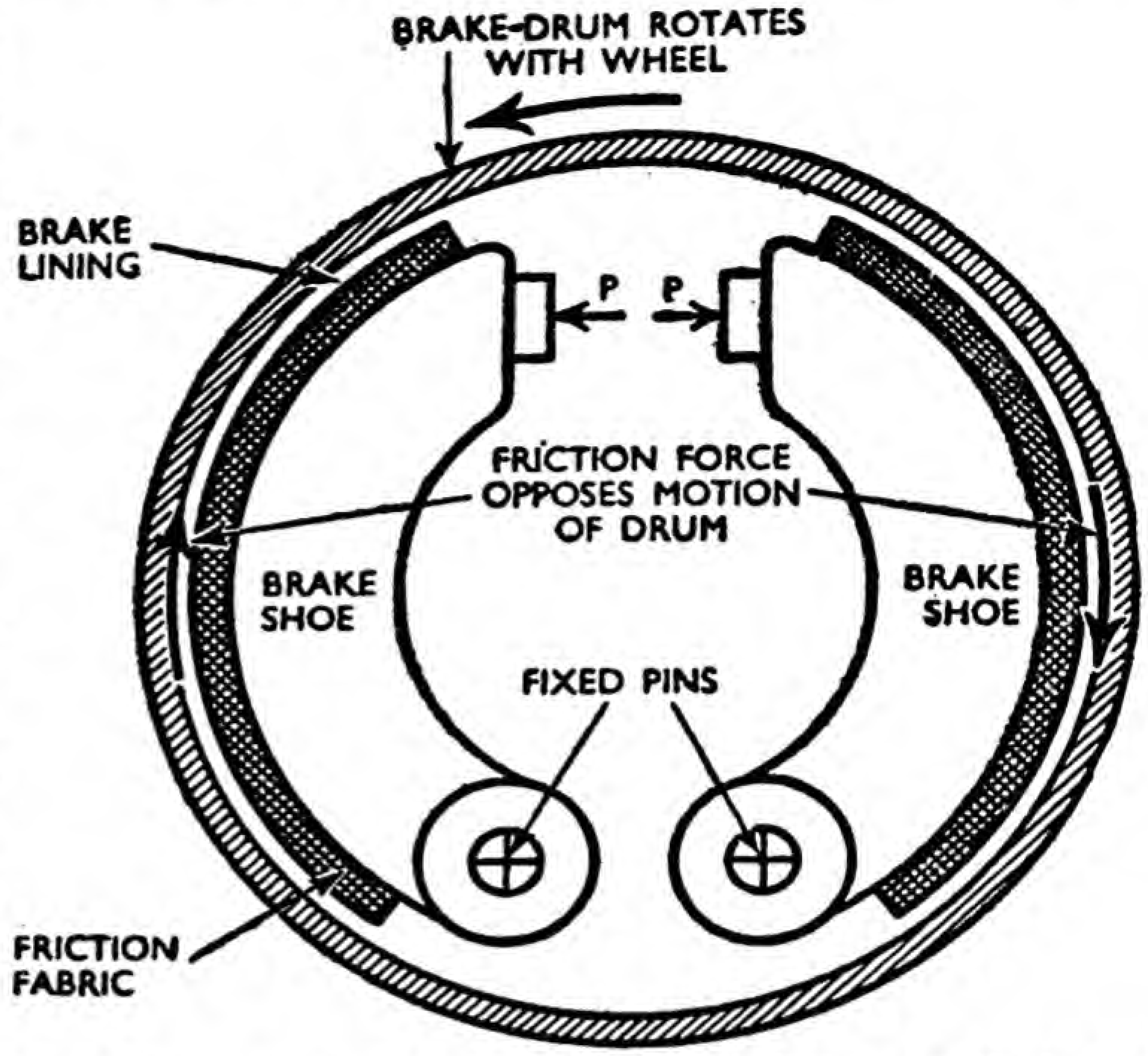


Fig. 10. Brake shoes lined with friction fabric are fitted inside the brake-drum. They are pivoted at the lower ends, and the upper ends are forced apart, thus pressing them against the lined surface of the drum. The pressure on the brake-drum causes a frictional force which opposes the motion of the brake-drum.

Angle Size is Important

If we turn the screw the other way, the load will be lowered. The direction in which we apply the effort E, either to push the bar round or to hold it back, will depend upon whether the angle of the thread B is greater or less than the angle of friction φ. If B is greater than φ, the load will lower itself unless the bar is held back. If B is less than φ, the screw will remain at rest until it is turned.

Most threads used are of the latter type so that they hold the load. When a nut is tightened on a bolt the friction is made of use to prevent the nut slackening off, because the angle of the thread is less than the angle of friction. Friction is not always a disadvantage. Without friction, brakes, belt or rope drives, friction clutches and, in fact, many of our daily actions would be impossible. Vehicles could not run on roads or rails if friction were absent.

Friction Brake

The friction brake for an automobile is shown in Fig. 10. The brake-drum rotates with the wheel of the car and a friction force is applied to it by means of two brake shoes. These shoes are fitted with strips of friction fabric called linings, partly to increase the coefficient of friction, and partly because wear is bound to occur and the linings can easily be replaced.

The shoes are pivoted on fixed pins at one end and are forced apart at the other end so that the

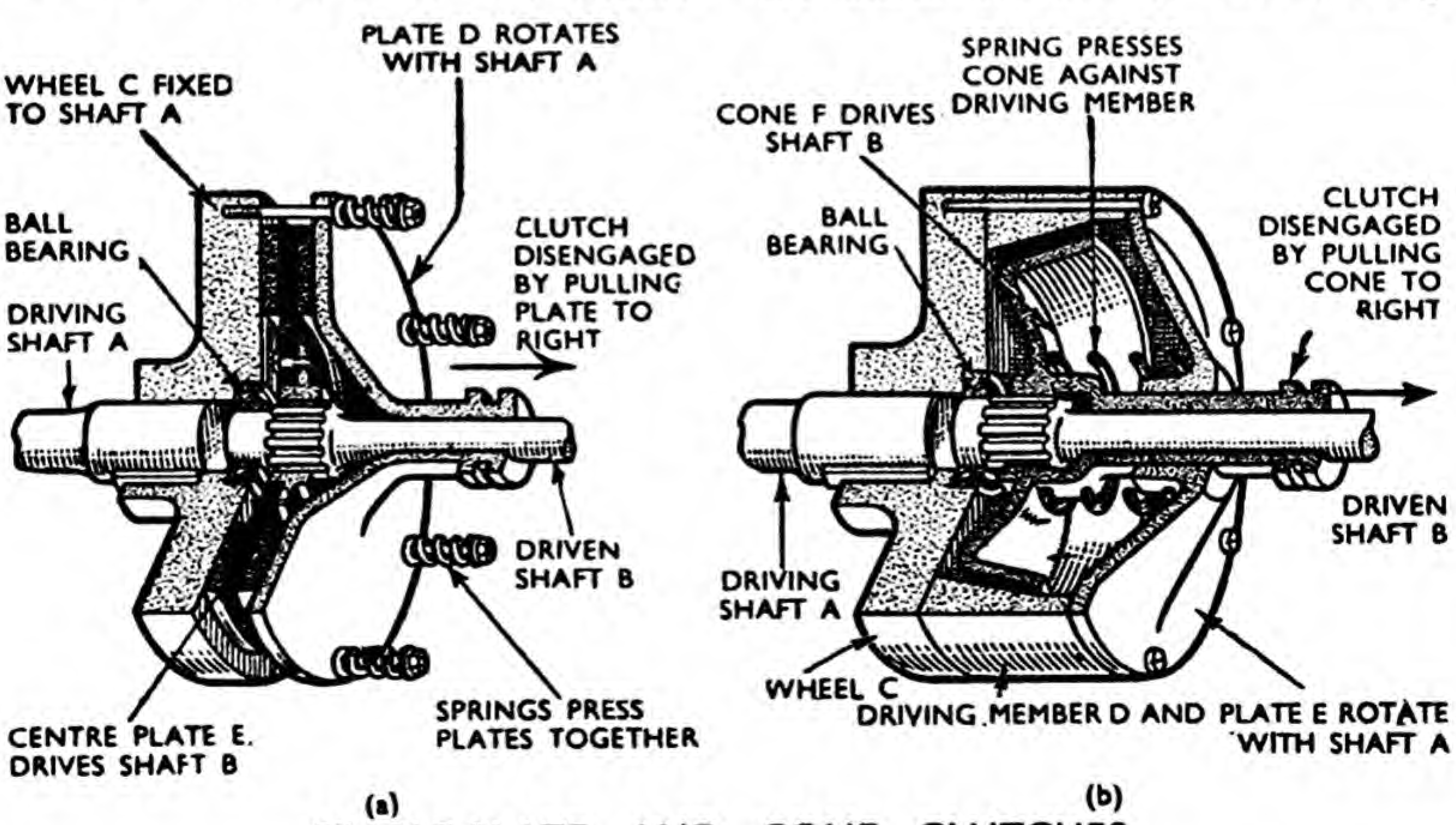
linings are pressed against the inside surface of the brake-drum. This produces a friction force which opposes the motion of the drum and so brings it, together with the wheel and the vehicle, to rest.

In Fig. 11 are illustrated two kinds of clutch, both of which use friction to transmit power from one shaft *A* to another shaft *B*. In the case of the plate clutch (Fig. 11(a)), a wheel *C* is firmly fixed to the driving shaft and carries with it a loose plate *D*. The centre plate *E* is squeezed between them by a number of springs, and drives the shaft *B*. If the plate *D* is pulled to the right, the springs are compressed and separate the plates and the wheel. The centre plate *E*, and with it the driven shaft *B*, can then remain at rest while shaft *A* is rotating.

Pieces of friction fabric or cork are often riveted to the centre plate in order to increase the coefficient of friction. In the clutch shown, there are two sliding surfaces, so that the force turning the driven shaft will be twice the coefficient of friction multiplied by the total spring force. This can be increased by having a number of plates alternately connected to the wheel and to the driven shaft. We then have what is called a multi-plate clutch.

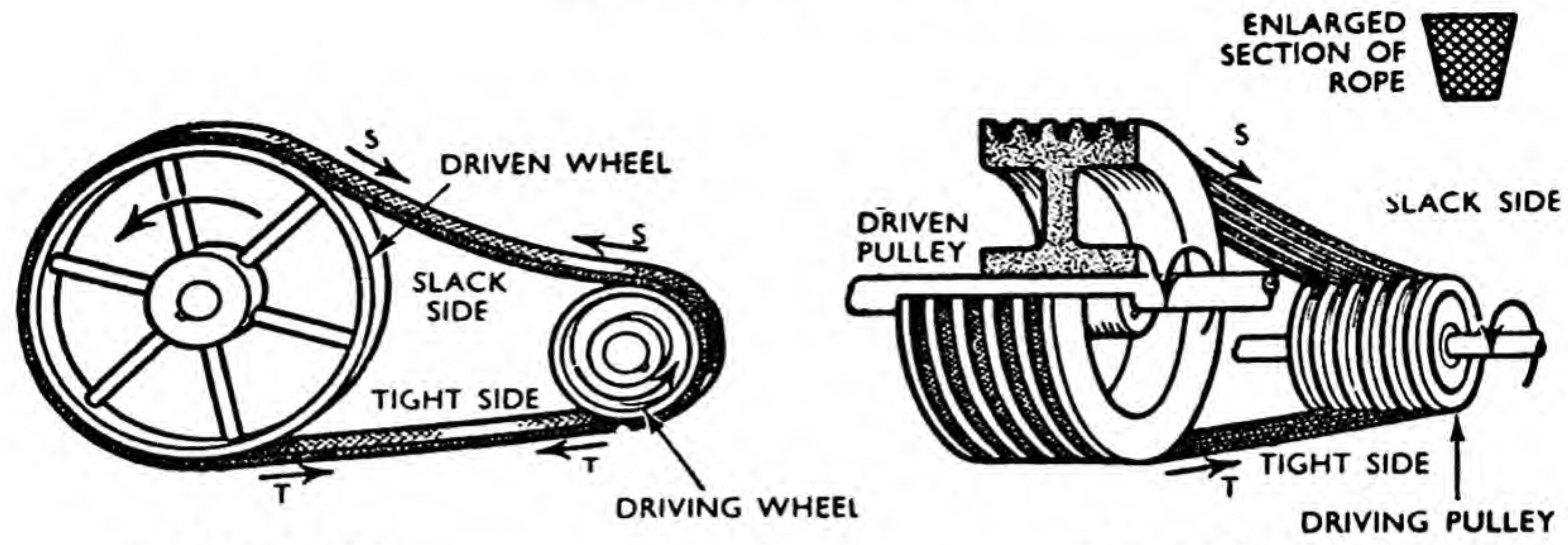
Cone Clutch

The cone clutch (Fig. 11(b)) enables more power to be transmitted than a similar single-plate clutch without using extremely stiff springs. The wheel *C*, the driving member *D* and the outer plate *E* are all firmly fixed to the driving shaft *A*. The cone *F*, which is a wheel with a conical rim, can slide along shaft *B*, but rotates with it. A spring presses the cone against the inner surface of the driving



SINGLE-PLATE AND CONE CLUTCHES

Fig. 11. These clutches are similar in that they act through setting up frictional forces. (*a*) The plate clutch transmits motion from shaft A to shaft B when the springs are allowed to press the centre plate E against the wheel C. (*b*) The action is similar in the case of the cone clutch, but the contact surfaces are conical and not flat.



FLAT BELT DRIVE (a) V ROPE DRIVE (b)

BELT AND PULLEY TRANSMISSION

Fig. 12. (*a*) Power is frequently transmitted by means of belts and pulleys. If the directions of rotation were opposite to that shown, the tight and slack sides of the belt would be reversed and the contact between the belt and the pulleys would be reduced, so diminishing the power which could be transmitted. (*b*) For transmitting large powers, V ropes and grooved pulleys are used.

member. Owing to the wedging action, the normal pressure between the sliding conical surfaces is much greater than the force produced by the spring.

With the type of cone usually employed, the normal pressure is about five times the spring force, so that the friction force, which drives the shaft *B*, is about five times the coefficient of friction multiplied by the spring force. This clutch is disengaged by pulling the cone to the right. This will compress the spring and separate the conical friction surfaces.

Rate of Lowering Controlled

If a heavy load is going to be lowered by means of a rope, it is possible to control the rate of lowering by quite a moderate force if the rope is first passed a few turns round a fixed circular bar. If a few tests are made it will be found that the ratio of the load we can lower to the force required depends upon the number of turns of the rope round the bar, and the coefficient of friction between the rope and the bar. If this coefficient of friction is 0·2 and the rope makes two complete turns round the bar, it is possible to lower a load of 370 lb. by applying a force of 30 lb. to the rope. By applying a force of 30 lb. to the end of the rope after it has made only half a turn round the bar, we shall be able to lower a load of only 56 lb.; but, with the same force and three turns round, provided the bar is strong enough, it is possible to lower 1,290 lb.

This principle is employed when power is transmitted through a belt, as shown in Fig. 12(a). The belt passes round both pulleys, but the angle of contact is less than one turn; in the case shown it is rather less than half a turn round the driving pulley and rather more than half a turn round the driven pulley.

Tension Ratio

When power is being transmitted from the driving pulley to the driven, one side of the belt is tight with tension *T*, and the other side is

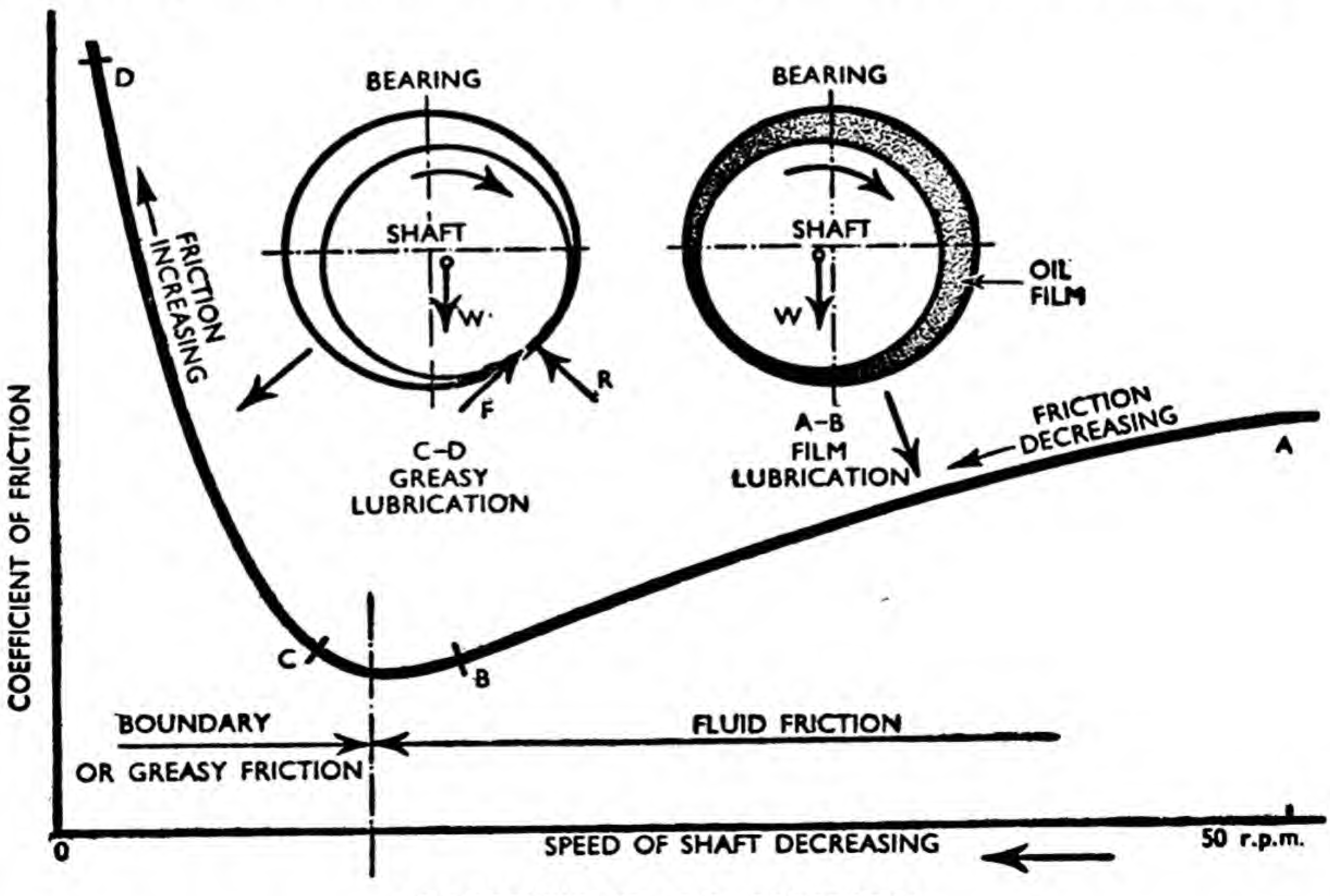
slack with tension S. The ratio of T to S depends upon the coefficient of friction between the belt and the rim of the pulley and upon the angle of contact. The effective force on the driven pulley is the difference between the tensions T and S, and the power transmitted will be given by the effective force multiplied by the velocity of the belt.

Instead of a flat belt and pulleys it is often found that a drive of this kind is obtained by the use of grooved pulleys with a number of V ropes running in the grooves, as shown in Fig. 12(b). The advantage of this system is that the wedging action of the ropes in the grooves increases the effective friction, and so increases the ratio of the tension in the tight side to the tension in the slack side. This effect, together with the number of ropes used, allows a considerable power to be transmitted.

The laws of dry friction have been considered in some detail, but in many cases where sliding occurs, especially in machine and engine parts, the surfaces have been lubricated. The primary effect of the lubricant is to decrease the coefficient of friction and to prevent the rapid wear and the waste of power which occurs when the surfaces are dry. The presence of the lubricant has an additional effect, however, in that the laws of dry friction do not then represent the behaviour of the friction between the surfaces.

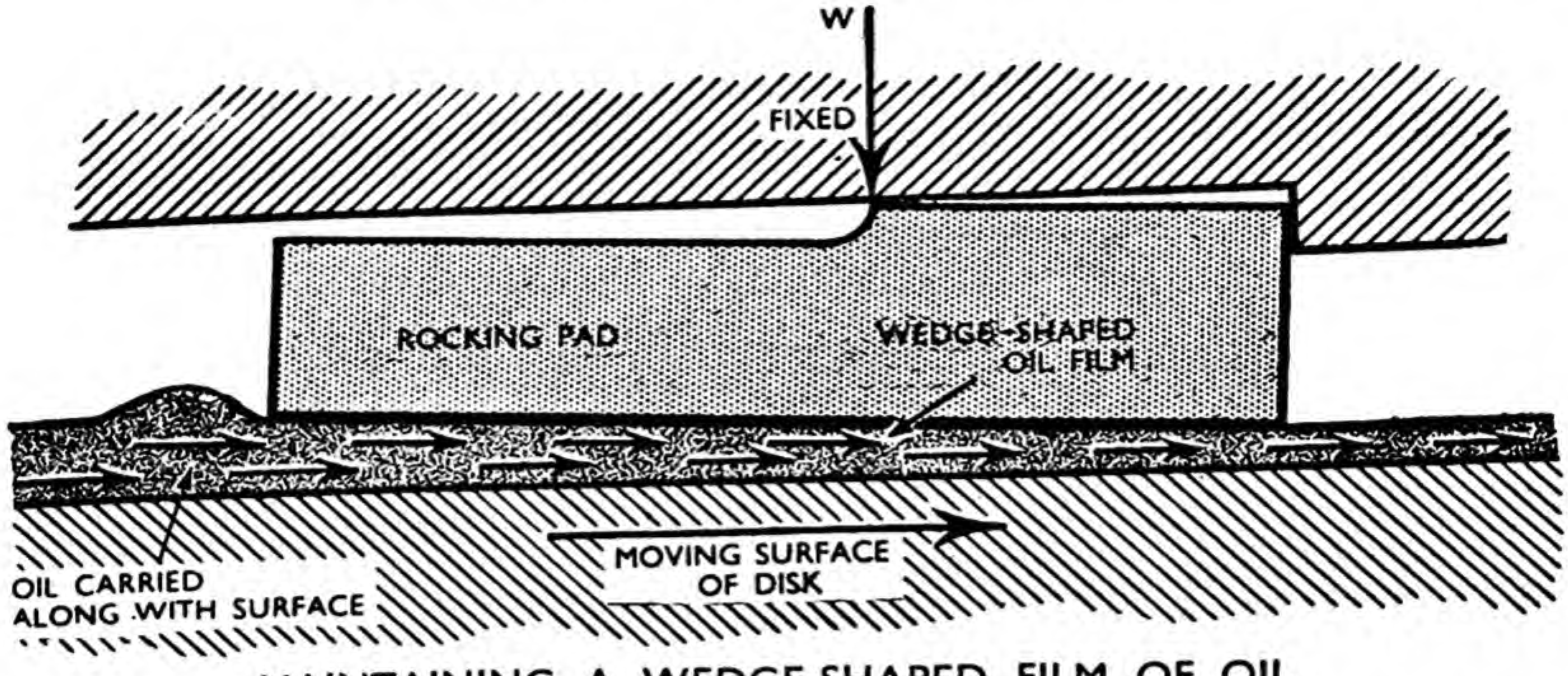
Rotating Shafts

A shaft rotating in bearings is the example most usually met in practice. If there is a loaded shaft running freely in well-lubricated bear-



COEFFICIENT OF FRICTION

Fig. 13. In the case of a shaft brought to rest by friction at the bearing, the amount of the friction decreases until a certain speed is reached. Below that speed, the amount of the friction will rise sharply.



MAINTAINING A WEDGE-SHAPED FILM OF OIL

Fig. 14. The Michel thrust block is largely used as a bearing in modern marine practice. The wedge-shaped film of oil is able to withstand a heavy load. The special feature of this Michel bearing is that as the speed increases, the load-carrying capacity also increases.

ings, it will gradually come to rest under the action of the friction. The friction can be determined by experiment as the speed falls, and from these results a curve may be plotted like that in Fig. 13, which shows how the coefficient of friction changes as the speed changes from a value such as 50 r.p.m. to zero.

As the speed falls, it would be found that the friction decreased gradually from *A* to *B*, but that below a certain speed, the friction increased as shown by the line *CD*, to some value off the graph when the shaft is at rest.

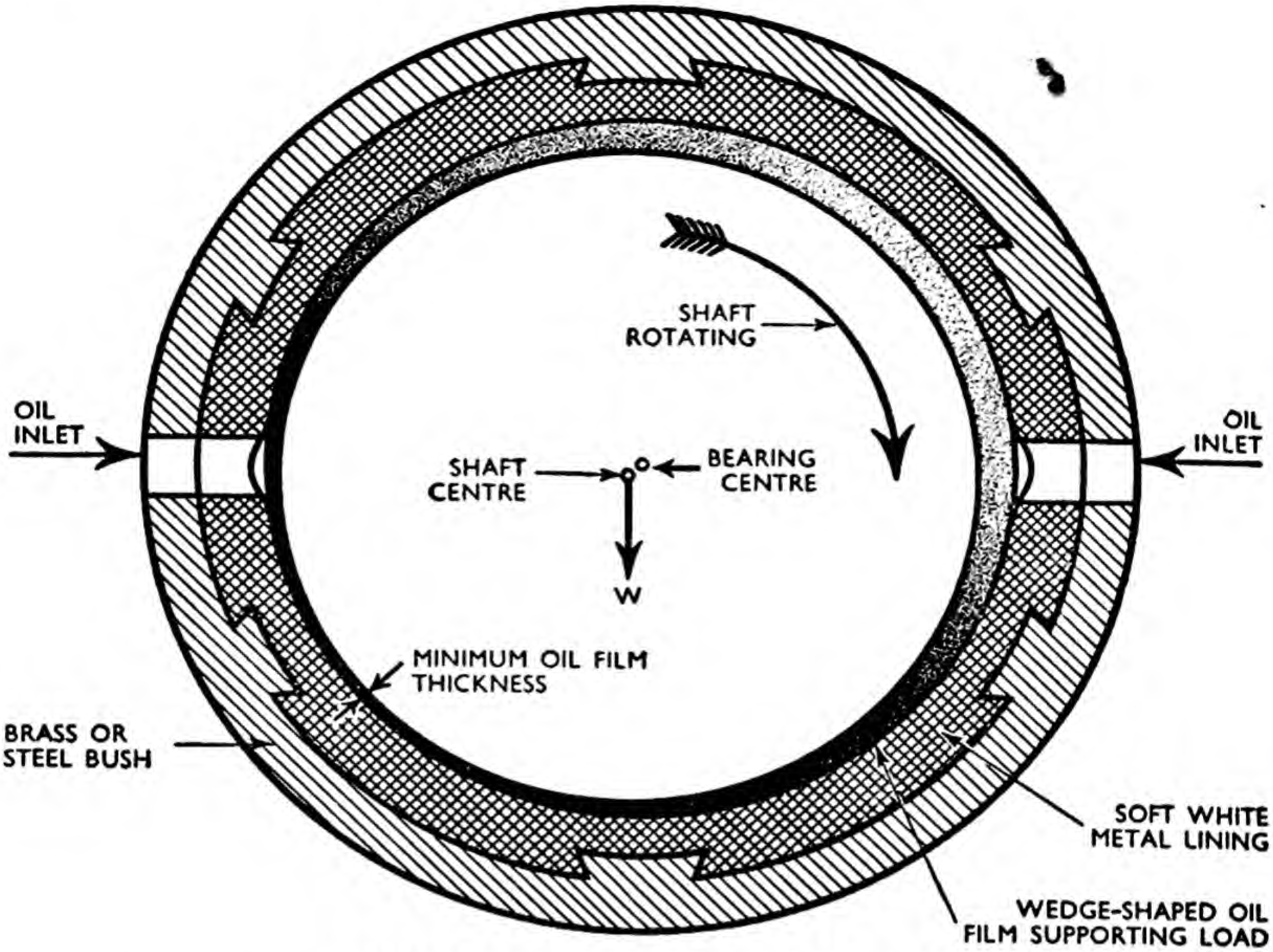
Varying Friction

Evidently the conditions in the bearing depend upon the speed of the shaft, and the type of friction existing at high speed is different from that at low speed. As a result of further experiment it is known that at high speed, when the line *AB* gives the friction, the shaft is supported on a thin film of oil. At low speeds, on the line *CD*, there is metallic contact, but the presence of the lubricant produces what is known as boundary or greasy friction.

The shaft is smaller than the bearing in which it runs by an amount called the clearance ; this has been exaggerated in the diagrams in order to show clearly what happens. In the small pictures of the shaft in Fig. 13, the position of the shaft in the bearing for the conditions *AB* with film lubrication, and *CD* with greasy lubrication may be seen. In the first case the centre of the shaft is to the left of the centre of the bearing, and in the second case it is to the right. With greasy lubrication, the load on the shaft *W* is supported by the two forces *R* and *F* acting on the surface of the shaft. With film lubrication the load is supported by the pressure set up in the film of oil.

Michel Thrust Block

The pressure in the oil film depends upon its shape. In Fig. 14, a diagram of the Michel thrust block is shown. A pad or block of metal is held by a fixed casing, but is able to tilt by rocking about its



SUPPORTING A LOAD ON OIL

Fig. 15. The ordinary cylindrical, or journal, bearing, well supplied with lubricant, builds up a wedge-shaped film of oil capable of carrying a considerable load without metallic contact between the shaft and bearing surfaces.

stepped edge. Beneath it is a moving surface which carries along with it a quantity of oil. This surface is part of a rotating disk attached to a shaft, and the oil is dragged under the pad by the motion of the surface. If the pad is tilted so that the oil film is wedge-shaped, a very considerable pressure may be set up which is sufficient to carry the load W. If the pad were fixed parallel to the moving surface, no such pressure could be produced, and the oil film would break down.

Effectiveness of Oil Film

The effectiveness of the oil film depends on the properties of the oil, on the speed of sliding and on its shape. The presence of the wedge-shaped oil film in a cylindrical bearing is shown in Fig. 15. This is a section through a typical high-speed bearing such as is used for turbines and propeller shafts of ships. The clearance space, which is exaggerated, is usually filled with oil from the two supply inlets, and is shaded in the diagram. The deeper shading at the bottom of the bearing shows the high-pressure wedge-shaped oil film which supports the load. Practically no wear takes place in these bearings when they are running. Actual contact between the shaft and the white metal occurs only when the speed is low, that is, when the shaft is being started or is coming to rest.

CHAPTER 6

PRACTICAL MECHANISMS

ENGINE MECHANISMS. MOTION OF CONNECTING RODS. ENGINE GUIDES. STROKE AND THROW. ROTARY AND RECIPROCATING MOTION. QUICK-RETURN APPLICATIONS. TOGGLE JOINTS. INSTANTANEOUS CENTRE OF ROTATION. THE PANTOGRAPH. CAMS. ELEMENTS OF TOOTHED GEARING. INVOLUTE TEETH. RATCHETS. CENTRIFUGAL GOVERNORS. PORTER GOVERNOR. SPRING-LOADED GOVERNORS. SPECIAL GOVERNORS.

ONE of the big problems in the development of the steam engine and its application to the driving of machinery, was the design of a satisfactory means of converting the reciprocating motion of the piston to the rotary motion required by the spinning machines of the mills.

James Watt devised a number of ways of doing this in addition to the crank, although at first a beam was always used, following the practice of the earlier pumping engines. Eventually, the beam was eliminated, and the engine mechanism shown in Fig. 1 became the most widely used arrangement.

In this, the piston *P* is caused to reciprocate in the cylinder *A* by the pressure of the steam acting alternately on each side of it, the spent steam being exhausted from one side as fresh steam is admitted to the other. This backward and forward motion of the piston in the cylinder is communicated to the crosshead *H* by the piston rod *R*, and the motion of the crosshead is, in turn, communicated to crank *C* by the connecting rod *HC*.

We can see that, throughout the motion, one end of the connecting rod must have the circular motion of the crank, whilst the other end reciprocates with the crosshead, rather like the motion of the forearm when turning a handle, the elbow taking the place of the crosshead. In order to make this rocking motion possible, the connecting rod must be pivoted or hinged at both ends. It pivots on the crosshead pin at *H* and on the crank pin at *C*. The eyes in the ends of the connecting rod, which fit on the crosshead pin and the crank pin, are called the small end and the big end respectively.

An undesirable consequence of

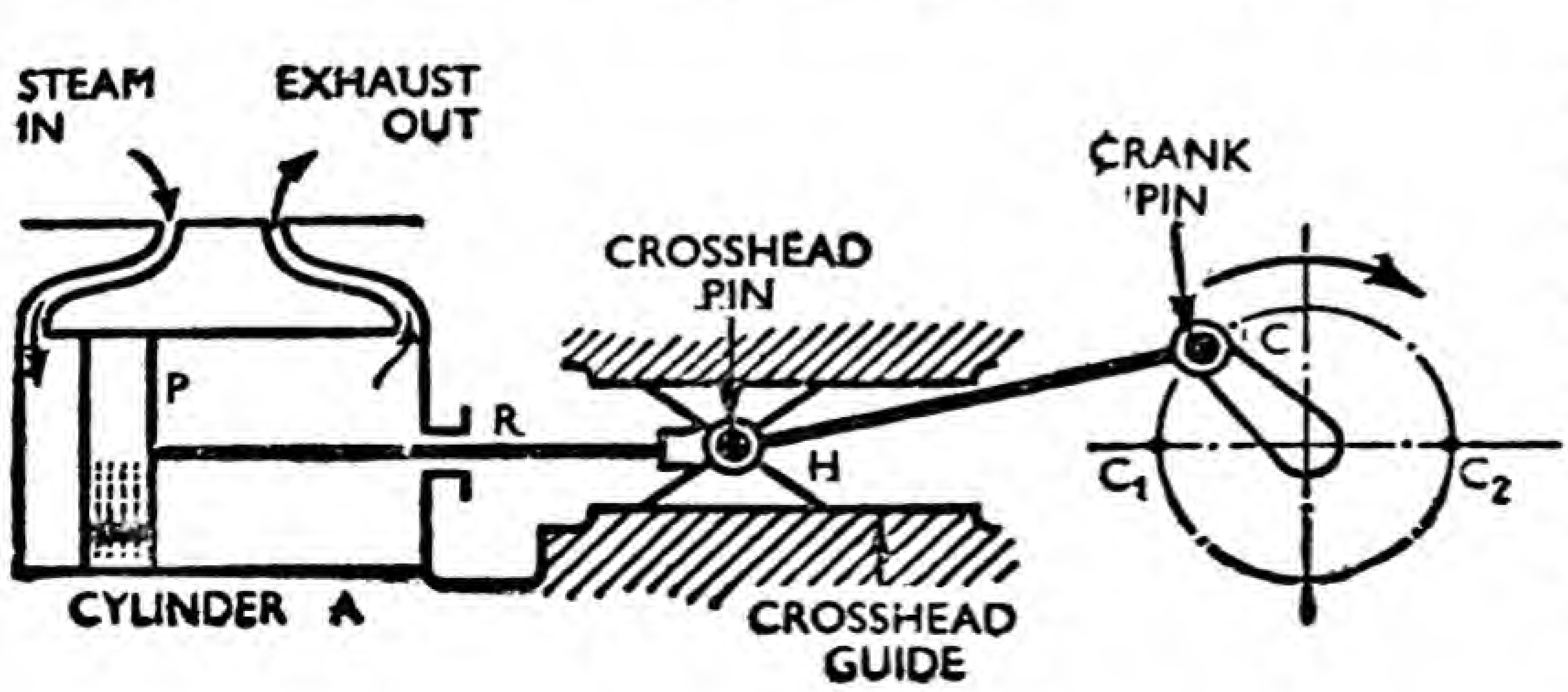


Fig. 1. The steam engine mechanism with reciprocating motion of piston *P* producing rotary motion of crank *C*, is made possible by connecting rod *HC* rocking from side to side in the manner of the forearm about the elbow joint.

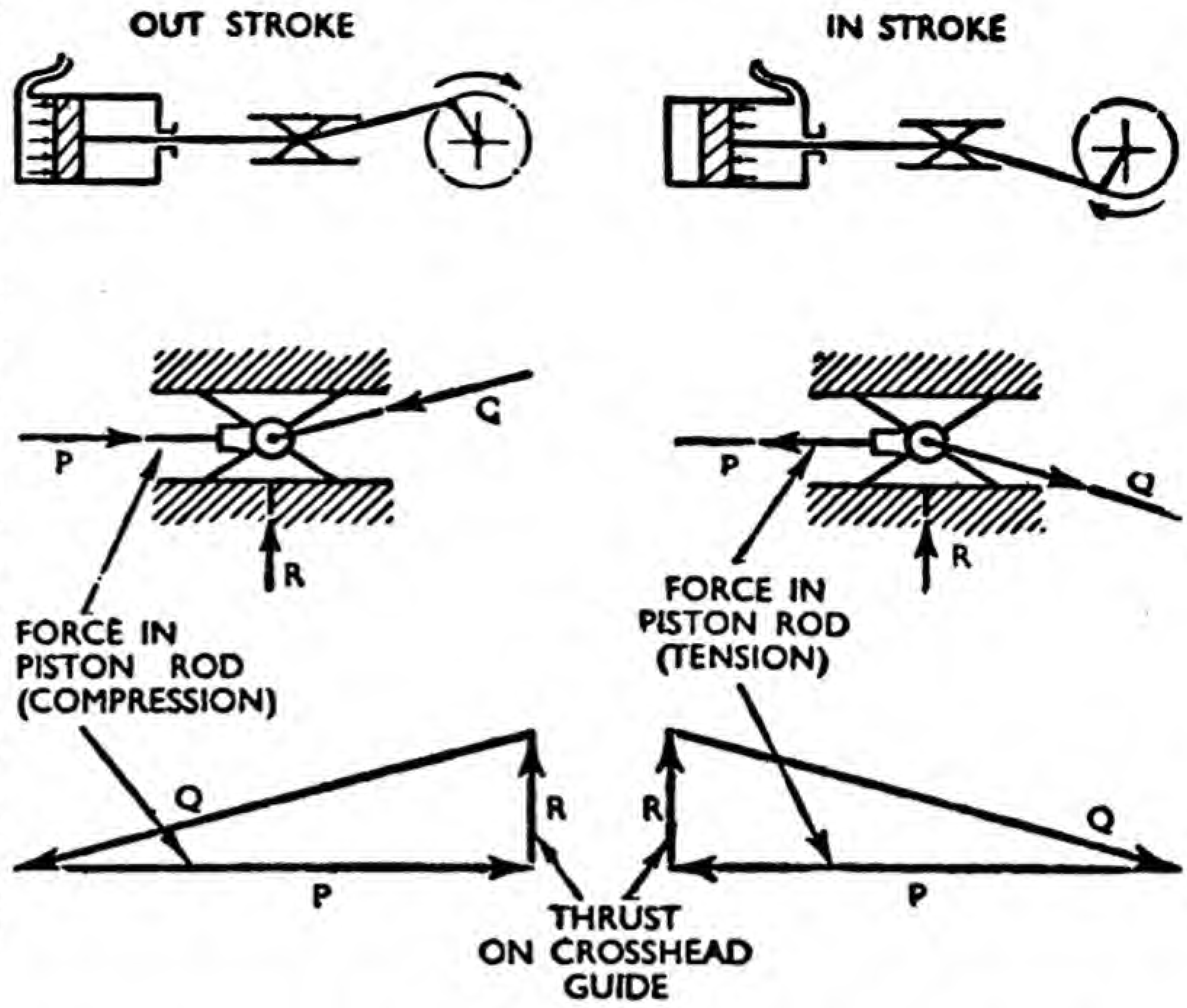


Fig. 2. Purpose of crosshead and its guide is to resist sideways forces which result from rocking motion of connecting rod. Three forces meet at centre of crosshead; force P in the piston rod, force Q in the connecting rod, and thrust R from the crosshead guide. Triangles of forces for P, Q and R show that during both out and in strokes, thrust R acts from the lower guide. Only when direction of rotation is reversed, does thrust act from upper guide.

the rocking motion of the connecting rod is that a side thrust is exerted on the crosshead. This makes it necessary for the crosshead to be provided with large bearing surfaces, called slide blocks, which press against fixed guides. At first sight it might appear, because the connecting rod is inclined first to one side and then the other, that the two guides would take the thrust alternately, but the force diagrams in Fig. 2 show that the thrust is always against the same guide.

Many engines have two guides, however, because when the direction of rotation is reversed, the side thrust is also reversed. The guides, therefore, are sometimes referred to as the ahead and astern guides. Some engines, particularly loco-

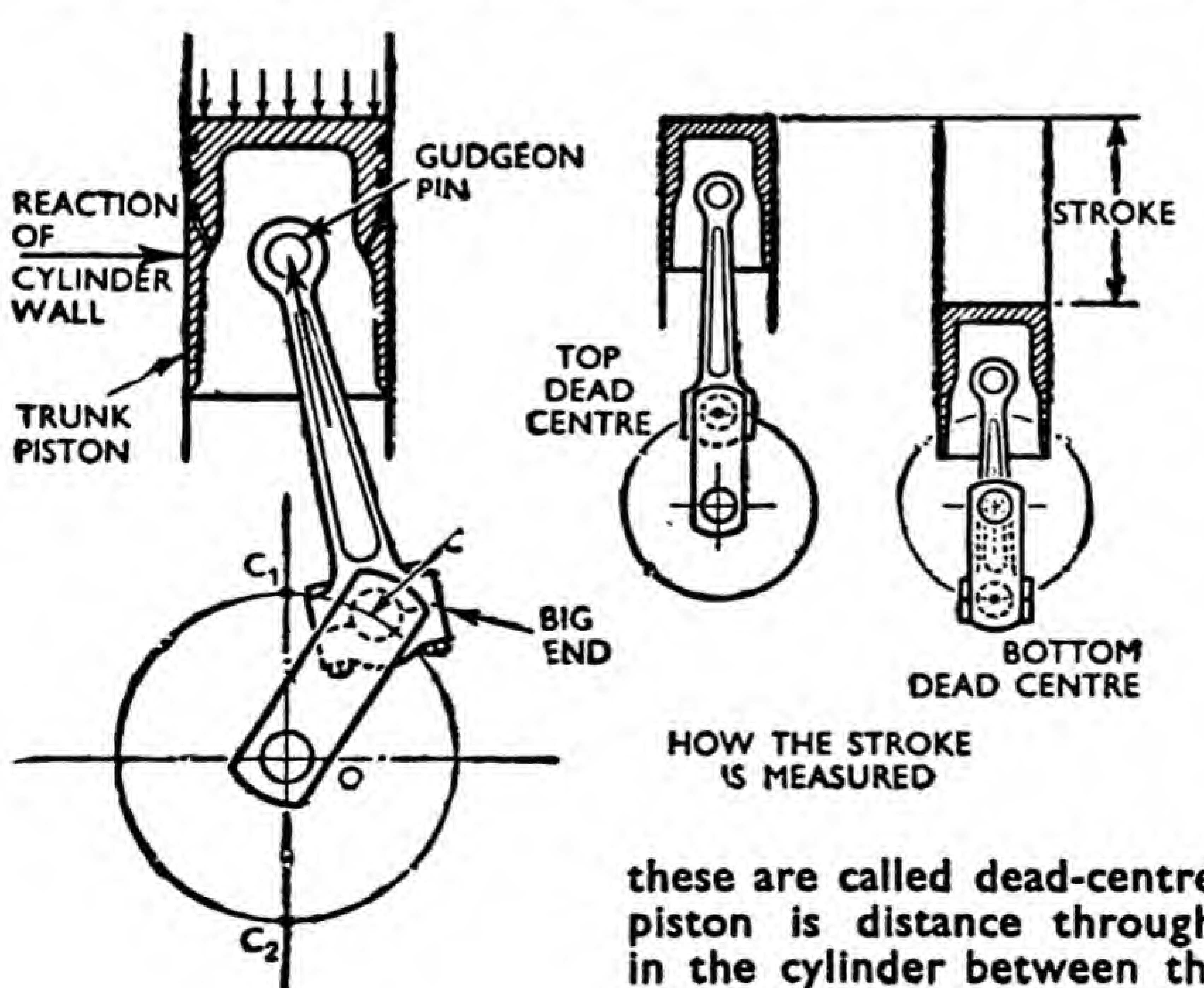
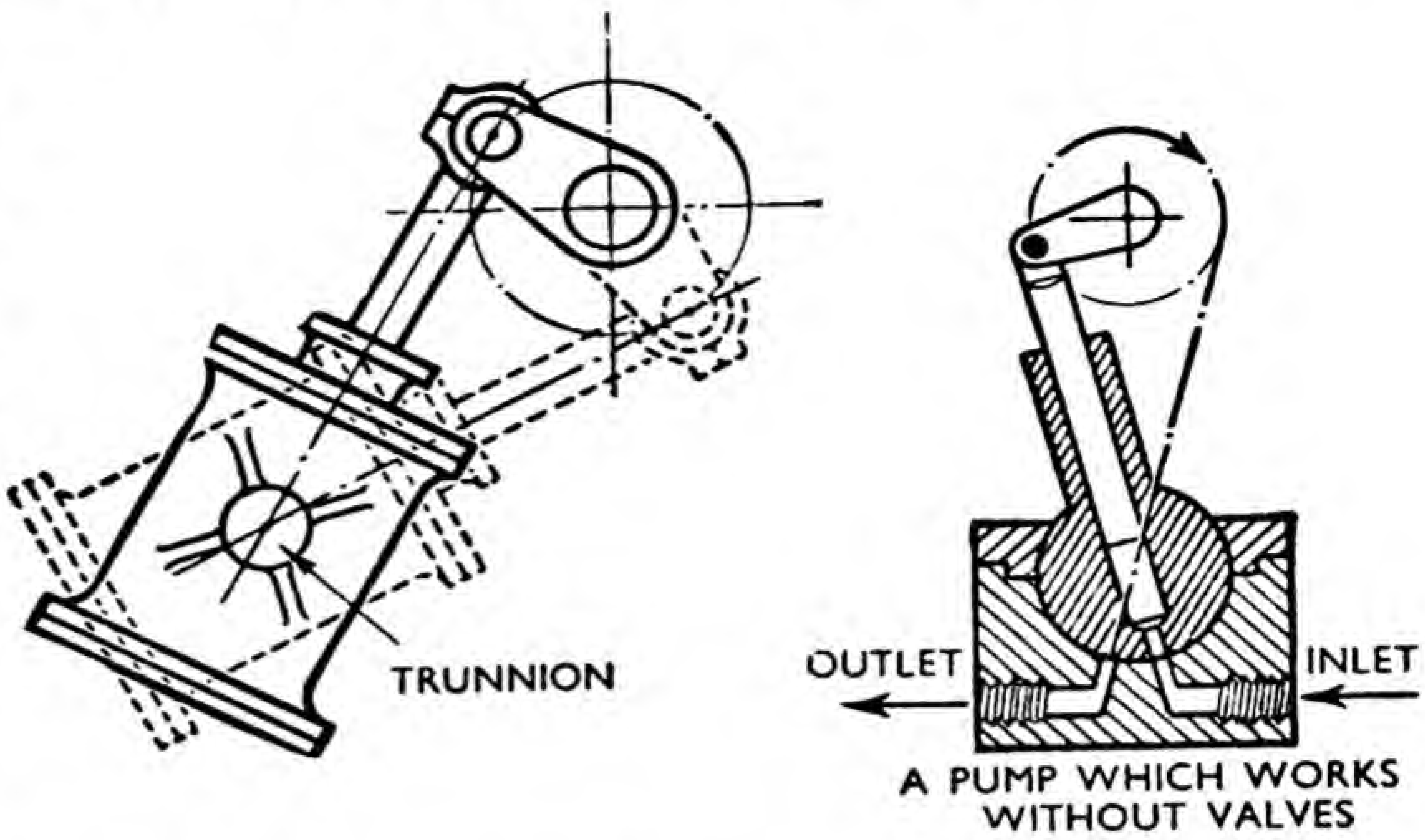


Fig. 3. Trunk piston serves both as piston and crosshead. It sustains steam or gas pressure on the crown, and the guide thrust on the side. It is longer than normal pistons to provide a large thrust surface in contact with cylinder wall which acts as guide. When crank pin is in position C_1 or C_2, piston is momentarily at rest in cylinder, and these are called dead-centre positions. Stroke of piston is distance through which piston slides in the cylinder between these extreme positions.

Fig. 4. In oscillating cylinder engines, entire cylinder assembly rocks about the trunnion, dispensing with a connecting rod. Steam must be led into and out of the moving cylinder, and sometimes the cylinder motion itself is used to do this automatically. Similar arrangements are used in oscillating cylinder pumps.



motives, have only one guide, although they are reversible. It will always be found in these instances, however, that the guide and slide block or slipper are so arranged that they can take the thrust in both directions.

Single-acting Engine

We call an engine like the one described above, a double-acting engine, because the steam or gas acts alternately on each side of the piston. Many engines are only single-acting as shown in Fig. 3, that is, the steam or gas acts only on one side of the piston. It is then possible to dispense with the piston rod, and to pivot the small end of the connecting rod directly to the piston.

This simplifies the mechanism, but the piston has to do the double job of acting as crosshead as well, and it is accordingly made longer, so that the side thrust from the connecting rod is distributed over a large area of the cylinder wall which acts as the guide. Such a piston is called a trunk piston, and it is the kind invariably used in the petrol engines of motor cars and aircraft. It is also used in larger marine oil engines. The pin in the piston on which the small end of the connecting rod pivots is called the gudgeon pin.

In another type of engine, the connecting rod is dispensed with, and the piston rod is connected directly to the crank pin. Because there is no connecting rod to take up the rocking motion, it is now necessary for the entire cylinder to rock, and such an engine, therefore, is called an oscillating-cylinder engine. These engines were widely used at one time as, for example, in paddle steamers, but the arrangement is now seldom used, except in small compressed air motors and in some types of pump, where the rocking motion of the cylinder can be employed to open and close automatically the inlet and outlet ports, as shown in Fig. 4.

Whether the cylinder is fixed or oscillating, the distance through which the piston slides in the cylinder is called the stroke, and this is usually equal to the diameter of the circle described by the crank pin. The crank radius *OC* (Fig. 3) is often known as the throw of the crank, and hence we have the relationship :—

stroke of piston $=$ 2 $\times$ throw of crank.

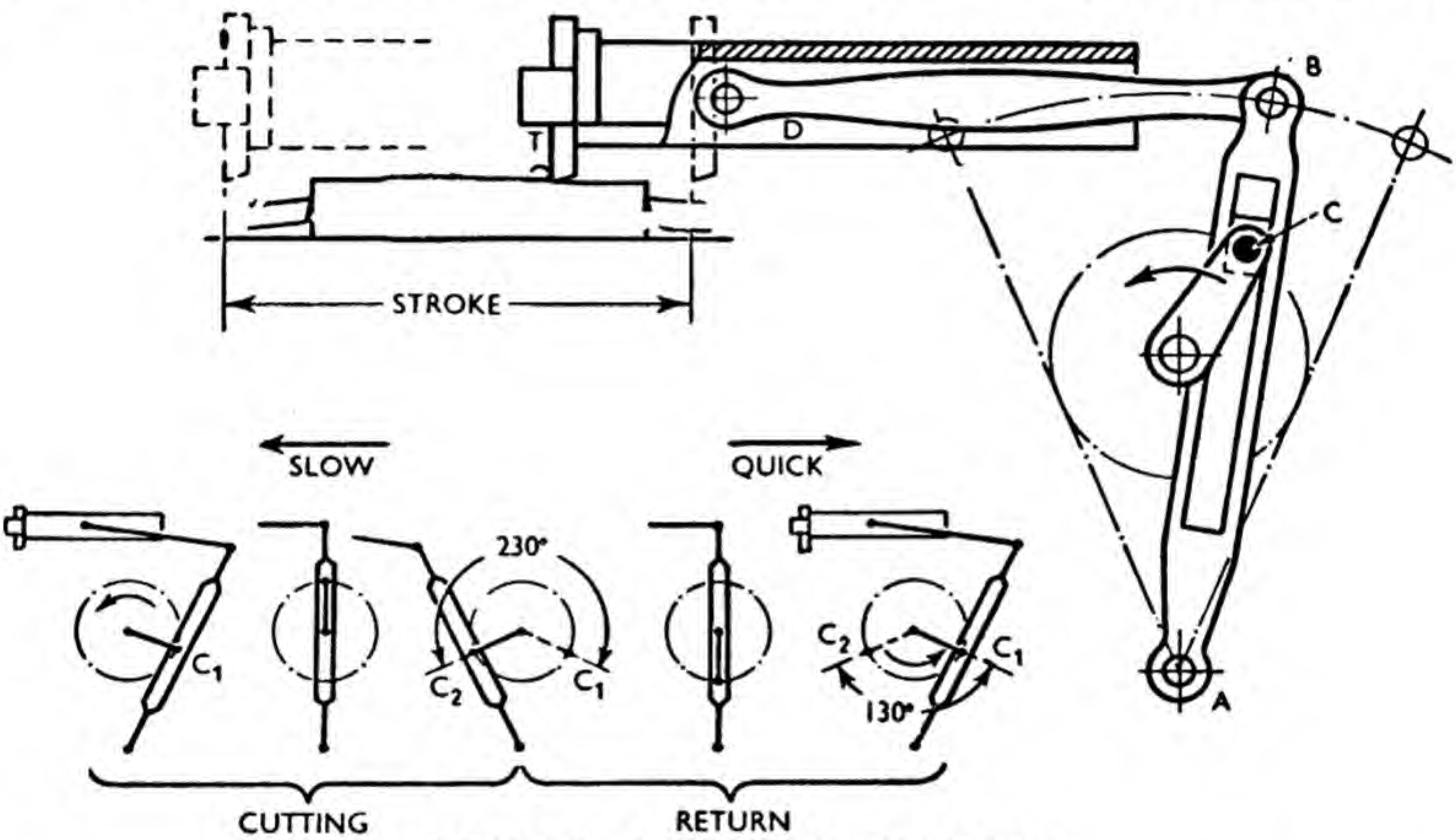
This is not quite true, however, in some multi-cylinder engines, notably

radial aero engines where, owing to the different arrangement of the connecting rods, the stroke of the pistons is rather greater than twice the throw of the crank.

We can also see from the diagrams that, at two points during each revolution, when the crank pin is at C_1 and C_2, there will be no turning moment on the crank however great the force on the piston, because the connecting rod and the crank are in line. These points are called the dead centres, and they correspond to the ends of the piston stroke. Thus, with a vertical engine like that of a motor car, we say that the piston is on top-dead-centre when it is in the extreme position at the top of the cylinder, and on bottom-dead-centre when it is in a similar position at the other end of the cylinder.

If it were not for the momentum of the rotating parts, the engine would tend to come to rest in the dead-centre positions, because there is then no turning moment. For this reason, a flywheel is fitted to carry the engine over the dead centres, and thus to assist steady running. If the engine has two cylinders, which is the case in many locomotives, the two cranks may be arranged at right angles to each other, so that both cranks cannot be on the dead centre simultaneously. This ensures that the engine can start with the cranks in any position. Similarly, if the engine has three cylinders, the cranks are usually at an angle of 120 deg. to each other.

Frequently, it is necessary to reverse the process, that is to convert rotary motion into reciprocating motion, and for this the crank and connecting rod mechanism is also used. Compressors and



DRIVING A SHAPING MACHINE

Fig. 5. In the shaping machine, the cutting stroke is made slowly and the return stroke quickly. This is made possible by a quick-return motion. During the cutting stroke, the crank pin *C* moves in the upper part of the slotted link *AB*, which rocks slowly from right to left. On the return stroke the crank pin moves in the lower portion of the slotted link, which rocks back quickly from left to right.

pumps are very familiar examples, but there are many other applications especially among machine tools, viz., punching and shearing machines, and the giant modern presses which can stamp out the whole side of a motor-car body at a single movement. There are shaping and slotting machines, too, in which the cutting tool reciprocates over the work, taking a shaving off at each stroke.

In these machines, the speed of the cutting tool must be chosen to suit the material being worked, viz., slow for cast iron and fast for brass. But no cutting takes place on the return stroke, and, therefore, it is desirable to withdraw the tool as quickly as possible, ready for the next cutting stroke. For this a quick-return motion is used. One of the most widely used types is shown in Fig. 5.

In this mechanism, instead of the connecting rod being driven directly by the crank pin, it is attached to one end of a slotted link, which in turn is driven by the crank, For this reason it is usually called the crank-and-slotted-lever quick-return motion.

Now let us see how this arrangement works. The crank pin C rotates at uniform speed, and as it slides in the slot in the slotted link AB, it causes this link to rock from side to side about the pivot at A. The connecting rod BD, which is attached at B, the other end of the slotted link, communicates this rocking motion to the cutting tool T which moves on fixed guides in the manner of an engine crosshead. So far we have accounted for the backward and forward motion of the tool.

How does it come about that with the crankshaft rotating at uniform speed, the tool makes the return stroke quicker than the cutting stroke? Now look at the small diagrams and you will notice immediately that the cutting stroke starts when the crank pin is at C_1, before it has reached the horizontal centre line, and ends when the crank pin is at C_2, in a similar position on the other side. The return stroke is then made whilst the crank rotates through the very much smaller angle from C_2 to C_1 to complete one revolution ready for the next cutting stroke.

In the diagrams, the angle from C_1 to C_2 is about 230 deg., and from C_2 to C_1 about 130 deg., so that the time taken for the return stroke is little more than one half that for the cutting stroke.

Crank-pin Moment

Another way of looking at this is to consider the moment which the force at the crank pin exerts about the pivot A of the slotted link. During the cutting stroke, when a large force is needed at the cutting tool, the crank pin is working in the upper part of the slot, and the moment about A is large. During the return stroke, when there is no cutting action, the crank pin is in the lower part of the slot, and the moment is small, but the speed of the crank pin is the same for both strokes.

The diagrams of Fig. 5 also show the relationship of the stroke of the cutting tool to the throw of the crank, and it will be noticed that, if the throw of the crank is made smaller, the stroke of the cutting tool will be made smaller in proportion. These machines, therefore, are arranged so that the throw of the crank can be adjusted to suit the size of the work being

machined. The crank pin is usually a separate piece mounted in a disk so that it can be clamped firmly in position by a large nut after it has been set at the desired radius.

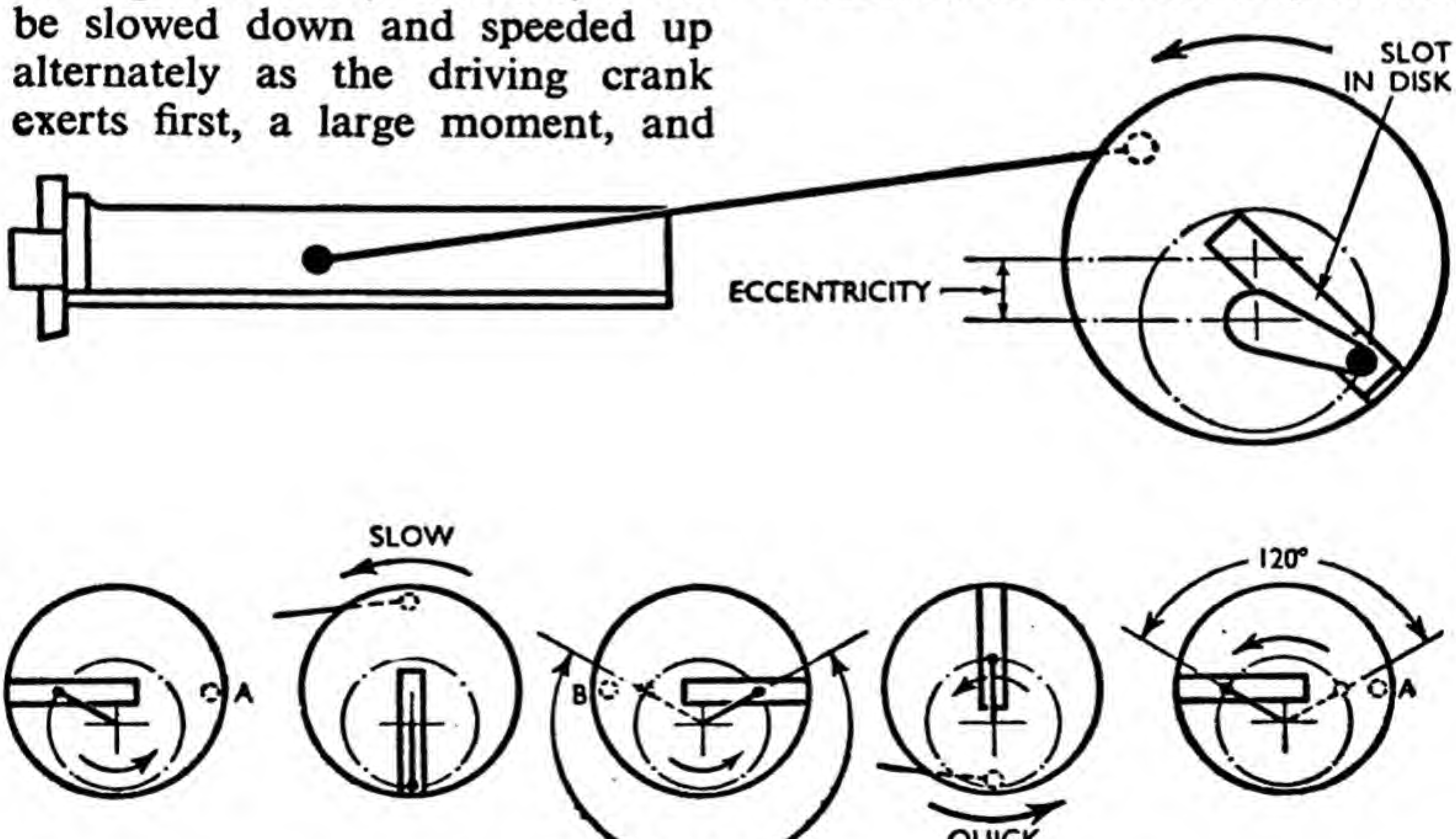
Whitworth Motion

Another type of quick-return motion is the Whitworth motion, shown in Fig. 6. Here also, the driving crank pin works in a slot, but this time the slot makes a complete rotation instead of merely rocking backward and forward. The slot is usually cut in a disk, which also serves as the driving member for the connecting rod, which imparts the motion to the tool.

The essential feature of this mechanism is that the crank and the slotted disk rotate about different centres. In other words, the slotted disk is eccentric to the driving crank and, therefore, it will be slowed down and speeded up alternately as the driving crank exerts first, a large moment, and then a small moment about the centre of the disk.

It will be observed from Fig. 6 that, if the crank and the disk both revolved on the same axis, both would rotate uniformly, and the crank pin would always occupy the same position in the slot, so that there would be no sliding. If the slotted disk is now made to rotate about a centre which is displaced from the axis of the driving crank, the motion of the disk will become irregular, although the crank continues to rotate uniformly. The crank pin will also have to slide in the slotted disk to allow for the eccentricity. The greater the eccentricity, the greater will be the amount of sliding, and the more pronounced will be the quick-return effect.

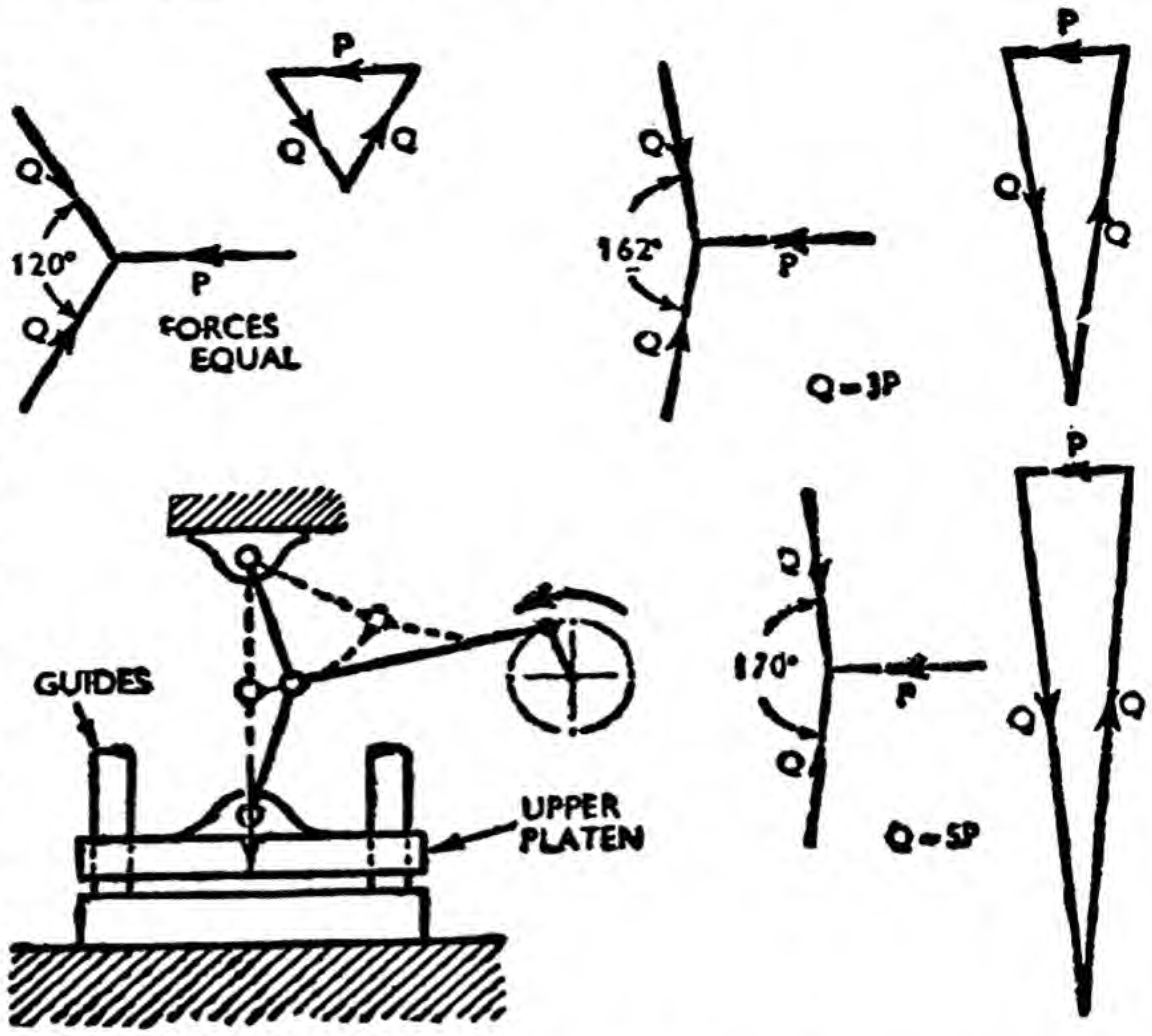
The cutting and return strokes of the tool are determined, of course, by the dead-centre positions



WHITWORTH QUICK-RETURN MOTION

Fig. 6. In the Whitworth quick-return motion, crank pin engages a slot which is cut in a disk. Disk rotates about an axis which is eccentric to axis of rotation of crank. Owing to this eccentricity, the disk is alternately slowed down, from *A* to *B*, and speeded up from *B* to *A*, for cutting and return strokes respectively. The greater the eccentricity, the more pronounced is the quick-return effect.

Fig. 7. When three forces acting at a point are in equilibrium, each force is proportional to the sine of the angle between the other two. When the angle between two of the forces approaches 180 deg., a small force *P* can balance two very large forces *Q*. A toggle press is an application. Two short stiff links are hinged together, and a connecting rod driven by a crank exerts a thrust at the hinge. Moving table is pressed down with tremendous force as hinged links are forced into line by the final squeeze.



of the slotted disk, and the small diagrams show clearly how the driving crank rotates through a large angle, on the cutting stroke, and hence takes a long time, and a much smaller angle on the quick-return stroke. On the cutting stroke, the slotted disk has to rotate through 180 deg. from *A* to *B*, while the crank rotates through about 240 deg., whereas on the return stroke, the slotted disk has to get back from *B* to *A* while the crank rotates through only about 120 deg.

We have seen in Chapter 4 that it is a fundamental property of machines that a small effort applied to a part of the machine moving at high speed can overcome a large resistance at a part which is moving very slowly at the same time. The quick-return motion is a convenient way of getting the high speed when there is no cutting force.

In another type of machine, the 'arge press, we require different conditions, a very great force for only a small portion of the stroke. For this purpose we use the toggle (Fig. 7). The principle is that of three forces acting at a point. When the three links which transmit the forces are equally inclined to each other, that is, at 120 deg., the forces in the links are equal, as the triangle of forces clearly shows. But as the angle between two of the links increases, the force in these links increases, until, when the two links are almost in line, the force may be twenty or even fifty times the force in the third link.

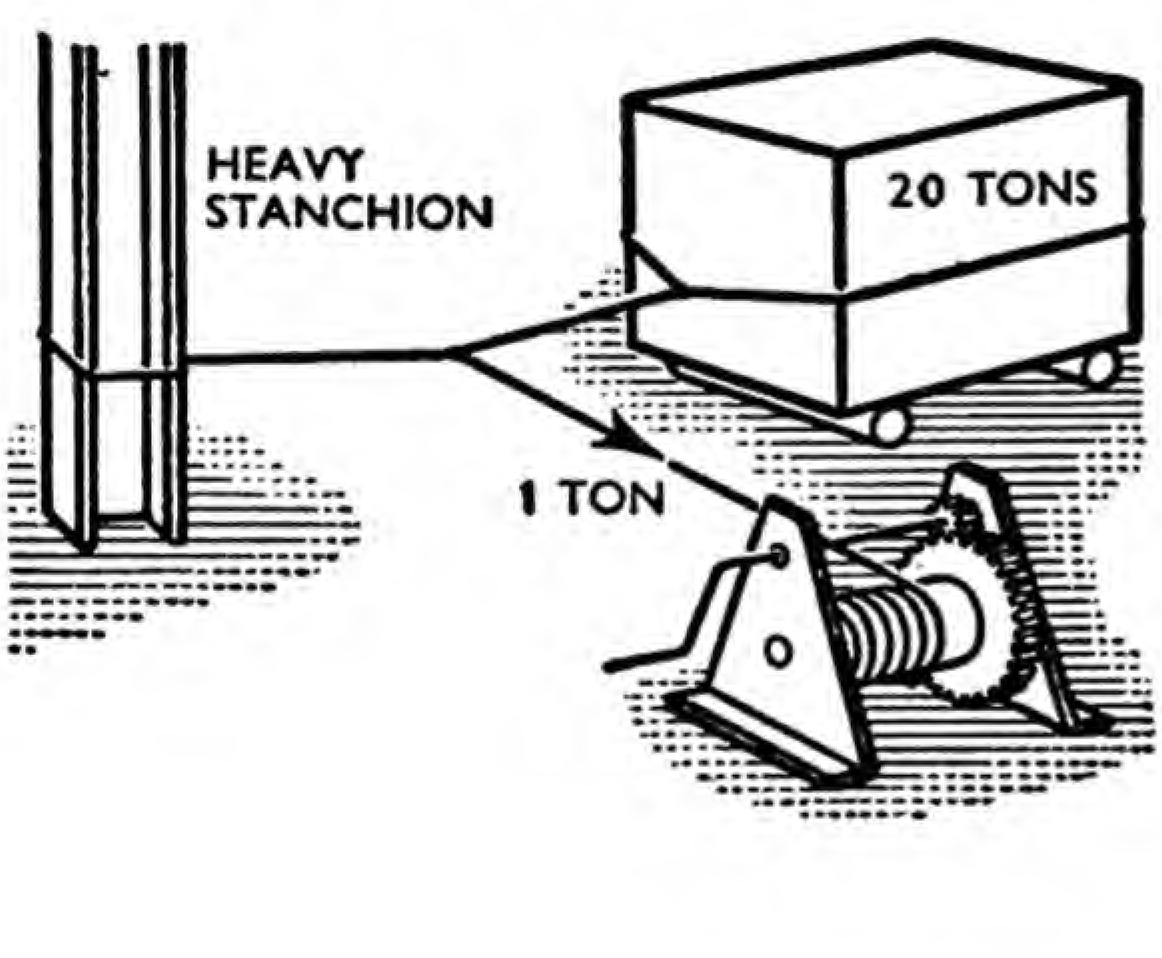
This is just what is wanted to give the final squeeze in a powerful press. As the upper platen of the press, as the moving table is called, descends on to the die in the lower fixed platen, no very great force is required at first, but the force increases as the metal sheet between the platens is forced into the impressions in the die. Then comes the final squeeze, amounting to hundreds of tons, and the sheet

being formed takes up all the detail of the impression with perfect sharpness.

The press is adjusted so that the links of the toggles come into line just at the moment when the two platens and the sheet between them are squeezed solidly together. Theoretically, when the links come into line, the force is infinite, but in practice the links always yield slightly however solidly the press is made, and so limit the maximum value of the force.

When the links are pushed still further, so that they go over the centre, this yield or give in the links is relaxed first, before the platens start to move apart. This property is often useful to make the toggle self-locking, although not in the press shown in Fig. 7. Take, for example, a toggle clamp. Here, force is required to squeeze the parts together and then to hold them in that position for as long as desired. The toggle is arranged in that case to go over the centre so that it locks, but not so far over that it releases the pressure.

The toggle can also be used in tension. For example, suppose we have to move a heavy machine weighing, say, 20 tons, and we have only a 1-ton winch (Fig. 8). How is it to be done? The answer is, by a toggle. The main hauling wire or chain secured to the load is made fast to a heavy stanchion or pillar, in line with the movement required. The wire from the winch is then secured to the centre of the hauling chain so that a pull can be exerted at right angles. As the load moves, the effectiveness of the toggle will decrease, owing to the two halves of the hauling chain becoming inclined to each other at a smaller and smaller angle. Therefore, the load must be moved a short distance at a time, and the hauling chain made taut before each move.



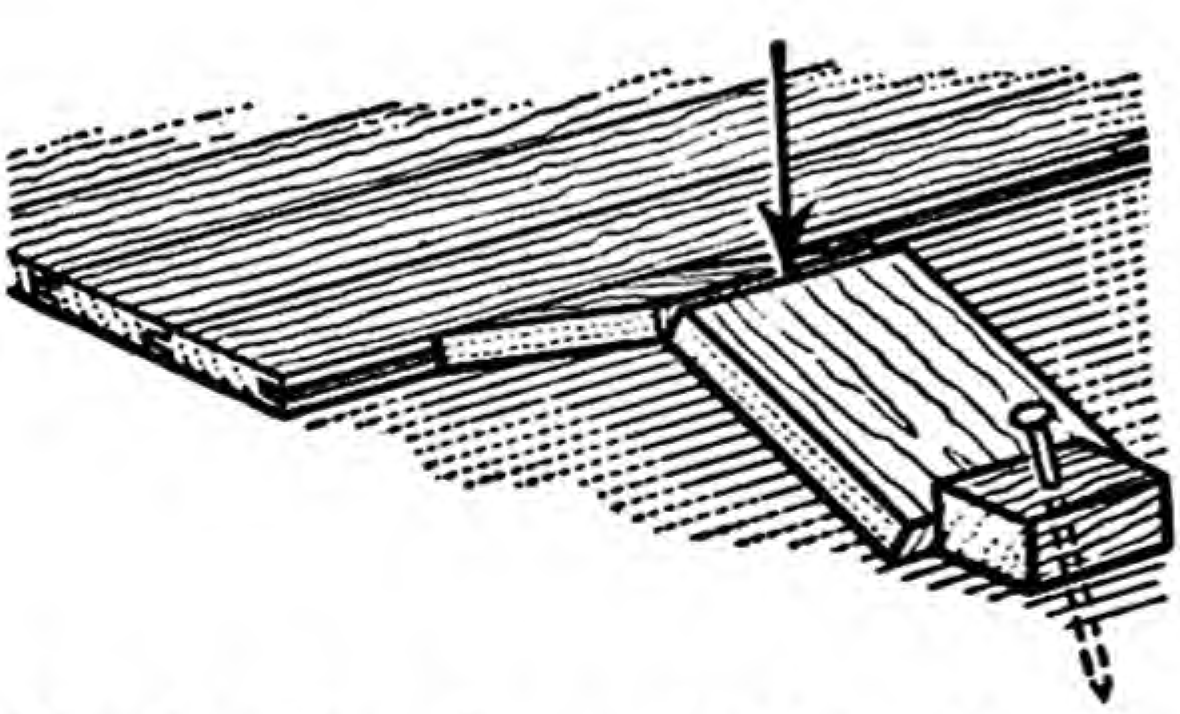


Fig. 8. Two improvised toggles are shown above. Hauling power of a small winch is much increased if its pull is exerted at right angles to the taut hauling cable. Two short lengths of board hinged together are all that are required to make a simple but effective floor cramp.

Velocities of Mechanisms

We have seen, in the engine mechanism for example, that the small end of the connecting rod reciprocates in a straight line whilst the big end moves in a circular path. What kind of motion has a point mid-way between the big and small ends? Again, with the toggle press, how fast is the top platen moving when it is, say, one inch away from the die?

To answer these questions it must be possible to find the velocity of

any point on a link at any instant. Fortunately, there is a very simple way of doing this. Suppose we consider a ladder *AB* resting in the angle between a wall and the floor as shown in Fig. 9. The upper end *A* can only slide vertically down the wall, whereas the lower end *B* can only slide along the floor. What is the motion of a point *C* on the ladder, say, a quarter of the length from the top?

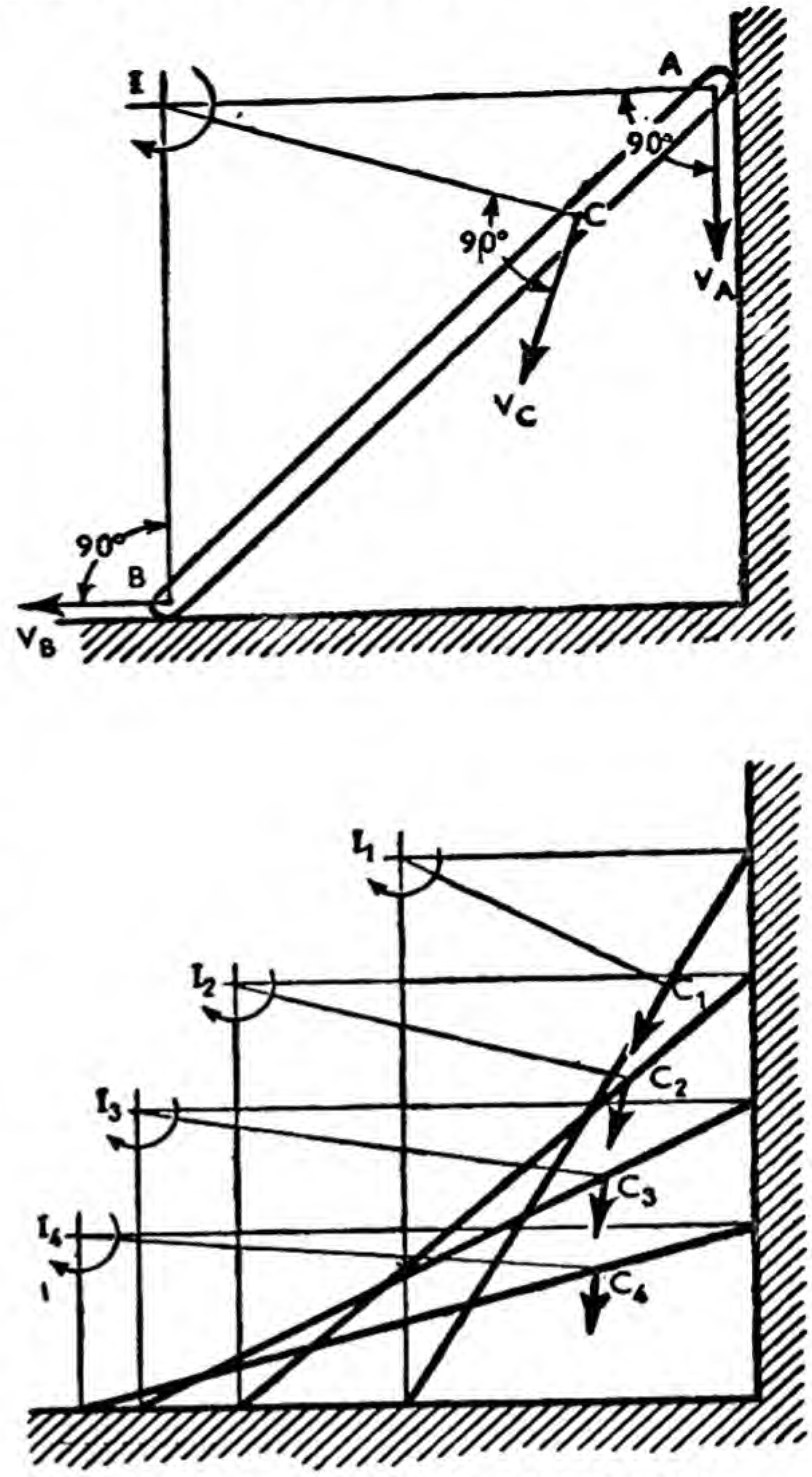
It is as shown in the figure; that is, as the ladder slides down and outward, its motion as a whole consists of a small rotation about the centre *I*. Therefore, *C* moves in a direction perpendicular to the line joining *C* to the centre of rotation.

Centre of Rotation

The question now is, how do we find the centre of rotation *I*? End *A* moves parallel to the wall, and, therefore, its motion can be considered as an infinitely small rotation about a centre on the line through *A*, perpendicular to the wall.

Similarly, end *B* moves parallel to the floor, and its motion can be considered as an infinitely small rotation about a centre on the line through *B*, perpendicular to the floor. The intersection of the two perpendiculars gives the centre *I*, about which the ladder as a whole can be considered to be rotating *at that instant*.

The centre *I* is called the instantaneous centre of rotation. From the lower diagram in Fig. 9, it is clear that as the ladder moves into a new position, the instantaneous centre changes its position too, but the velocity of any point on the ladder is always in a direction perpendicular to the line joining it to the instantaneous



THE INSTANTANEOUS CENTRES FOR DIFFERENT POSITIONS OF THE LADDER

Fig. 9. Ladder *AB*, sliding between a wall and the floor, has a motion which consists of a small rotation about an imaginary centre *I*, which is called the instantaneous centre of rotation. In all positions, the centre *I* is at the intersection of the lines drawn through *A* and *B*, perpendicular to the wall and the floor respectively.

centre of rotation in that particular position. Actually, if the motion of *C* was worked out in detail, we should find that its path is a quadrant of an ellipse.

We can also determine the magnitude of the velocity of *C* as well as its direction, if the velocity of *A* or *B* is known. Because all points on the ladder are rotating at the same rate around the centre *I*,

their linear velocities will be proportional to their radii from I, viz.,

$$\frac{\text{Velocity of } C}{\text{Velocity of } A} = \frac{IC}{IA}$$

and,

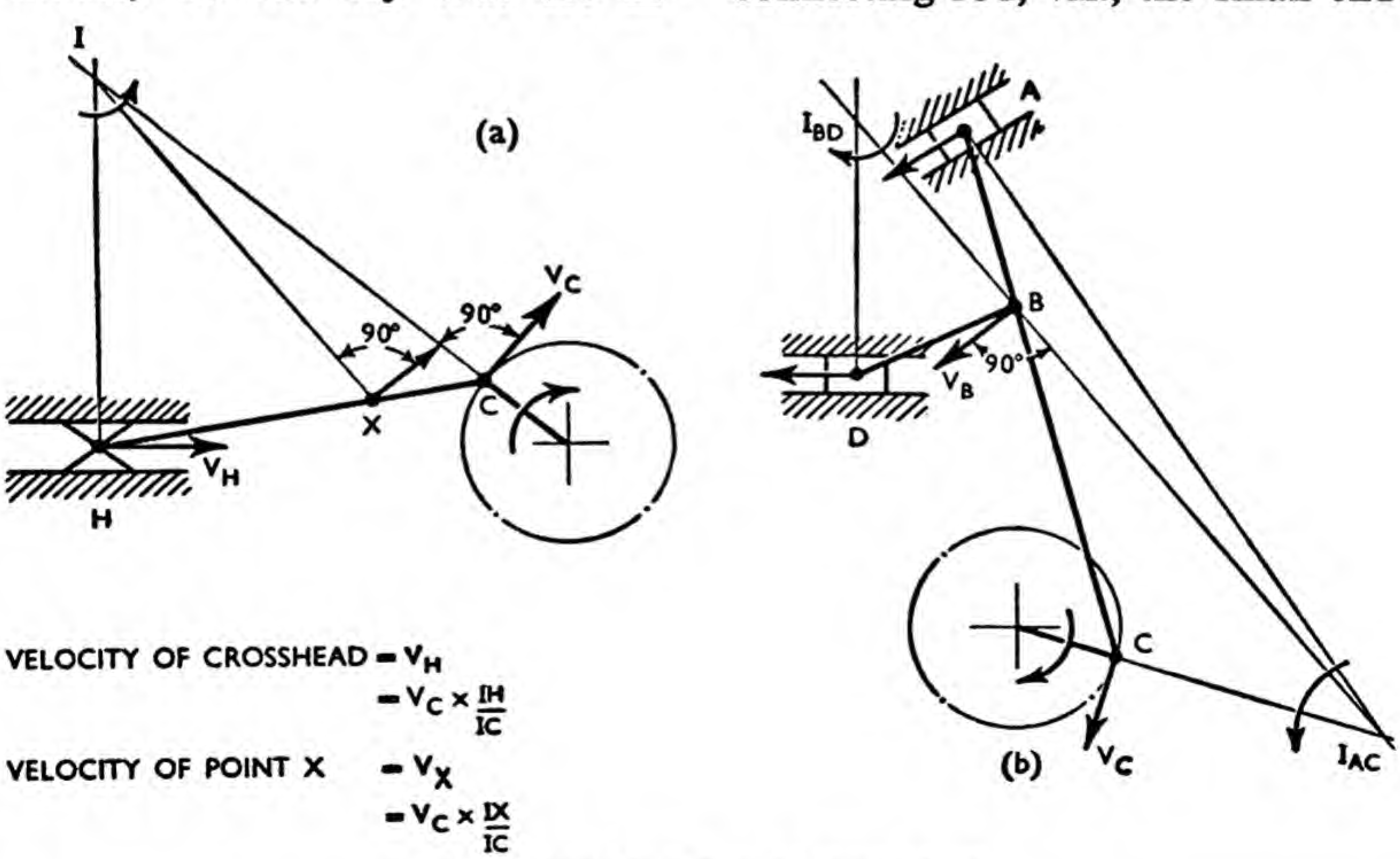
$$\begin{aligned}\text{Velocity of } C &= \text{Velocity of } A \times \frac{IC}{IA} \\ &= \text{Velocity of } B \times \frac{IC}{IB}\end{aligned}$$

We can now apply exactly the same method to the engine mechanism, to find, say, the velocity of the crosshead or of a point on the connecting rod. First the instantaneous centre for the connecting rod must be found by considering the motion of its two ends. The end H (Fig. 10(a)) moves with the crosshead in a straight line and, therefore, as for the foot of the ladder, we can say that the instantaneous centre will lie on a line perpendicular to its path.

At first sight, the crank pin seems rather different because it is actually rotating about the point O. But if we bear in mind that due to this rotation about O, the linear velocity of the crank pin at any instant is tangential to the circle at the point C, we can then forget all about O, and say that the instantaneous centre lies on a line perpendicular to the linear velocity of C, viz., perpendicular to the tangent at C, just as if C were moving in a straight path.

These two perpendiculars are shown in Fig. 10 intersecting at the point I, which is the instantaneous centre of rotation for the connecting rod in the position illustrated.

The velocity of any point on the connecting rod, viz., the small end



VELOCITIES OF MECHANISMS

Fig. 10. Motion of mechanisms is readily investigated by the instantaneous centre method. In the engine mechanism (*a*), the connecting rod *HC* is momentarily rotating about the centre *I*. Similarly in the valve mechanism (*b*) the link *ABC* is momentarily rotating about the centre I_{AC}, and, therefore, the motion of point *B* is tangential to a circle centred at I_{AC}. End *D* slides in a slot, so that I_{BD} is the instantaneous centre for the link *BD*. The reader may like to calculate how fast *D* is moving if the velocity of *C* is 10 ft. per sec.

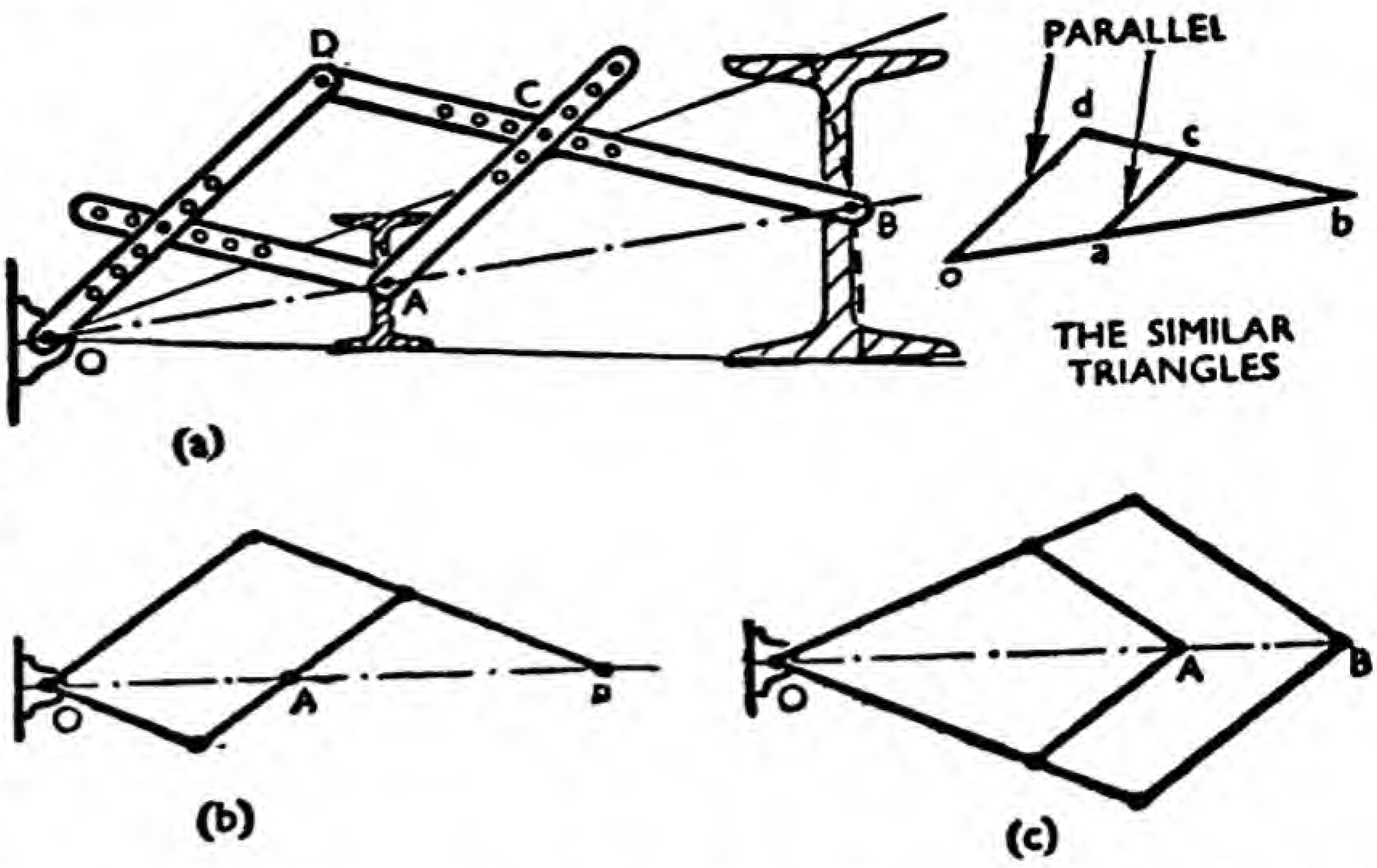


Fig. 11. The pantograph is a linkage for copying designs. By placing a pencil at *A*, a reduced scale of the outline at *B* is obtained. Conversely, an enlarged scale of *A* is produced if the pencil is placed at *B*. The scale of reproduction is in the ratio of *OA* to *OB*, and the links are perforated so that this can be varied. (*b*) and (*c*) are alternative linkages.

H or a point *X*, part way along its length, is then proportional to its radius from *I*. In this case, the linear velocity of *C* can usually be determined from the throw of the crank and the speed of rotation, so that it is easy to calculate the other velocities by proportion. Fig. 10(b) shows a more complicated mechanism, but the same method is used. We have to proceed in stages. First the ends of the link *AC* are considered, and the centre I_{AC} is obtained. This gives us the velocity of *B*, and then I_{BD} can be found, and hence the velocity of *D*.

The Pantograph

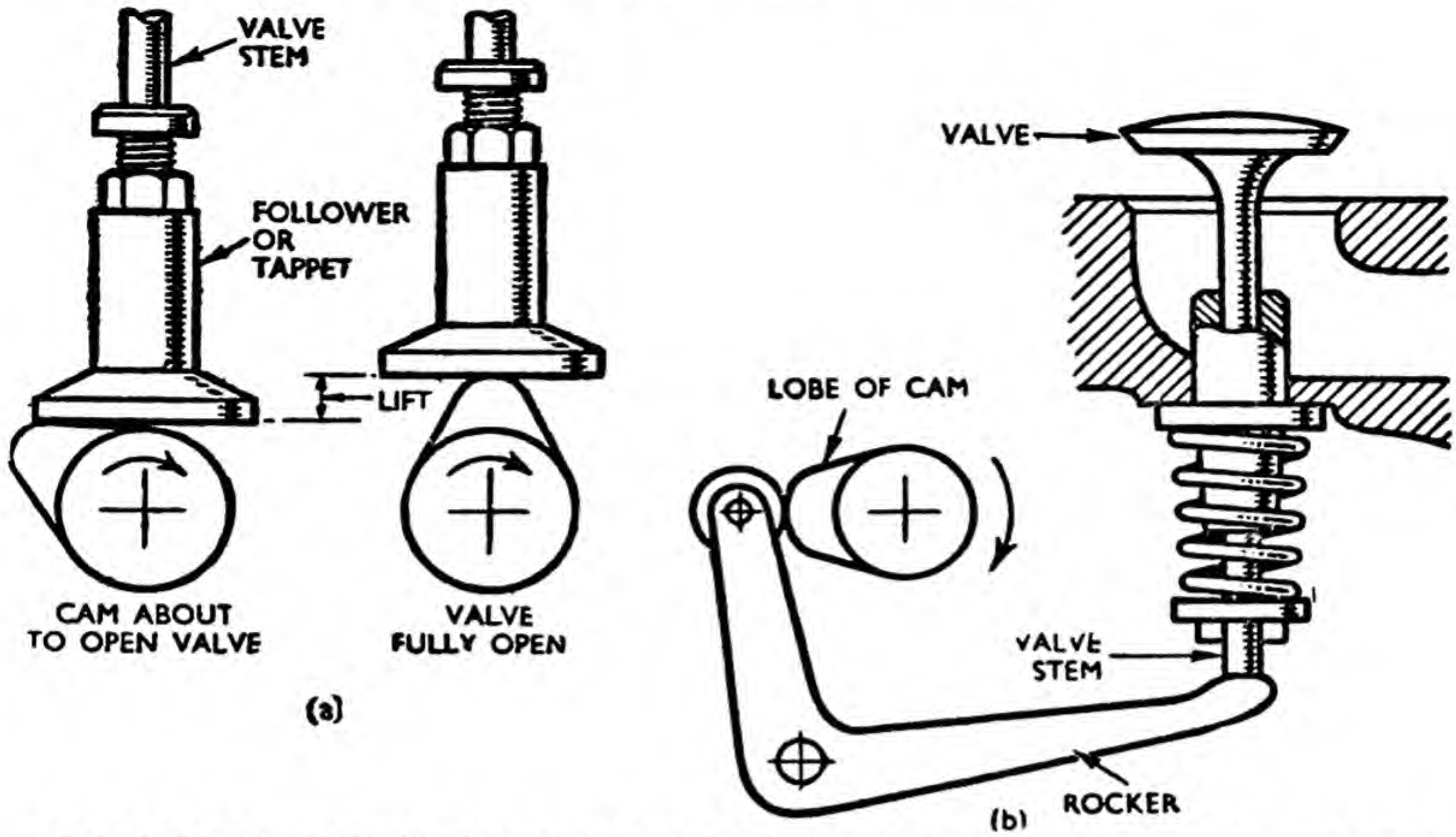
There are many arrangements of links for special purposes, and a very useful one is shown in Fig. 11(a). This is known as the pantograph, and it is used for copying designs either to an enlarged or a reduced scale, depending upon whether the tracing point is put at *B* or *A*. The linkage must be so arranged that the three points *O*, *A* and *B* are in a straight line. The scale of the enlargement or reduction will then be in the ratio of *OB* to *OA*. By means of holes provided in the links, the position of *A* may be adjusted along the line *OB*, so as to give different ratios.

To understand the geometry of the pantograph, we have only to consider the similar triangles *abc* and *obd*. As the linkage is moved into different positions, these triangles will change their shape, but they will always be similar to each other, and the lengths of all their sides will be in the ratio of *od* to *ac*. Thus, when we set the length of *AC*, by choosing the appropriate holes in the links, we automatically fix the ratio of *OB* to *AB*.

Other forms of the pantograph are shown in Figs. 11(b) and (c), and in all three arrangements, if desired, we can put the pivot at *A* instead of at *O*, and then put the tracing point at *O*. The scale of the drawing will then be in the ratio of *OA* to *AB*.

Cam Mechanism

Let us consider the cam mechanism for operating the valves of the petrol engine. Think what it has to do. It has to open the valves by a definite amount at a particular moment in the cycle, keep them open for a definite time, and then close them again precisely at the right instant! This is



HOW CAMS ARE USED TO OPERATE VALVES OF A PETROL ENGINE

Fig. 12. Valves of internal combustion engines are usually operated by rotating cams. (*a*) Here is the common arrangement where the lobe of the cam lifts a follower, or tappet, which, in turn, lifts the valve. (*b*) Illustrates the use of a rocker to transmit the motion when the axis of the camshaft is not in line with the valve stem. The shape of the lobe controls both the amount of lift and the rate and duration of the opening.

typical of the many special jobs for which cams are used.

Now look at Fig. 12, and you will see that it is all done by choosing a suitable shape or, as it is usually called, profile of the cam. In this instance the cam rotates uniformly and, as the valve is to remain closed for almost three-quarters of the time, the profile of the cam is cylindrical over an arc of about 250 deg.

There is, then, a projection or lobe which controls the opening and closing of the valve. Note that the height of this lobe governs the amount of opening or lift of the valve, and that the position of the lobe controls the instant at which the valve commences to open. Also note that the valve may be kept open for as long as desired by altering the profile of the lobe, as at (b). Fig. 12 shows also how the cam works against a roller so as to reduce friction, and operates the valve through a lever or rocker.

Sometimes cams have two or more lobes so that more than one operation occurs during each revolution. The cam for operating the contact-breaker of the motor-car ignition distributor is a very well-known example (Fig. 13(a)). For a

Fig. 13. (*a*) Cams operating the contact breaker of motor car ignition have several lobes. (*b*) In machine tools, reciprocating or sliding cams are used.

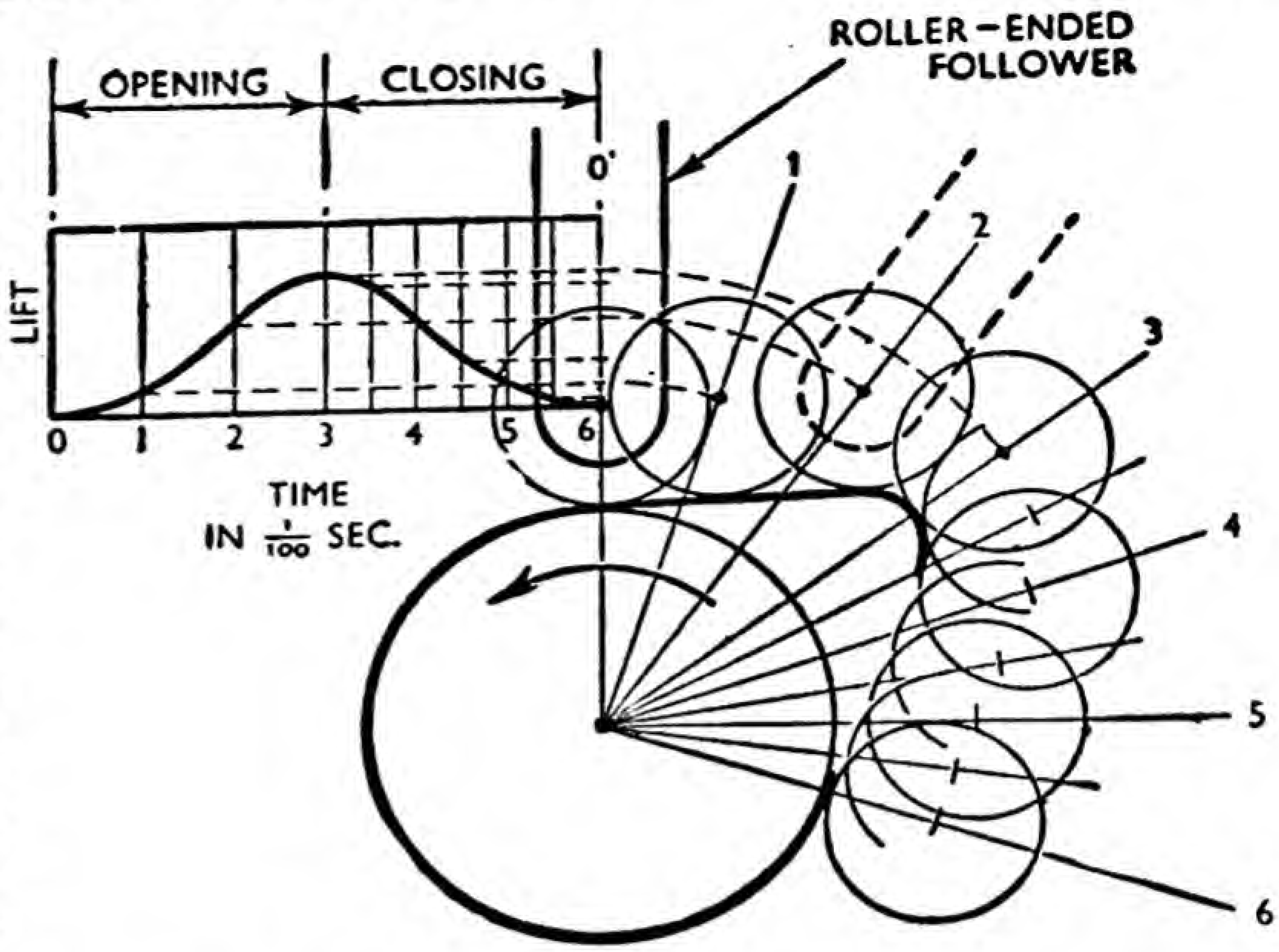


Fig. 14. How to develop the profile of a cam. The cam is kept stationary, and the follower is rotated round the cam in the direction opposite to that of rotation. In each successive position during opening and closing (shown by the radial lines 1, 2, 3, 4, etc.), the follower is given its correct lift. The curve which touches the follower in each position is the required cam profile.

four-cylinder engine, it looks like a square shaft with the corners rounded. In reality it is a cam with four lobes, each rounded corner forming one lobe, and causing the contacts to open and close.

Correct Profile

Now let us see how the correct cam profile for any particular motion is obtained. First of all we need a graph of the motion on a time base, such as is shown in Fig. 14. Each of the divisions of the base represents 1/100 sec., and the graph shows that the valve has to be opened ½ in. in 0·03 sec. and then must close again in 0·03 sec. Next it must be considered how much the cam rotates during the opening and closing of the valve. Suppose it rotates at 300 r.p.m., that is five revolutions per second. It will turn through 18 deg. in each 1/100 sec., so we mark off a series of radial lines, 1, 2, 3, 4, 5, 6, at 18-deg. intervals to represent the rotation of the cam during each period of 1/100 sec.

Now actually, of course, the valve merely oscillates up and down through ½ in. while the cam rotates, but to draw the profile it is necessary to keep the cam fixed, and the right relative motion must be obtained by rotating the valve round the cam, and moving it into its correct position to correspond with each 1/100-sec. interval as given by the graph. This will give us seven points on the profile, and we can get as many intermediate points from the graph as we wish. This has been done (Fig. 14) for the closing period, and you will notice that the short arcs representing the roller in each position come so close together that it is easy to draw the cam profile as a smooth curve touching all the arcs.

Sometimes the cam slides instead of rotating (Fig. 13(b)), or is mounted on the surface of a cylindrical drum so that it causes motion parallel to the axis instead of radially, but the method of getting the profile is very similar to that outlined above. Cams of this kind are widely used on automatic machine tools to control the movement of the cutter, and so to obtain any shape desired.

Many readers may often have

wondered what was the secret of the *brain* of the amazing automatic machines which can turn out thousands of accurate parts for, perhaps, safety razors, or for giant aero engines. The secret is a set of cams which feed the cutters in and withdraw them again precisely at the right instant, each cutter having its own special job to do.

Toothed Gearing

We have seen that, with the cam, the precise time of opening of the valve, or whatever it is that the cam controls, depends upon the position

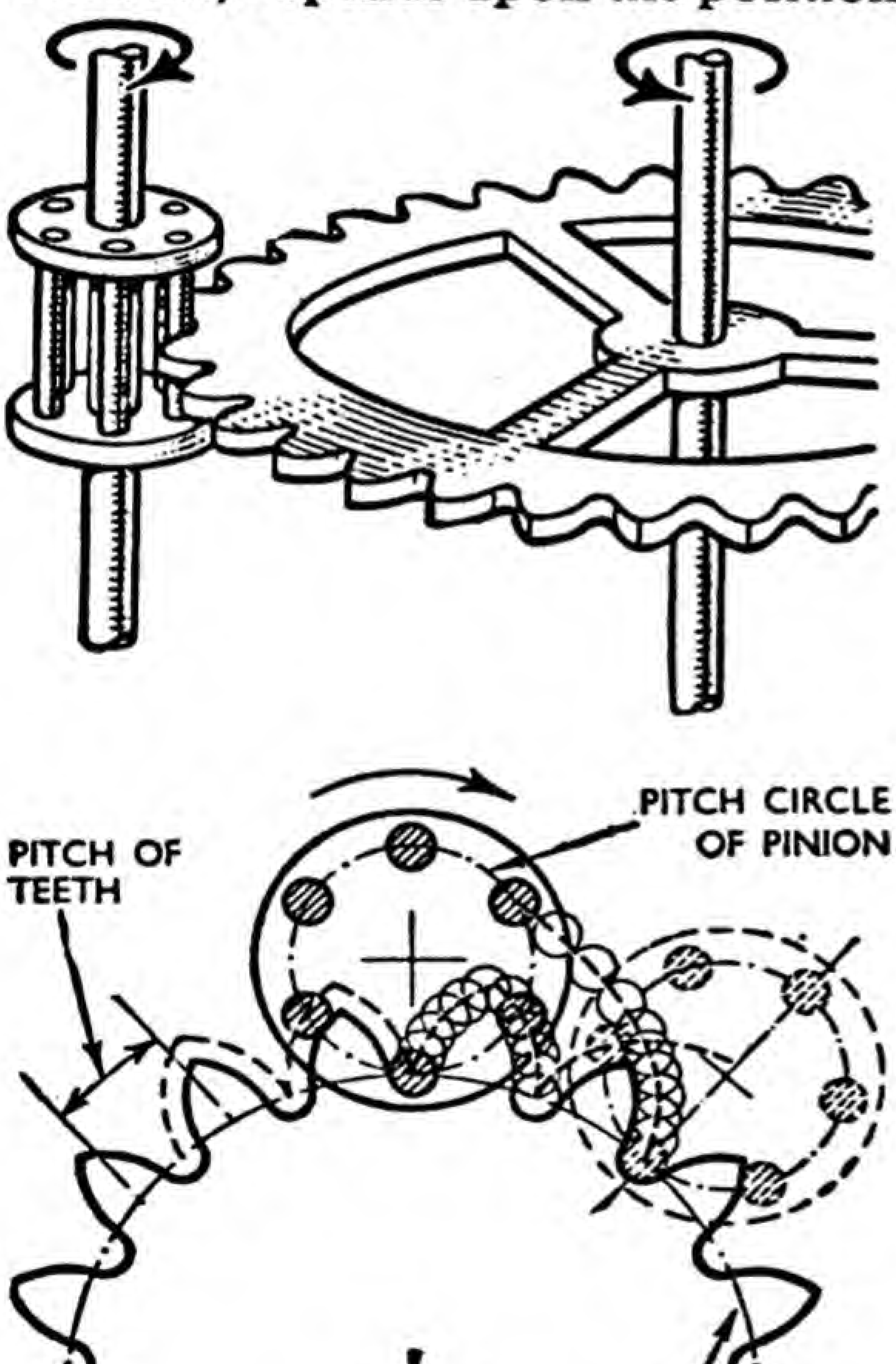


Fig. 15. In clockwork mechanisms, the pinion frequently has circular pins which act as teeth. This is called a lantern pinion. The method of drawing the profile of the teeth of the mating gearwheel is similar to that used for cams. The lantern pinion is rolled around the pitch circle of the gearwheel, and the pins trace out the profile of the teeth.

of the lobe. It follows that if the valve is always to be opened precisely at the correct instant, there must be no possibility of slip in the mechanism driving the cam. For a positive drive such as this, either toothed gearing or a chain drive is used. It is the interlocking of the teeth or cogs which prevents slip occurring, and the shape of the cogs is important if the gears are to work smoothly.

In one form of gear, often found in clockwork mechanisms, one set of cogs are circular pins, and Fig. 15 shows how we can find the proper shape of the teeth of the other wheel which is to mesh with the lantern pinion, as the wheel with the pins is called. We use the same method that was used for the cam, that is, keep the wheel fixed and roll the lantern pinion round it, moving the pins a definite amount each time. The shape of the teeth must be such that they touch the pins in each position. You will notice that the pins are equidistantly spaced on a circle.

This is called the pitch circle of the lantern pinion, and the distance between the centres of the pins is called the pitch, or more correctly, the circular pitch, because it is measured along the circumference of the pitch circle. In the same way, the distance between the centres of the wheel teeth is called the pitch of the wheel teeth, and this must be equal to the pitch of the pins of the lantern pinion, because the pins must fit exactly between the teeth.

The pitch of the teeth is also measured along the circumference of the pitch circle of the wheel, and it will be noticed that these two pitch circles touch each other. In fact, the pitch circles correspond to

two friction wheels which, if they rotated without slip, would have the same motion as the gear wheels. It is immediately apparent that if the lantern has ten pins, and the wheel has twenty teeth, the pitch circle of the wheel will have twice the circumference of the pitch circle of the lantern, and as the wheels rotate a tooth for a tooth, one revolution of the wheel will involve two revolutions of the lantern pinion.

The profile of the teeth obtained in Fig. 15 is a part of a curve known as a cycloid, and this curve is often used for the profiles of the teeth of both wheels, instead of one being a lantern pinion, and such wheels are said to have cycloidal teeth. This profile is not the most convenient if the teeth have to be machined to shape (machine-cut gears) as all accurate gears are nowadays, and a slightly different curve, an involute, is used.

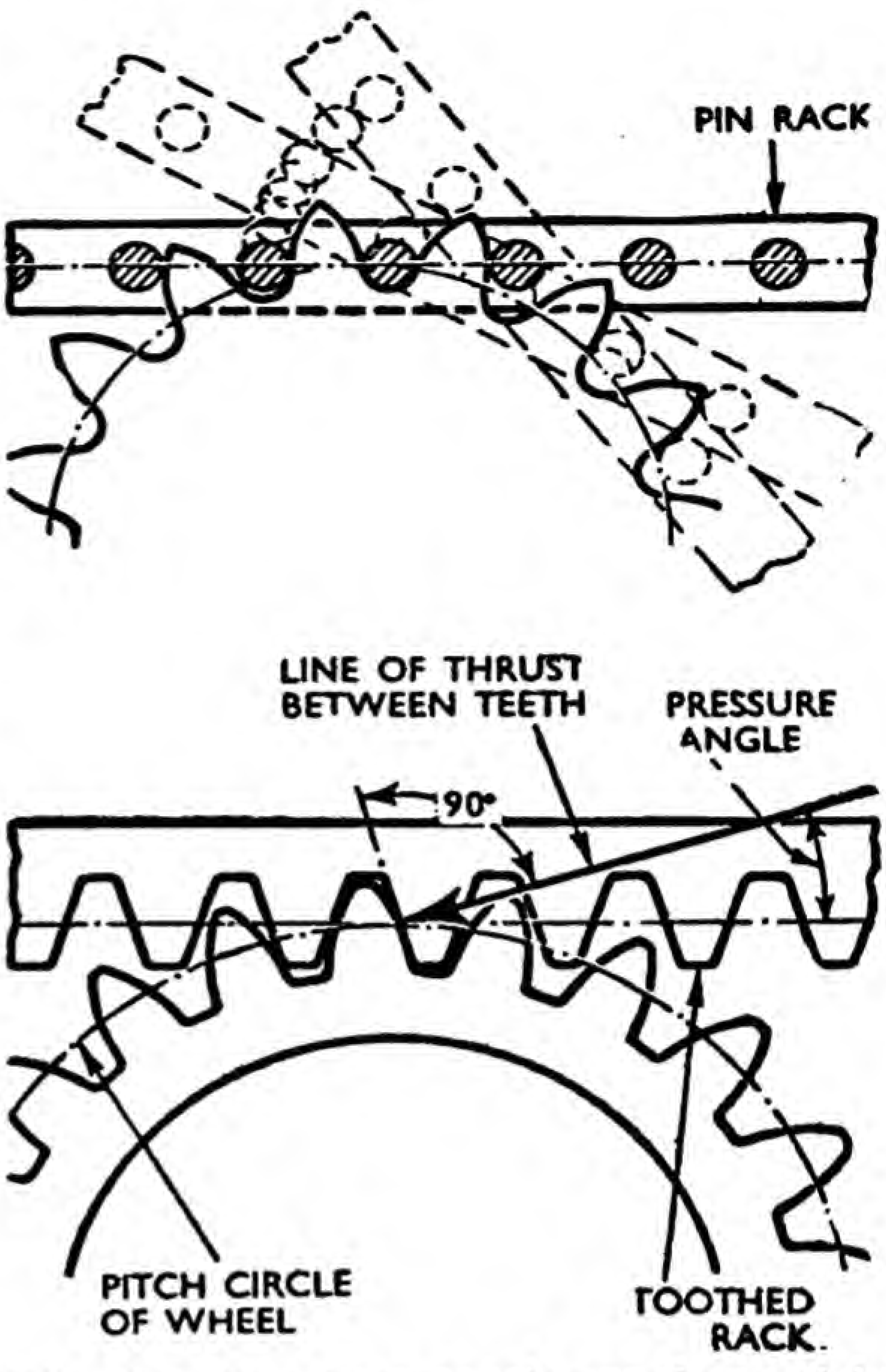


Fig. 16. Involute teeth are formed by using a straight rack instead of a pinion. For the teeth of commercial gear-wheels, the rack has straight-sided teeth instead of the circular pins. The thrust between the teeth is along a straight line which is at right angles to the rack teeth.

Involute Teeth

If, instead of the lantern pinion, we have a straight rack with circular pins arranged like the rungs of a ladder, and we repeat the process of Fig. 15, the profile of the teeth on the wheel will then be involute. This is shown in Fig. 16, and the teeth are exactly like those on a bicycle sprocket. Instead of circular pins in the rack, we can use straight-sided teeth, and the teeth of the wheel are still of involute form. These are, in fact, the kind of gear teeth that are most widely used today.

For the straight-sided teeth of the rack, we may use a large angle of taper to give short, strong teeth, or a smaller angle to give more slender teeth. This angle is important because it controls the direction of the thrust between the teeth, which is always at right angles to the tooth profile. This angle, therefore, is called the pressure angle, and the value is usually from 15 deg.–20 deg. It will be seen from Fig. 16 that the line of this thrust between the teeth passes through the point at which the pitch line of the rack touches the pitch circle of the wheel. This must be true for all gears if they are to mesh correctly, viz., the normal to the teeth at the point of contact must pass through the pitch point.

If the rotary motion is not required to be continuous, or if it is

desired to cause motion in one direction, but to prevent it in the other, then some form of ratchet, as shown in Fig. 17, is used. You will notice that it consists of a toothed wheel with a spring-loaded or weighted finger which engages the teeth on one stroke, but slips over them on the return stroke.

Ratchets

Thus, in the first diagram, which shows the drive to a mechanical lubricator, the toothed wheel is rotated clockwise for a fraction of a revolution when the operating rod moves to the left, but is not moved at all on the return stroke, the finger merely slipping over the teeth.

It is also possible to control the speed of rotation, for the finger may be arranged to move the wheel through 1, 2, 3 or more teeth on each stroke. It will be seen that in this instance the shape of the teeth is such that only clockwise rotation can be obtained, but in the next diagram, which shows a ratchet drill, either clockwise or counter-clockwise rotation can be obtained as desired. For this, we use a two-fingered pawl, as the finger is called, so that by engaging the finger on the left, the drill is rotated in the normal right-hand, or clockwise, direction, when the handle is worked backward and forward, but is rotated in the left hand direction when the finger on the right is engaged.

Another application of the same principle, is the free-wheel of the bicycle where, although the ratchet may be used to drive the rear wheel by a series of short backward and forward movements of the pedals, its main function is to permit the wheel to overrun without driving the pedals. The same principle is used in other applications to permit rotation in one direction, but to prevent it in the other, such as in the winding mechanism of a clock, where a ratchet is used to prevent the spring unwinding itself except by driving the gears.

Centrifugal Governors

A familiar sight on old steam engines was the centrifugal governor, with its massive rotating steel balls. Its purpose is to control the speed of the engine, and it depends upon a very simple principle, the conical pendulum described in Chapter 4.

In the Watt governor (Fig. 18), the balls are suspended by links which are pivoted on the axis of rotation, and as they rotate they are subjected to a downward force due to their own weight, and the

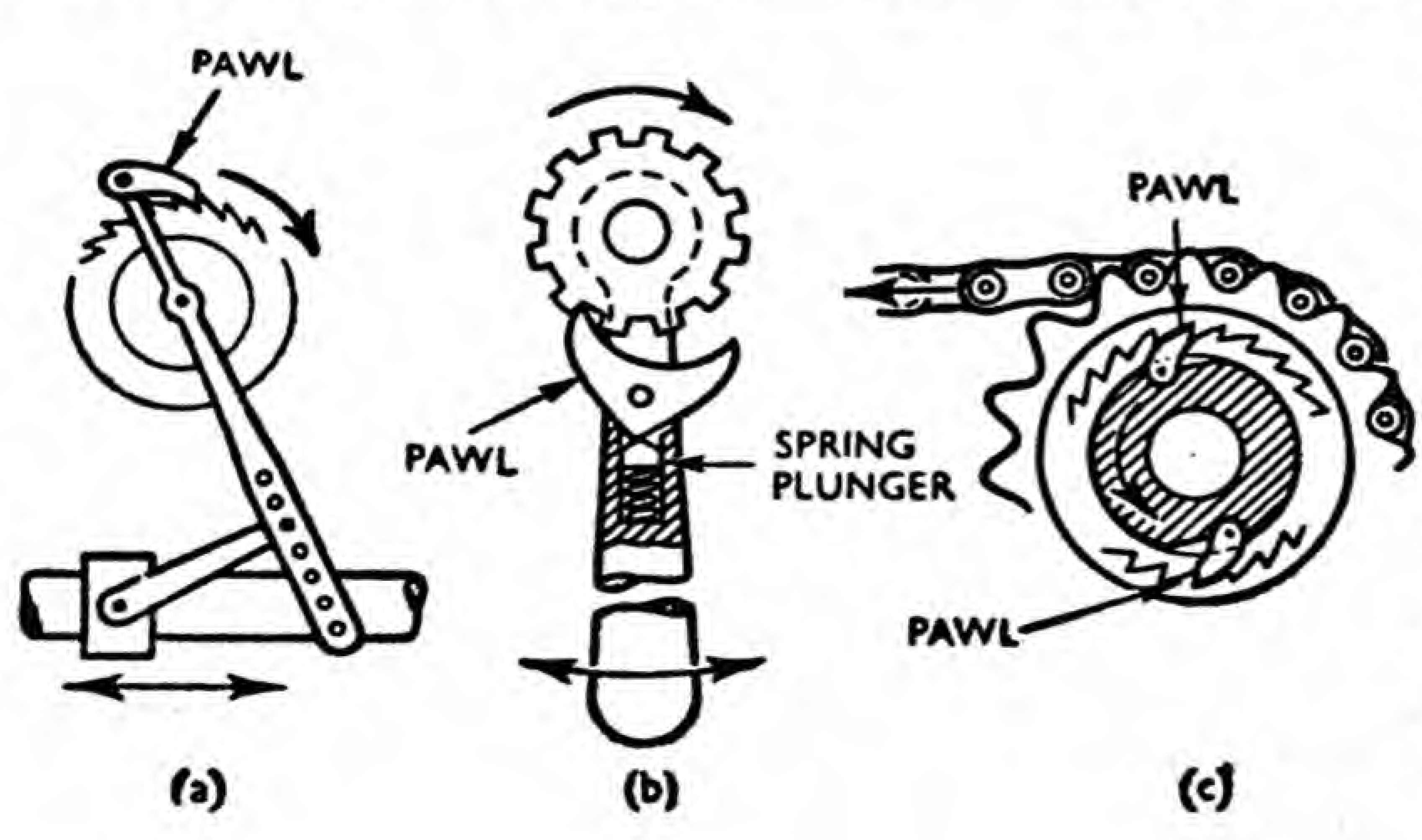


Fig. 17. Ratchets are used to convert reciprocating or oscillating motion into unidirectional motion. (*a*) Shows simple ratchet where the pawl drives the toothed wheel clockwise on one stroke, slipping over teeth on return. (*b*) Two-way ratchet working either way as desired. (*c*) Ratchet of bicycle free wheel (old type).

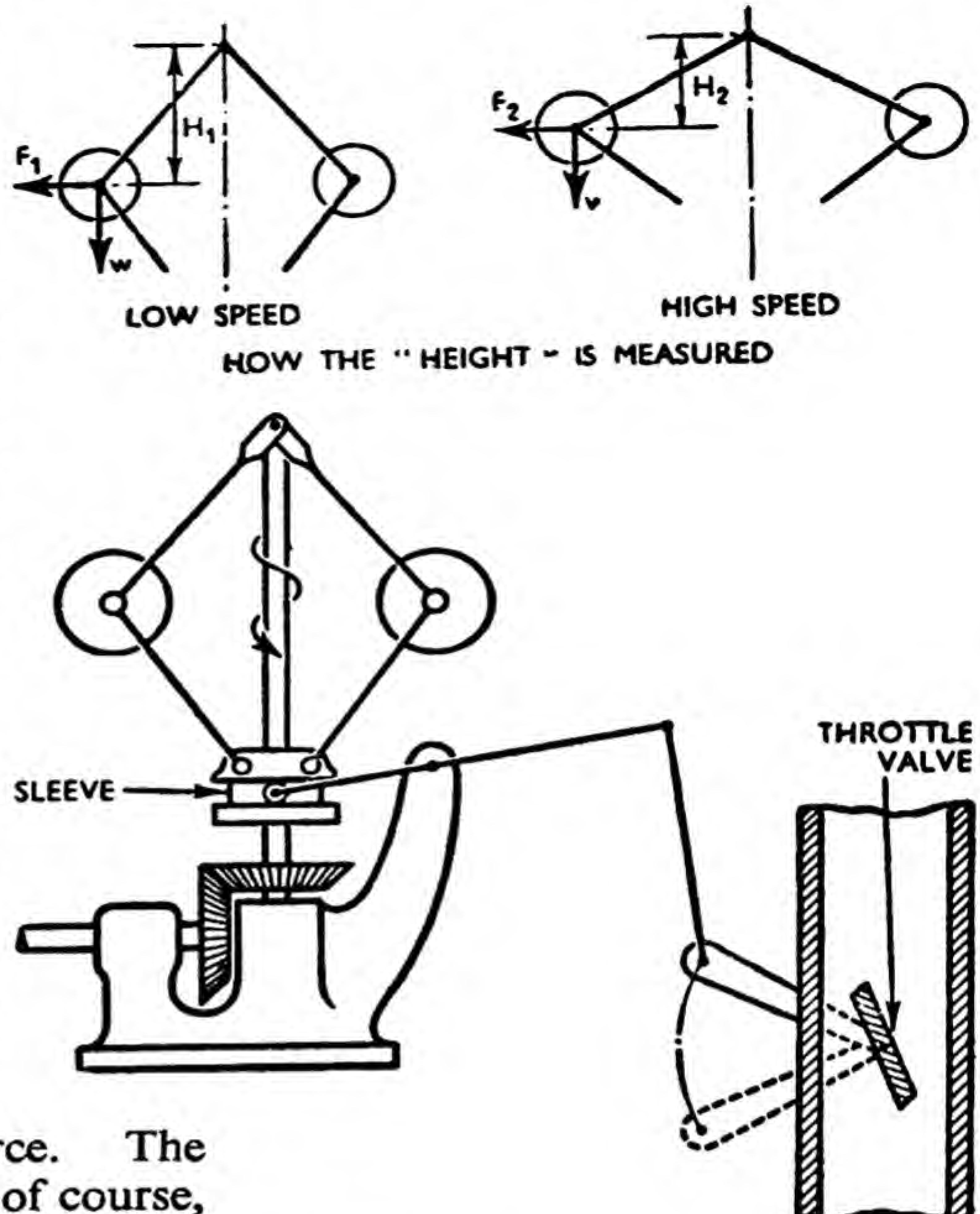


Fig. 18. The Watt governor operates on the principle of the conical pendulum. The centrifugal force acting on the heavy balls causes them to move outward and upward as the speed increases; that is, causes the height of the governor to decrease with increasing speed. The height is measured from the plane of rotation of the balls to the intersection of the arms. An increase in the centrifugal force from F_1 to F_2 causes the height to decrease from H_1 to H_2. This decrease in height is utilized to move a sleeve which is free to slide on the governor spindle. By means of a suitable linkage, the movement of the sleeve closes the throttle valve and so reduces the steam supply until the speed falls to its normal value.

outward centrifugal force. The weight remains constant, of course, whereas the centrifugal force increases as the speed increases, so that the balls rise higher the higher the speed. As the balls rise they cause the sleeve to rise, so partly closing the steam valve and preventing the speed rising further.

From the triangle of forces it can be shown that the height of the governor, viz., the height of the conical pendulum, is given by :—

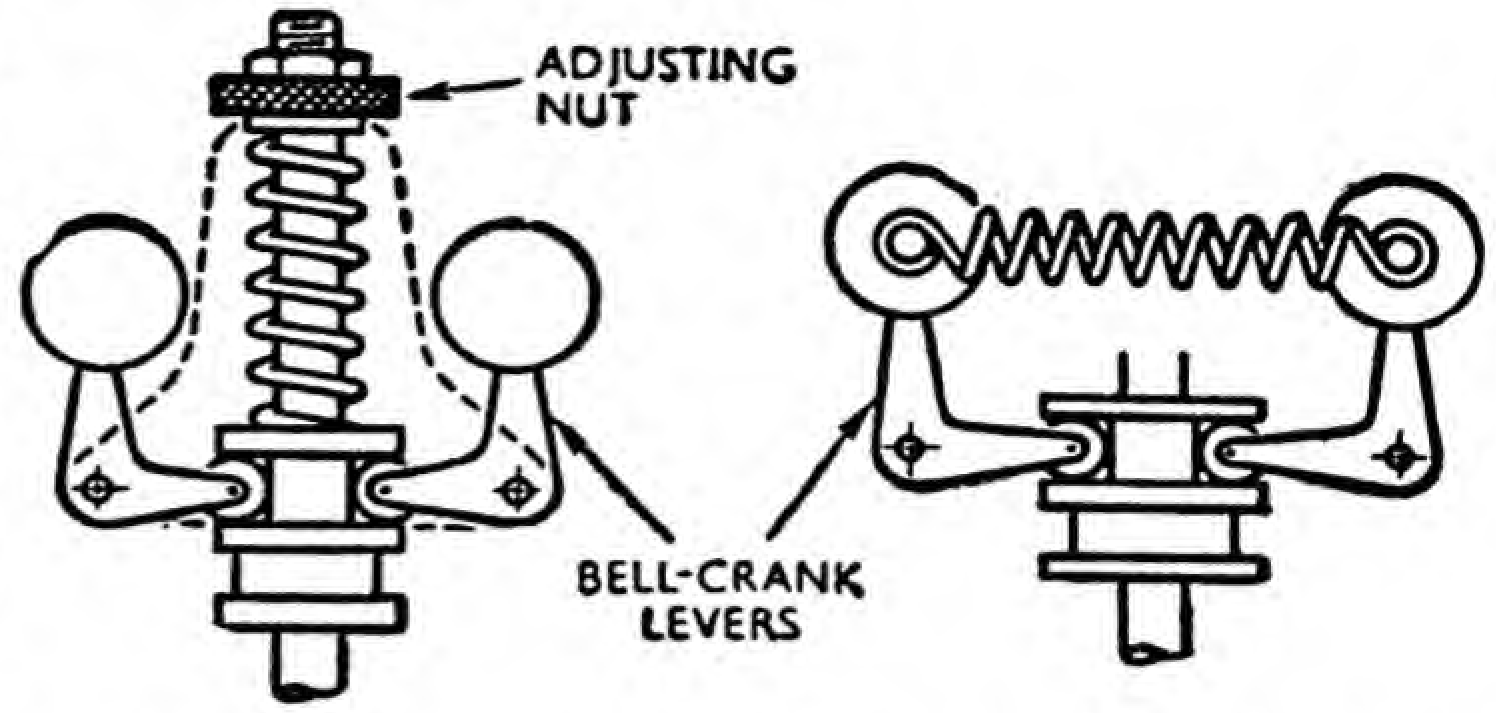
$$\text{height } H \text{ in inches} = \frac{35,300}{N^2}$$

where N is the speed of rotation of the governor in revolutions per minute. Thus, at a speed of 60 r.p.m. the height would be almost 10 in., whereas at 100 r.p.m. it would be little more than $3\frac{1}{2}$ in., so that the balls would rise almost $6\frac{1}{2}$ in. as the speed rose from 60 to 100 r.p.m. If this is worked out, it will be found that for speeds above about 150 r.p.m., the links are almost in line, and this is one of the disadvantages of the Watt governor, it is not suitable for high speeds.

For higher speeds, we must prevent the sleeve from rising too easily, and, therefore, we put a weight on it, or spring-load it, so that the centrifugal force on the balls has to overcome the weight or the pressure of the spring before it can raise the sleeve. Such a weighted governor is called a Porter governor. An advantage of using a spring is that by means of an adjusting screw, the speed at which the sleeve rises sufficiently to close the valve can be regulated.

It is not only the steam or internal combustion engine that requires to be controlled by a

Fig. 20. Spring - loaded governors are suitable for a wide range of speeds. Balls are usually mounted on bell-crank levers which raise the sleeve as the balls move outward. For high speeds, very stiff springs are required, and it is an advantage to attach these directly between the balls.



governor. The gramophone motor, for example, has a governor (Fig. 19). Its function is to ensure that the turntable revolves at a constant speed, which is usually 78 r.p.m., whether the spring is fully wound or partly unwound, or whether the needle is at the edge of the record or near the centre.

It has to be of a very compact form, and the spring control is incorporated in the links, which are made of spring steel strip. As the weights fly outward, the links bend and so bring the sleeve into contact with a fixed friction disk. This increases resistance and so slows the motor down. It will be noted that, as the speed falls, the spring links straighten out again, and the resistance at the friction disk is reduced.

There are many other kinds of spring-loaded governors, some suited to extremely high speeds, such as for oil engines and steam turbines. Two types are shown in Fig. 20, and it will be noticed that the balls are mounted on the ends of cranked levers called bell-crank levers. One arm of each bell-crank lever moves with its ball about the fulcrum, and the other moves the sleeve. The higher the speed of the governor, the stiffer must be the springs in order to resist the centrifugal force.

The speed of the engine or turbine usually increases somewhat as the load decreases, since the balls or weights must move farther apart in order to reduce the opening of the valve. If, as in the case of a turbo-generator, it is desirable that the speed should be kept constant for all loads, a very sensitive governor must be used.

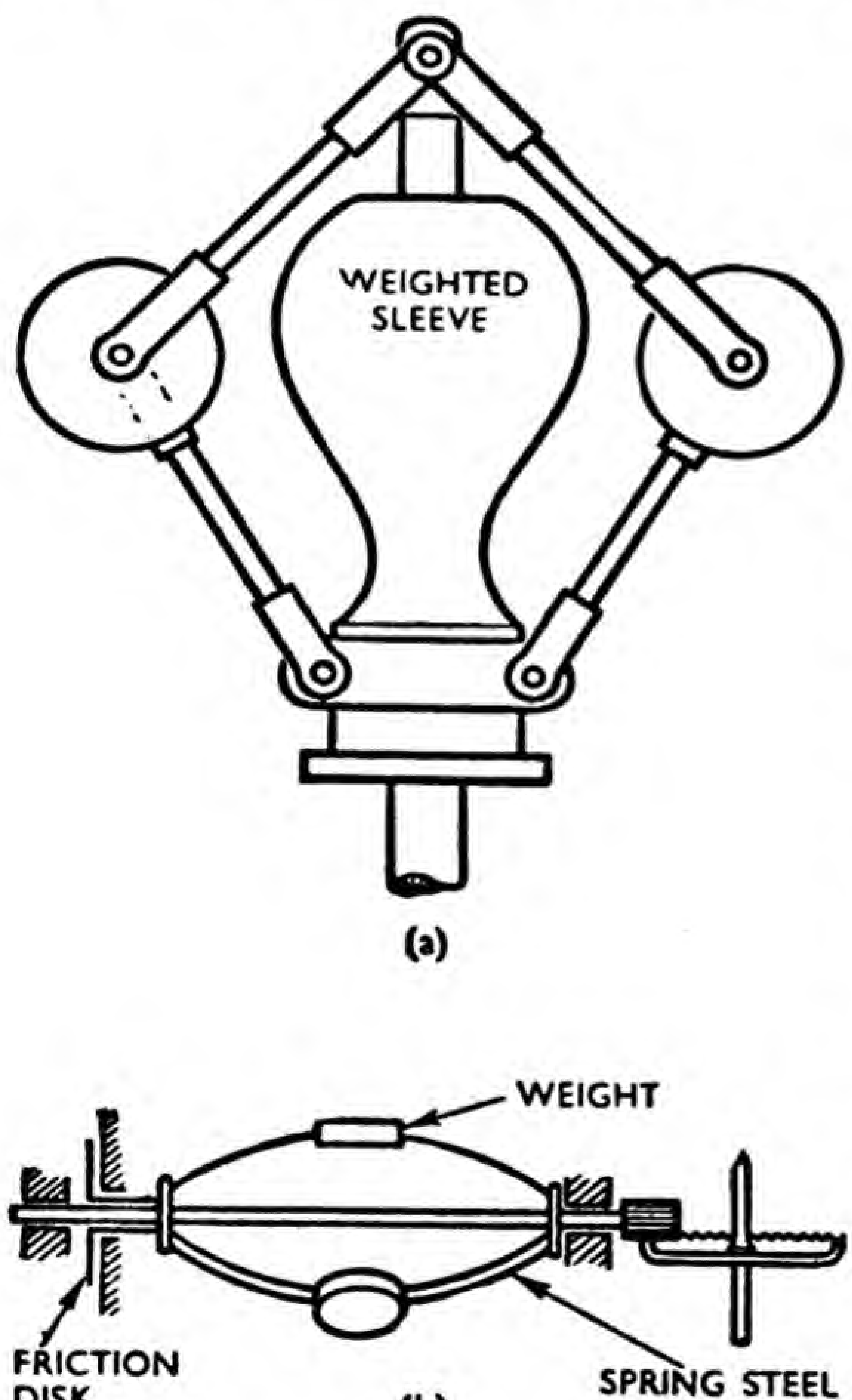


Fig. 19. (*a*) Porter governor, used for higher speeds. This has a heavy weight on the sleeve so that a greater centrifugal force is required to cause the sleeve to rise. (*b*) The speed of the gramophone is kept steady by a governor with springy arms and friction disk, which varies the resistance.

CHAPTER 7

FORMS OF ENERGY

MECHANICAL, THERMAL AND ELECTRICAL ENERGY. DOING WORK. POTENTIAL ENERGY. KINETIC ENERGY OF FALLING BODIES. CHANGE OF ENERGY. JOULE'S EQUIVALENT. TRANSFORMING HEAT. BOARD OF TRADE UNIT. AMOUNT OF ENERGY LOST IN TRANSFORMATION. IMPRACTICABILITY OF PERPETUAL MOTION. STORING ENERGY. CALCULATING KINETIC ENERGY. RADIUS OF GYRATION. MOMENT OF INERTIA. CYCLICAL FLUCTUATION.

IT is an accepted fact that, whenever anything is moved, there is a resistance to be overcome by the application of a force. We need to apply a force to a parcel in order to lift it, or to a lawn-mower in order to push it across the lawn. The movement of a ship, an automobile or an aeroplane can only be brought about by overcoming the resistance to motion. The same is true of the many kinds of machines used in industry; all are working against resistances, and the force required, which is very large in some cases, is obtained from manual labour, from engines or electric motors, or from windmills or water mills.

Mechanical Work

In moving anything against a resistance work must be done, whether manual or mechanical. The popular use of the word work is rather loose, as in the case of manual work as opposed to brain work. In engineering, however, the term work has a restricted meaning. It is defined in such a way that it becomes a measurable quantity.

If a weight of 1 lb. is resting on the table, and it is lifted vertically a distance of 1 ft., then the force required to overcome the gravitational resistance is 1 lb. acting upward, and this force will move vertically through a distance of 1 ft. In doing this a certain amount of work is being done. Twice this amount of work will be necessary to lift the 1-lb. weight a distance of 2 ft., or to lift a 2-lb. weight a distance of 1 ft. (Fig. 1).

In brief, the amount of work done depends both upon the resistance to movement and upon the distance moved. This idea of work as a means of measuring can be used if it is agreed to take as a unit the work done in raising a weight of 1 lb. a vertical distance of 1 ft. A suitable name for this unit is the foot-pound.

Other suitable units which are used when convenient are the inch-pound and the centimetre-gram. If the force applied to a body and the distance this body moves are known, then the amount of work done is the product of the force multiplied by the distance.

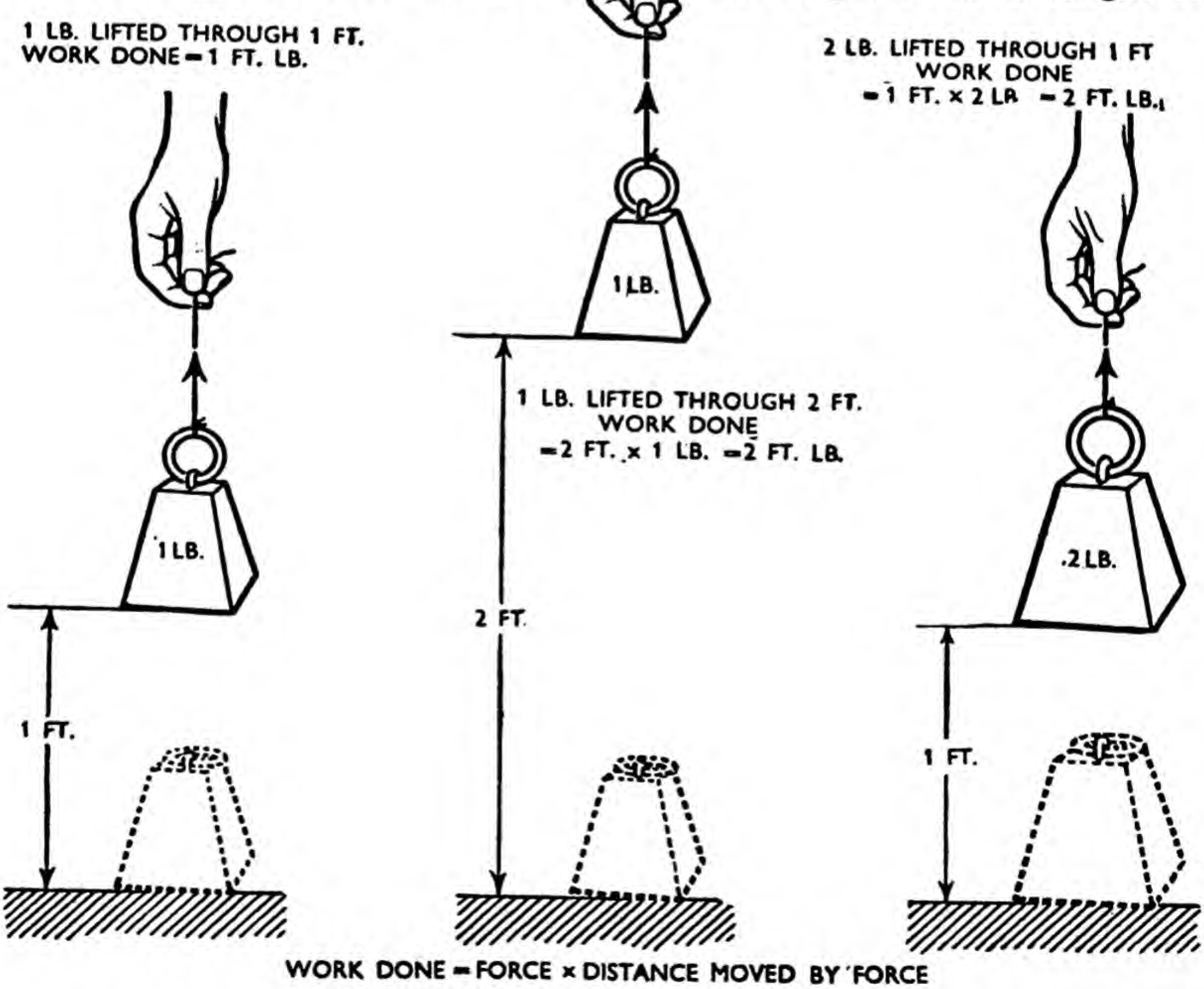
For instance, if two people take a lift from the ground floor of a building to an upper floor, work must be done to raise them through the required distance. If one

person weighs 140 lb. and the other weighs 160 lb., and the height from the ground floor to their destination is 50 ft., then the amount of work done on one is 50 ft. multiplied by 140 lb. and is equal to 7,000 ft.-lb., while the work done on the other is 50 ft. multiplied by 160 lb. which is equal to 8,000 ft.-lb.

Effective Work

Of course, if they go up the stairs they must do the work themselves, but it is important to notice here that the effective amount of work done will be exactly the same as before. One must do 7,000 ft.-lb. of work and the other must do 8,000 ft.-lb. of work, as the vertical distance moved is still 50 ft., though the total distance travelled in going up several flights of stairs may be much greater. In finding the work done, the distance moved must always be measured in the direction of the force. (Fig. 2).

If a weight of 2 lb. is lifted a distance of 3 ft. from the floor on to a table, 6 ft.-lb. of work on it must be done. By allowing the weight to overcome a resistance in returning to the floor it could do 6 ft.-lb. of work. This could be achieved by attaching the weight to a cord and allowing it to drive a small machine, such as a Meccano model or a sewing machine. In other words, when the weight is at a height of



WORK DONE = FORCE × DISTANCE MOVED BY FORCE

MEASURING THE UNIT OF WORK

Fig. 1. The measuring of the unit of work is shown on the left. In the centre and on the right are two ways in which the same amount of work (viz., 2 ft.-lb.) may be done.

3 ft. above the floor, it is capable of doing 6 ft.-lb. of work.

The capacity for doing work is called energy. The energy possessed by a body is measured by the amount of work it is capable of doing before losing this energy, so that the energy of the 2-lb. weight on the table is 6 ft.-lb., while the energy it possesses when resting on the floor is nothing (Fig. 3).

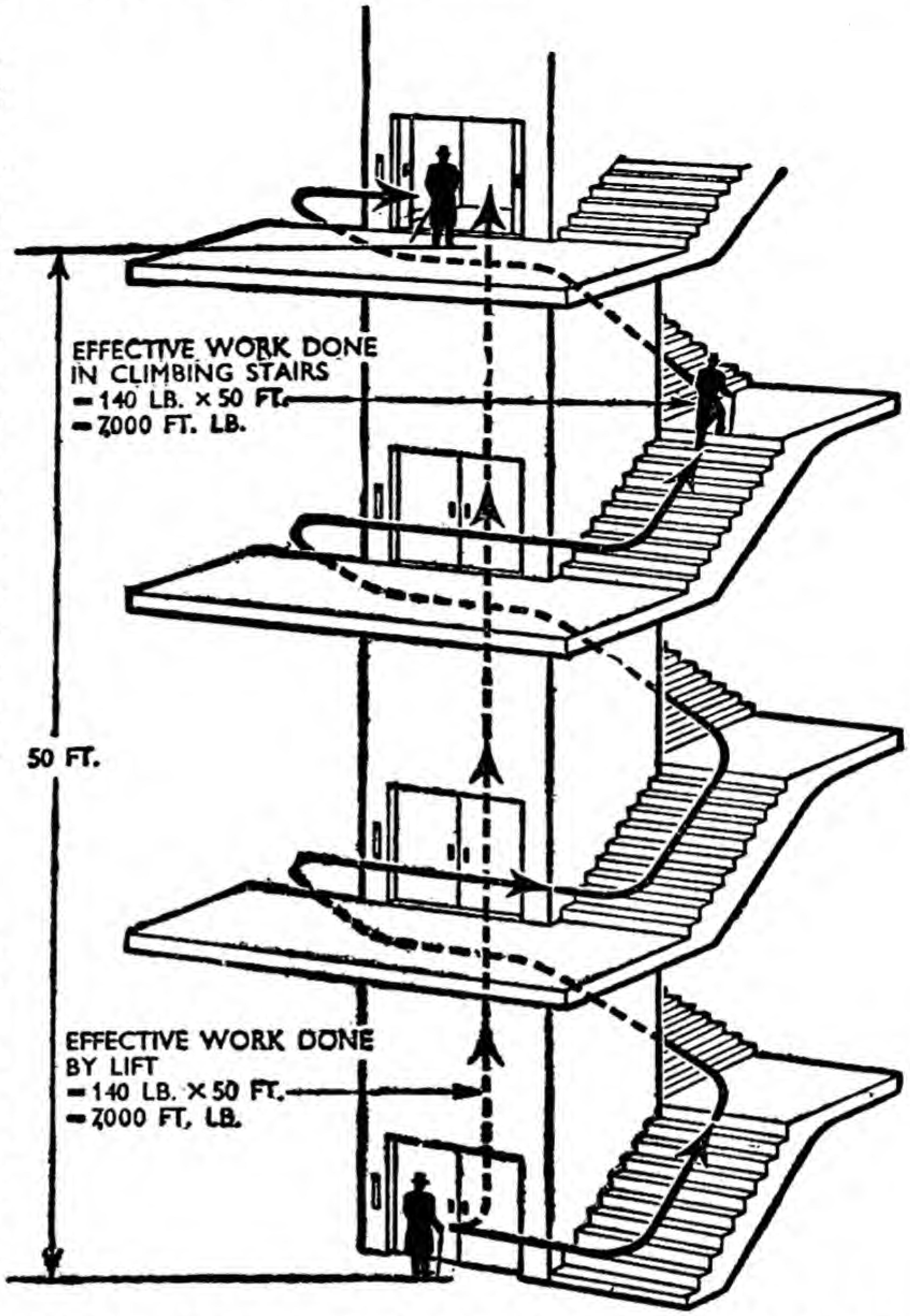


Fig. 2. The man in the illustration wishes to go up from the ground floor to the top floor of the building. He has the choice of two routes. He can walk up the stairs or go up in the lift. The work done is the same in both cases, which is his weight (140 lb.) times the vertical distance (50 ft.) between the floors.

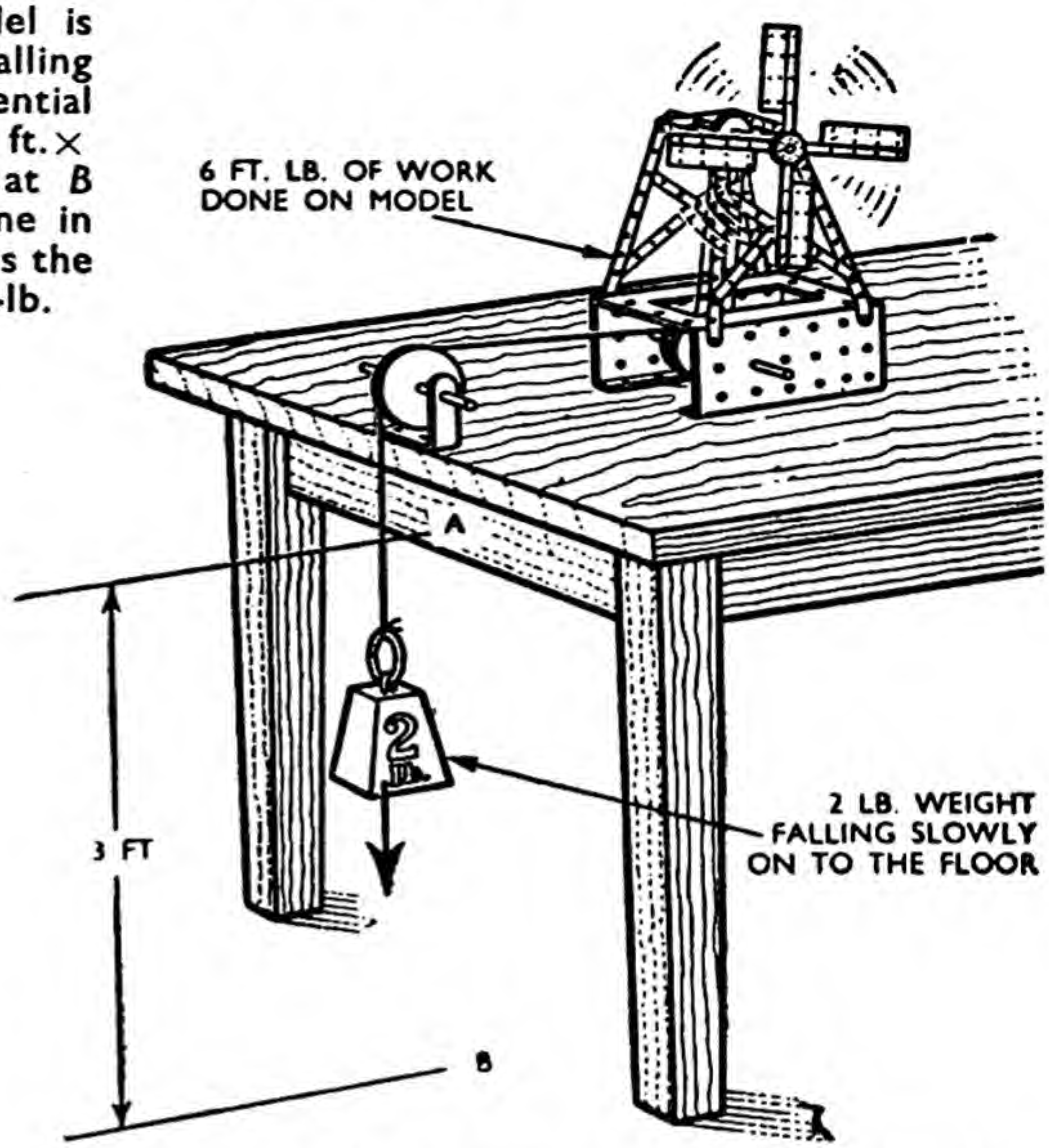
Potential Energy

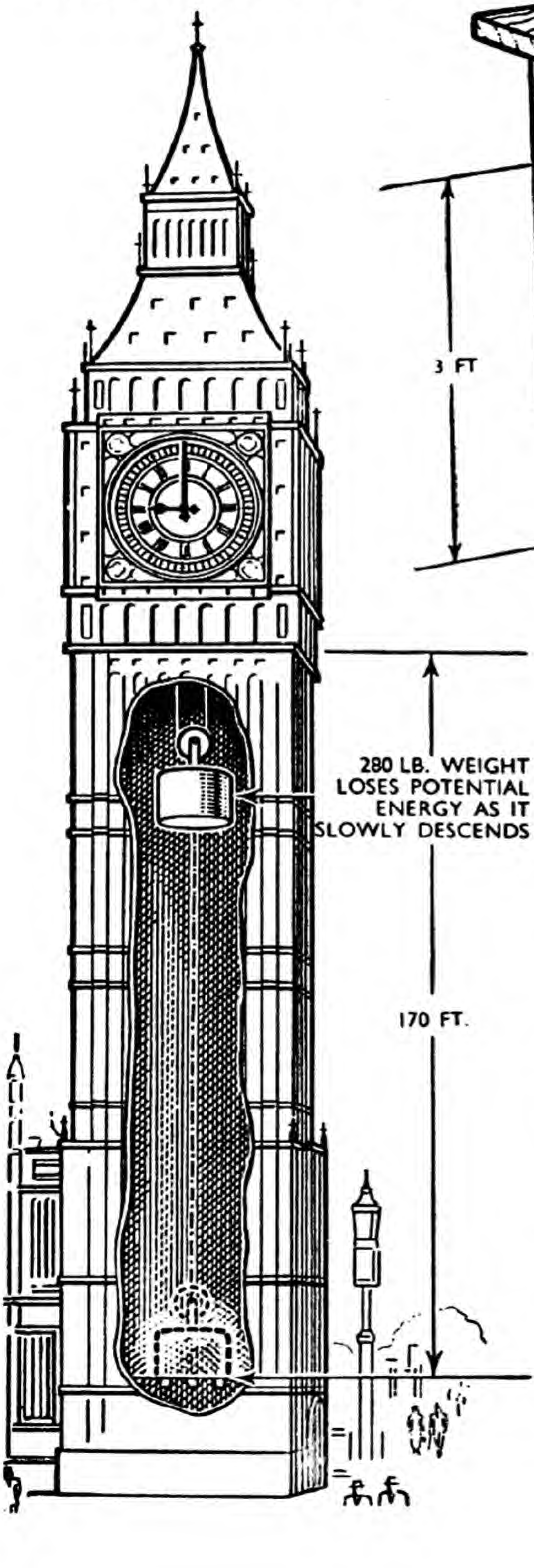
Energy which depends upon the position of a body is called potential energy. In order to keep going the mechanism of the clock Big Ben, work must be done, and in order to obtain this work, use must be made of the potential energy of a heavy weight hanging at the end of a long wire rope. When the clock is wound up, the weight is raised to its highest position and it then possesses potential energy. As the weight slowly falls, unwinding the rope, its energy is used in driving the clock, overcoming the resistance of the mechanism and keeping the pendulum moving. When the weight falls to its lowest position, all its potential energy has been used and the clock has to be wound up again (Fig. 4).

The energy possessed by the mainspring of a watch, or by the spring of a clockwork gramophone mechanism, is another form of potential or stored energy, since the spring, when wound up, is capable of doing work in overcoming the resistance to motion.

The work done in driving a nail

Fig. 3 (*right*). The model is operated by a weight falling from *A* to *B*. At *A* the potential energy of the weight is 3 ft. × 2 lb. = 6 ft.-lb., whereas at *B* it is zero. The work done in operating the model is thus the difference, which is 6 ft.-lb.



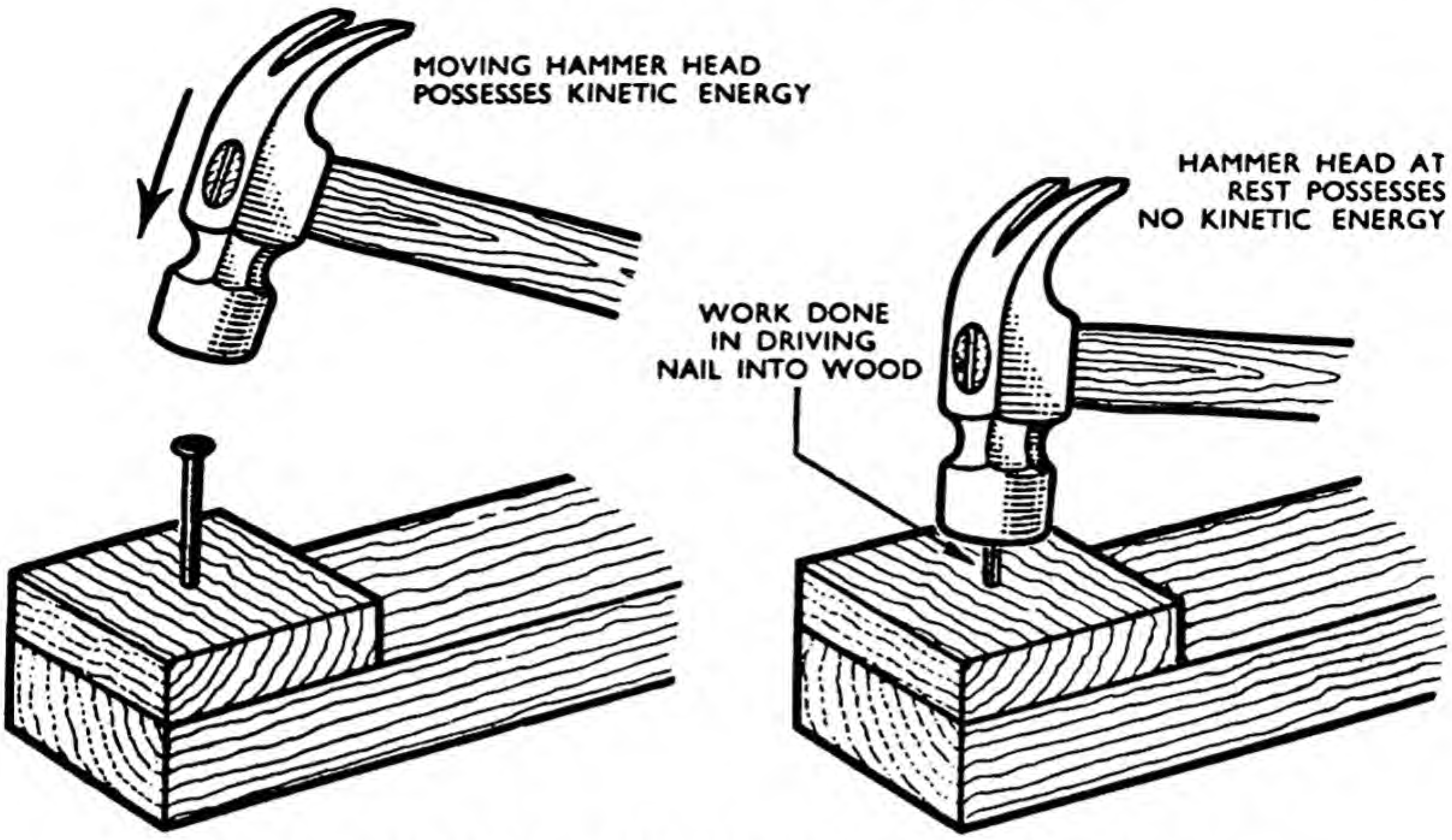


into a piece of wood is considerable. If the head of a hammer is merely laid on top of the nail, or even if the hammer is pushed hard, it will be impossible to drive it in. But, if the hammer head is brought down smartly on to the nail with a blow, in the usual manner, the nail will be driven into the wood.

Energy Available

A definite amount of work will be done, much more than the potential energy possessed by the hammer head when it is resting on the nail. The energy available for driving the

Fig. 4 (*left*). Big Ben is operated by means of the energy lost by a hanging weight as it slowly descends a distance of 170 ft. inside the tower. The clock must ultimately be wound up again by raising the 280-lb. weight to its upper position. It is necessary to do this once every four days, and raising by hand takes twenty minutes.



KINETIC ENERGY POSSESSED BY A RAISED HAMMER

Fig. 5. To drive a nail into a piece of wood, the hammer must possess kinetic energy which can be utilized in doing the work of driving the nail home.

nail in depends, not on the position of the hammer head, but on its motion. The greater the speed at the moment of impact, the greater the distance the nail will be driven into the wood (Fig. 5).

Kinetic Energy

This kind of energy, depending upon the speed of the body, is called kinetic energy. Whenever a body is moving, it possesses kinetic energy; when it is at rest, it possesses no kinetic energy. If a cricket ball is moving with a speed of 25 ft. per sec. it possesses about 10 ft.-lb. of kinetic energy. If it is caught by a fielder, he must do 10 ft.-lb. of work in order to bring the ball to rest. If the hand moves 2 ft. during the catch, the mean force on the hand will be 10 ft.-lb. divided by 2 ft., which is 5 lb. If he attempts to stop the ball in a distance of 1 in., the mean force on his hands will be 120 lb., since the same amount of work has to be done. Obviously in this case injury may result.

Mechanical energy can be easily changed from one form to the other. Fig. 6 shows a bicycle at the top of a hill where the energy, which it and its rider possesses, is that due to position, that is, potential energy. If the rider mounts the bicycle and freewheels down the hill, he will acquire a considerable velocity at the foot of the dip. The kinetic energy acquired at the foot of the hill will be exactly equal to the potential energy lost, provided there is no energy lost in friction.

This kinetic energy can be transformed into potential energy again if the rider allows himself to be carried up the opposite hill without pedalling. If there were no loss of energy he would rise to the same height as that at which he started. In practice, however, there is always a frictional loss, so that all the energy can never be transformed from one form into the

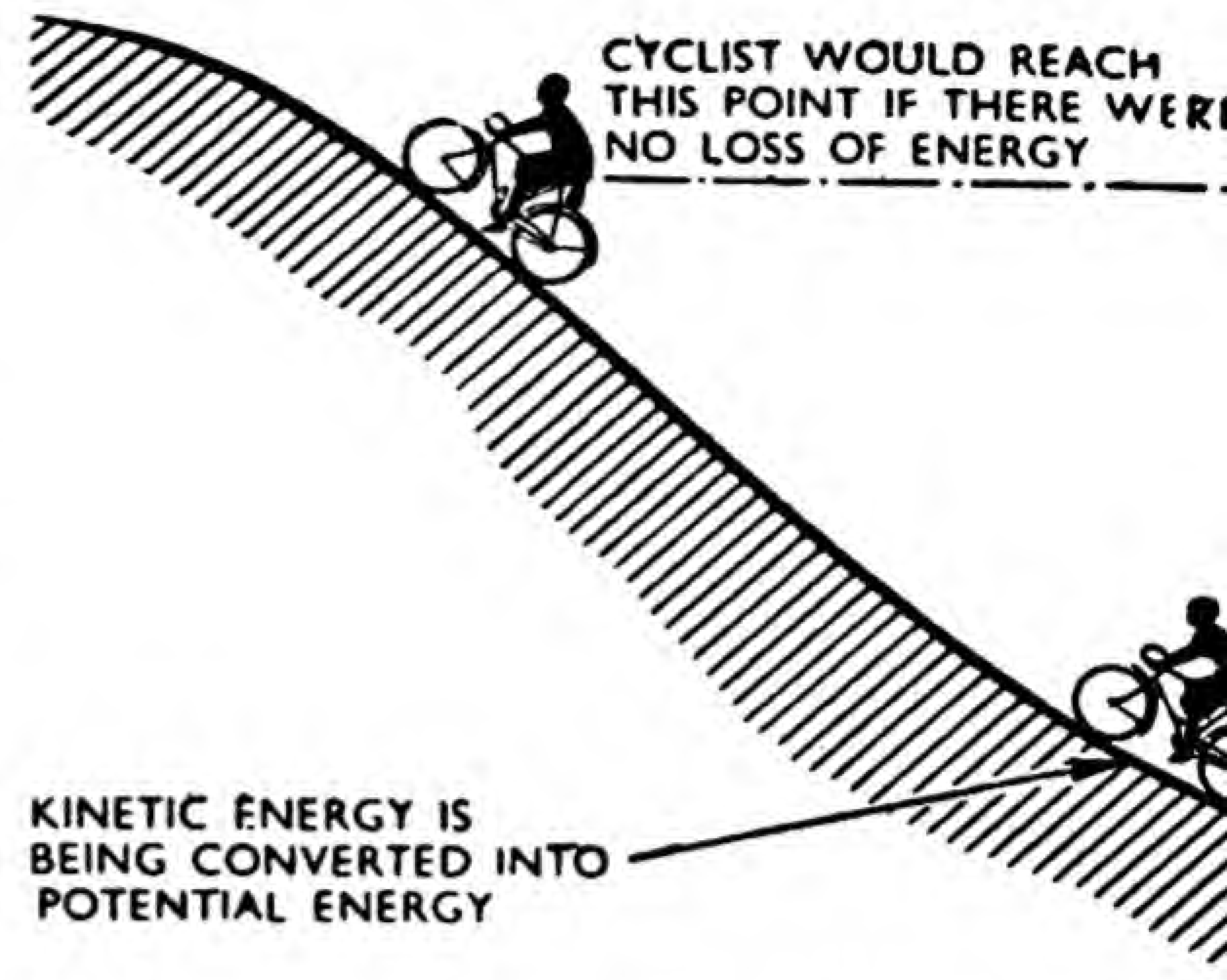
other. No energy is destroyed, however, the missing energy having been converted into another form of energy, viz., heat.

Pendulum Experiment

We can make a pendulum by suspending a heavy weight at one end of a cord. If the weight is displaced to one side we do a little work on it and give it potential energy, the weight is now at a higher level than before. If now the weight is released, it swings towards its mean position, acquiring kinetic energy at the expense of its potential energy (Fig. 7). When it reaches the mean position all its energy is kinetic, and it now begins to rise on the opposite side, losing kinetic energy and gaining potential energy.

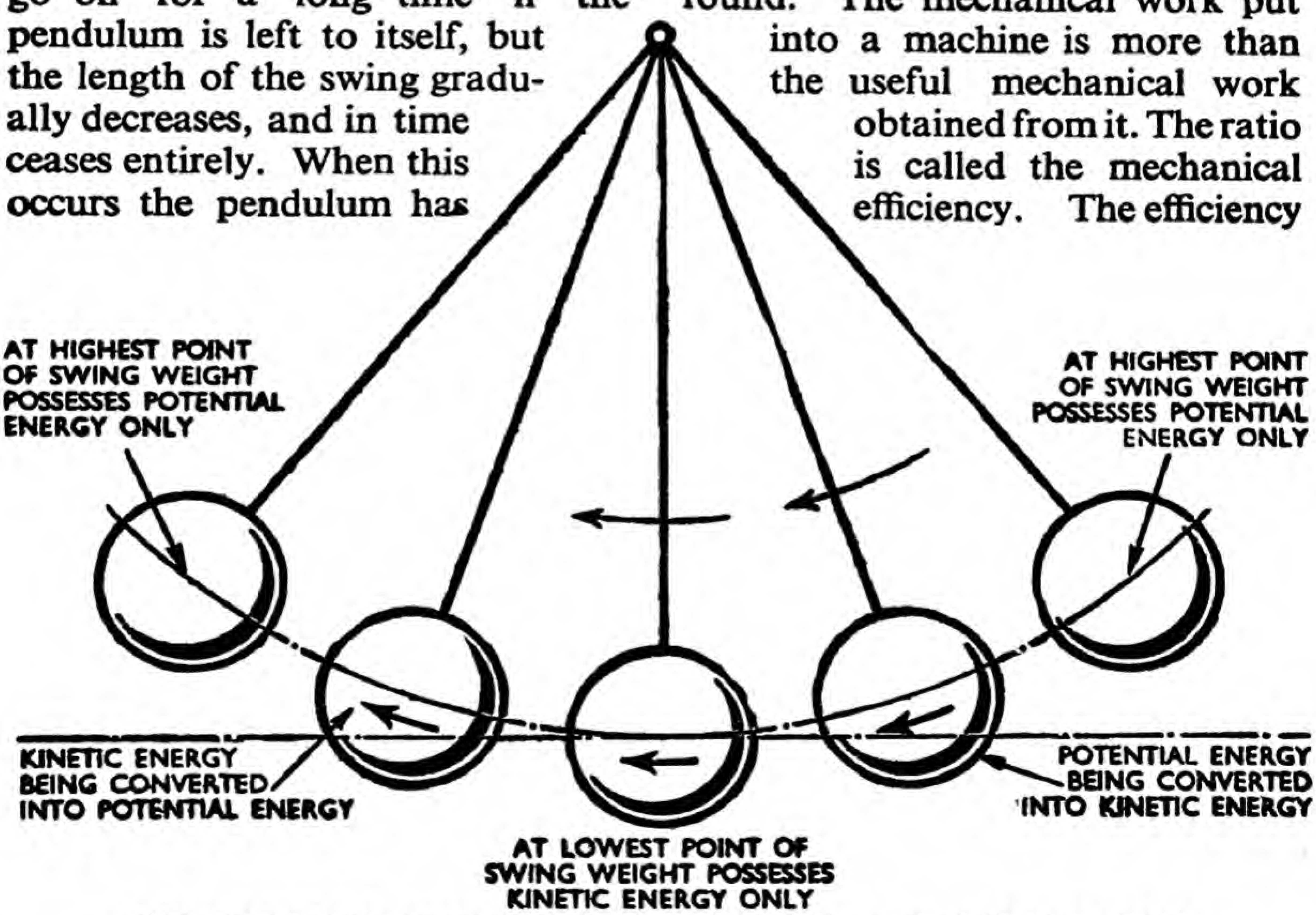
This interchange of energy will go on for a long time if the pendulum is left to itself, but the length of the swing gradually decreases, and in time ceases entirely. When this occurs the pendulum has lost the mechanical energy which it originally possessed.

Friction is present in all machines and always a loss of energy is found. The mechanical work put into a machine is more than the useful mechanical work obtained from it. The ratio is called the mechanical efficiency. The efficiency



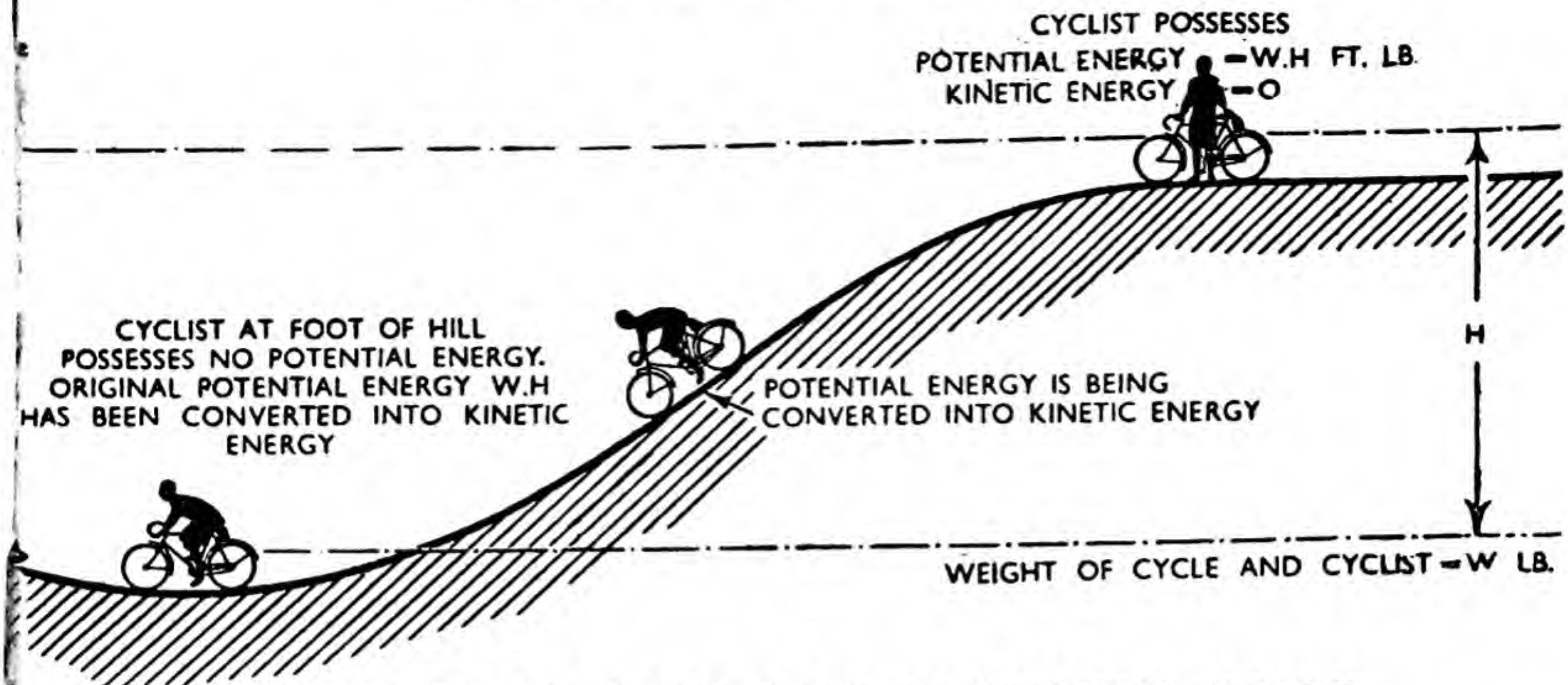
ENERGY OF CYCLIST CHANGES FRO

Fig. 6. When the cyclist is stationary at the top of the hill, he possesses potential energy only. As he descends,



CONTINUOUS CHANGE OF ENERGY IN A SWINGING BOB

Fig. 7. In the case of a pendulum swinging about its point of support, there is a constant change in the proportions of potential and kinetic energy possessed by the bob, just as there was in the case of the cyclist in the previous figure.



OTENTIAL TO KINETIC ENERGY AND BACK TO POTENTIAL ENERGY

the potential energy is converted into kinetic energy, and when he reaches the bottom, he possesses only kinetic energy, and no potential energy. When he ascends on the other side, the kinetic energy is converted back into potential energy.

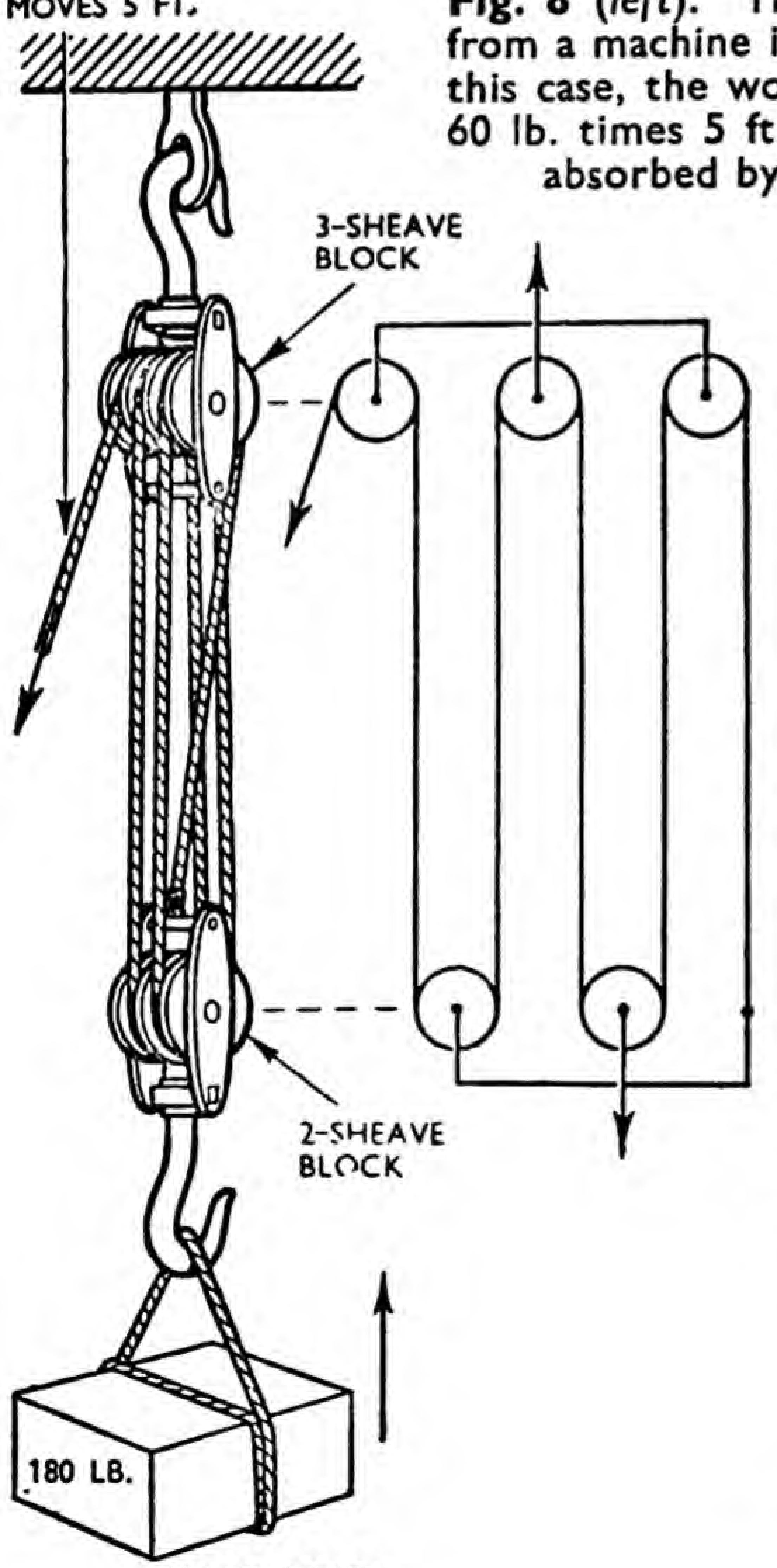


Fig. 8 (*left*). The amount of mechanical work obtained from a machine is always less than that supplied to it. In this case, the work done on the machine by the effort is 60 lb. times 5 ft., which is 300 ft.-lb. Hence the energy absorbed by the machine is 300 − 180 = 120 ft.-lb.

of a rope block and tackle as shown in Fig. 8 is about 0·6. In this particular set, the end of the rope must be pulled a distance of 5 ft. if the load is to be lifted a distance of 1 ft.

If the force on the end of the rope is 60 lb. and it moves a distance of 5 ft., then the work that must be done is 60 lb. multiplied by 5 ft., which is equal to 300 ft.-lb. The amount of work got out of the machine will be 0·6 multiplied by 300 ft.-lb., viz., 180 ft.-lb., so that if the load rises a distance of 1 ft., it will be possible to lift a load of 180 lb. The work lost in overcoming the frictional resistances, in this case, is 120 ft.-lb.

Energy Transformed

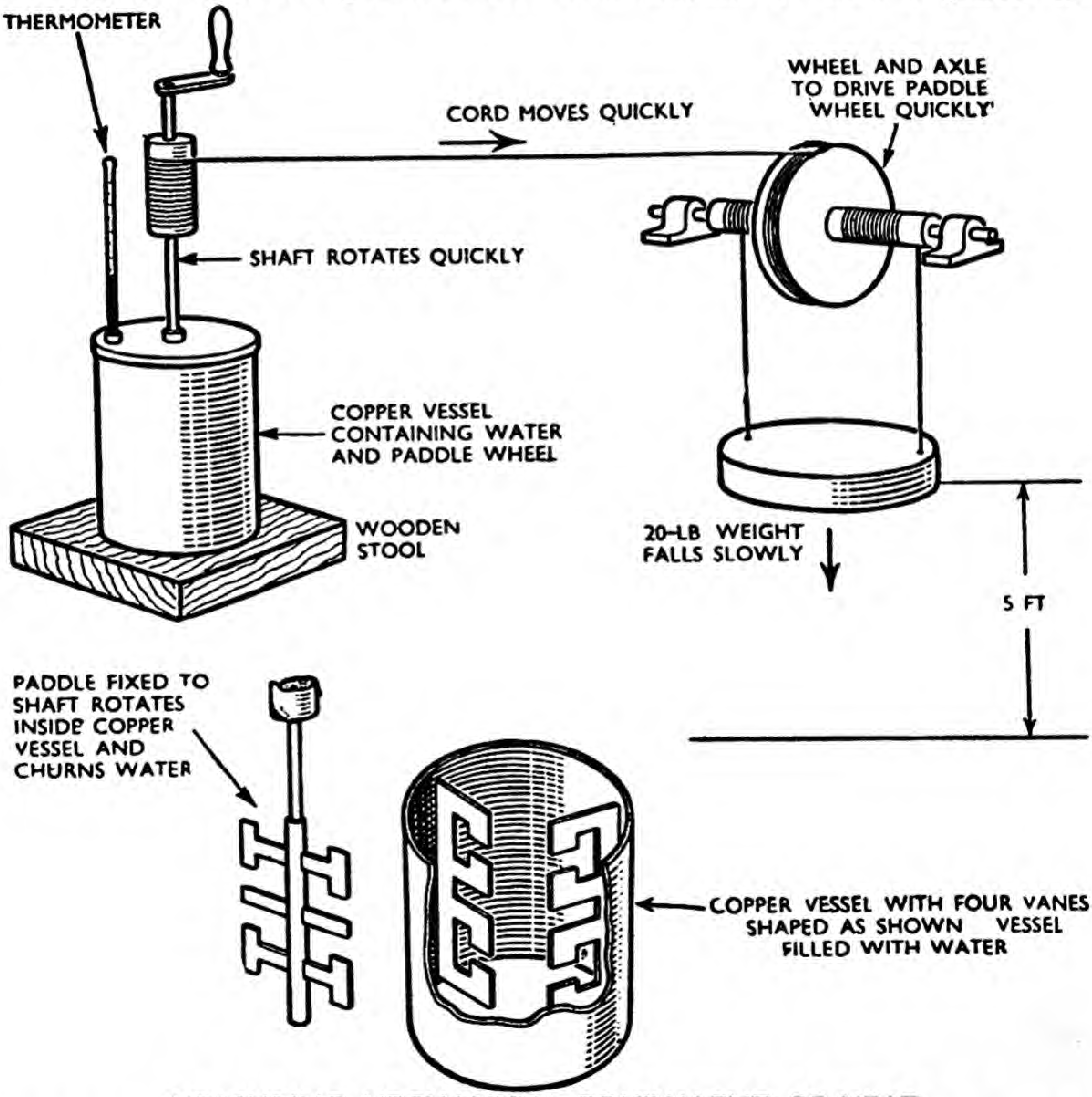
In all these cases the energy lost has been transformed into heat. The amount of heat generated is not usually very large except when

the machine is working continuously. The bearings of a machine shaft, for instance, tend to get warm due to the friction losses. If the brakes of a car are used to control the speed down a long incline it is found that the brake-drums get hot.

We can regard heat as being a form of energy. The change of mechanical energy into heat energy is easy, as shown in the above examples, but the change from heat energy to mechanical energy can only be brought about by means of a heat engine, such as a steam engine or a petrol engine.

Amount of Heat Required

The unit of heat is the amount of heat required to raise the temperature of 1 lb. of water by 1 deg. Fahrenheit ; this is called a British thermal unit (B.Th.U.). Similarly a Centigrade heat unit (C.H.U.) is the amount of heat required to



MEASURING MECHANICAL EQUIVALENT OF HEAT

Fig. 9. Joule's apparatus for measuring the mechanical equivalent of heat is illustrated above. A known quantity of water contained in the copper vessel is churned up by the rotating paddle, which is driven by a wheel and axle operated by a falling weight. The work done by the latter is converted to thermal energy, which can be measured by the rise in temperature of the water.

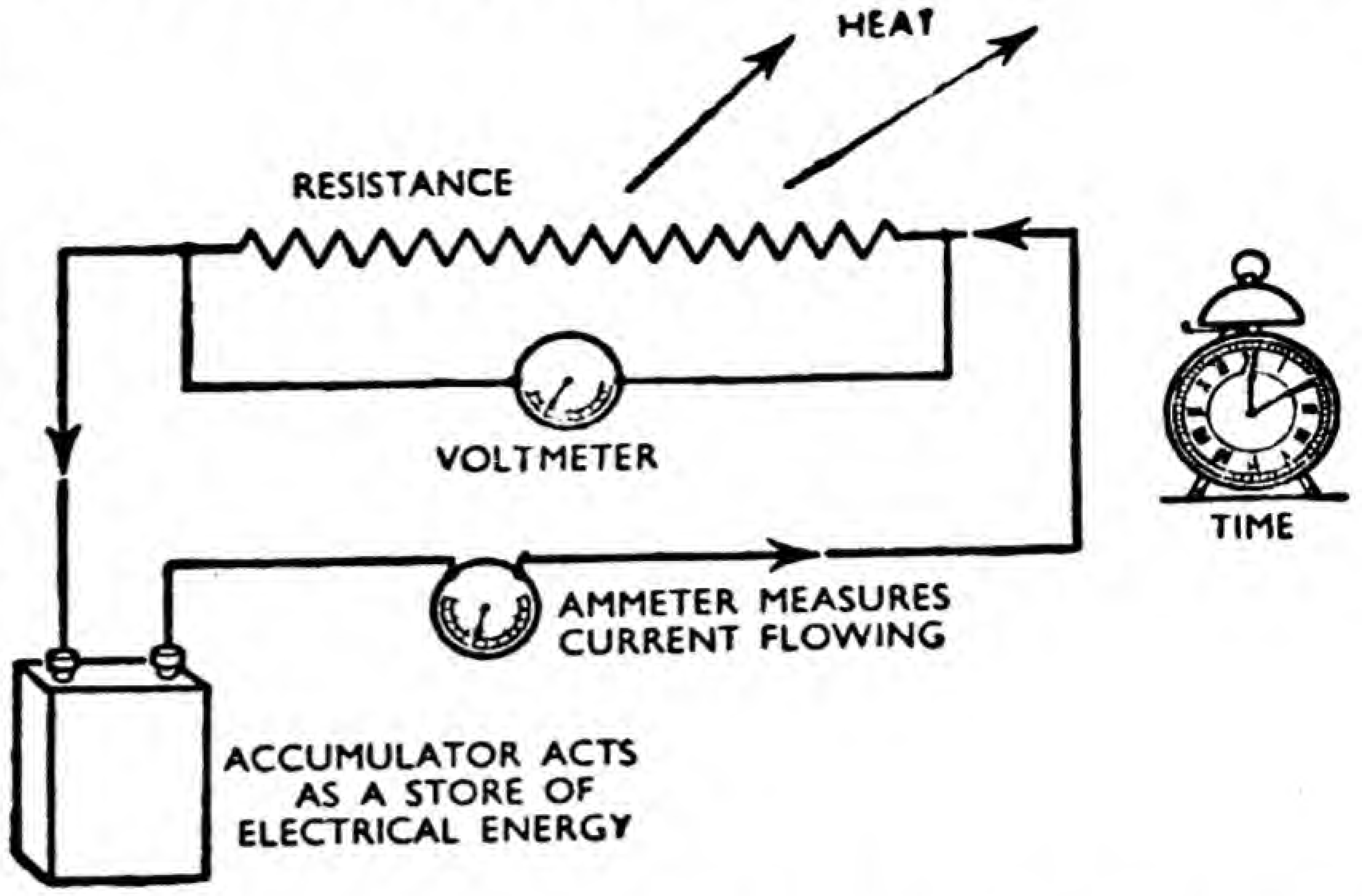


Fig. 10. The electrical energy stored in an accumulator passes along a wire to the resistance coil, where it is changed into thermal energy. We find that the heat given off is equal to the amount of electrical energy used, and is equal to the product of amperes multiplied by the volts multiplied by the time during which the current is flowing.

raise the temperature of 1 lb. of water by 1 deg. Centigrade, and a calorie is the amount of heat required to raise the temperature of 1 gram of water by 1 deg. Centigrade.

The relation between the units of heat energy and the units of mechanical energy was discovered by Joule in 1847. This is now called Joule's equivalent J, and is given very closely by the statement: 778 ft.-lb. of mechanical work will generate heat equivalent to one B.Th.U. This is best looked upon as a rate of exchange or transformation, viz., $\frac{778 \text{ ft.-lb.}}{1 \text{ B.Th.U.}}$, generally denoted by J, or the reciprocal, $\frac{1 \text{ B.Th.U.}}{778 \text{ ft.-lb.}} = \frac{1}{J}$. The apparatus used by Joule is illustrated in Fig. 9.

Transforming Thermal Energy

If 1 lb. of coal is burnt in the furnace of a steam boiler, the amount of heat energy which is made available is about 12,000 B.Th.U., and if this could be completely transformed into mechanical energy we should get 12,000 B.Th.U. $\times \frac{778 \text{ ft.-lb.}}{1 \text{ B.Th.U.}}$ or 9,336,000 ft.-lb. of work. In practice, it is rarely possible to transform more than 20 per cent of the heat, so that the maximum amount of mechanical work obtained from 1 lb. of coal would be 1,867,500 ft.-lb. This value, however, does not give any indication of the power generated.

Power is the name given to the rate at which work is done. James Watt found that a horse could do work at the rate of 33,000 ft.-lb. per min. The unit of power is, therefore, the horse-power, and a steam engine which develops 10 h.p. will do 330,000 ft.-lb. of work in 1 min. or $330{,}000 \ \frac{\text{ft.-lb.}}{\text{min.}} \times \frac{60 \text{ min.}}{1 \text{ hour}}$ or 19,800,000 ft.-lb. of work in one hour. Thus, the engine, when developing 10 h.p., will require $19{,}800{,}000 \ \frac{\text{ft.-lb.}}{\text{hour}}$ divided by $1{,}867{,}000 \frac{\text{ft.-lb.}}{1 \text{ lb. coal}}$, or 10·6 lb. of coal per hour, with the above assumptions.

The engine could be used to drive an electric generator. The mechanical energy passing along the shaft would then be changed into electrical energy. Electricity as a form of energy is also indicated by the

fact that it can be used to produce heat when an electrical radiator or cooking stove is switched on. Electrical energy supplied to a motor can also be changed into mechanical energy.

Unit of Electrical Power

The unit of electrical power is the watt (W), and the number of watts in a given case is the product of the amperes and the voltage. A kilowatt is 1,000 W., so that if we have a current of 10 amp. flowing in a circuit and 100 V. across the terminals, we shall have a power of 10 amp. multiplied by 100 V., viz., 1,000 amp.-V. or 1 kW. Fig. 10 illustrates how the electrical energy stored in an accumulator can be transformed into thermal energy by passing it through a resistance wire.

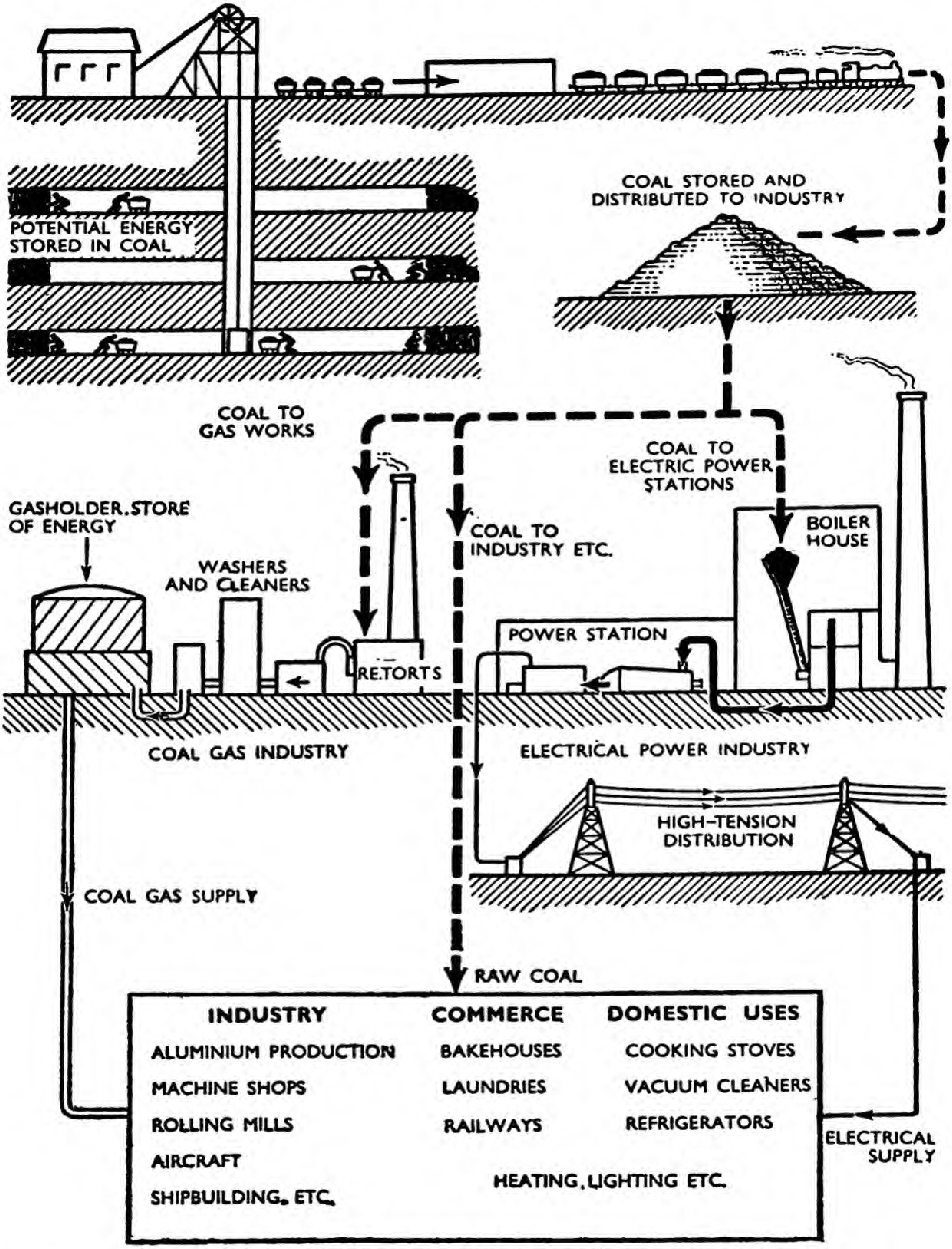
The unit of energy which is paid for when electricity is used, is the kilowatt-hour (kWh) ; this is the energy used when one kW. is maintained for one hour. It is called the Board of Trade unit. The rate of exchange between electrical and mechanical power is 746 W. per h.p., and the rate of exchange between electrical and heat energy is 3,412 B.Th.U. per kWh.

In this age of machines, energy is one of the most important commodities. Energy must be purchased in order to keep machines and appliances running, for driving lathes, looms and all other machines in industry ; for railway, road, air and sea transport ; for heating our homes and cooking our food. It is immaterial whether it is bought in the form of coal, gas, petrol or electricity, or whether it is obtained from water power, but each operation requires a certain amount of energy, irrespective of the units in which it is measured, mechanical, heat or electrical.

The table below gives a list of the

ENERGY EQUIVALENTS FOR VARIOUS SOURCES

Source of energy	Amount of energy obtained expressed in		
	B.Th.U.	Ft.-lb.	kWh.
1 lb. of coal	12,000	9,330,000	3·5
1 pint of petrol	14,500	11,300,000	4·25
1 cu. ft. of coal gas	475	369,000	0·138
1 therm of coal gas	100,000	77,800,000	29·2
1 unit of electricity	3,412	2,660,000	1·0
1 h.p. for one hour	2,545	1,980,000	0·746
1,000,000 gals. of water at a height of 1 ft.	12,900	10,000,000	3·75



REMOVING POTENTIAL ENERGY FROM THE EARTH

Fig. 11. The potential energy which is stored in the earth in the form of coal, is available for conversion to other forms of energy after the coal has been mined. The energy can be distributed to consumers in the form of either gas or electricity. The largest consumer is industry, which also uses raw coal for certain of its processes. Illustrated above is an impression of the various processes through which coal is conducted so that energy in the three different forms (raw coal, gas and electricity) may be passed on to consumers.

various forms in which energy may be purchased and the amount of energy which is obtained is expressed in the three kinds of unit.

The energy shown in the table above is the commodity bought and used, the power is the rate at which the energy is used and the rate at which it must be obtained.

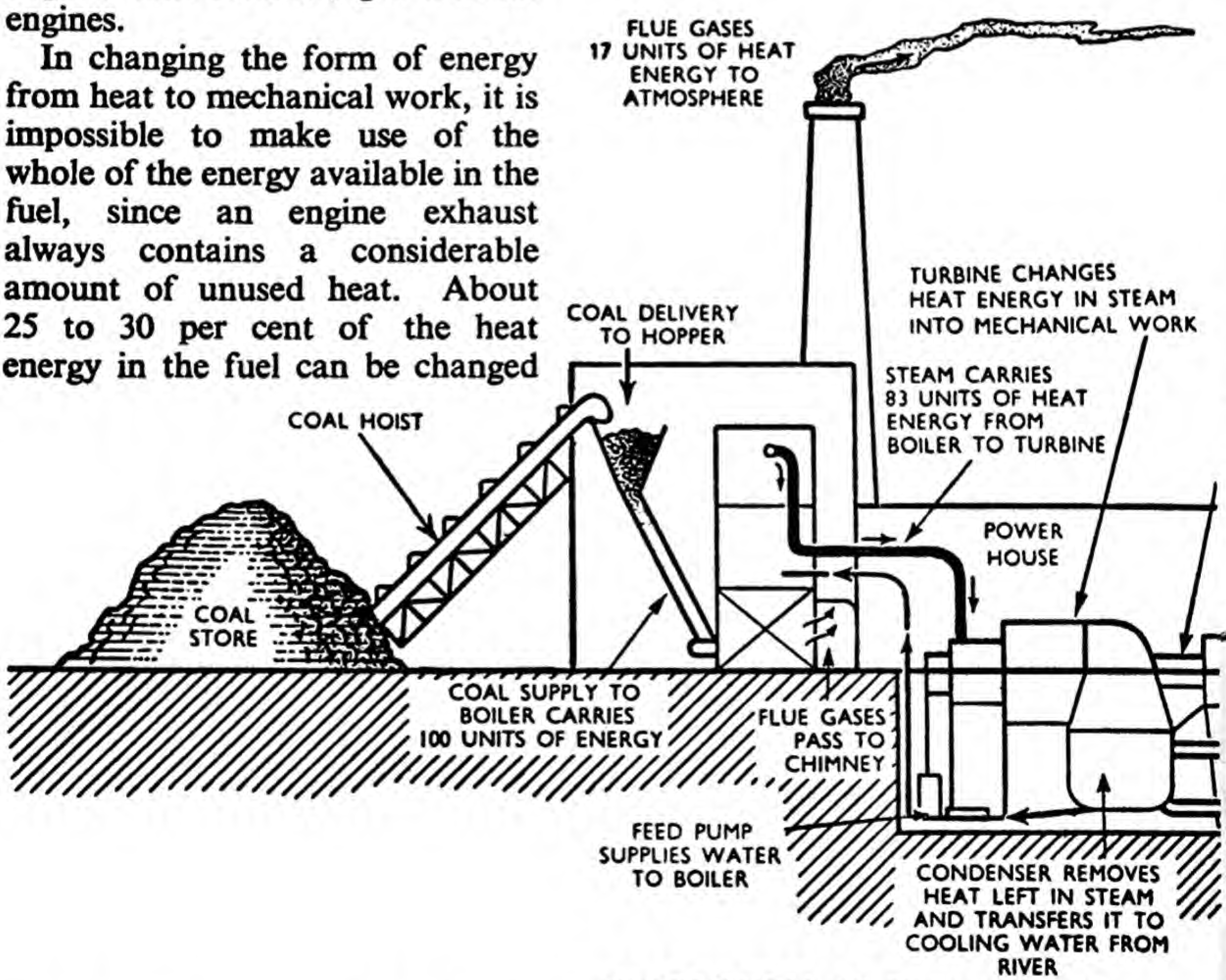
Importance of Coal

In this country, the greatest source of energy is coal. Fig. 11 shows, in the form of a diagram, how the vast amount of energy used today is distributed in the convenient form of gas and electricity. Most of the power is derived from heat engines, steam engines and turbines, gas and oil engines.

In changing the form of energy from heat to mechanical work, it is impossible to make use of the whole of the energy available in the fuel, since an engine exhaust always contains a considerable amount of unused heat. About 25 to 30 per cent of the heat energy in the fuel can be changed to mechanical work, the rest remains in the form of heat energy and is lost, at any rate as far as the production of power is concerned.

It is of interest, however, to follow the changes of energy which occur in the use of one of these prime movers. The example we have taken in Fig. 12 is the production of electrical energy in a modern power station.

Coal is burned in the furnace of a boiler where the heat energy released flows through the walls of the heating surface to the water inside the tubes and drums. The heat energy is taken up by the water, and in the process the water



CONVERSION OF ENERGY FROM

Fig. 12. The conversion of the energy contained in coal into electrical energy is illustrated diagrammatically above. The coal supplies heat to the boilers and converts water to steam, which, in turn, is used to drive the turbines.

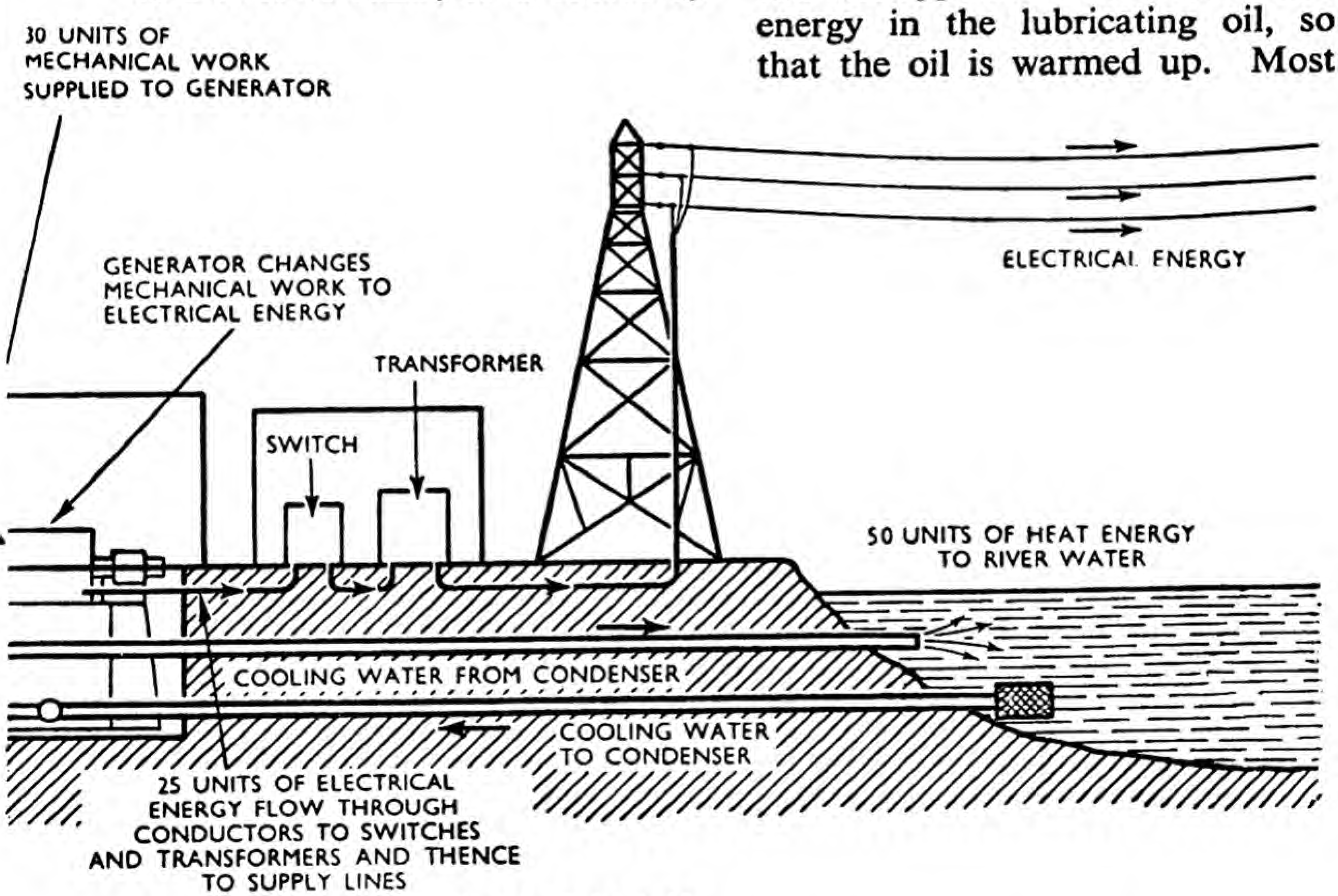
is changed into high-pressure steam. This high-pressure steam is passed through pipes to the inlet of a turbine where it is allowed to expand to a very low pressure.

The expansion of the steam produces mechanical work on the turbine shaft which is used to drive the electric generator. The steam leaving the turbine then passes to a condenser where heat is removed from it so that it changes back to the liquid condition, the heat removed being taken up by cooling water flowing along the inside of a large number of tubes through the condenser.

The condensed steam is returned to the boiler by means of a feed pump, and the cooling water, which has been warmed up in its passage through the condenser, is returned to a river or canal, or is cooled by means of one of the huge cooling towers which may be seen near some power stations. The mechanical energy passing along the turbine shaft is changed into electrical energy in the generator.

Amount of Energy Lost

In the diagram, it has been assumed that one hundred units of heat energy have been supplied to the boiler in the form of coal. About seventeen of these units are lost, most of them in the flue gases and smoke leaving the boiler chimney, the other eighty-three units are taken up by the steam. Of these eighty-three units supplied to the turbine, about thirty are converted into mechanical work. About three units are lost in overcoming the friction of the bearings ; and this loss appears in the form of heat energy in the lubricating oil, so that the oil is warmed up. Most



COAL INTO ELECTRICAL ENERGY

These are coupled to special machines generating electricity. As can be seen from the energy balance on page 168, out of 100 units of energy supplied, only 25 units pass into the supply lines as electrical energy, the remainder being lost in various ways.

of the remaining fifty units of energy are carried into the condenser with the expanded steam and are transferred to the cooling water as heat.

This energy then is lost in heating up the condenser cooling water. Of the thirty units of energy leaving the turbine and entering the generator, about three units are lost in bearing friction, and, therefore, appear as heat in the lubricating oil. Another two units are lost in heating up the coils and windings of the generator. In large machines it is usual to circulate air through the windings in order to maintain a reasonable temperature, so that this loss appears as heat given to the air.

Of the original one hundred units of energy supplied to the plant, only the remaining twenty-five are changed into electrical energy for use on leaving the station.

If a balance sheet for the energy were drawn up in a manner similar to that applied to cash it would appear as shown below.

It can be seen from this energy account that the total amount of energy being supplied to the plant is equal to the total amount of energy leaving the plant. The energy which is spoken of as lost does not disappear, it has merely changed its form so that it is no longer possible to convert it into mechanical, or electrical, energy. All the losses which appear in the energy account given below leave the plant in the form of heat. This is generally true of all energy losses, they nearly always appear ultimately as heat.

Total Energy Unaltered

In all the operations, whether large and complicated like the production of electricity or gas, or small and simple like the dropping of a brick, the sum total of energy at the end of the operation is the same as it was at the beginning. It may have changed its form, it may have become of little or no use, but it still exists. This is known as the principle of the conservation of energy.

A so-called perpetual motion

ENERGY BALANCE

Energy supplied to plant (credit units)		Energy leaving plant (debit units)	
Energy supplied in form of coal.	100	Energy converted into electricity.	25
		Loss to condenser cooling water.	50
		Loss in bearing friction :	
		(*a*) in turbine	3
		(*b*) in generator	3
		Loss in generator	2
		Loss from boiler	17
Total	100	Total	100

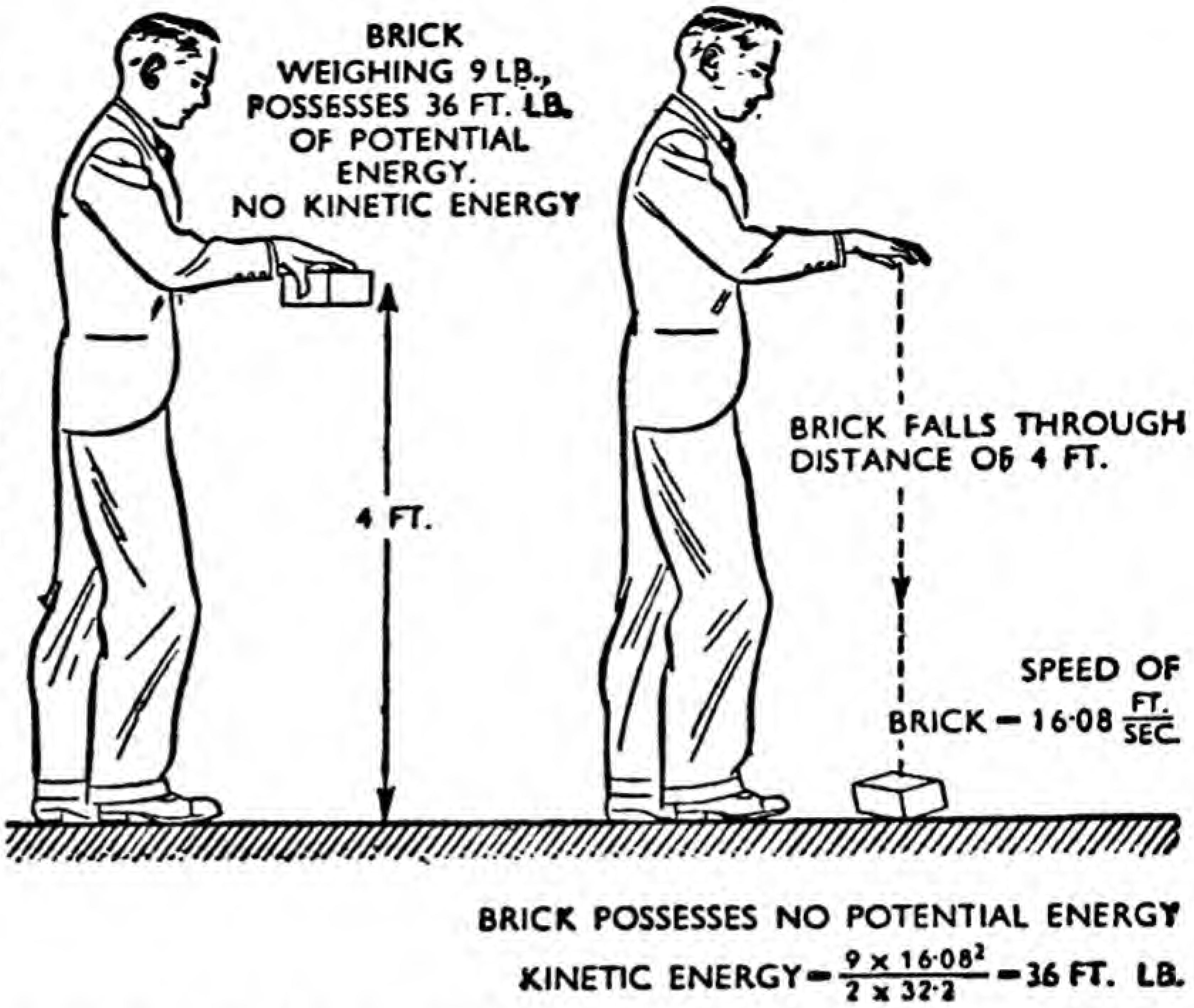


Fig. 13. This drawing illustrates the conversion of 36 ft.-lb. of potential energy, possessed by the brick in the elevated position, to the equivalent amount of kinetic energy, possessed by the brick when it is just reaching the ground after having been released. The text shows how to calculate the speed of fall of the brick.

machine is one which continues to do external work without being supplied with energy, i.e., no driving effort is applied and no fuel is burned.

For centuries, men made an unceasing search for a perpetual-motion machine, a search which was always fruitless in spite of the time, money and thought devoted to it. Gradually, several of our greatest early scientists began to assume that perpetual motion was impossible, and from this assumed impossibility they discovered new knowledge and developed the science of mechanics. We cannot prove that perpetual motion is impossible, it is a matter of observation and deduction, but the assumption that it is in fact impossible, is part of the law of the conservation of energy.

Storing Energy

Motion is always resisted by some kind of friction and therefore work must be done in order to keep anything moving. This work cannot be done without the expenditure of energy in some form, so that motion is impossible without the expenditure of energy. In winding up a clock a certain amount of work must be done. This work is stored as potential energy and an exactly equal amount of work will be done on the clockwork as it runs down. If we had stood by the clock and turned it by hand for eight days we should do the same amount of work that is done in winding up the clock. The storing of potential energy is a matter of convenience. In fact, we put energy into a bank and draw it out as required.

If a brick weighing 9 lb. is held in the hand at a height of 4 ft. above the ground, it will possess 9 lb. multiplied by 4 ft., or 36 ft.-lb., of potential energy. If the brick now falls freely to the ground, the potential energy is converted into kinetic energy (Fig. 13). The amount of kinetic energy gained will be exactly equal to the amount of potential energy lost, provided that there is no work done in overcoming friction. The only possible

friction loss in this case is that between the brick and the air, which at the speeds attained in this example will be very small. Thus, the amount of kinetic energy possessed by the brick just before it strikes the ground must be equal to the potential energy lost, which is 36 ft.-lb.

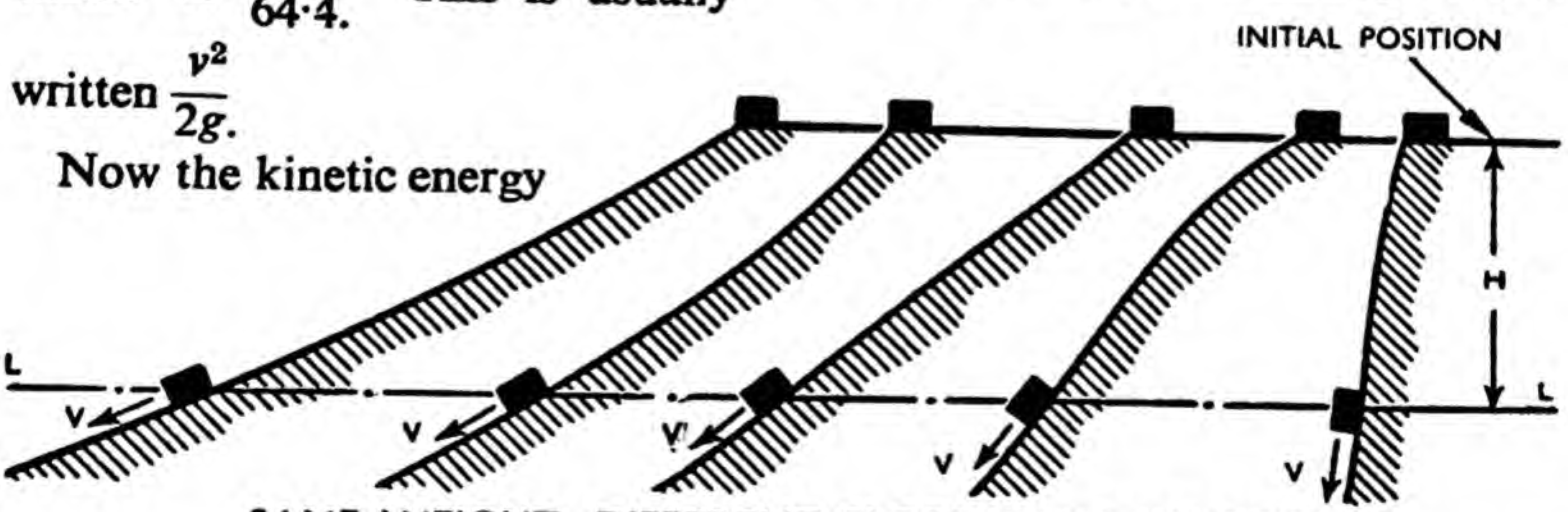
Acceleration Due to Gravity

Now the acceleration with which the brick falls to the ground is that due to gravity, namely 32·2 ft. per sec. per sec. If the velocity acquired at the end of a fall of height H is v ft. per sec., the average velocity will be $\frac{v}{2}$, since the velocity increases uniformly with time. The time taken to fall a distance H will be the velocity acquired divided by the acceleration, viz., time of fall equals $\frac{v}{32{\cdot}2}$ sec. The distance H must be equal to the mean velocity multiplied by the time taken, hence H is equal to $\frac{v}{2}$ multiplied by $\frac{v}{32{\cdot}2}$, which is $\frac{v^2}{64{\cdot}4}$. This is usually written $\frac{v^2}{2g}$.

Now the kinetic energy gained is equal to the potential energy lost, which is equal to W multiplied by H, where W is the weight of the body, but H is equal to $\frac{v^2}{2g}$, so that the kinetic energy gained is equal to W multiplied by $\frac{v^2}{2g}$. That is, the kinetic energy possessed by a body is equal to the weight of the body multiplied by the square of its velocity and divided by twice the acceleration due to gravity. For the 9-lb. brick falling through 4 ft., the velocity at the end of the fall is the square root of $2g$ multiplied by the height, viz., the square root of 64·4 multiplied by 4, which is 16·08 ft. per sec. Its kinetic energy, therefore, is given by the equation :—

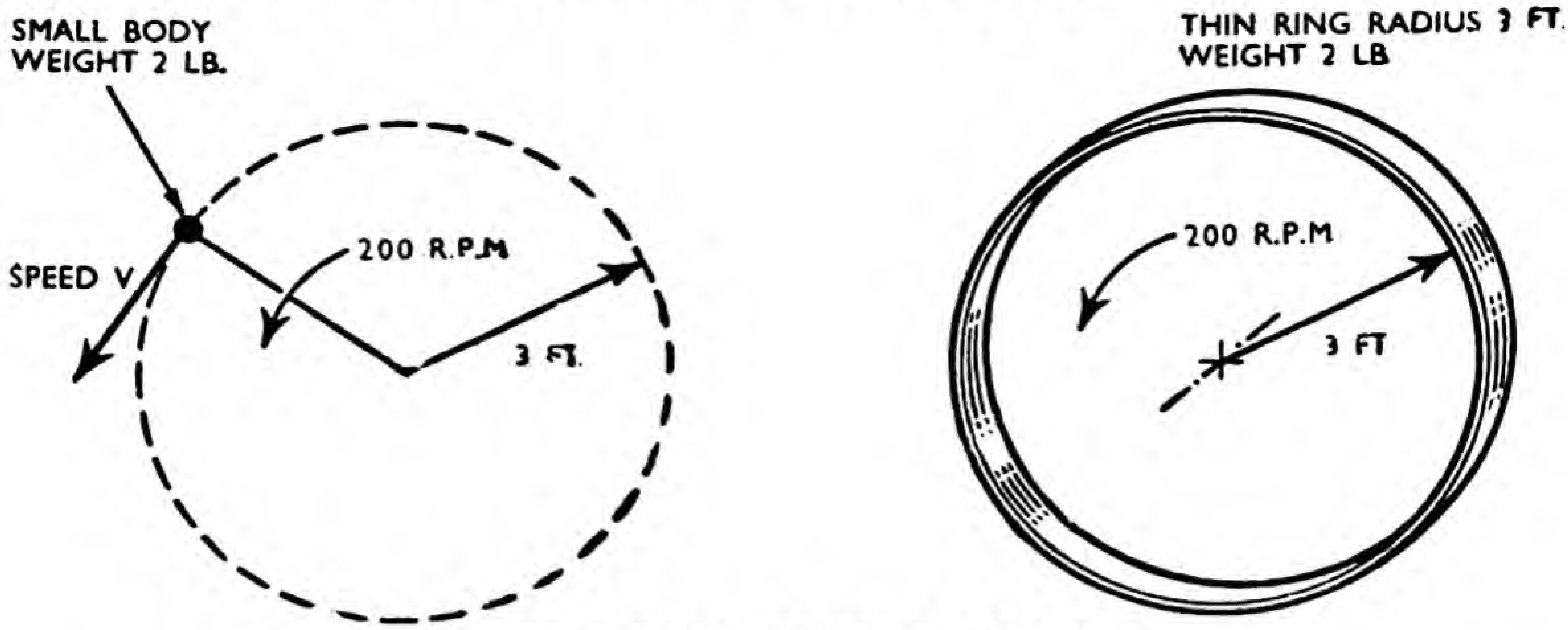
$$\text{K.E.} = \frac{9 \text{ lb.} \times (16{\cdot}08 \text{ ft./sec.})^2}{2 \times 32{\cdot}2 \text{ ft./sec./sec.}}$$
$$= 36 \text{ ft.-lb.}$$

If the body is allowed to slide down a smooth frictionless plane instead of falling freely, the potential energy lost will depend upon the weight of the body and the vertical distance through which



SAME WEIGHT, DIFFERENT SLOPE, SAME VELOCITY

Fig. 14. In the last figure, the brick fell vertically. This figure shows that the velocity V acquired after a vertical fall H is the same, whatever the path of the fall. Five blocks of the same weight W start from an initial position in which they all possess the same amount of potential energy. They slide down *smooth* planes of different slopes. When each block crosses the line LL, it has lost potential energy equal to WH and gained kinetic energy equal to $Wv^2/2g$, and moves with velocity V. Since the potential energy lost in all cases is the same, the kinetic energy of all blocks at LL is equal, so that the blocks all move with the same velocity as they cross the line LL.



KINETIC ENERGY OF ROTATION

Fig. 15. The kinetic energy of rotation of the small body on the left is the same as that of the thin ring on the right. In both cases the angular speed $\omega = 2\pi N/60 = 20{\cdot}9$ radians per sec. The kinetic energy of rotation $= WR^2\omega^2/2g = \dfrac{2 \times (3)^2 \times (20{\cdot}9)^2}{64{\cdot}4} = 124$ ft.-lb.

it has travelled. Hence the gain in kinetic energy and the velocity which the body acquires, will depend upon the vertical distance through which the body falls, and not upon the distance travelled along the plane.

In Fig. 14 a number of bodies are shown sliding down planes of different slopes. If all the bodies started from the same height, they would all have the same velocity when they arrived at the bottom. The body on the steepest slope would arrive at the bottom before any of the others, because the distance travelled would be less.

The kinetic energy of a body depends upon its speed of motion irrespective of its direction. If a body of weight W lb. rotates round the circumference of a circle of radius R ft. at a constant speed of N r.p.m., its speed will be constant and equal to the circumference of the circle multiplied by the number of revolutions in one sec. (Fig. 15). If the speed is V ft. per sec., then V will be equal to $2\pi R$ multiplied by $\dfrac{N}{60}$ ft. per sec. This may be written $2\pi\dfrac{N}{60}$ multiplied by R.

The quantity $2\pi\dfrac{N}{60}$, which is in radians per sec., is called the angular velocity of the body, and is usually represented by the Greek letter omega, ω. Thus the linear speed of a body rotating in a circle of radius R ft. is the product of the angular velocity ω and the radius of the circle.

Now the kinetic energy of this body is obtained from the weight of the body multiplied by the square of its linear speed and divided by twice the acceleration due to gravity. Putting this in the form of an equation: $\text{K.E.} = \dfrac{W.V^2}{2g}$.

But since V is equal to ωR when the body is rotating in a circle, its kinetic energy will be given by

$$\text{K.E.} = \frac{W.\omega^2.R^2.}{2g}$$

$$\text{or K.E.} = \frac{1}{2} \times \frac{W.R^2}{g}\,\omega^2.$$

It has been assumed that the whole weight of the body is concentrated at the radius R from the centre of rotation. This is quite

satisfactory as long as the body is small compared with the radius, or is in the form of a thin ring of radius R. If, as is usual, the body is a rotating wheel, with shaft, hub, spokes and a rim, then each part will possess an amount of kinetic energy depending upon its weight and radius.

Total Kinetic Energy

There is one particular radius, somewhere between the shaft and the rim, where it can be considered the whole weight of the wheel is concentrated. If this radius is substituted in the equation above, it will be possible to find the total kinetic energy of the wheel. This radius is called the radius of gyration, and, following the usual practice, it will be denoted by the letter k. An equation can now be written down to give the kinetic

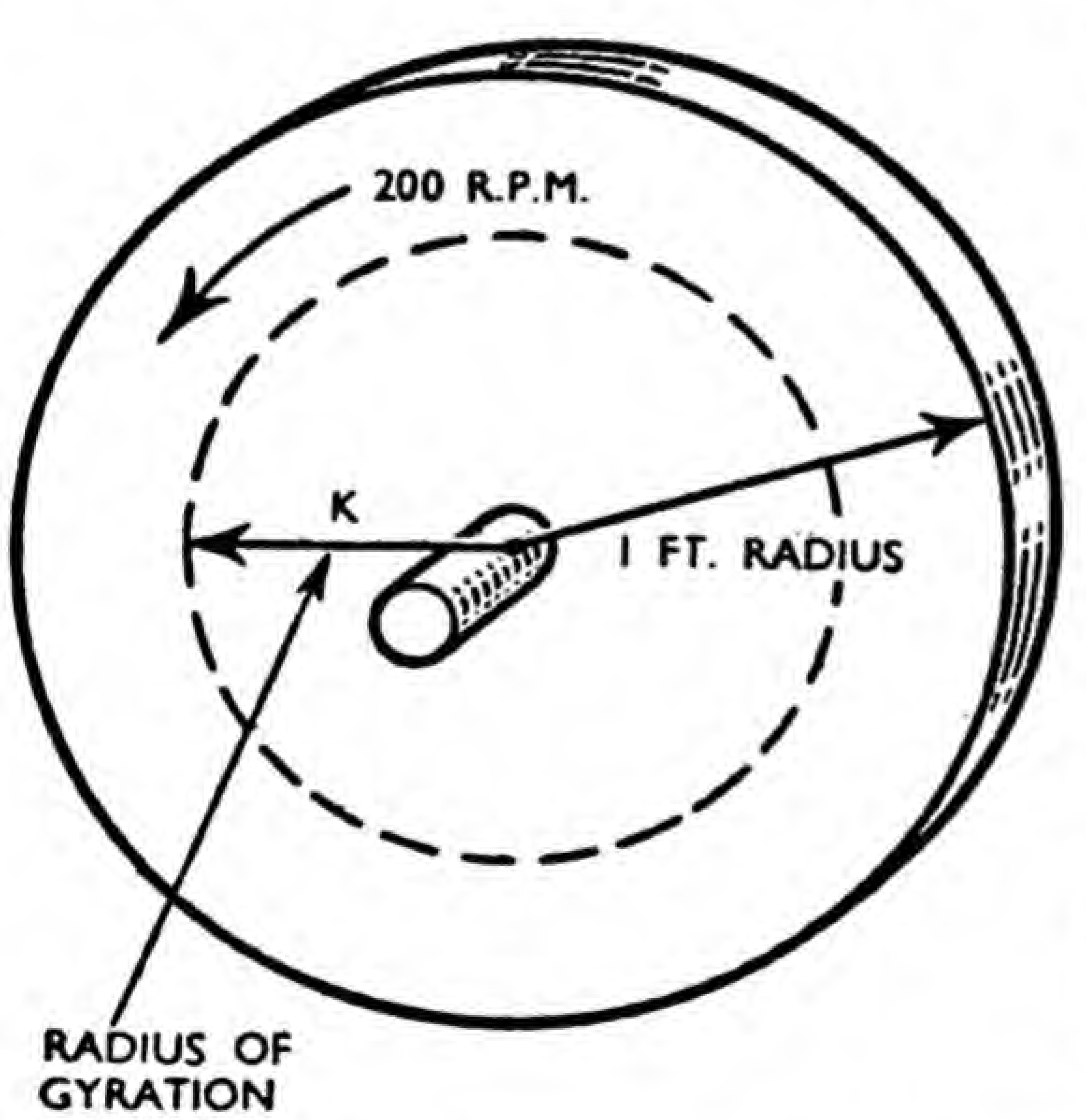


Fig. 16. The kinetic energy of rotation of the solid disk illustrated can be calculated as follows:—weight of wheel $=400$ lb., radius R of disk$=1$ ft., speed of rotation $N=200$ r.p.m. For this disk, $K^2=R^2/2=\frac{1}{2}$ ft.^{2}. Hence the moment of inertia of the disk$=400\times\frac{1}{2}=200$ lb.-ft.^{2} The angular speed $=2\pi N/60=20{\cdot}9$ radians per second. Therefore, kinetic energy of rotation $=I\omega^2/2g=200\times 20{\cdot}9^2/2\times 32{\cdot}2=1{,}350$ ft.-lb.

energy of any rotating body. This will be:—K.E. $=\frac{1}{2}\times\frac{W.k^2}{g}\,\omega^2$.

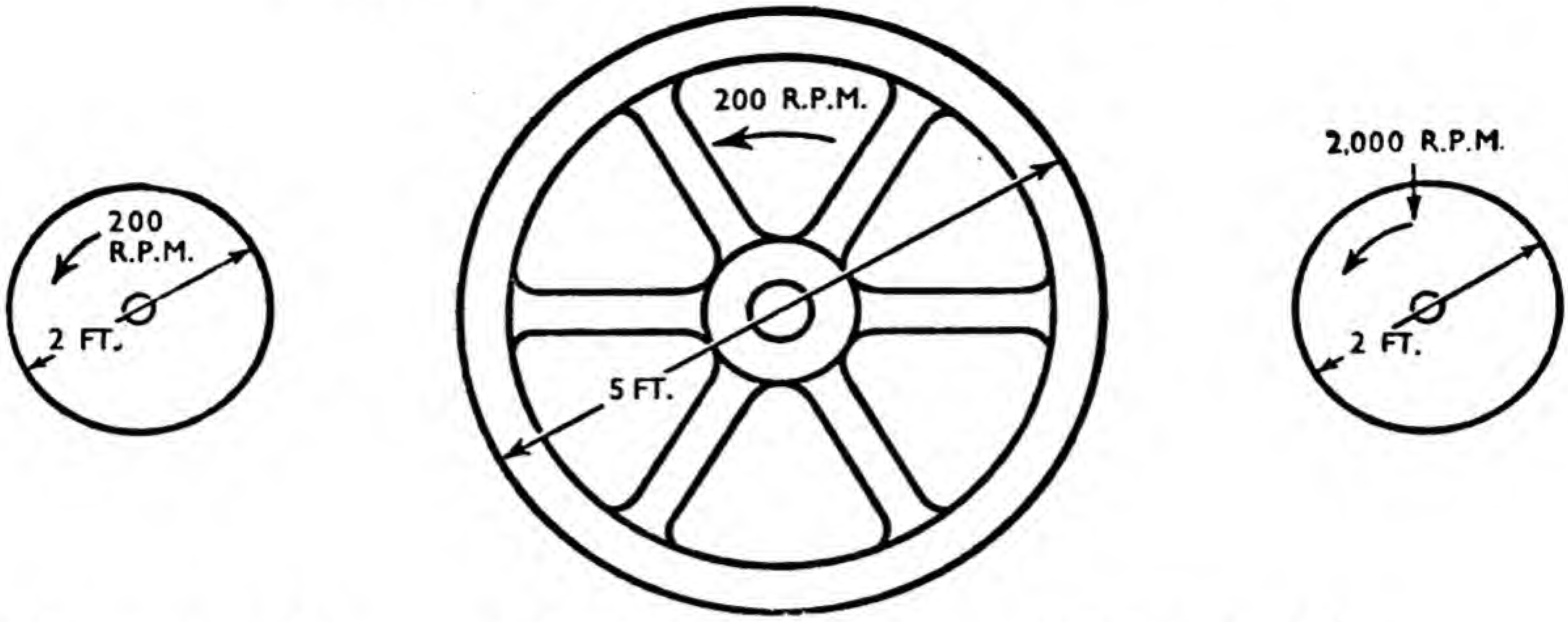
Now the weight W and the radius of gyration k of the body depend only upon the size and shape of the body, so that Wk^2 is also determined by the size and shape. This factor is called the moment of inertia of the body, and is denoted by the letter I. Substituting I for Wk^2 in the expression for the kinetic energy of a rotating body, the equation then obtained will be:—K.E. $=\frac{I.\omega^2}{2g}$ (Fig. 16).

Putting this into the form of words, it can be said that the kinetic energy of a rotating body is the moment of inertia of the body multiplied by the square of its angular velocity and divided by twice the acceleration due to gravity. This is very similar to the expression for the kinetic energy of a body moving in a straight line.

A rotating wheel is a very convenient method of storing energy in the form of kinetic energy of rotation, and the amount of energy stored in this way may be made as small or as large as required, since it is determined by the size and shape of the wheel, and by the speed of rotation. Some idea of the amount of kinetic energy possessed by rotating wheels may be obtained from Fig. 17.

Kinetic energy of rotation has been used in many ways, from driving a toy automobile, to operating presses and punching machines. Perhaps the most important and interesting example is the use of flywheels for steam and internal-combustion engines.

Most engines drive some machine which requires a steady turning effort to keep it working, as for



Diameter =2 ft.	Diameter =5 ft.	Diameter =2 ft.
Weight =400 lb.	Weight =1,500 lb.	Weight =400 lb.
Moment of inertia =200 lb.-ft.^{2}.	Moment of inertia =6,000 lb.-ft.^{2}.	Moment of inertia =200 lb.-ft.^{2}.
Speed =200 r.p.m.	Speed =200 r.p.m.	Speed =2,000 r.p.m.
Kinetic energy =1,350 ft.-lb.	Kinetic energy =40,800 ft.-lb.	Kinetic energy =135,000 ft.-lb.

Fig. 17. Rotating wheels are very convenient methods for storing kinetic energy. Illustrated above are three wheels with an indication of their capacities for storing energy.

example, an electric generator. The turning effort supplied by all reciprocating engines varies as the shaft rotates so that the speed changes and a cyclical fluctuation is obtained. Most of us have felt this change of speed in a train driven by a steam locomotive when it is entering or leaving a station.

Store of Energy

At one part of the cycle, the engine is delivering more energy than is required, and at another part of the cycle it is delivering less. If a flywheel is fitted to the engine, then the surplus energy is stored in the form of kinetic energy of rotation when the speed increases, and it delivers energy to the drive when the speed decreases. Thus, the fluctuation of the energy supplied by the engine is corrected by the change in kinetic energy possessed by the flywheel.

The greater the moment of inertia of the flywheel, the smaller will be the change of speed required to take up a certain amount of energy, and the fluctuation of speed will be reduced in this way. The size of flywheel required depends, to some extent, upon what the engine is driving, but mainly upon the type of engine and the speed at which it is running. For instance, a slow-speed steam engine requires a very large flywheel, while a six-cylinder petrol engine giving the same power, but running at a much higher speed, will need only a small flywheel.

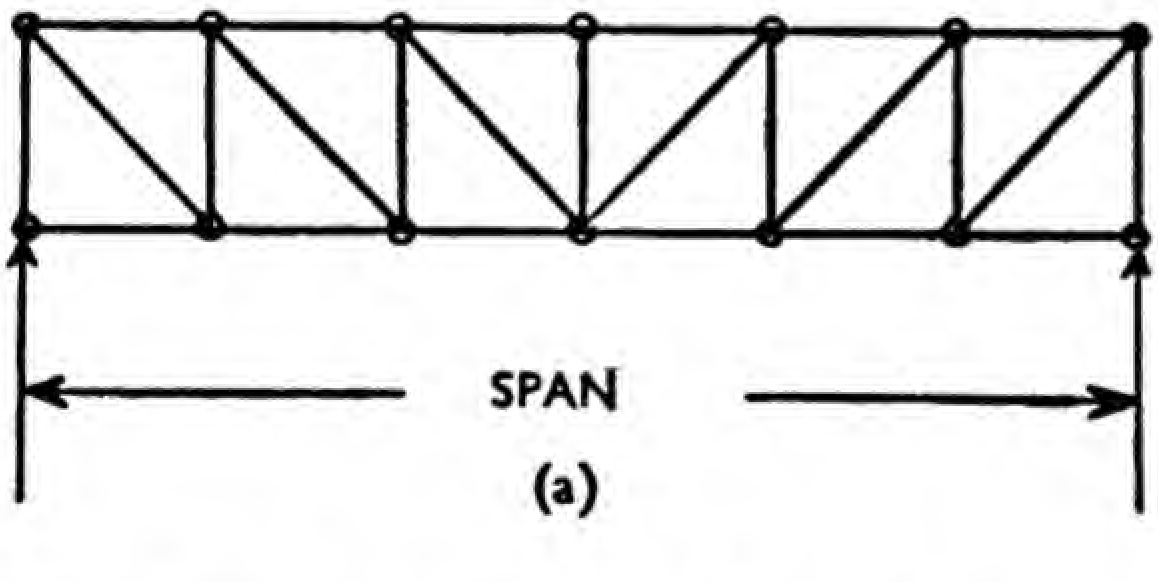
On the other hand, a single-cylinder petrol engine running on the four-stroke cycle, will not run at all without a flywheel, owing to the very great changes in the turning effort during the cycle. This is due to the fact that the engine must be supplied with energy in order to get rid of the burnt gases and to draw in and compress the fresh charge. This energy, we find, is supplied by the flywheel.

CHAPTER 8

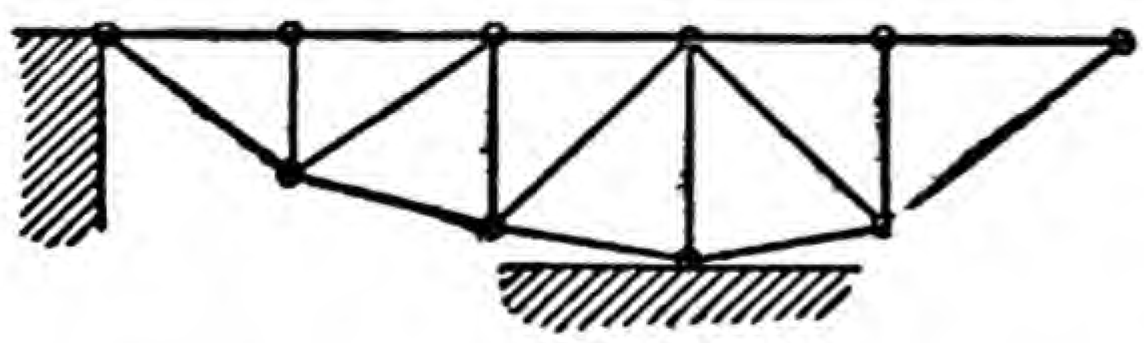
STRENGTH OF MATERIALS

RATIO OF DEAD TO LIVE LOAD. SHEARING FORCE. UNIFORMLY DISTRIBUTED LOAD. BENDING MOMENTS. CRITICAL SECTIONS AND SAFE LOADS. PROPERTIES OF CROSS-SECTIONS. MOMENTS OF INERTIA. MODULUS OF ELASTICITY. COMPOUND BARS. TEMPERATURE STRESS. SHEARING STRESS AND STRAIN. RIVETED JOINTS. LOADS AND FORCES IN EQUILIBRIUM. TENSION AND COMPRESSION. FLEXURE FORMULA. HORIZONTAL SHEAR. DEFLECTION. WELDED JOINTS. TORSION. ANGLE OF TWIST. CLOSE-COILED HELICAL SPRINGS. MATERIALS AND WORKING STRESSES.

WHEN a bus travels over a bridge there is no visible indication that the structure has received any extra load. The bridge is apparently quite inactive. As the bus moves along the roadway, each portion of the structure must, however, take some share in resisting the effect of the passage of the vehicle. On some parts of the bridge, the effect of the passage of the bus may be sudden and short lived. Other portions may experience an extra load during the whole of the period that the bus is on the bridge.



(a)



(b)

Fig. 1. Although bridge and roof structures are usually riveted at the joints, it is common practice to design them as if they were jointed by smooth steel pins passing through eyes in the steel members. In the past, steel structures were actually built in this way without riveting, but the practice has ceased.

General Principles

This chapter deals with the effects of loading of this kind ; with internal forces in structures like bridges or grandstands. It shows how engineers proceed in their calculations, and how they ensure, for example, that when a great celebrity appears at the local cinema, every portion of the balcony will be adequately proportioned to carry the crowds.

A technical subject of this kind must be approached step by step, each portion of the work being thoroughly understood before proceeding to the next. In the present discussion there are two main sections.

(*a*) The effect of the total loading on each of the individual parts of the structure. It is obvious that the engineer must be able to estimate, fairly accurately, how the external loads are distributed.

(*b*) How external loading induces internal effects in the material of which the structure is composed.

As a corollary to this section, the properties of the various materials used in construction must be studied. This is discussed in Chapter 9.

Balance of Forces and Moments

The first important point to be noted is that the external loads and the supporting forces and moments acting on the structure must be in equilibrium, or, in other words, they must balance each other regardless of the shape or size of the structure on which they act. The problem of forces in equilibrium has been discussed in detail in Chapters 2 and 3, and it is suggested that these be revised before further study is made of the present subject. Assuming, however, that it is understood how forces act and react in order to keep a body in equilibrium, now consider the methods by which the structure itself can withstand the applied loads.

In the first place, the structure may be built up of a number of relatively slender bars arranged to form a series of triangles (Fig. 1). Such frames are usually considered to be constructed of members which are joined together at the ends by pins, or, as in Fig. 1, at the points shown by circles. The force experienced by each member of the frame is thus either a direct push or a direct pull. The values of these forces may be determined from the loading on the structure, as explained in Chapter 3.

The second type of load-carrying structure is composed of one or more relatively stiff members which can resist bending. Such beams bend slightly under the loading, but care can be taken to ensure that this deflection is not excessive.

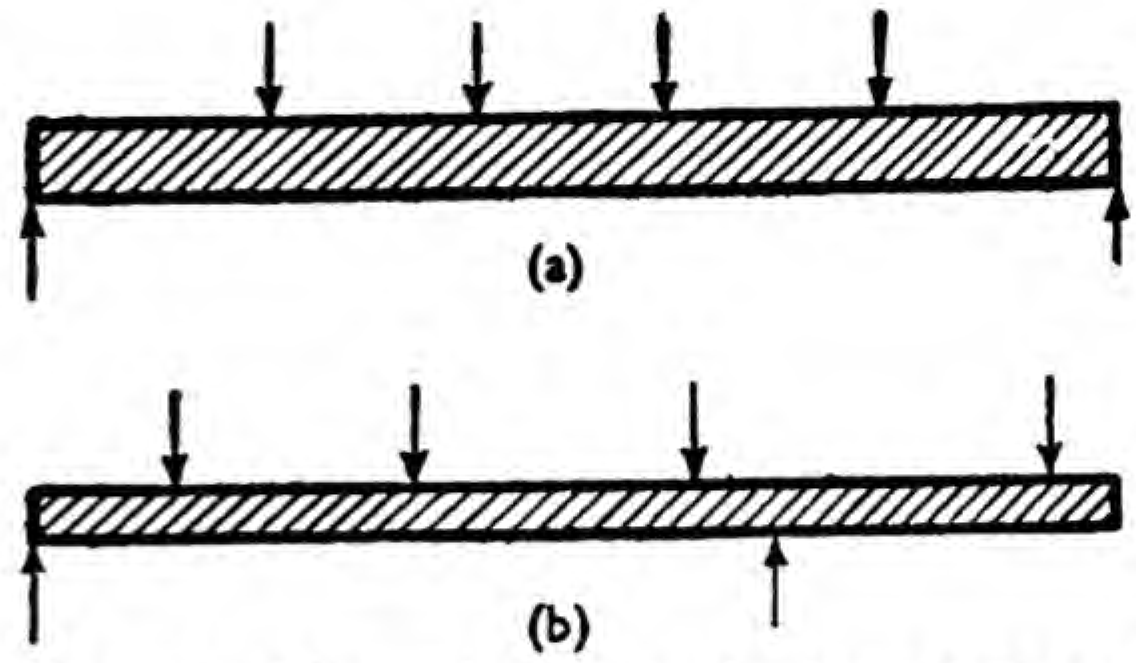


Fig. 2. The pin-jointed structures of Fig. 1 resist external loading by tension and compression in a series of straight relatively slender members. Beams, however, resist external loading by developing shearing and bending stresses.

Fig. 2 shows how structures of the second type carry a load.

It may be well to realize at this stage that the difference between structures of the first and second types is only one of degree. The structures shown in Fig. 1 are not fundamentally different from the beams of Fig. 2. They are merely built to a larger scale. Fig. 1(a), for example, may be considered to be the beam of Fig. 2(a) with triangular portions cut away, leaving only the essential amount of metal in the form of slender bars. When loads must be carried over wide spans, such elimination is necessary in order to save weight.

Summary

To summarize, the following five statements must be remembered :—

(*a*) the external forces on the structure must balance each other ;

(*b*) the structure may be built up of relatively slender pin-jointed members ;

(*c*) the structure may be made of relatively stiff members which can resist bending ;

(*d*) a structure of the first type can usually be related to one of the second type. The difference in appearance is usually caused

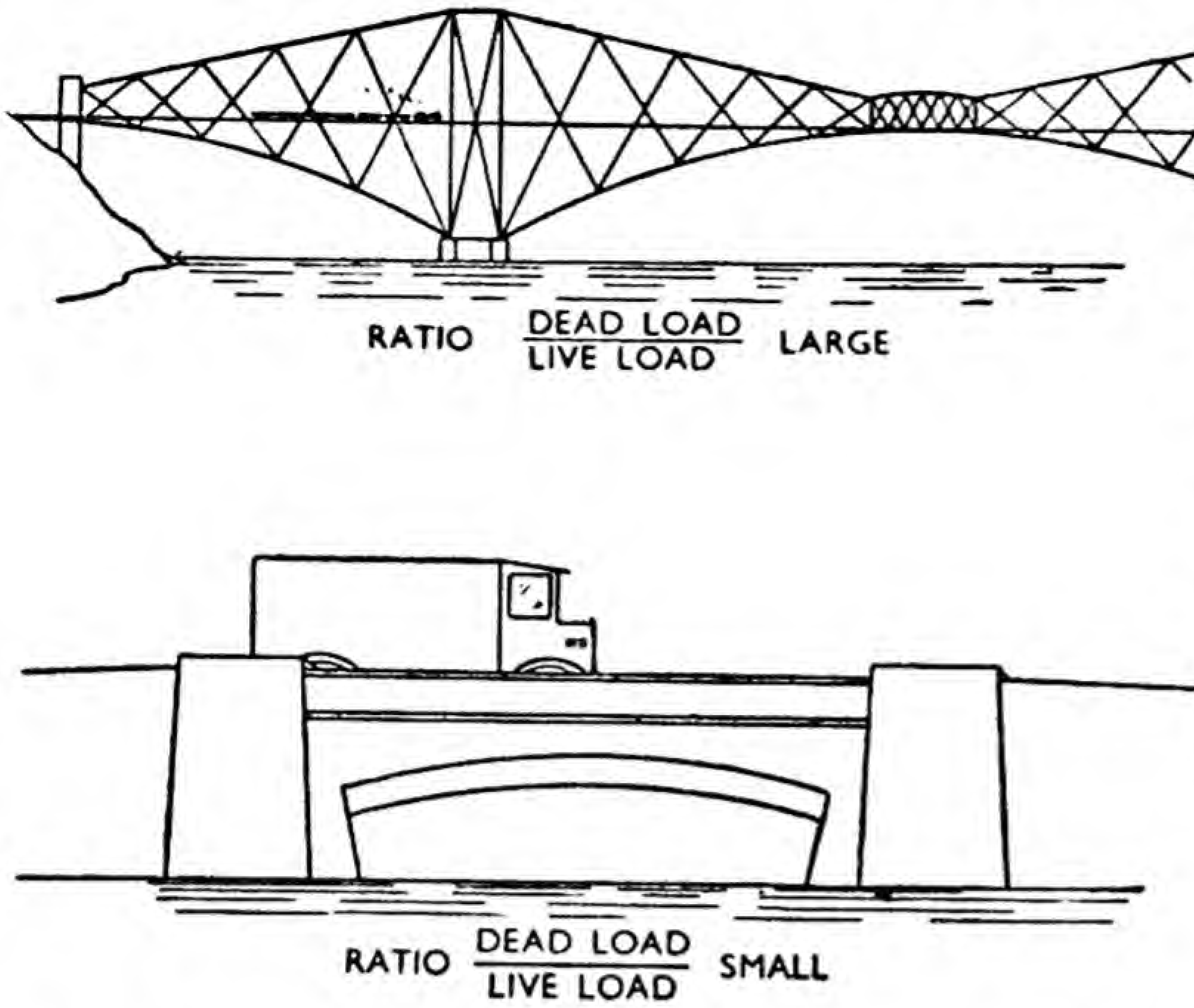


Fig. 3. The magnitude of the effect produced on a structure by a moving load depends partly on the ratio of the dead load (or weight of the structure) to the live load. On the Forth Bridge, for example, the moving load adds relatively little to the stresses caused by the weight of the bridge itself.

by the difference in scale of the two structures ;

(*e*) the present chapter deals with the second type only.

Dead and Live Load

External loads may be classified into two broad groups according to whether they act continuously, or only intermittently. The first of these loadings is called the dead load, and the intermittent or moving load is called the live load.

The dead load consists of the total weight of the structure itself. The live load consists of the movable objects which may, or may not, be present at any given instant. An express train crossing a bridge is obviously a live load, but live loads need not necessarily be in motion. A motionless audience at a lecture is a live load on the building. The office safe which has stood in the same corner for many years is also a live load, for, conceivably, it might be moved.

The passage of an express train or a car is an instance of a rapidly applied load, and this extra effect is known as impact and must be allowed for in design. Such allowance is, however, outside the scope of this chapter.

The ratio between dead load and live load is also very important in the design of beams or frames. In

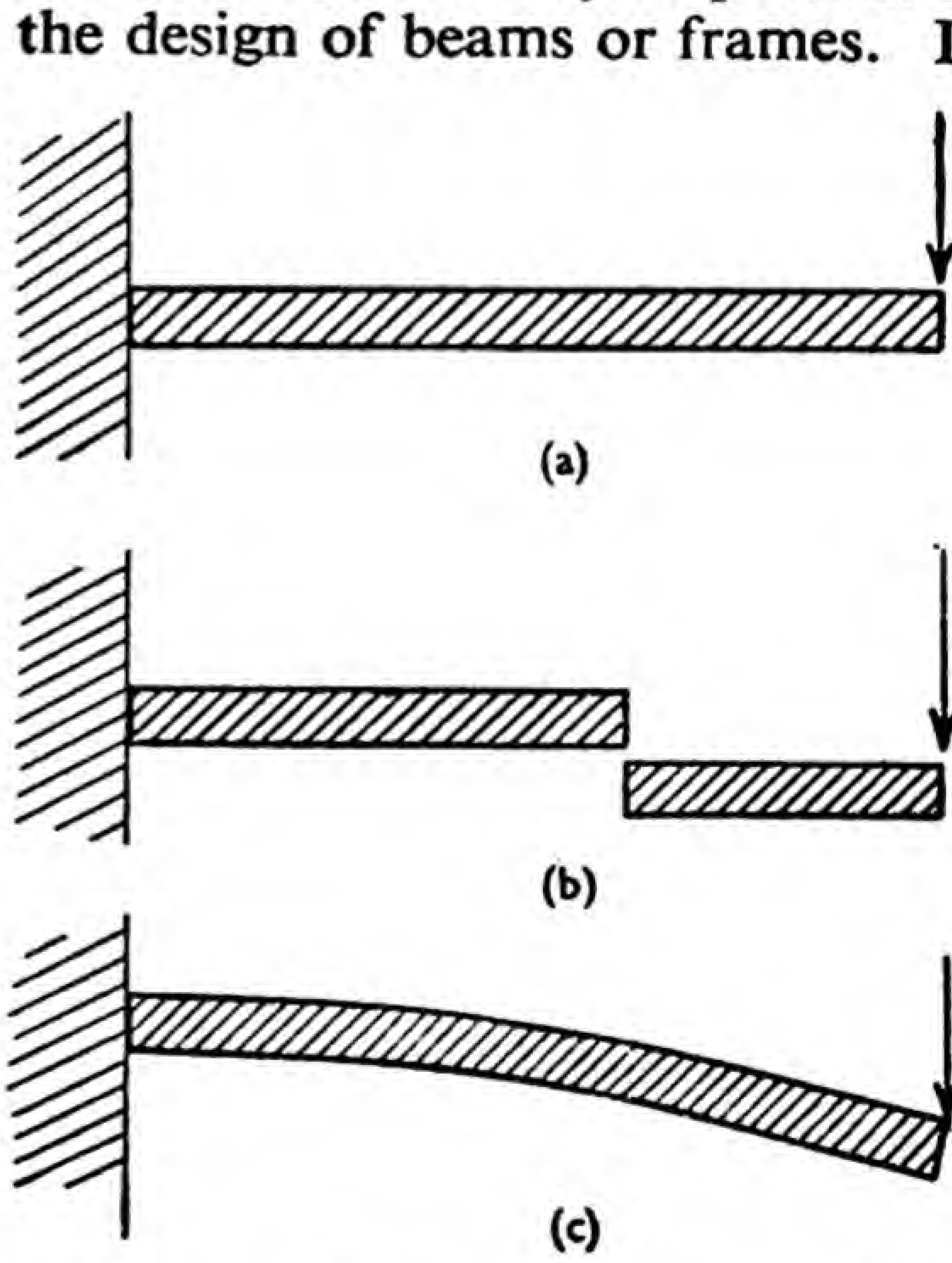


Fig. 4. A beam, held at one end and projecting like a balcony, is called a cantilever. In any beam there are both shearing and bending tendencies. In design these effects are considered separately as a matter of convenience and because shearing is resisted chiefly by one part of the beam section and bending by another.

the Forth Bridge, for example, the ratio of dead to live load is very large, whereas on a bridge crossing a canal and carrying heavy traffic, the ratio of dead to live load may be small (Fig. 3).

Consider now what a load on a beam is trying to do. Fig. 4(a) shows a beam, built into a wall, and loaded with a concentrated load at the free end. This might represent someone standing on the outer edge of a balcony. Such projecting beams, unsupported at the outer end, are known as cantilevers.

Imagining the beam to be sawn through at any point on its length, it is obvious that the outer portion of the length will drop, relative to the inner portion (Fig. 4 (b)). Again, imagine the beam to be made of a flexible material ; Fig. 4(c) shows the bending effect which may be produced. The loading on the beam is constantly attempting to carry out both these effects simultaneously at all points along the length of the cantilever. How far it succeeds depends on the strength and stiffness of the beam.

The imposed effect of the load can, however, be studied without knowing whether the beam is sufficiently strong or not. The following important statement can be made at this stage : the beams that are being studied may be represented by straight lines, without any reference to the sectional areas of the parts of which they will be finally constructed. The members of the frames illustrated in Chapter 3 were, in the same way, depicted by straight lines. The size and shape of the members of the beams will be determined in what has been called the second section of this discussion.

It is rather inconvenient to try to think of the two effects shown in Fig. 4 as occurring simultaneously. They *do* occur at the same time, but, for convenience, they are always considered separately.

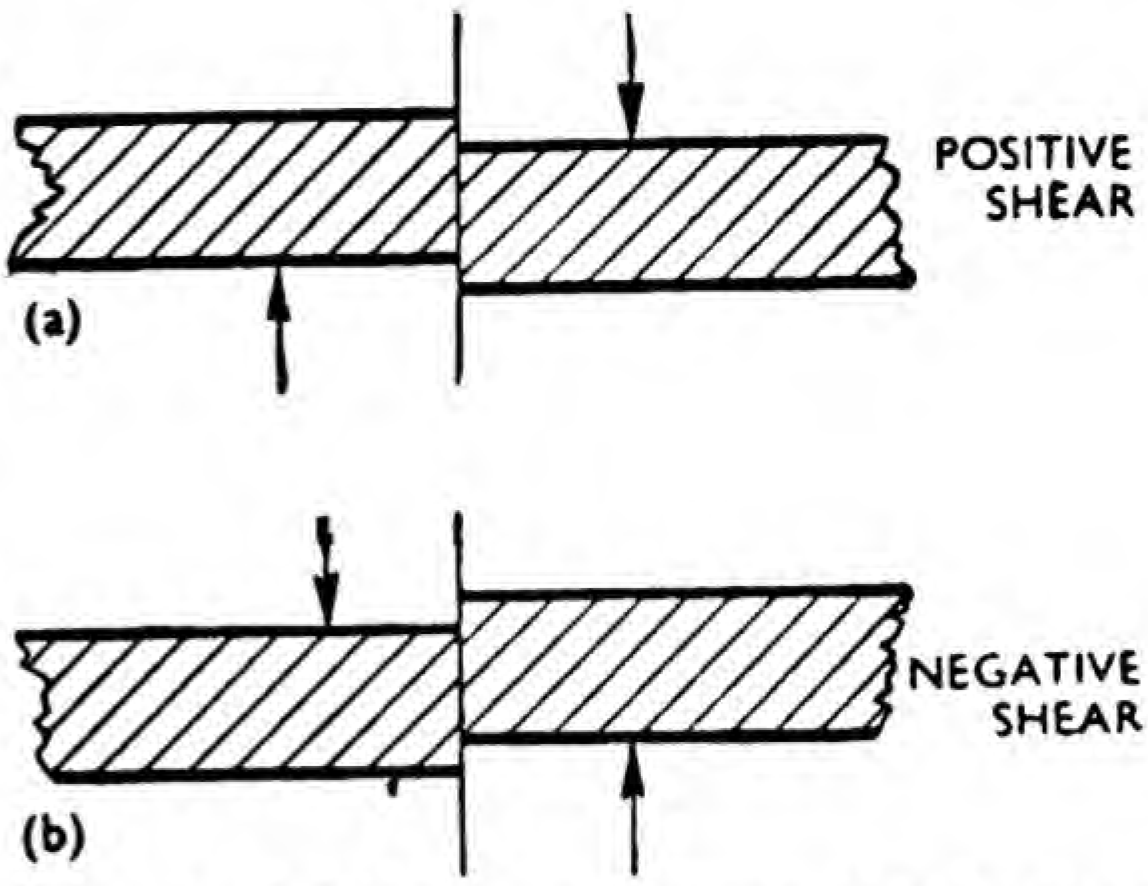


Fig. 5. (*a*) If, due to shear, the portion of a beam to the right of a section tends to go down, relative to the portion on the left, the shear force is considered to be positive. (*b*) With reverse tendency, the shear force is negative.

Shearing Force

The effect shown in Fig. 4(b) is known as shear, and can be illustrated in a very simple manner. Let one's two hands be placed together, palm to palm, the fingers pointing upwards. Now, if the left hand is moved up and the right hand down, the palms will slide over each other. This represents the way in which two portions of the beam tend to slip over each other. Of course, shear may occur in the opposite sense, the right hand going up and the left hand down. If one movement is positive, the other is considered to be negative (Fig. 5).

In this simple experiment it will be noticed that the force or effort required to push one hand up is the same as that required to pull the other hand down. Thus, the shearing force at the section of a beam which is imagined to be cut

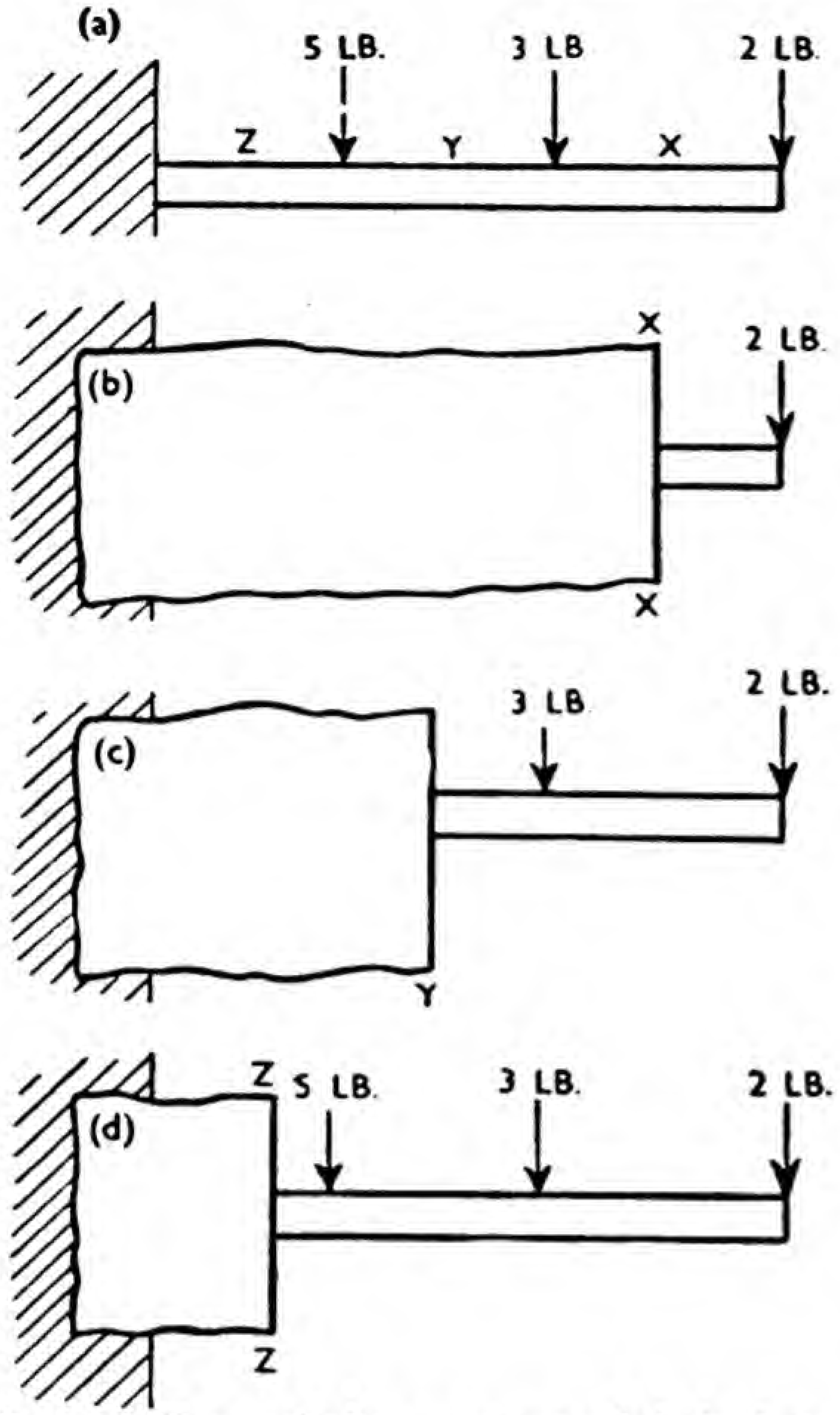


Fig. 6. One of the most useful dodges in calculating shearing force is to use a piece of paper which completely covers the diagram of the beam. This paper is then gradually drawn along, uncovering the loads in turn. Only the loads which can actually be seen are then used in the calculations.

(Fig. 4(b)), is the same whether it is calculated from the left-hand side or from the right-hand side.

So far, the nature of shearing force has been considered, but not its magnitude. How to find the value of the shearing force at any section of a beam must now be determined. From the simple experiment described above, it can be realized that the value of the shearing force at a certain point is merely the total upward or downward force exerted by the right-hand or left-hand side of the beam at that point.

Fig. 6 illustrates a cantilever carrying three loads. It is required to find the shearing force at three points, *X*, *Y* and *Z*. Remembering that shearing force is the total upward or downward effect *on one side of the section*, it is better to ensure that only the loads affecting one side of the section will be seen. For this purpose a piece of paper, or something equally opaque, is used. It is most important at this stage not to rely on one's imagination, but to cover up the portion of the beam or cantilever not required.

Considering first the section *X*, the paper is placed so that everything on the left of *X* is covered, then the upward or downward effect of the loading *which can be*

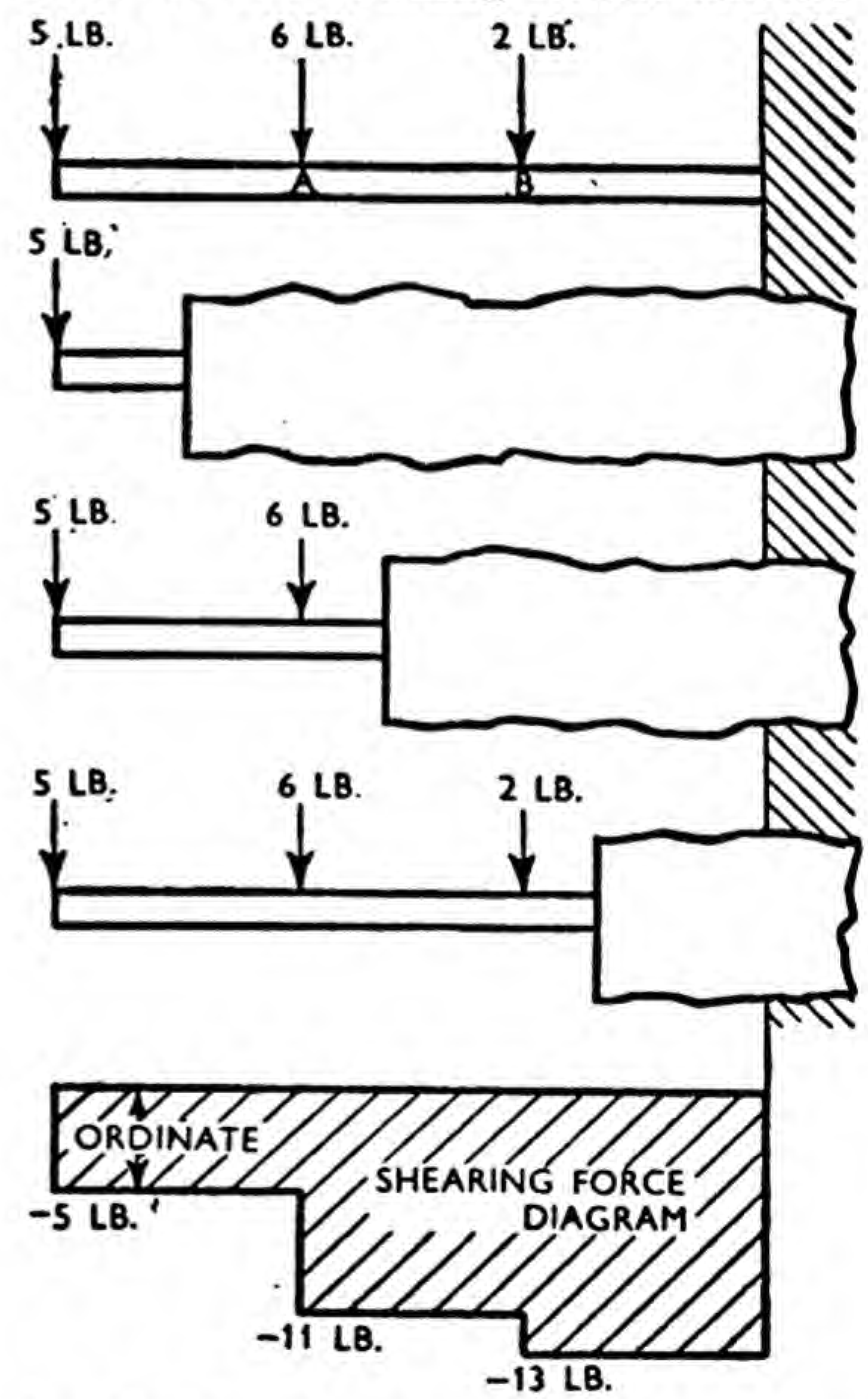


Fig. 7. The depth or thickness of a shearing-force diagram at any section of a beam is a measure of the value of the shearing force at that point. In more definite terms, the ordinate of the shearing-force diagram represents the shearing force to a given scale. When the loads are concentrated, this diagram appears as a series of steps.

seen is investigated. In this instance, the only force which can be seen is 2 lb. acting downward. The shearing force is, thus, 2 lb.

Now, moving the paper edge to the point *Y*, it is found that all the loading on the beam is obliterated except the two loads of 2 lb. and 3 lb. They are both acting down, so their effects must be added, the total shearing force being 5 lb. In the same way, the shearing force at *Z* is found to be 10 lb.

Pictorial Method of Recording

It is important, of course, in discussing the strength of a beam or cantilever, that the designer should have before him a picture of how the shearing force on the beam varies from point to point. In Fig. 6 three different values of the shearing force were found at three different sections, but such piecemeal investigation is clumsy and inefficient. Using Fig. 7 and proceeding to draw a shearing-force diagram, it will be noted that this is merely a pictorial method of recording such figures as were found from Fig. 6.

First commence by covering the whole of the cantilever except for the 5-lb. load at the left-hand end. The shearing force at the edge of the paper is, thus, 5 lb. The paper is now drawn gradually towards the right. Until the point *A* is reached, only the 5-lb. load is seen, and the value of the shearing force remains 5 lb. The shearing-force diagram for this portion of the beam has a thickness or height representing 5 lb. This happens to be in the negative direction, because the beam tends to go downward on the left of the paper edge (Fig. 5).

When the edge of the paper comes to the point *A*, the 6-lb. load

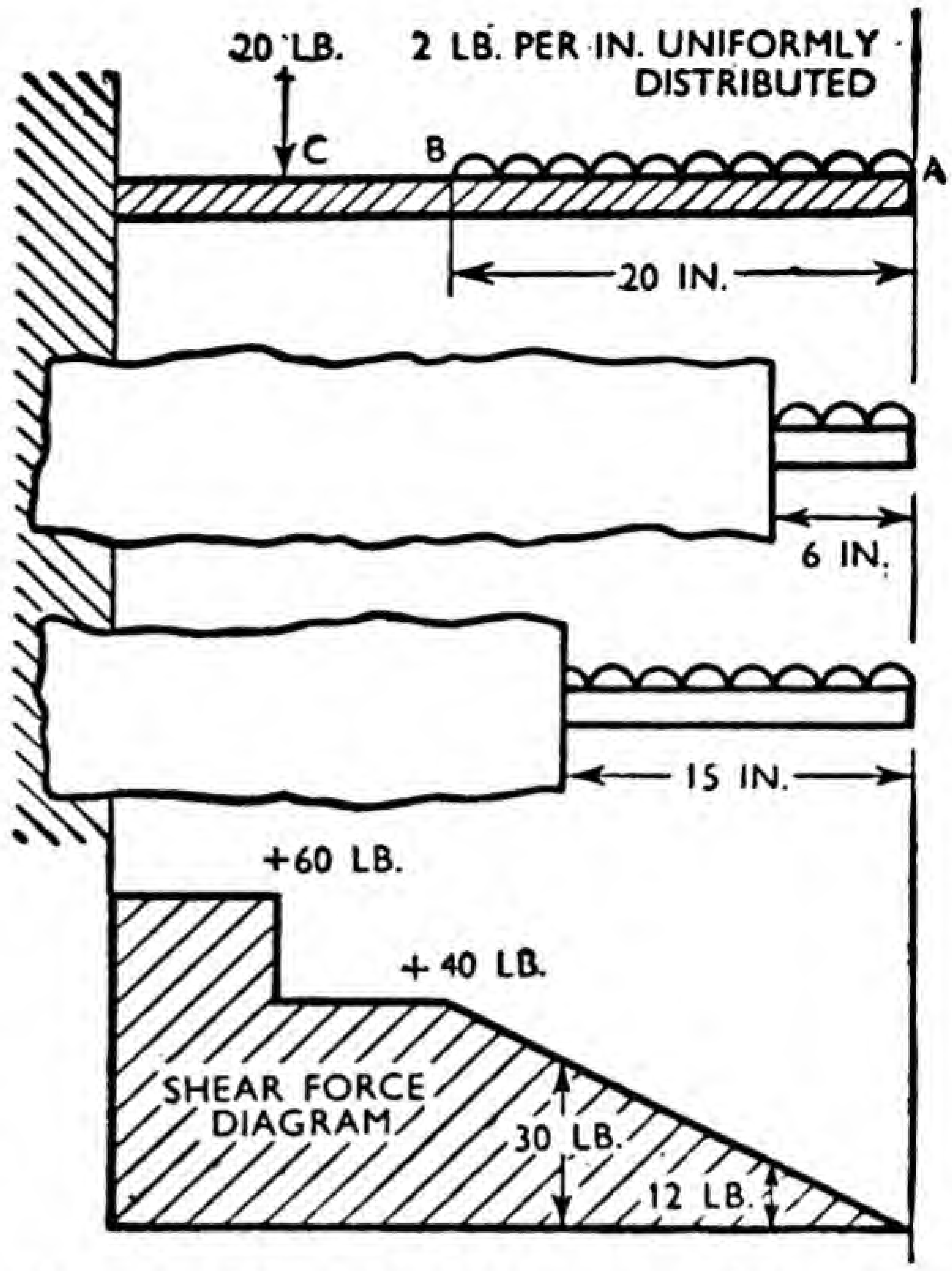


Fig. 8. When the load on a beam is uniformly distributed, it can be thought of as a series of loads extremely close together. The steps of the shearing force diagram of Fig. 7 are then so small and close together that they become a sloping straight line.

suddenly appears. The shearing-force diagram, therefore, must have a sudden change in thickness. The depth now becomes $5 + 6 = 11$ lb. In all sections of the beam between *A* and *B*, the shearing force, or sum of the forces on the left of the section, is 11 lb. The rest of the diagram is completed in a similar fashion.

Uniformly Distributed Load

In Fig. 8 another kind of loading is shown, a type of which is often encountered both as a dead load and as a live load. This is called a uniformly distributed load. On every inch of the first 20 in. of this cantilever, there is a load of 2 lb., so that the total load is 2 lb. per in. $\times$ 20 in. $= 40$ lb.

When the piece of paper covers

all the cantilever except the tip A, it will be seen that the shearing force, or downward effect of the loading, is zero because there is no concentrated load acting at that point. However, as the length AB is uncovered, inch by inch, a value of 2 lb. shearing force must be added for every inch exposed. For example, when 6 in. have been uncovered, a downward load of 12 lb. acting on the right of the section is obtained, which is equivalent to a positive shear of 12 lb. When 15 in. have been uncovered, the shearing force is 30 lb.

When the edge of the paper reaches the point B, the whole of the 40 lb. has been exposed, and the shear remains constant at 40 lb. until the edge of the paper uncovers the load at C. Here, the shear increases suddenly as it did in the previous problem, and remains at 60 lb. until the end of the beam.

It has thus been noticed that the shearing-force diagram changes by steps when concentrated loads are acting, and by sloping straight lines when uniformly distributed loads are acting.

Fig. 9 shows a beam, supported at both ends by simple props, and carrying both a concentrated load and a uniformly distributed load. The values of the supporting forces are shown in the figure, and these must be taken into account when calculating the shearing force at any point. The forces on a structure do not merely comprise the vertical downward loading, but also the upward supporting forces. Using the edge of the paper as the recording mark, again study the upward or downward effect at the edge of the paper.

While the edge of the paper is between A and B, the only force visible is the reaction at A, which is 8 tons. The shearing force, therefore, is 8 tons, and is negative, because the force on the right of the paper is pointing up (Fig. 5).

When the 5-ton load is uncovered there are two forces visible beyond the edge of the paper. They are acting in opposite directions, and the shearing force, there-

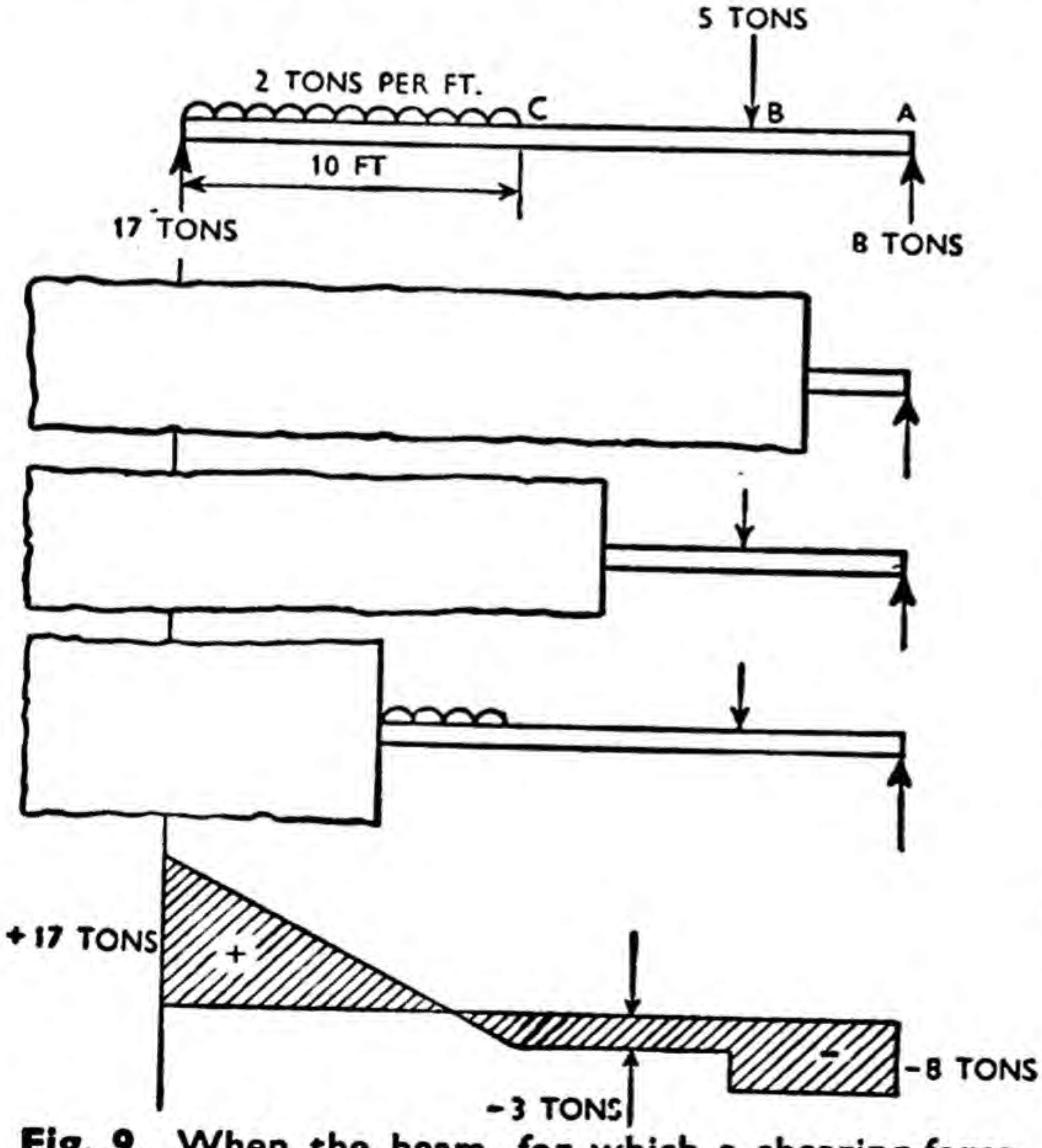


Fig. 9. When the beam, for which a shearing-force diagram is required, is supported by two reactions, these must first be calculated before work can be commenced on the diagram itself. The covering paper, then, first reveals one of the end reactions whose value defines the first ordinate of the diagram.

fore, is the difference of 8 tons and 5 tons, which is 3 tons. The net tendency is for the portion on the right to go up, and the shearing force is still negative.

When the edge of the paper passes C, it is found that 2 tons for every foot of beam exposed has been uncovered, and as this new load is acting downward, its effect must be added to that of the 5 tons. When 1 ft. 6 in. of the portion beyond C has been uncovered, 8 tons up is obtained, and $5 + 2 \times 1\frac{1}{2}$ down. These values are equal, and there is no shear at all at the section 1 ft. 6 in. from C. Thereafter, the shearing force becomes positive.

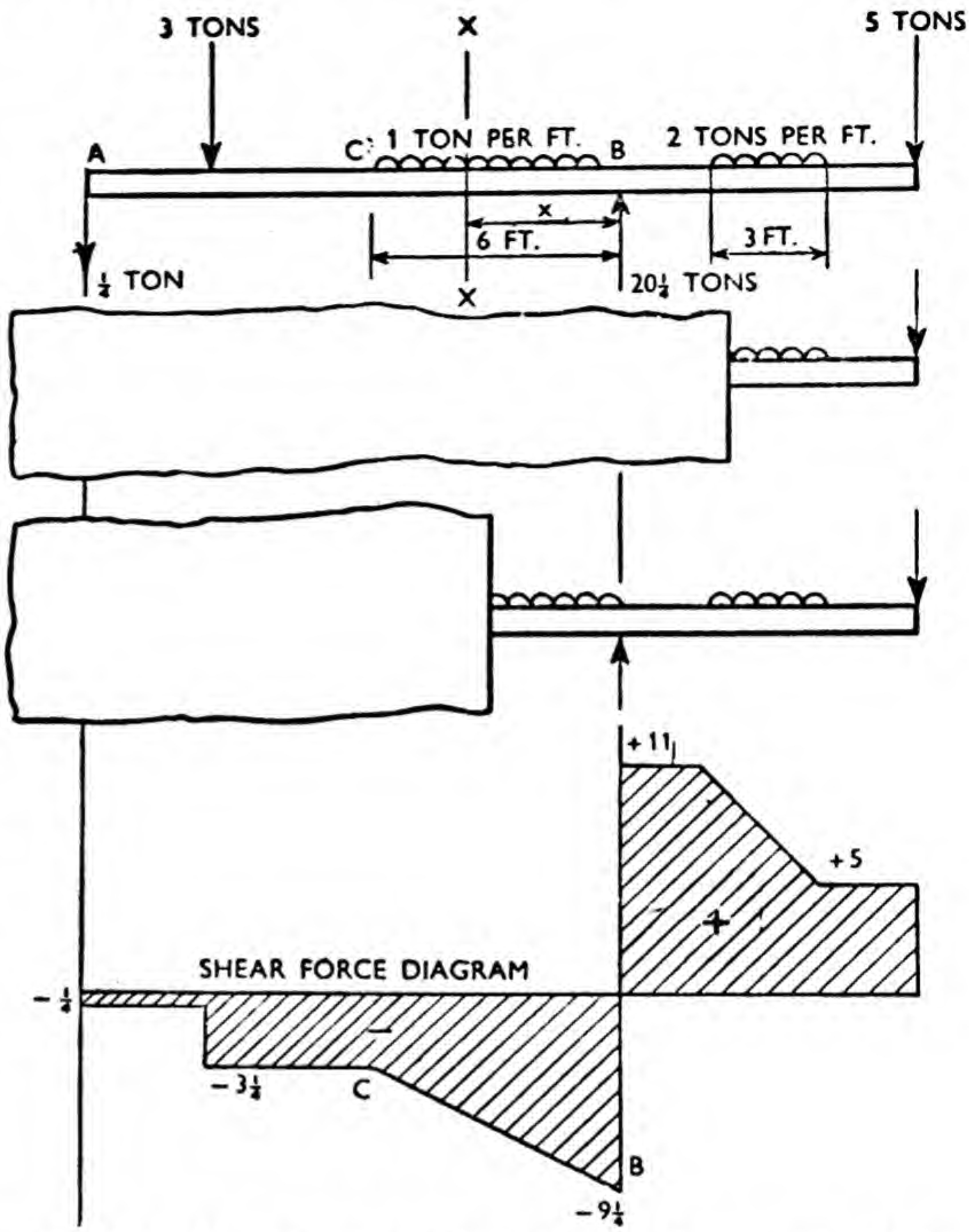


Fig. 10. This represents another exercise in the determination of shearing force. The structure is a combination of the cantilever of Fig. 8 with the simply supported beam of Fig. 9. The first step is to calculate the values of the supporting forces. The diagram is a combination of steps and sloping lines.

The final shearing force is found to be 17 tons, and this can be verified by reversing the opaque paper, covering up the right-hand portion of the beam, and calculating the shearing force, commencing at the left-hand end.

Fig. 10 shows a combination of beam and cantilever, and the loads are so arranged that the left-hand reaction must act downwards in order to keep the structure in equilibrium.

The shearing-force diagram is drawn in the same way as before, each load being uncovered in turn as the opaque paper moves from right to left across the beam. The sudden change from positive shear to negative shear at the right-hand support should be noted.

Algebraic Calculation

When the loading is more complicated, it is sometimes convenient to have an algebraic expression for the value of the shearing force. Such an expression is obtained for the shearing force at section XX, which is x ft. from the right-hand support B.

By covering the left-hand portion of the beam beyond XX, it is found

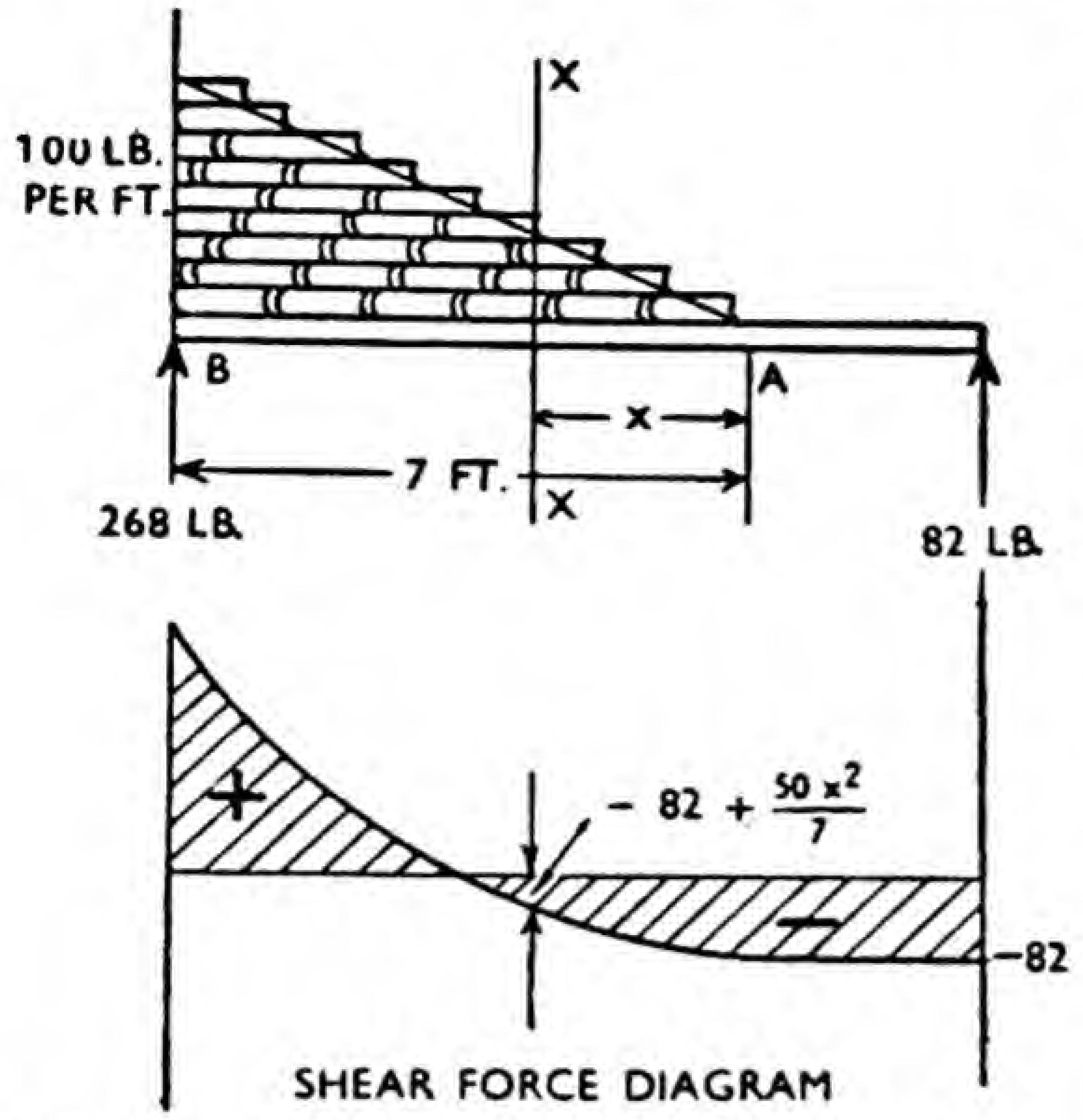


Fig. 11. When the load on a beam is not uniform, as it was in Figs. 9 and 10, the relevant portion of the shearing-force diagram is not a straight line. Here we have a triangular distribution of loading which results in a curved diagram with a change of sign within the load.

that the only forces exposed to view are :—

Down, 5 tons, (2×3) tons, and $(1 \times x)$ tons.

Up, $20\frac{1}{4}$ tons.

The shearing force at XX is thus :

$$+5+6+x-20\tfrac{1}{4}$$
$$= (x - 9\tfrac{1}{4}) \text{ tons.}$$

This expression is true for any point between B and C, and represents the equation to the sloping line between B and C in the shearing-force diagram.

As a final example of the method of obtaining the shearing-force diagram, consider Fig. 11. Here, the loading on the beam is not uniformly distributed, but of a triangular shape. It represents a pile of books on a library floor. At the left end of the beam, the load on the floor is 100 lb. for every foot of the beam. This load decreases to a zero value at 7 ft. from the end. At a distance x ft. from the point where the load is zero, the intensity of loading on the floor is in proportion to the length x. At XX, the intensity of loading is :—

$$\frac{x}{7} \times 100 \text{ lb. per ft.}$$

Now, in order to determine the shearing force at any point along the 7-ft. length, the total weight of books up to the section in question must be known. Put down the opaque paper to cover the whole of the beam to the left of XX. The only forces visible are 82 lb. acting up, and a triangular pile of books acting down. The total weight of the books exposed is the average weight of the books per foot multiplied by the number of feet covered.

Weight of books

$$= \tfrac{1}{2} \text{ of } \frac{100x}{7} \text{ lb. per ft.} \times x \text{ ft.}$$
$$= \frac{50x^2}{7} \text{ lb.}$$

Numerical Valuation

The shearing force at section XX is the net effect of all the loads to the right of the section. From A to B the shearing force is :—

$$-82 + \frac{50x^2}{7} \text{ lb.}$$

By giving x various numerical values a number of ordinates, or

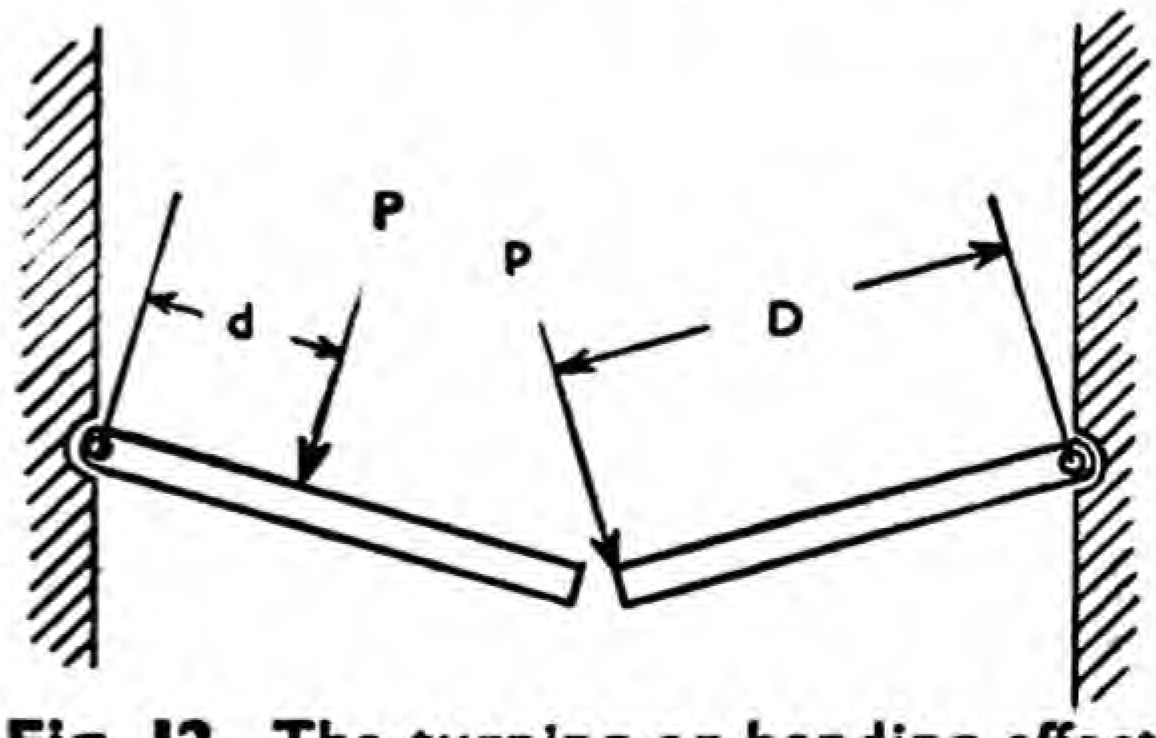
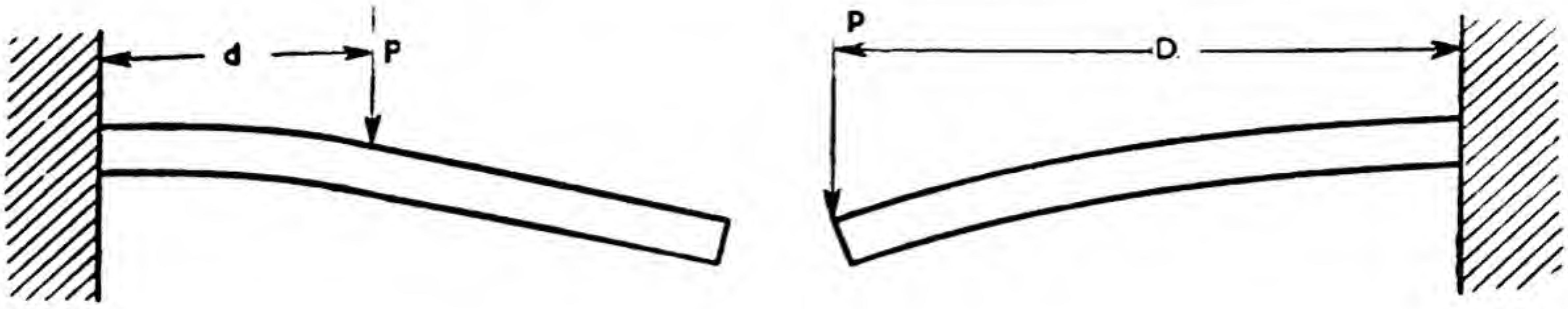
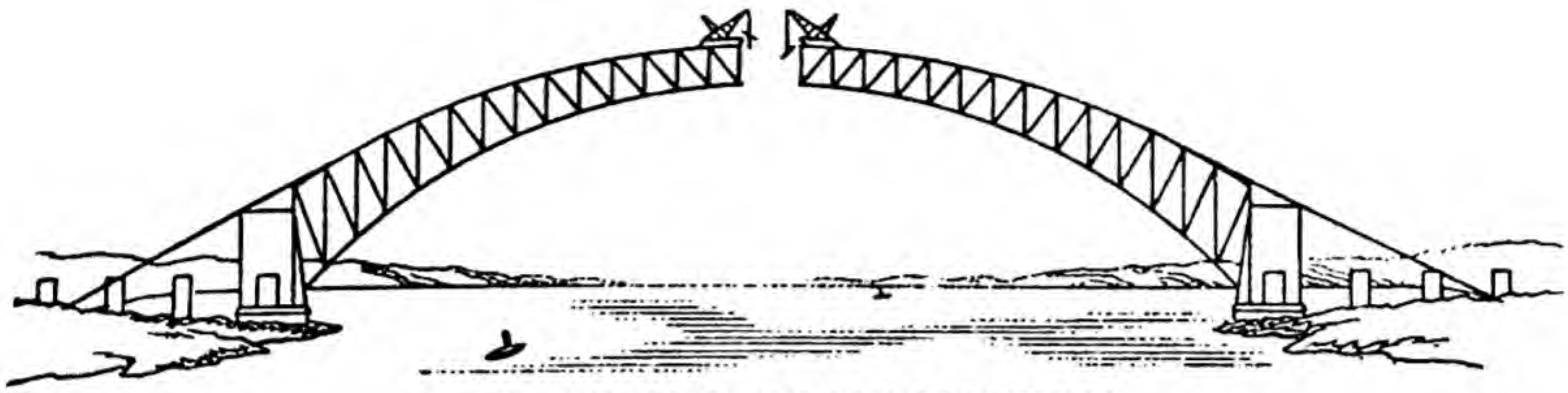


Fig. 12. The turning or bending effect of the moment of a force is well shown by the opening of a swing door. If the push is exerted far from the hinge, it can be much smaller than if it is exerted near the hinge.





CANTILEVER CONSTRUCTION

Fig. 13. Cantilevers are beams fixed at one end and unsupported at the other. Arch bridges have been built out from both abutments as cantilevers meeting in the centre of the span. During erection, they were self-supporting, being anchored by cables until the central closure was successfully accomplished

thicknesses, for the shearing-force diagram is obtained. The curve is a parabola (Fig. 11).

Bending Moment

Having studied the shearing effect of external loads, it is possible to pass to the second of the effects shown in Fig. 4. Bending of the beam occurs simultaneously with the tendency to shear, but may be considered separately.

The distinction between shear and bending, which must be appreciated at this stage, is that shear depends only on the loads acting, but bending depends on both loads and distances. The principle of moments has been studied in Chapter 2, but may well be quickly revised. Facility in determining the effect of bending in a beam depends on an understanding of moments.

A moment may be thought of as a turning effect, and an example is shown in Fig. 12. Here, are shown a pair of swing doors viewed from above. Two men are leaving together. One is pushing lightly with a force p and the other is pushing with a much bigger force P. The reason for the difference between the forces required to operate the doors, lies in the fact that they are applied at different distances from the hinges.

The moment, or turning effect of a force, is the product of the force and its distance from the axis round which the rotation is taking place; in this instance, it is the hinges. The moment of one force is p lb. $\times$ D ft. $= pD$ ft.-lb. The moment of the other force is P lb. $\times$ d ft. $= Pd$ ft.-lb.

Now, if there were no well-oiled hinges, the two men, in trying to open the doors, would exert bending all along the doors, which would now act as a pair of cantilevers (Fig. 13). It is a bending effect of this kind, caused by resistance to a turning moment, which must be studied here.

Commence with the important

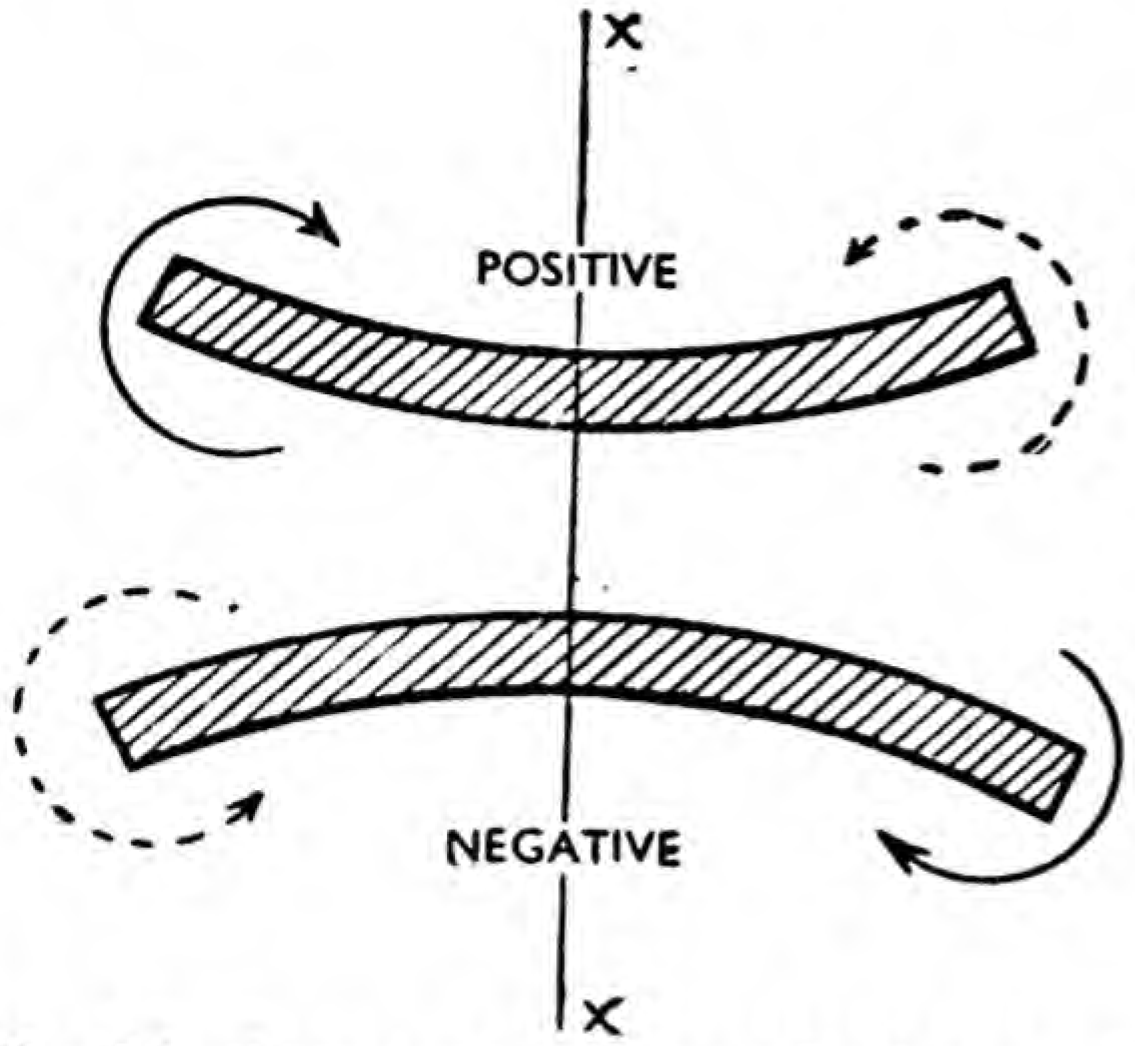


Fig. 14. Conventional signs for bending depend upon the curve of the beam. In this chapter, a sagging beam is said to be under positive bending, and a hog-back beam is said to be under negative bending.

statement that the bending effect or bending moment at any section of a beam is the sum of the moments of all the forces on one side of the section taken about that section.

Sign Convention

Fig. 5 showed the convention of signs for the up and down movements associated with shearing force. Fig. 14 shows the signs adopted for the effects of bending moments. If the bending moments on the beam cause it to bend with the convex side down or, in other words, the beam is sagging, the bending moment is said to be positive. If the convex side is up, the bending moment is said to be negative.

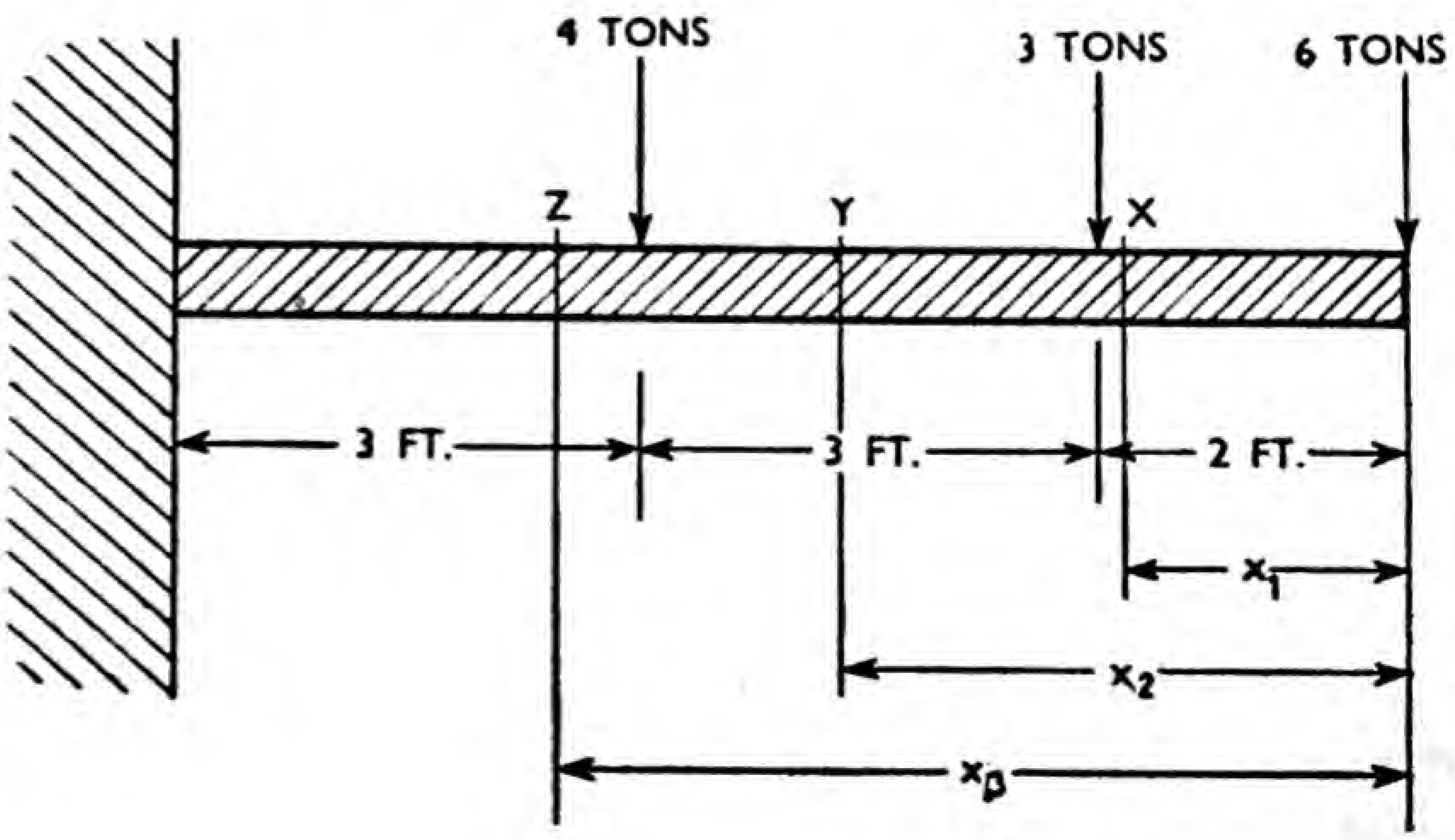
A piece of opaque paper is again a very useful tool. In determining bending moments, as in finding shearing forces, only that portion of the beam *on one side* of the section should be considered. To find the bending moment at *X*, for example (Fig. 15), cover up the left-hand portion of the cantilever, so that only the portion to the right of *X* is visible. The edge of the paper, *X*, can now be considered to be a hinge about which the visible forces are trying to bend the part of the beam to the right of *X*.

Place the point of a pencil on the 6-ton force, which is the only one to be seen, and move the pencil to the edge of the paper. The distance over which it passes is x_1, and the bending moment at *X* is, thus :—

$$6 \text{ tons} \times x_1 \text{ ft.} = 6x_1 \text{ ft.-tons}$$

The bending moment at *Y* is found by moving the paper so that the portion of the beam to the right of *Y* is uncovered. The two forces which are visible are 6 tons and 3 tons. Place the pencil point at 6 tons and move it to the edge of the paper. The distance over which it passes is x_2. Now, place

Fig. 15. The bending moment at any point in a beam must be determined before the beam can be designed to resist bending stresses. The bending moment at any section is found by adding the moments of all the forces acting on the right or left of the point concerned.



the pencil point at the 3-ton load and move it to Y. It passes over a distance $(x_2 - 2)$ ft. The bending moment at Y is :—

$$6x_2 + 3(x_2 - 2) \text{ ft.-tons}$$

The bending moment at any section between two of the loads on the beam thus increases in direct proportion to the x distances, and, as will be seen from Fig. 16, the bending-moment diagram is composed of sloping straight lines without any steps.

Fig. 16 illustrates how the bending moment may be calculated at critical points, and the bending-moment diagram built up. Since

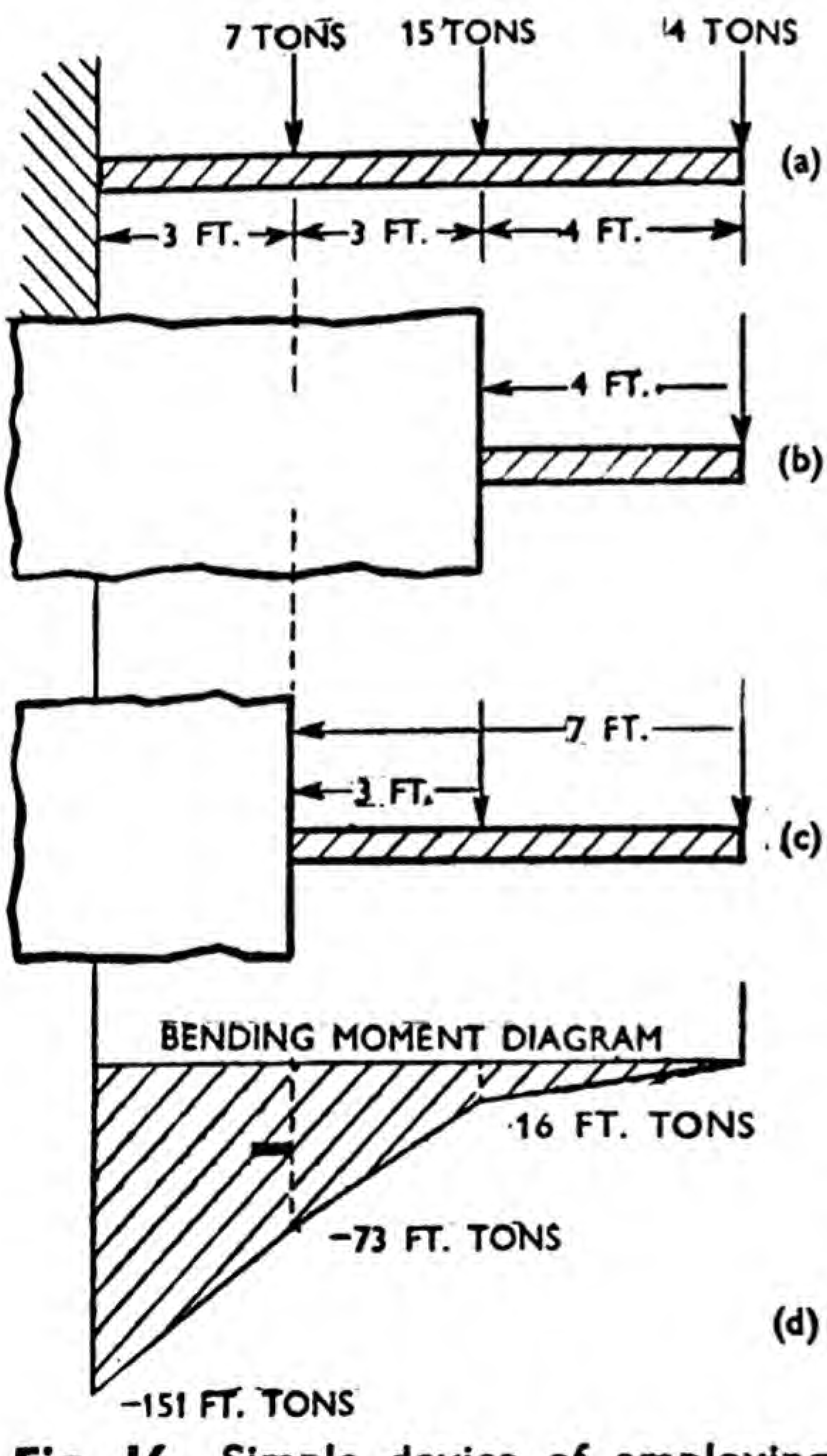


Fig. 16. Simple device of employing an opaque sheet of paper, which was used in discussing shearing force, is again useful. Bending moment at the section of the beam represented by the edge of the paper, is the algebraic sum of the moments of all the forces *which can be seen,* about the edge of the paper.

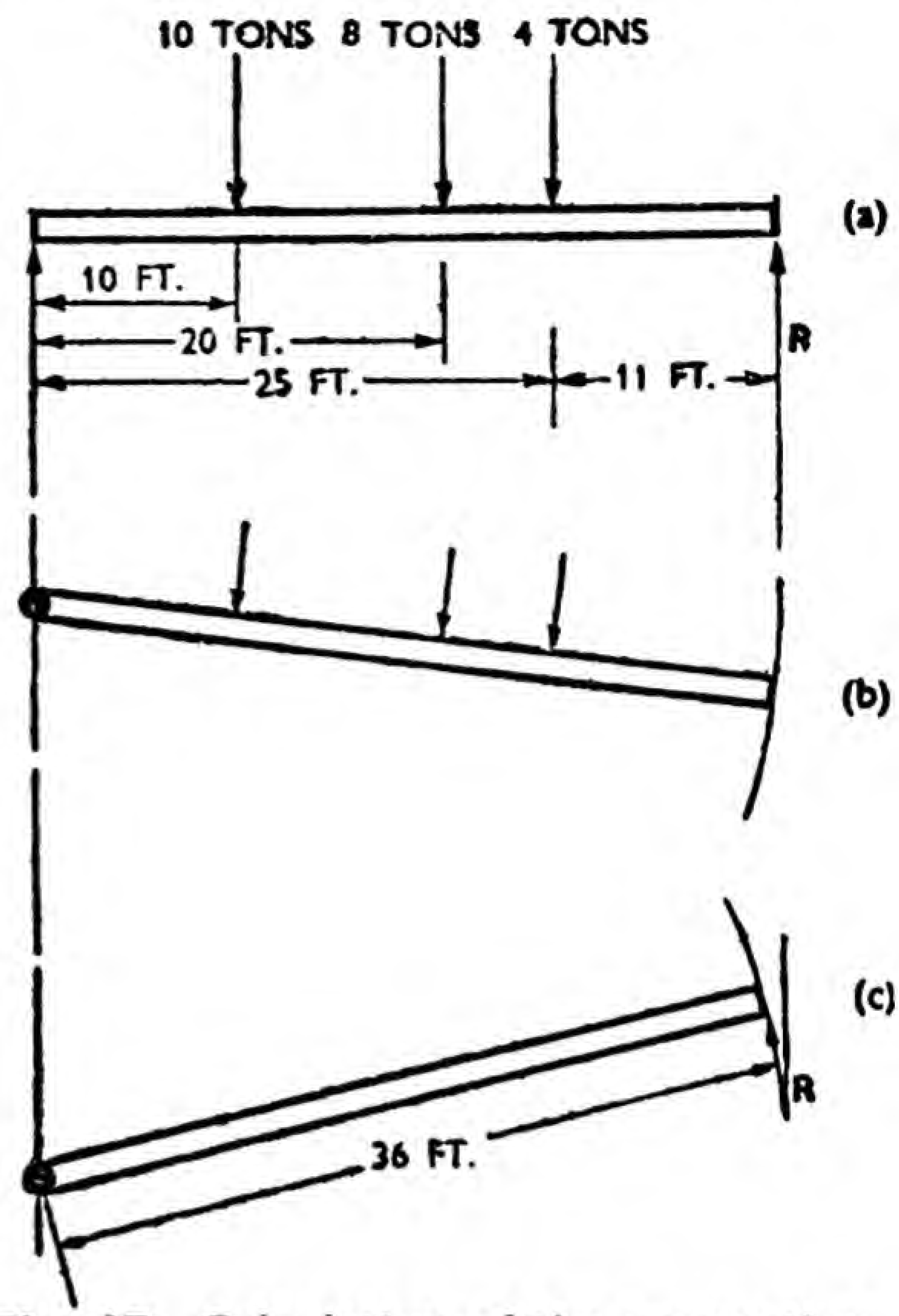


Fig. 17. Calculation of the supporting forces on a beam can be accomplished by imagining one of the supports to be a hinge. The loads then tend to rotate the beam in one direction and the support to rotate it in the opposite sense. These turning tendencies are equal.

the bending moment increases gradually as the paper is pulled back from the end of the beam to the point where the 15-ton load acts, this point is a critical section. Fig. 16(b) indicates that the bending moment there, is :—

$$4 \text{ tons} \times 4 \text{ ft.} = 16 \text{ ft.-tons.}$$

The next critical section is at the 7-ton load, and Fig. 16(c) shows that the bending moment there, is :—

$$4 \text{ tons} \times 7 \text{ ft.} + 15 \text{ tons} \times 3 \text{ ft.}$$
$$= 73 \text{ ft.-tons.}$$

Similarly at the end of the cantilever, the bending moment is found to be 151 ft.-tons. All these bending effects are negative.

When beams supported by simple props are considered (Figs. 2, 9, 10, 11), the problem of first determining the values of the

reactions or supporting forces is met. In Fig. 17, if the left-hand end is imagined as a hinge, and the turning effects of the various forces have been calculated, it is possible to determine the value of the supporting force R at the right-hand end.

Fig. 17(b) shows how the three loads tend to turn the beam about the hinge in a clockwise direction. The magnitude of this turning effect is represented by the moment of all the three forces about the left-hand end of the beam. This moment is :—

$$4 \text{ tons} \times 25 \text{ ft.} + 8 \text{ tons} \times 20 \text{ ft.} + 10 \text{ tons} \times 10 \text{ ft.} = 360 \text{ ft.-tons.}$$

Now the reaction R, if acting by itself, would tend to turn the beam

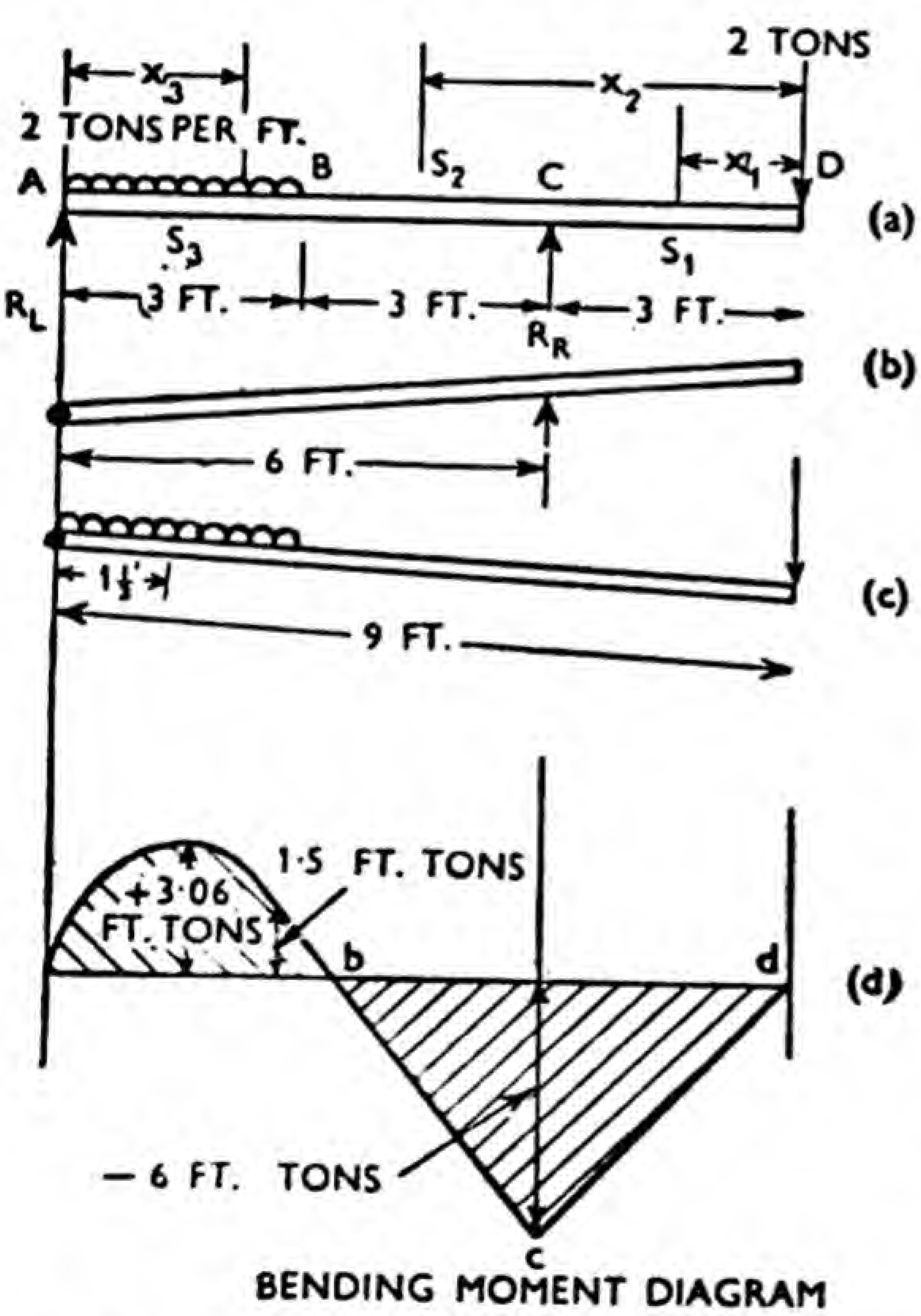


Fig. 18. First step in preparation for drawing the bending-moment diagram for a beam is to determine the supporting forces. The bending moment at any point on the beam is then the sum of the moments of all the forces on one side of the section, about that section. Note the point of contraflexure *b*.

the other way about the hinge as shown in Fig. 17(c). The moment of R about the left-hand end is :—

$$R \text{ tons} \times 36 \text{ ft.} = 36R \text{ ft.-tons.}$$

Value of Reaction

It is known, of course, that there is, in fact, no movement such as that shown in Fig. 17, so this fact can be written down numerically by saying that the turning effect of the loads is exactly equal to the turning effect of the supporting force R, as follows :—

$$360 \text{ ft.-tons} = 36R \text{ ft.-tons,}$$

which is equivalent to stating that R is 10 tons.

This method of finding a reaction, by taking moments of all the forces on the beam about the other reaction, is used for all beams of the types studied here.

In the problem of Fig. 18 it is possible to study all the necessary steps in their correct order.

First of all, it is necessary to find the values of the supporting forces R_L and R_R. This is done, as in the previous example, by considering the left support to be a hinge. R_R is trying to rotate the beam counter-clockwise, and the loading on the beam is trying to rotate it clockwise. These two effects are shown in Figs. 18(b) and 18(c). It is known that there is, in fact, no rotation, and that the moment of the one set of forces must be equal to the moment of the other force (R_R).

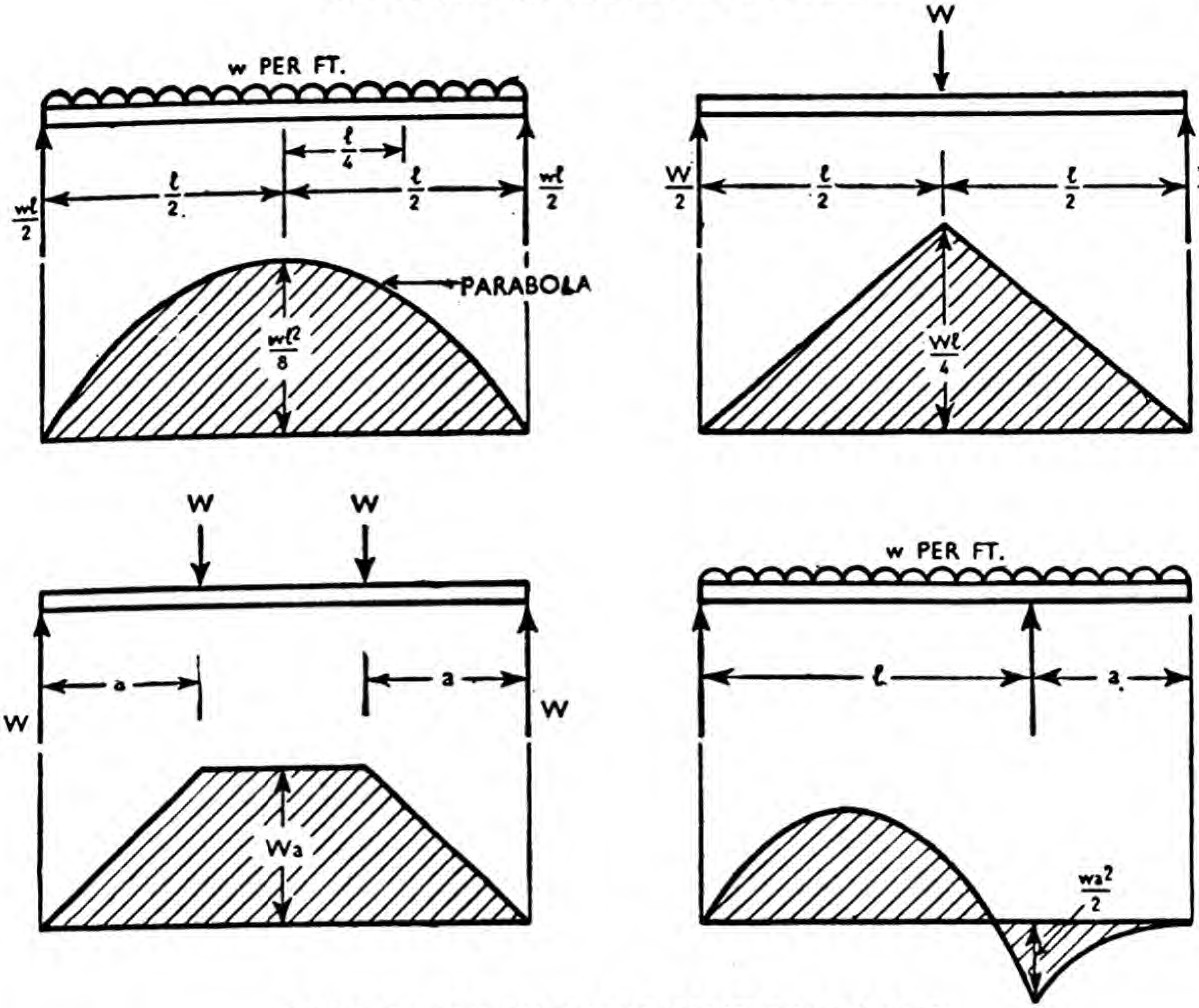
$$6 \text{ tons} \times 1\tfrac{1}{2} \text{ ft.} + 2 \text{ tons} \times 9 \text{ ft.} = R_R \times 6 \text{ ft.}$$

$$\text{or } R_R = 4\tfrac{1}{2} \text{ tons.}$$

R_L must be the difference between the downward loading and the upward force that has just been calculated. Thus :—

$$R_L = 8 - 4\tfrac{1}{2} = 3\tfrac{1}{2} \text{ tons.}$$

Secondly, knowing the down-



TYPICAL BENDING-MOMENT DIAGRAMS

Fig. 19. Certain types of loading are very frequently encountered, and it is convenient to memorize the shapes and dimensions of the appropriate shearing-force and bending-moment diagrams for these loadings. The commonest loading is probably the uniformly distributed load : a beam carrying a floor, for example, would be loaded in this way. Plate girders may carry cross beams which impose concentrated loads, as exemplified in the second and third diagrams.

ward and upward forces on the beam, the bending moment at any point on it may be calculated, and so the bending-moment diagram may be drawn. The beam, in this instance, can be divided into three critical sections, *AB*, *BC* and *CD*. In each of these the form of the bending-moment diagram is different because of the change in the type of loading. Let them be taken in turn.

CD. Cover up the whole of the beam on the left of the section defined by x_1. The only force visible is 2 tons, and its distance from the section (edge of paper) is x_1 ft. The bending moment anywhere in *CD*, therefore, is $-2x_1$ ft.-tons. When x_1 is zero, the bending moment is zero (point *d*) and when x_1 is 3 ft., the bending moment is $-$ 6 ft.-tons (point *c*). A straight line joins these two points.

BC. Cover up the whole of the beam on the left of the section defined by x_2. The only forces visible are 2 tons, and R_B. The moments of these forces about the edge of the paper, which is the section defined by x_2, are the forces multiplied by their respective distances from the section considered.

$$\text{Bending moment at } S_2 = 4\tfrac{1}{2} \text{ tons} \times (x_2 - 3) \text{ ft.} - 2 \text{ tons} \times x_2 \text{ ft.} = (2\tfrac{1}{2}x_2 - 13\tfrac{1}{2}) \text{ ft.-tons.}$$

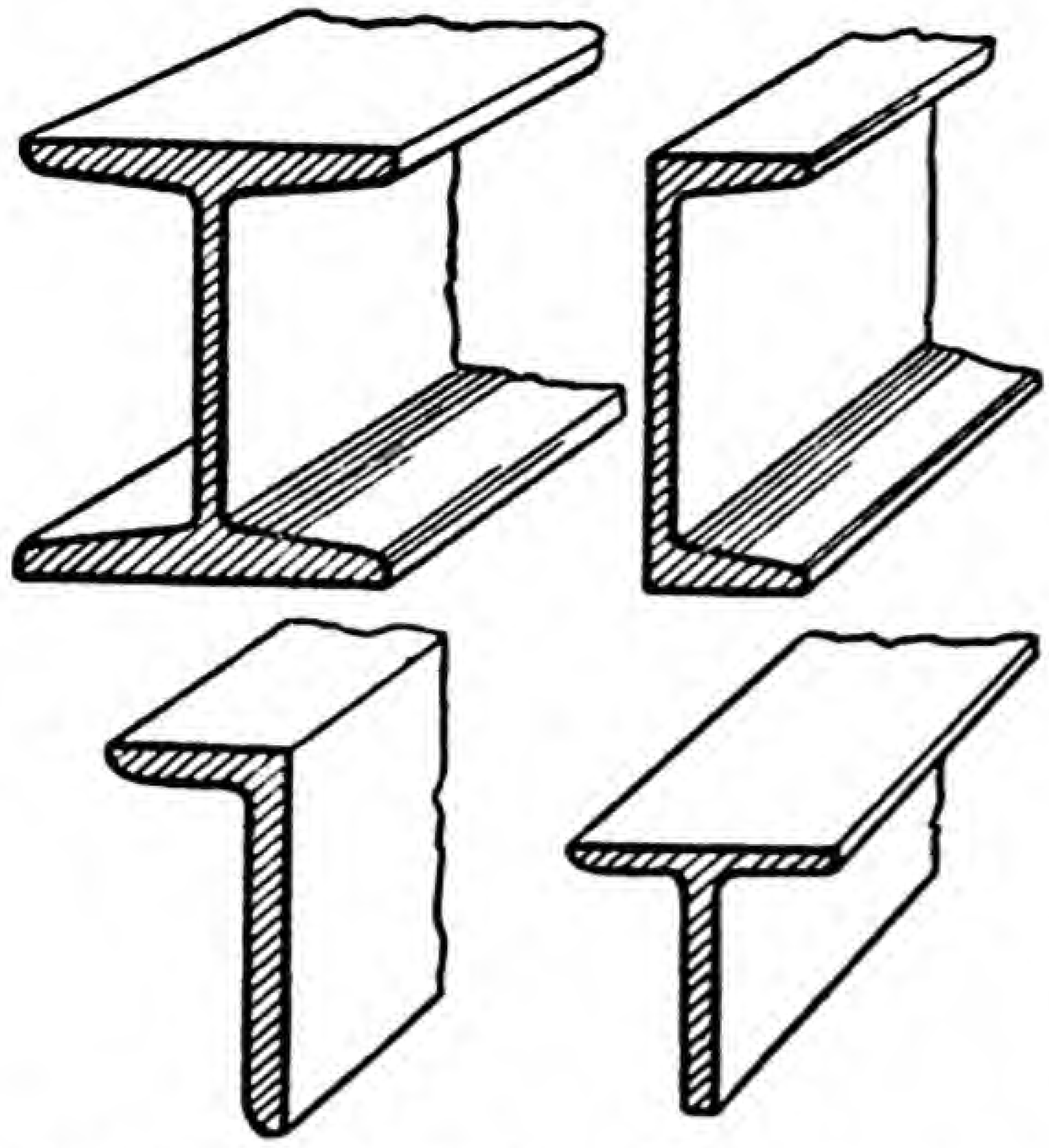


Fig. 20. A rectangular shape of cross-section, such as is used in timber joists, is not the most efficient or economic type of section. By cutting away portions of the rectangular shape, the beam may be made lighter and more efficient.

The value of x_2 can be anything between 3 ft. and 6 ft. If x_2 is 3 ft., the bending moment at S_2, which then becomes C, is $(2\frac{1}{2} \times 3 - 13\frac{1}{2}) = -6$ ft.-tons. This is the same value as was obtained from the first calculations. When x_2 is 6 ft., the bending moment at S_2, now at B, is $+ 1\frac{1}{2}$ ft.-tons (point b).

AB. Cover up the whole of the beam to the right of S_3. The only forces visible are R_L and 2 tons per ft. run on a length of x_3 feet. The moments of these forces must be taken about S_3. The whole of the uniformly distributed load is assumed to act through its centre of gravity, which is midway along x_3.

Bending moment at S_3

$$= R_L \times x_3 - 2x_3 \times \tfrac{1}{2}\, x_3$$
$$= 3\tfrac{1}{2}x_3 - x^2_3 \text{ ft.-tons.}$$

This is the equation of a parabola, and its form is shown in the bending-moment diagram (Fig. 18). The maximum point of such a bending-moment diagram occurs where the shearing force is zero. By using the methods already described for shearing force, it can be shown that the shearing force is zero when $x_3 = 1\frac{3}{4}$ ft. Substituting this value of x_3 in the expression for bending moment :—

Maximum bending-moment at any section between A and B :—

$$3\tfrac{1}{2} \times 1\tfrac{3}{4} - (1\tfrac{3}{4})^2 = 3{\cdot}06 \text{ ft.-tons.}$$

In practice, bending-moment diagrams are often required for quite simple loadings, and Fig. 19 shows some of these loadings and their respective bending-moment diagrams. By using the methods just described, the values of the maximum bending moments of Fig. 19 may be checked.

Internal Effects

So far the external effects of the loading on a structure have been considered. These effects were determined without any knowledge of the internal construction of the structure, or of the material of

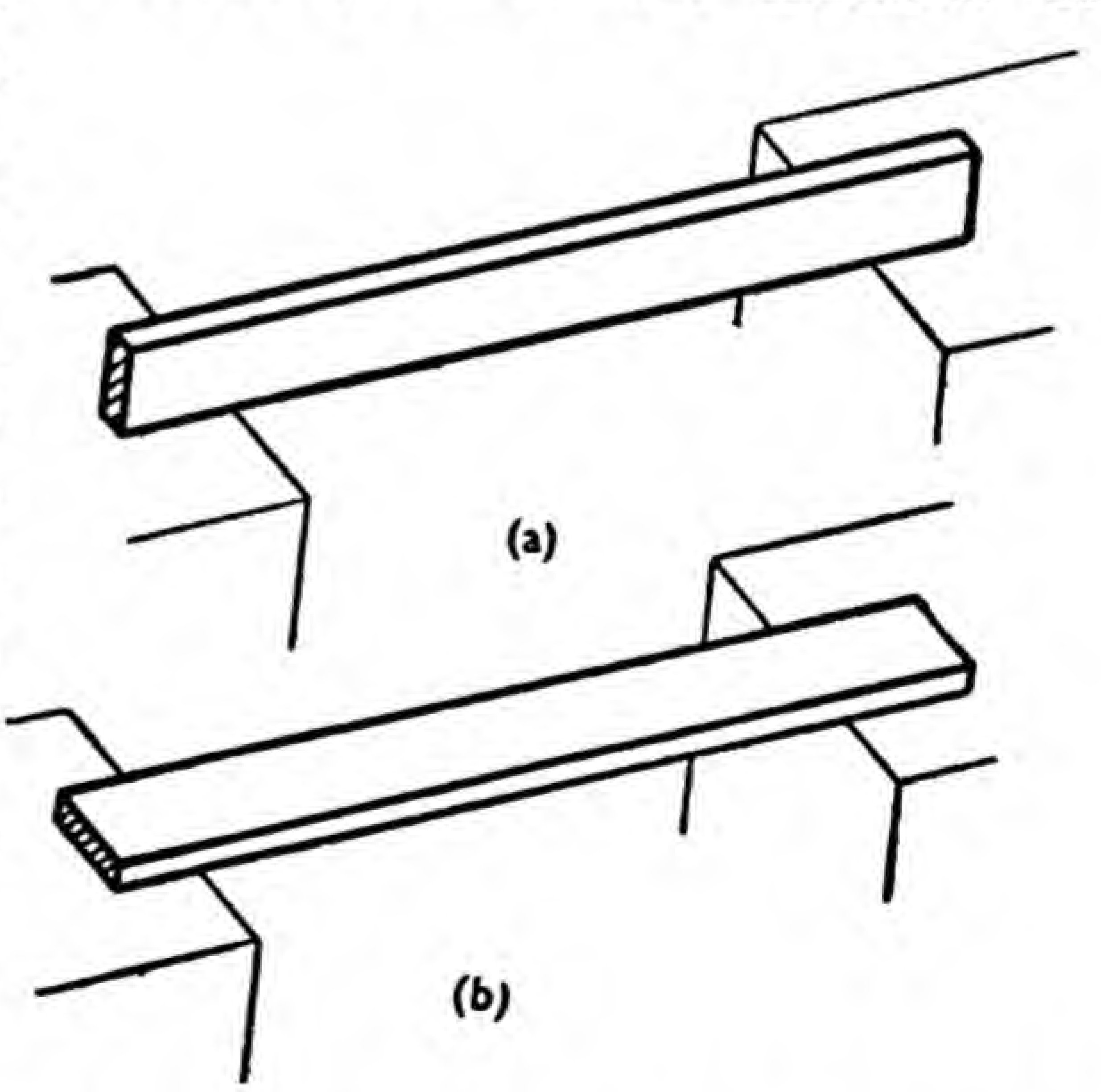


Fig. 21. The flexibility or stiffness of a beam depends on the value of its moment of inertia. If the depth of a joist is doubled, its moment of inertia increases to eight times its previous value and its deflection under load is only one-eighth of what it was before.

which it was composed. The beams were merely represented by straight lines.

When, however, it is required to find out the effects of the loading on the individual members and parts of members, something about their internal construction must be known. When a member is sawn through, the sawn end presents a cross-section of the member. It is on the size and shape of this cross-section that the properties of the member depend.

Area and Moment of Inertia

The first property of the cross-section which must be determined is its cross-sectional area, in other words the number of square inches passed through by the saw. The cross-section may be of a simple shape, such as circular or rectangular, but very often it is complex and may be shaped as shown in Fig. 20. The cross-sectional areas of such standard rolled sections are listed in specially published books of reference.

The second property which must be studied is the moment of inertia of the cross-sectional area. The value of the moment of inertia of a section may be considered to

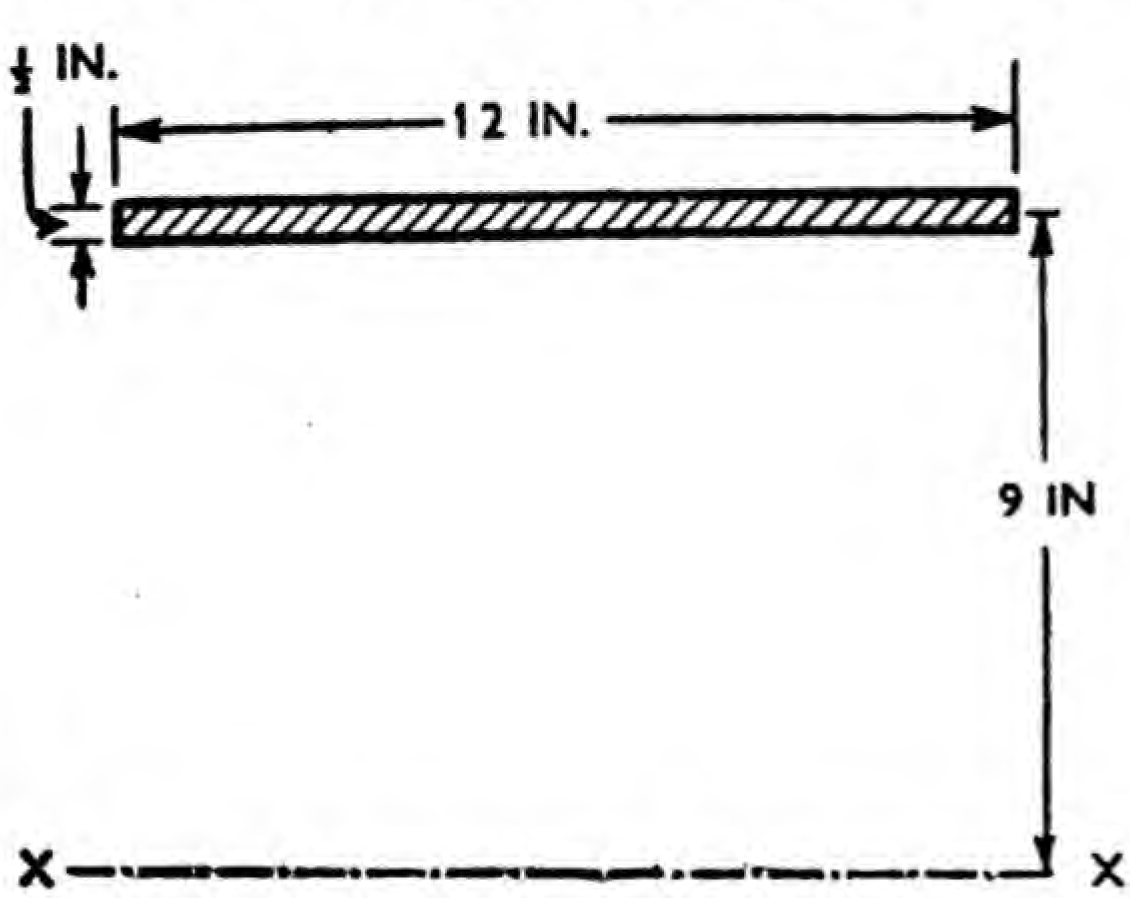


Fig. 22. The moment of inertia of an area about a given axis (*XX*) is the second moment of the area about that axis.

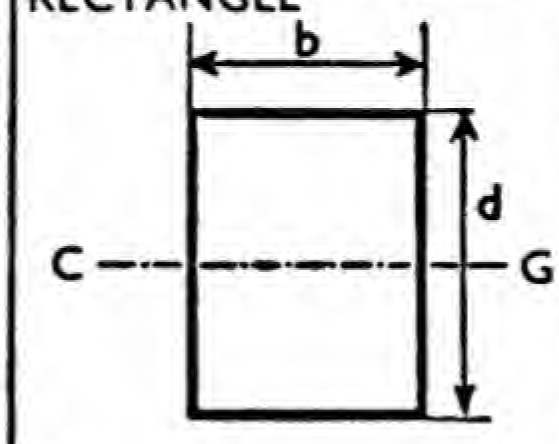
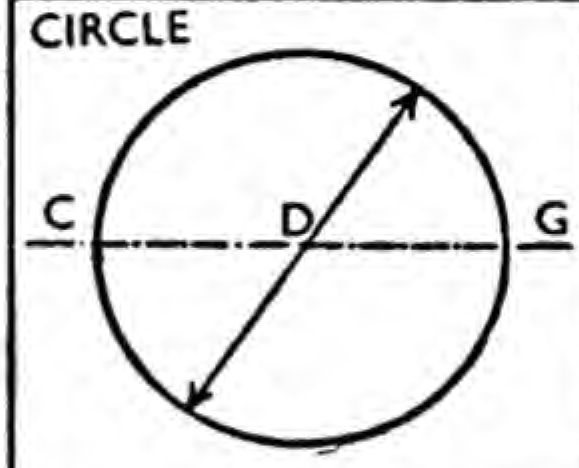
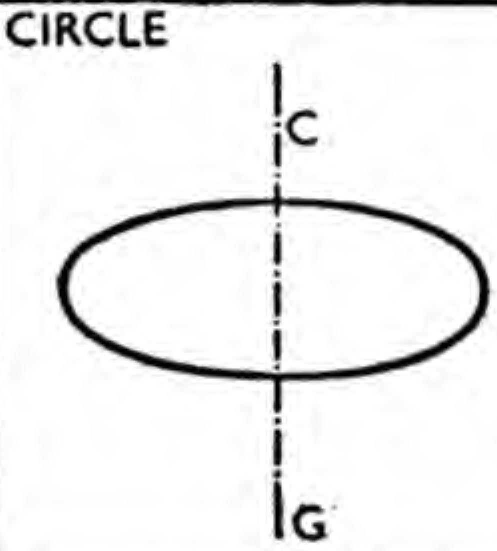
CROSS-SECTION	MOMENT OF INERTIA (I)
RECTANGLE 	$\frac{bd^3}{12}$
CIRCLE 	$\frac{\pi D^4}{64}$
CIRCLE 	$\frac{\pi D^4}{32}$ (POLAR MOMENT OF INERTIA J)

Fig. 23. The moments of inertia of frequently used sections should be memorized. Timber joists have a rectangular section; pulley shafts carrying the loads of driving belts and also transmitting power are subjected to bending ($\pi D^4/64$) and twisting ($\pi D^4/32$).

represent the resistance which that section will offer to bending.

Fig. 21 shows two beams made of rectangular planks of the same shape and size. It is known, however, that plank (a) will be much more resistant to bending than plank (b). The reason lies in the fact that the moment of inertia of the plank in the (a) position is much greater than the moment of inertia of the plank in the (b) position.

The moment of inertia of a section should be considered merely as a useful mathematical tool. It is usually measured in inches to the fourth power (in.^{4}), which has no visual significance.

The moment of inertia of an

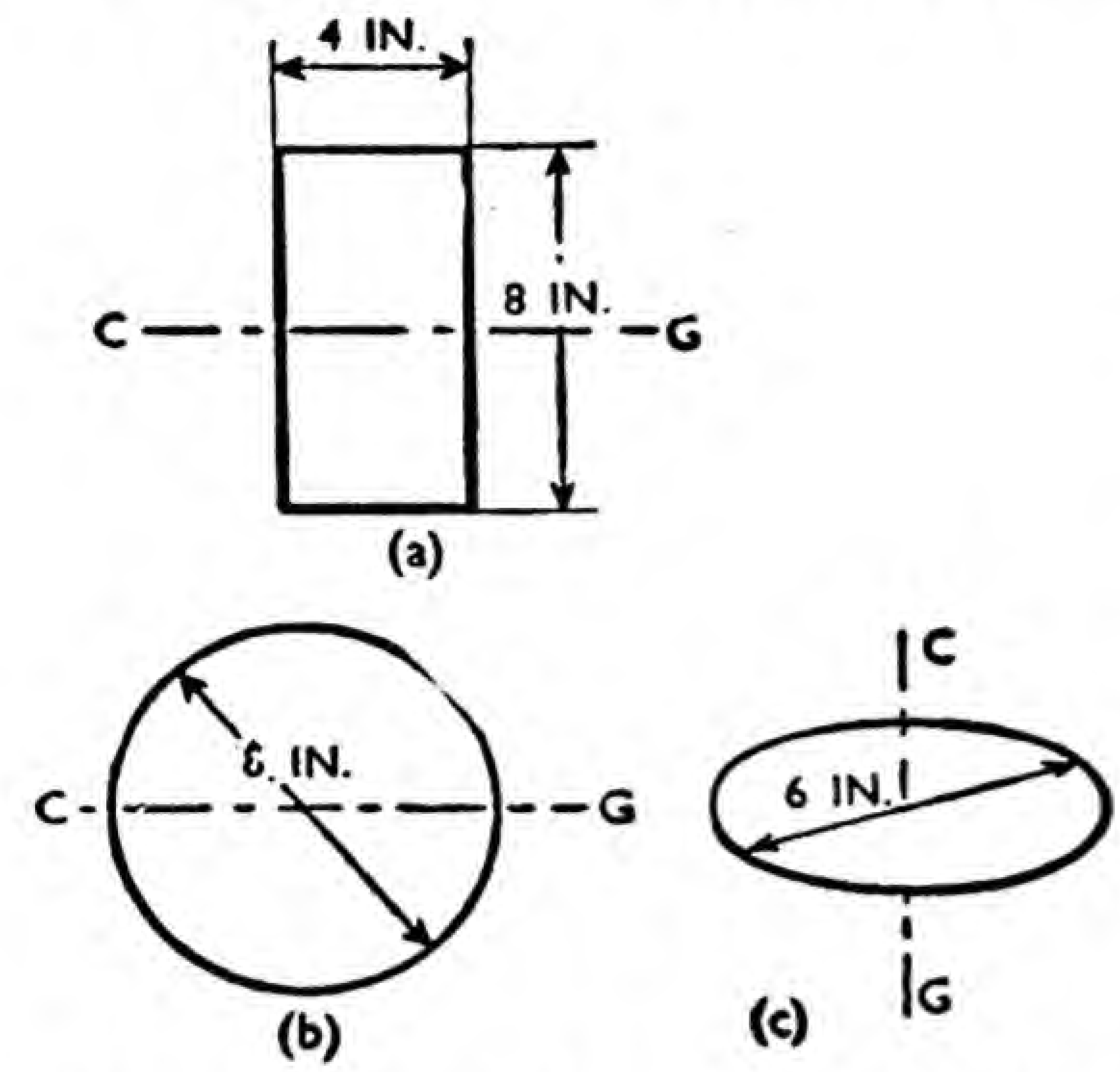


Fig. 24. When the moment of inertia of a section is being calculated as has been done for these three, it is important to decide on the position of the axis about which the bending or twisting is taking place.

area about a given axis, is the second moment of that area about the axis. Note that the axis about which the area is tending to bend or turn must always be defined. Fig. 22 shows an area of 6 sq. in. (in.^{2}) at a distance of 9 in. from the axis *XX*. The moment of the area about that axis is :—

$$6 \text{ in.}^2 \times 9 \text{ in.} = 54 \text{ in.}^3$$

Now if the second moment of the area about the axis is taken, the result is (multiplying twice by 9 in.) :—

$$6 \text{ in.}^2 \times 9 \text{ in.} \times 9 \text{ in.} = 486 \text{ in.}^4$$

This is the moment of inertia of this area about the axis *XX*.

The most important axis about which the moment of inertia of a section must be known is the axis, or axes, passing through its centroid, or centre of gravity. From these fundamental values, the moment of inertia of the area about other axes can be determined.

Fig. 23 shows some simple types of area for which the moment of inertia can be written down in symbols. Let these expressions be used to find the moments of inertia of sample areas. Fig. 24 shows the areas selected.

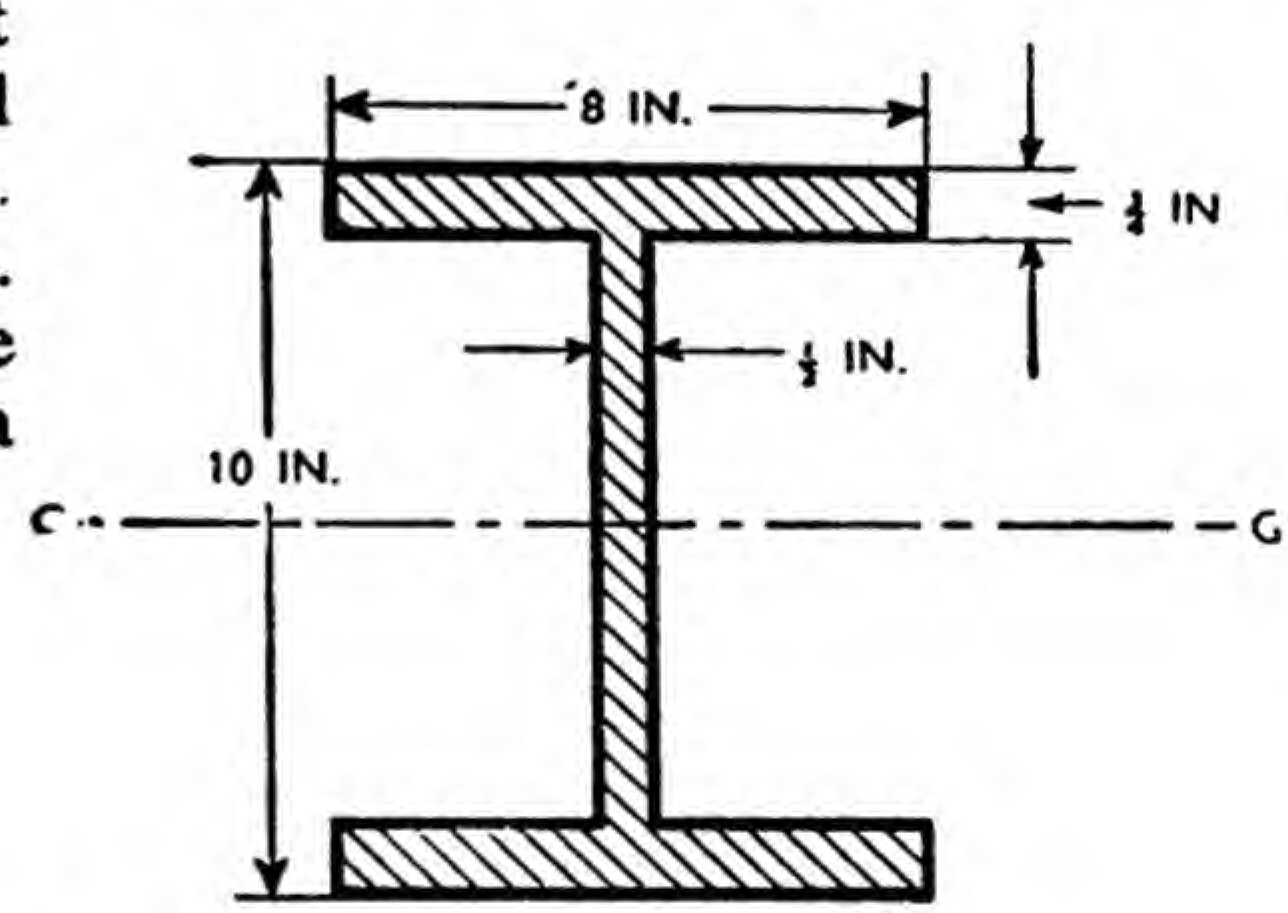
(1) The moment of inertia of the rectangle about an axis through its centre of gravity, and parallel to the 4-in. side, is :—

$$\frac{bd^3}{12} = \frac{4 \text{ in.} \times 8^3 \text{in.}^3}{12} = 170\tfrac{2}{3} \text{ in.}^4.$$

(2) The moment of inertia of the circle about a diameter is :—

$$\frac{\pi d^4}{64} = \frac{\pi \times 8^4 \text{in.}^4}{64} = 201 \text{ in.}^4.$$

(3) The moment of inertia of the circle about an axis perpendicular



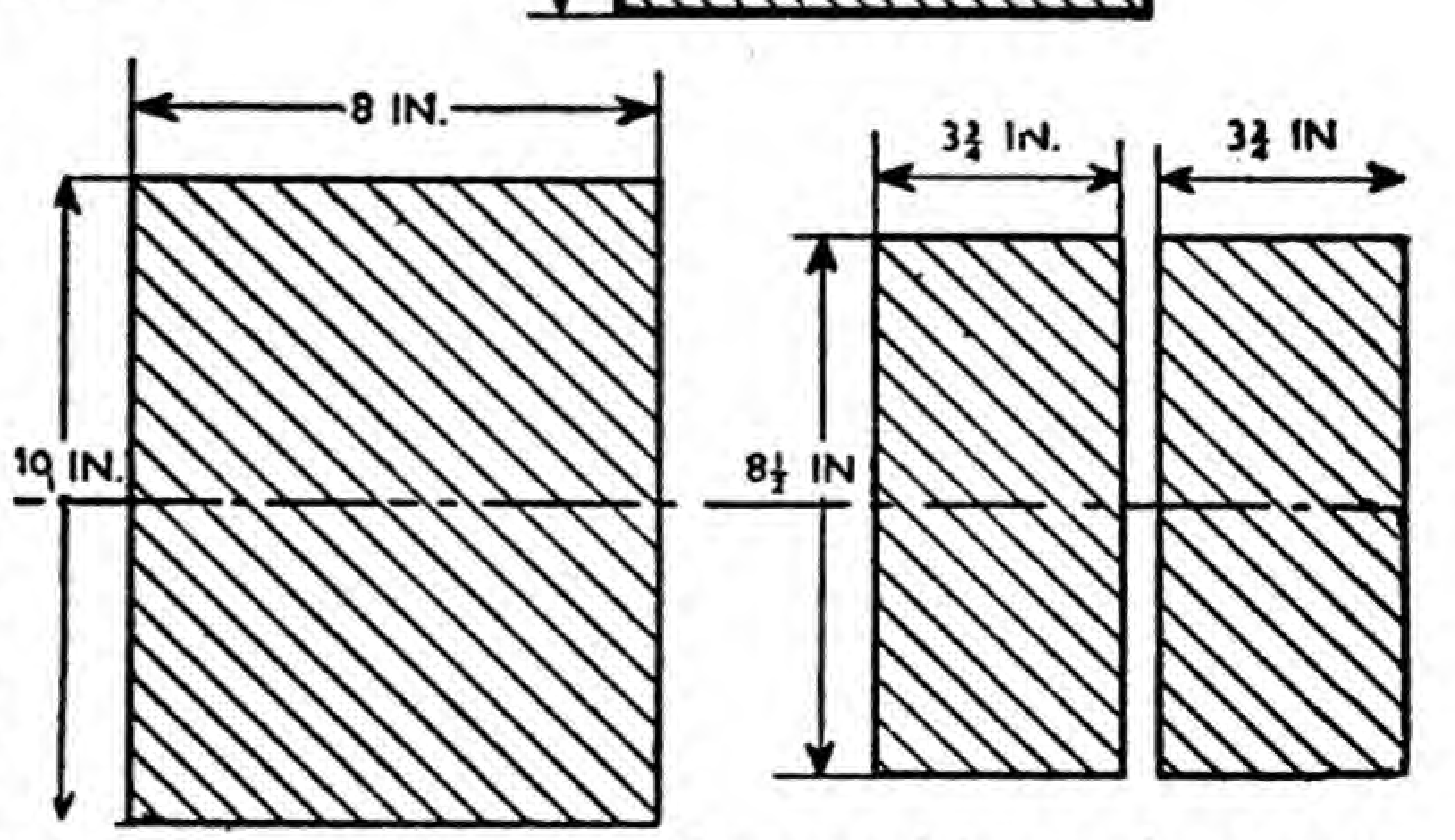


Fig. 25. An I beam is often used in steel structures. In its simplified form (shown here), it is obtained by cutting from a rectangular beam all the dead material which is not acting efficiently in resisting the loads on the beam.

to the plane of the circle and passing through the centre, is known as the polar moment of inertia. It is used when torsion (or twisting) of shafts is considered. The polar moment of inertia is :—

$$\frac{\pi d^4}{32} = \frac{\pi \times 6^4 \text{in.}^4}{32} = 127\tfrac{1}{3} \text{ in.}^4.$$

The Greek letter *pi* (π) which is used in these expressions, merely represents the ratio of the circumference of a circle to its diameter. The value of π is 3·142.

The I Beam

The commonest type of section used in constructing steel beams is the **I** shape, a simplification of which is shown in Fig. 25. To find the moment of inertia of the **I** section, the moment of inertia of the enclosing rectangle may first be found, and from this value is subtracted the moment of inertia of the two rectangles, $8\frac{1}{2}$ in. × $3\frac{3}{4}$ in., which must be removed to form the **I** section.

Moment of inertia of a rectangle, 10 in. × 8 in., about CG :—

$$\frac{8 \times 10^3}{12} = 666\tfrac{2}{3} \text{ in.}^4.$$

Moment of inertia of two rectangles, $8\frac{1}{2}$ in. × $3\frac{3}{4}$ in., about CG :

$$\frac{2 \times 3\frac{3}{4} \times 8\frac{1}{2}^3}{12} = 383{\cdot}8 \text{ in.}^4.$$

Moment of inertia of the **I** section is the difference of these two values :—

$$(666{\cdot}7 - 383{\cdot}8) \text{ in.}^4 = 282{\cdot}9 \text{ in.}^4.$$

Sometimes such **I** beams as are shown in Figs. 20 and 25 are not sufficiently strong to carry the imposed loading. In such a case, the moment of inertia may be increased by riveting flat plates to their upper and lower flanges.

Fig. 26 shows a compound beam of this type. Its moment of

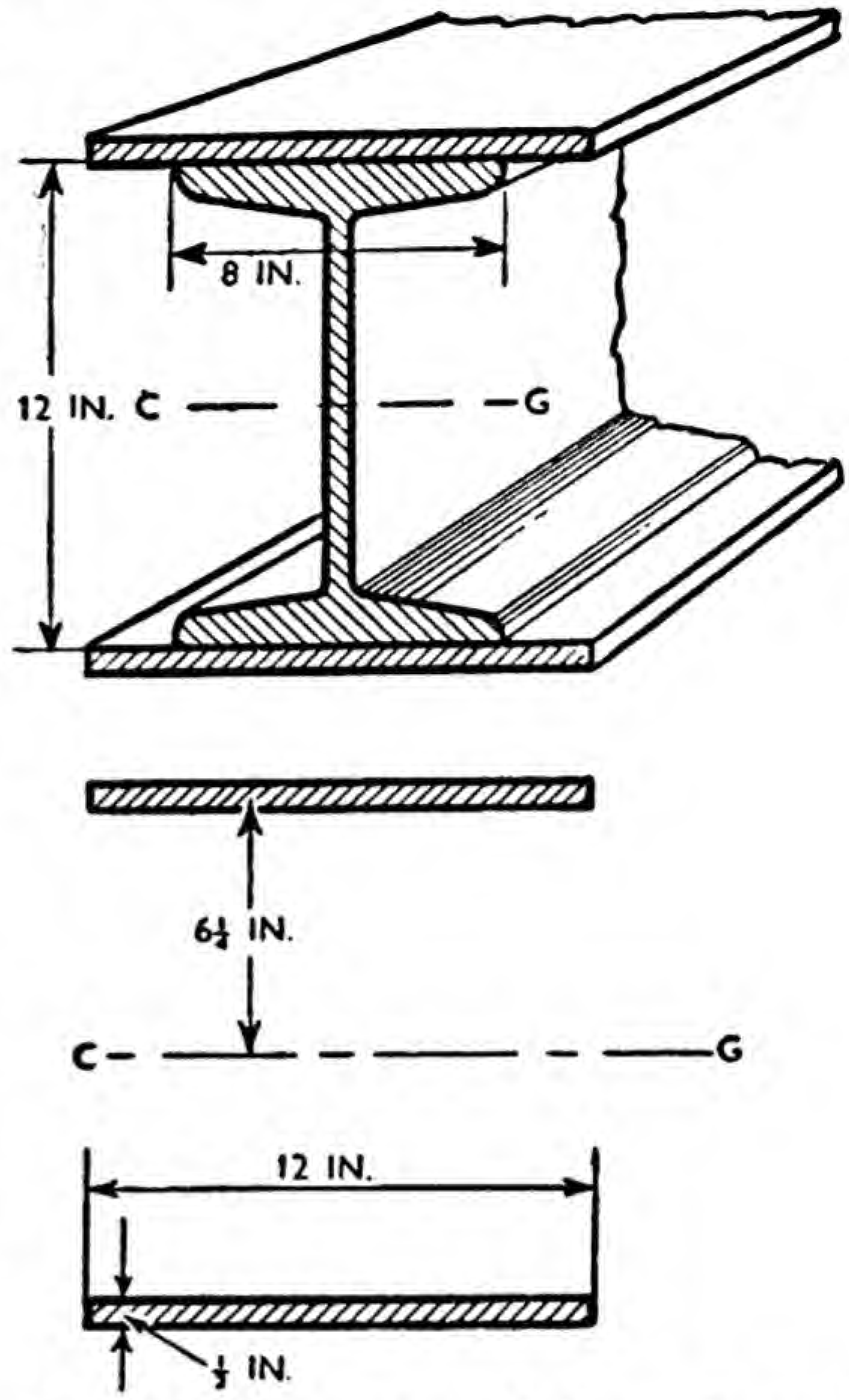


Fig. 26. A compound beam is more resistant to bending than a simple **I** beam. Its moment of inertia is obtained by adding the moment of inertia of the **I** beam to that of the plates (as calculated in Fig. 22).

inertia about CG consists of the moment of inertia of the **I** beam plus the moment of inertia of the added plates. Moments of inertia of standard **I** beams have been calculated and tabulated. It only remains to calculate the moment of inertia of the plates about CG. The moment of inertia required is the second moment of the area of the plates about CG, as was shown in Fig. 22 and, therefore, the final calculation becomes :—

Moment of inertia of **I** beam (from tables) :—437·3 in.^{4}

Moment of inertia of plates about CG :—

$2 \times$ area of plate $\times 6{\cdot}25^2$

$= 12 \times 6{\cdot}25^2$

$= 468{\cdot}8$ in.^{4}.

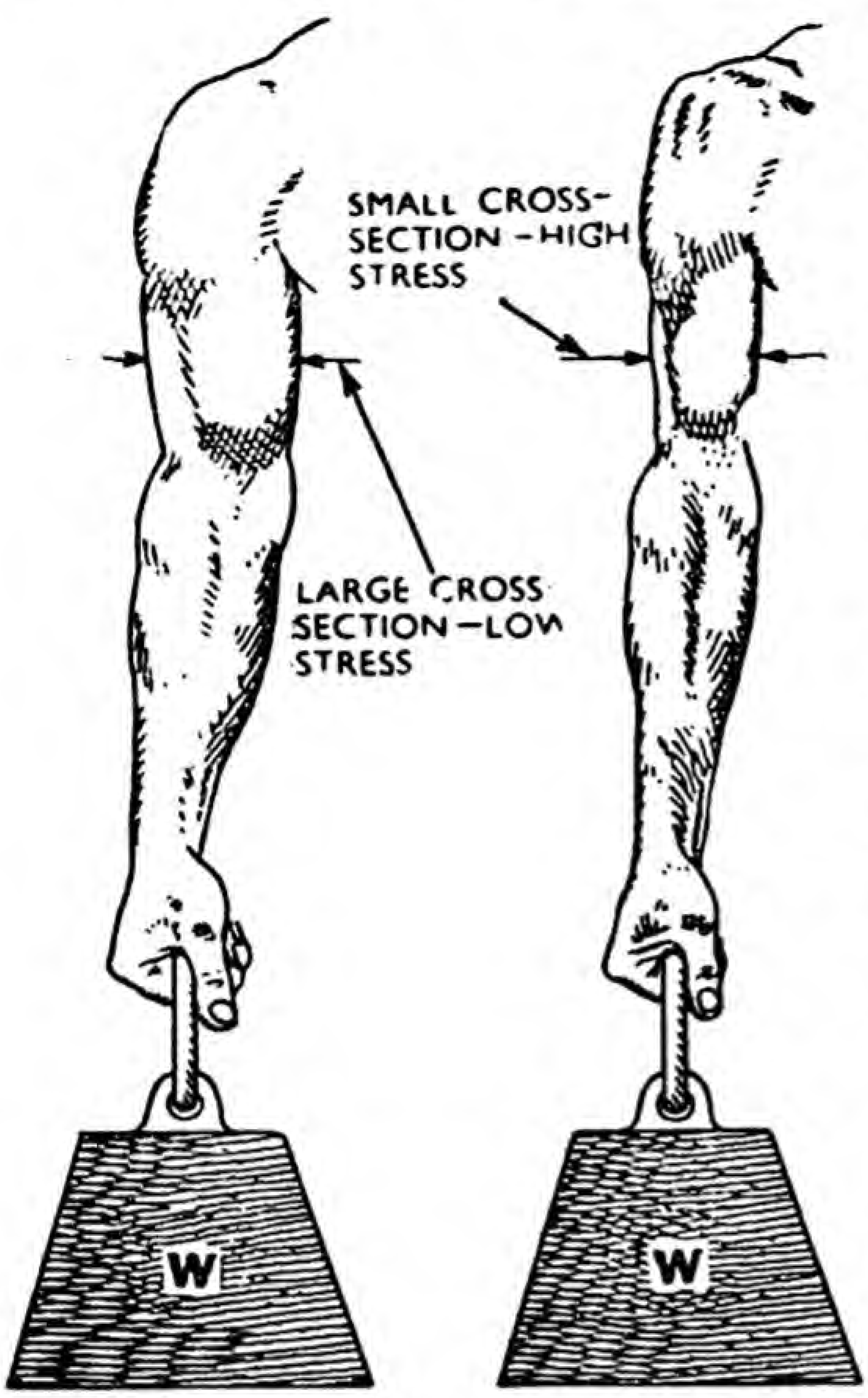


Fig. 27. The total load carried by any structure, or member of a structure, is not the sole indication of the internal stress to which the material is subjected. The intensity of stress in a member carrying a load in tension is measured by dividing the load by the cross-sectional area of the member.

Total moment of inertia of compound beam :—

$(437{\cdot}3 + 468{\cdot}8)$ in.$^4 = 906{\cdot}1$ in.^{4}.

Having discussed the properties of the cross-section of a member, it must now be considered how these properties are used to help in determining the internal effect of external loads. The cross-sectional area is used in determining stress. When, for example, a load or force of X tons is acting on a member of a pin-jointed frame, it is not certain whether this is a safe load until the *intensity* of loading on the cross-section has been discovered.

Fig. 27 shows that a load of W lb. may be taken up either by a member of large cross-sectional area, or by a member of small cross-sectional area. The intensity of load in the first instance is much smaller than in the second. The man with big arm muscles feels less stress than the man with muscles of smaller cross-sectional area.

Intensity of Stress

The intensity of load, then, is called stress and is found by dividing the load on the member by the number of square inches, or square feet, in the cross-section. The result is in tons per sq. in., lb. per sq. ft., etc., giving the load which must be carried by each unit of area. Whether such a stress is a safe one depends upon the material being used.

Direct stress may be either a pull

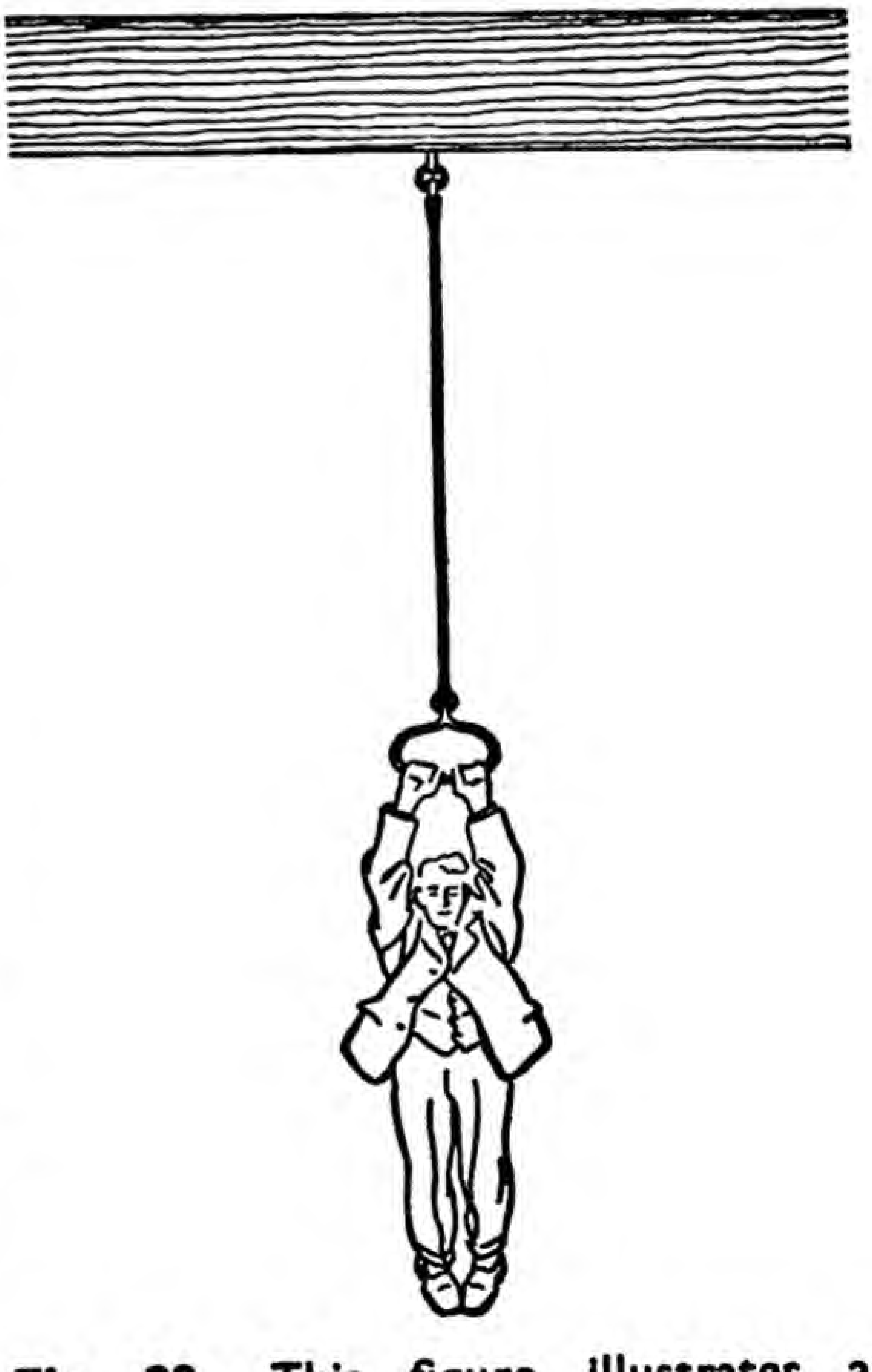


Fig. 28. This figure illustrates a numerical example of direct tensile stress. The man weighs only 16 stone but the stress on the wire is about eight tons per sq. in. because the cross-sectional area of the wire is so small.

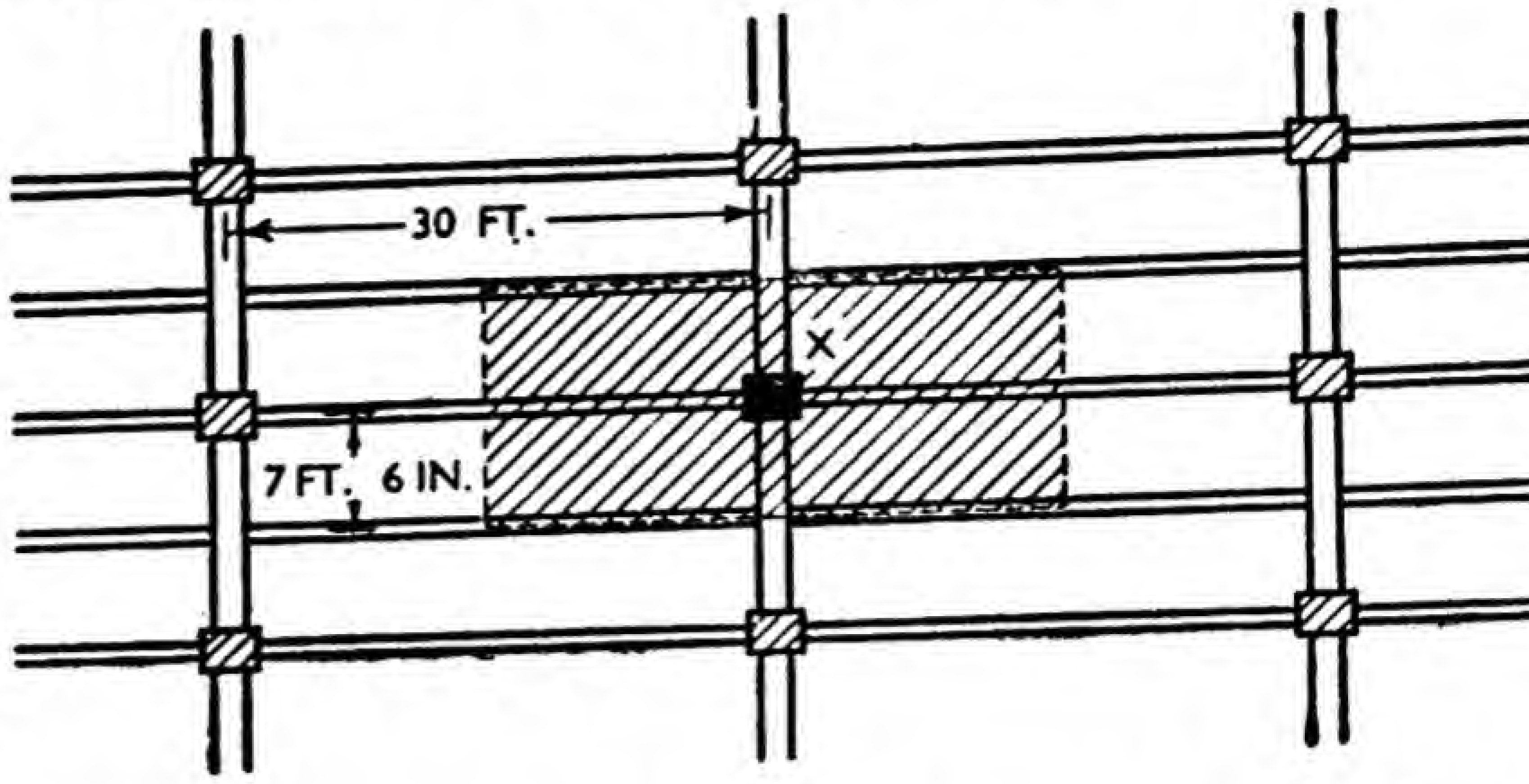


Fig. 29. Load on a reinforced concrete floor must be transferred, step by step, to the ground. First, the slab passes the load to secondary beams and these, in turn, rest on main beams and columns. The area of floor which can be assumed to be carried by one column is shown shaded.

(tension) or a push (compression), and in problems concerning this type of stress, only two quantities need be found. The first is the load and the second is the cross-sectional area. It is important that the words stress and strain should not be confused. In everyday conversation, the two are often used synonymously, but in mechanics they signify entirely different quantities, as will be shown later.

Fig. 28 shows a 16-stone man hanging on a wire $\frac{1}{8}$ in. in diameter. If the ultimate strength of the steel is 30 tons per sq. in., will the wire break?

The load must first be found, which, in this instance, is the weight of the man. 16 stones $= 16 \times 14$ lb. $= \frac{16 \times 14}{2240}$ tons, or 0·1 ton. The area of the wire in cross-section must then be determined. The cross-section is a circle, $\frac{1}{8}$ in. in diameter, which represents 0·0123 sq. in. The applied stress caused by the weight of the man acting on the wire is the load divided by the cross-sectional area :—

$$= \frac{0{\cdot}1 \text{ ton}}{0{\cdot}0123 \text{ sq. in.}} = 8{\cdot}1 \text{ tons per sq. in.}$$

So far, it is assumed that the wire does not break. It is only at this point that the material of which the wire is made need be considered. It is obvious that the stress imposed on the wire must be less than the breaking strength of the steel. The behaviour of steel under tension is explained in Chapter 9.

Typical Example

Fig. 29 shows the floor of a factory building with its various beams and columns as viewed from above. The columns are all square in cross-section and may carry a safe stress of 600 lb. per sq. in. What size of column is required?

First of all, the area of floor supported by each column must be found, and, if each is to do its share in carrying the load, it can be seen from the diagram that each column carries an area of floor equal in shape and size to the shaded area carried by column *X*. This area is 30 ft. × 15 ft., or 450 sq. ft. Suppose the machinery carried by the floor has an average weight of 100 lb. per sq. ft. of floor area, and the weight of the beams and the floor itself (dead load) is about 50 lb. per sq. ft. of floor area, the following calculation for the total load carried by the shaded area of floor is obtained :—

Total load, 150 lb. per ft.^{2} × 450 ft.^{2} = 67,500 lb.

The allowable stress on the

column X is 600 lb. per sq. in. Remembering that load divided by area is equal to stress, the cross-sectional area of the column can be found from the statement :—

$$\frac{67{,}500 \text{ lb.}}{\text{sq. in. of cross-section}} = 600 \text{ lb. per in.}^2$$

Cross-sectional area =

$$\frac{67{,}500 \text{ lb.}}{600 \text{ lb. per in.}^2} = 112{\cdot}5 \text{ sq. in.}$$

A column 10-in. square would have a cross-sectional area of 100 sq. in., and a column 11 in. square would have 121 sq. in. cross-sectional area. The required size is thus between 10 and 11 inches square and is, in fact, just over 10·6 inches square. In practice, it would probably be made 11 inches square.

Linear Strain

The effect of a tensile or compressive force, i.e., a direct force, is to cause a change in length of the member carrying the load. This change is extremely small, and can be measured only by sensitive instruments. However minute as it is at working loads, this deformation is extremely important.

Just as stress has been defined as the load per unit of area, strain is defined as the deformation per unit of length. Deformation under tensile or compressive loads is often called linear strain, and is measured as inches per inch, or feet per foot. Strain is thus merely a ratio. It can be said that for a stress of p lb. per sq. in. the member suffers a strain of one-millionth, meaning that the length is altered by one-millionth of its original value.

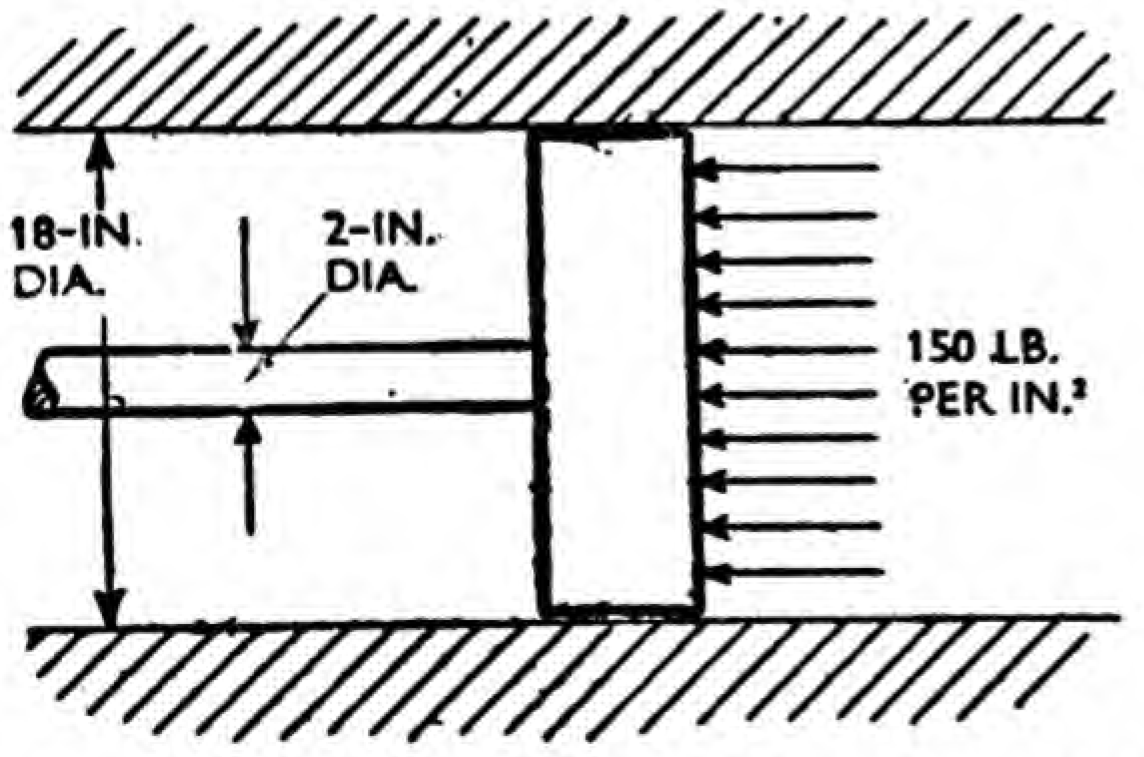


Fig. 30. This problem illustrates the relationship between stress and strain in the piston rod of a steam engine. The force is calculated from the steam pressure and the piston area, and the strain in the piston rod can then be found.

For the more important materials used in construction, it has been found that strain is proportional to stress. Thus, if the load per unit area on a section is doubled, the strain is doubled. If the strain is reduced to one-third, the stress, or load per unit area, is reduced to one-third of its former value. The ratio between direct stress and linear strain is known as a modulus, usually called the modulus of elasticity, or Young's modulus.

This modulus of elasticity may be considered to be constant for any one material, and thus it is possible to estimate in advance how much this material will alter in length when subjected to a given stress.

The relationship between the three quantities is :—

$$\frac{\text{Stress}}{\text{Strain}} = \text{Young's modulus (or modulus of elasticity).}$$

The symbol used for Young's modulus is E, and it is measured in the same units as stress, tons per sq. in.; lb. per sq. ft., etc.

The steam-engine piston shown in Fig. 30 has a piston rod 4 ft. long. The face of the piston is subjected to a pressure of 150 lb. per sq. in. What change will occur in the length of the piston rod?

The first thing to determine is the force exerted by the steam. The 150 lb. per sq. in. is a stress. This

stress, multiplied by the area of the piston, gives the total thrust exerted by the steam.

$$\text{Area of piston} = \frac{\pi \times 18^2}{4}$$
$$= 254{\cdot}5 \text{ sq. in.}$$

$$\text{Total thrust} = 150 \times 254{\cdot}5$$
$$= 38{,}175 \text{ lb.} = 17{\cdot}04 \text{ tons.}$$

This thrust is carried by the piston rod. The stress on the cross-section of the piston rod is, therefore :—

$$\frac{\text{Total thrust}}{\text{Area of rod}} = \frac{17{\cdot}04}{\pi}$$
$$= 5{\cdot}42 \text{ tons per sq. in.}$$

If the rod is made of steel, as it is normally, it is found that :—

Stress = Strain × Modulus of elasticity for steel.

The modulus of elasticity for steel is 30×10^6 lb. per sq. in. or 13,500 tons per sq. in. It is found then :—

$$\text{Strain} = \frac{\text{Stress}}{E} = \frac{5{\cdot}42}{13{,}500}$$
$$= 0{\cdot}0004017.$$

Strain is the elongation per unit of length, so the rod changes in length by 48 in. × 0·0004017, or 0·019 in.

Compound Material

Quite frequently two materials may be required to act together as component parts of one object. The behaviour of combined bars of this kind is not quite so simple as is that of the steel piston rod in the last example.

Fig. 31 shows two different materials riveted together into one bar. If a tensile load is applied to that bar, it is obvious that the stretch of both materials must be the same, since they are riveted together. Suppose the amount of stretch is a length *d*, as shown in Fig. 31. Remembering that stress is proportional to strain, and that strain is the deformation per unit length, separate relationships for brass and for steel may be written down as follows :—

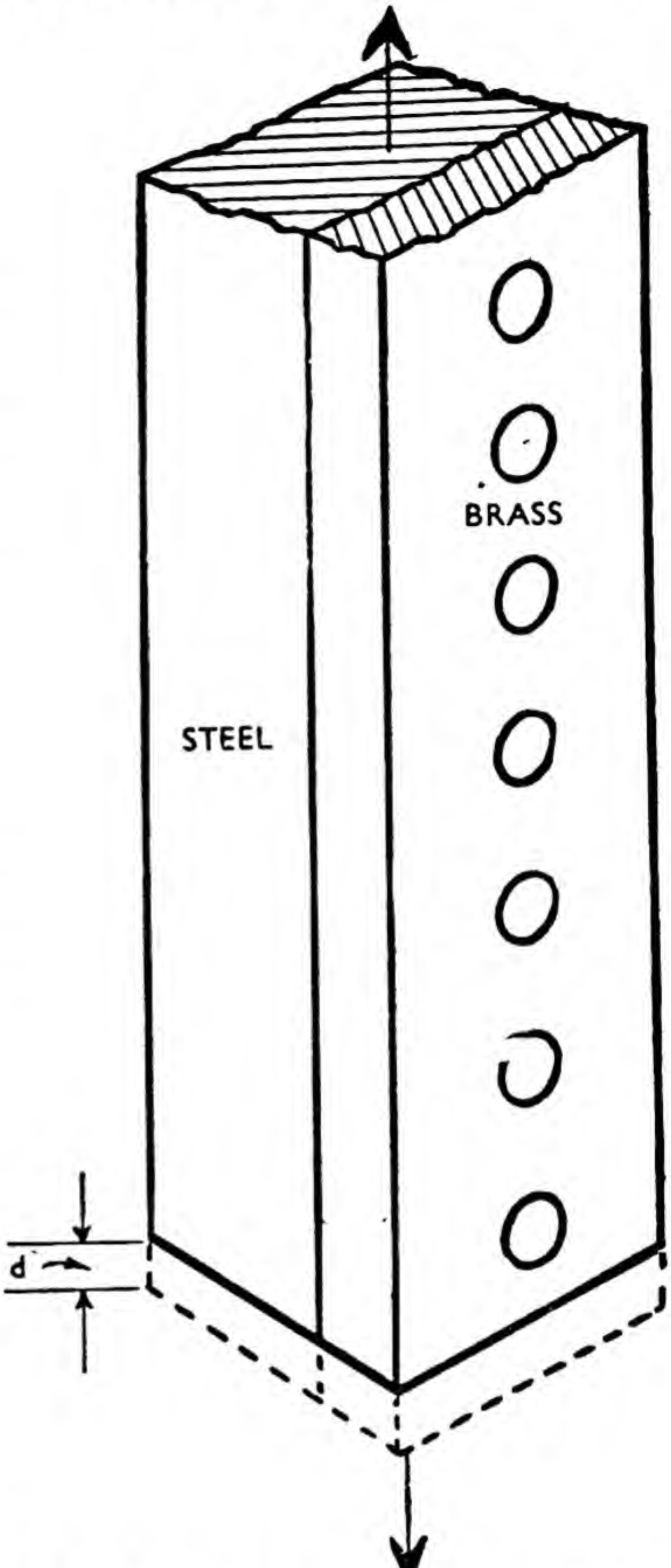


Fig. 31. When two different materials carry a load jointly, the stress in each of the materials is in proportion to its modulus of elasticity. Here, the steel is subjected to a higher stress than the brass.

$$\frac{\text{Stress}}{\text{Strain}} = \frac{f_{\text{brass}}}{e_{\text{brass}}} = E_{\text{brass}}$$

$$\therefore e_{\text{brass}} = \frac{f_{\text{brass}}}{E_{\text{brass}}}$$

$$\frac{\text{Stress}}{\text{Strain}} = \frac{f_{\text{steel}}}{e_{\text{steel}}} = E_{\text{steel}}$$

$$\therefore e_{\text{steel}} = \frac{f_{\text{steel}}}{E_{\text{steel}}}.$$

Since the elongation of the steel rod equals the elongation of the brass rod, and the lengths of the two rods are equal, then the strains of the two materials are equal, or

$$e_{\text{brass}} = e_{\text{steel}}.$$

If this is true, then from the relationships written down above:

$$\frac{f_{\text{brass}}}{E_{\text{brass}}} = \frac{f_{\text{steel}}}{E_{\text{steel}}} \text{ or } \frac{f_{\text{brass}}}{f_{\text{steel}}} = \frac{E_{\text{brass}}}{E_{\text{steel}}}.$$

Putting this into words, it may be said that the stresses in a bar of two or more materials, rigidly connected together, are in the same ratio as their moduli of elasticity. Since the values of the moduli of elasticity for materials used in construction are known, it is, therefore, possible to calculate the ratio of the stresses in the materials used.

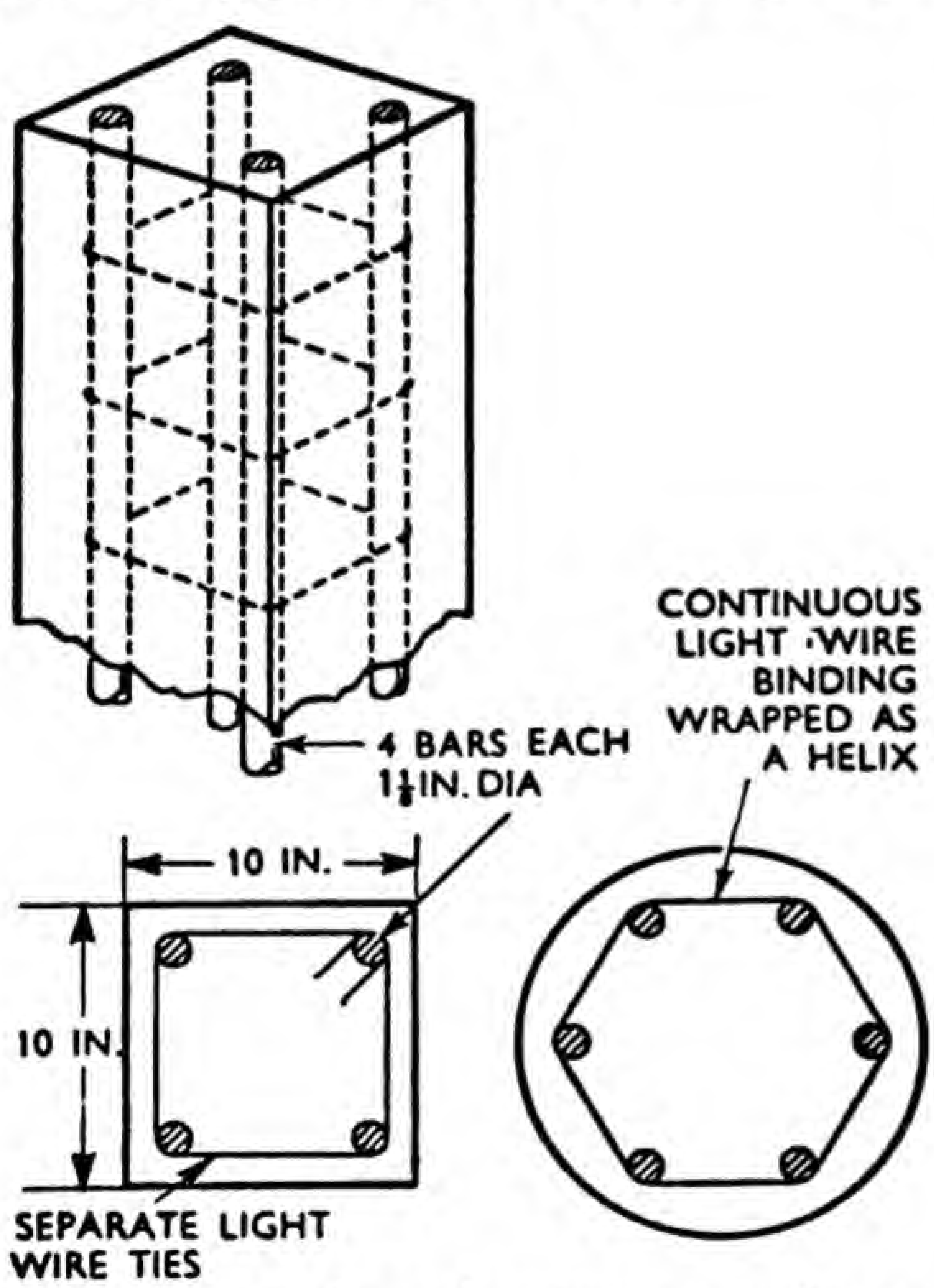


Fig. 32. Reinforced concrete columns show how two different materials can act together. The vertical steel rods and the surrounding concrete share the compressive load. The vertical rods may be lightly held in place by wire stirrups, or bound by a continuous helix of wire if the column is of circular or octagonal section.

An important application of this principle in the erection of buildings is in the calculation of the stresses in reinforced concrete columns. Here there is a combination of concrete and steel, the steel bars being arranged in the manner shown in Fig. 32.

Reinforced Concrete Column

Suppose a square concrete column is reinforced with four longitudinal bars (Fig. 32) each $1\frac{1}{8}$ in. in diameter, and the stress in the concrete must not exceed 600 lb. per sq. in. compression. What load can the column support?

The modulus of elasticity for steel is 30×10^6 lb. per sq. in., and that for concrete approximately 2×10^6 lb. per sq. in. The ratio :—

$$\frac{E_{\text{steel}}}{E_{\text{concrete}}} = \frac{30 \times 10^6}{2 \times 10^6} = 15.$$

Remembering that the ratio of the stresses in a bar of two different materials is the same as that of the moduli of elasticity, the modular ratio, 15 in this instance, is also the ratio of the stresses in steel and concrete.

The stress in the steel, therefore, is $15 \times$ stress in the concrete :—

$15 \times 600 = 9{,}000$ lb. per sq. in.

The area of the steel is that of 4 circular rods, each $1\frac{1}{8}$ in. in diameter $= 4 \times 0{\cdot}994 = 3{\cdot}98$ sq. in.

The total area of the column is 100 sq. in.

The area of the concrete is $100 - 3{\cdot}98 = 96{\cdot}02$ sq. in.

The load on the column when the concrete stress is 600 lb. per sq. in. :—

Stress in concrete $\times$ Area of

concrete + Stress in steel × Area of steel
$= 600 \times 96{\cdot}02 + 9{,}000 \times 3{\cdot}98$
$= 93{,}432$ lb. $= 41{\cdot}7$ tons,
which is the total load the column can safely carry.

Temperature Stress

When any material is heated it expands. Metal bars are liable to lengthen under the heat of the sun, or when a building catches fire, and precautions must be adopted to prevent damage to any part of the structure which tends to restrict expansion. To study the forces exerted by bars under the influence of a rise in temperature, the extent of the elongation which would normally take place if the heated bar were free to take up its new length without restriction must first be known.

This elongation is measured in terms of a figure known as the coefficient of expansion, which is the expansion in inches per inch for each degree of rise in temperature. To find the total elongation of the bar, this coefficient must be multiplied by the number of degrees of rise in temperature, and also by the length of the bar. Table I gives some of the more useful coefficients of expansion.

TABLE I

Coefficients of Linear Expansion for various Materials per 1 deg. F.

Material	Coefficient
Brass	0·0000104
Cast Iron	0·0000059
Steel	0·0000063
Brick	0·0000031
Concrete	0·0000080
Timber	about 0·0000030

A standard 18 in. × 6 in. **I** beam, (cross-sectional area, 16·18 sq. in.) of the type shown in Figs. 20 and 26 is built in between two walls 30 ft. apart, as shown in Fig. 33. If the building catches fire, and the temperature of the beam rises 150 deg. F., what outward force does the beam exert on the walls if they each yield $\frac{1}{16}$ in.?

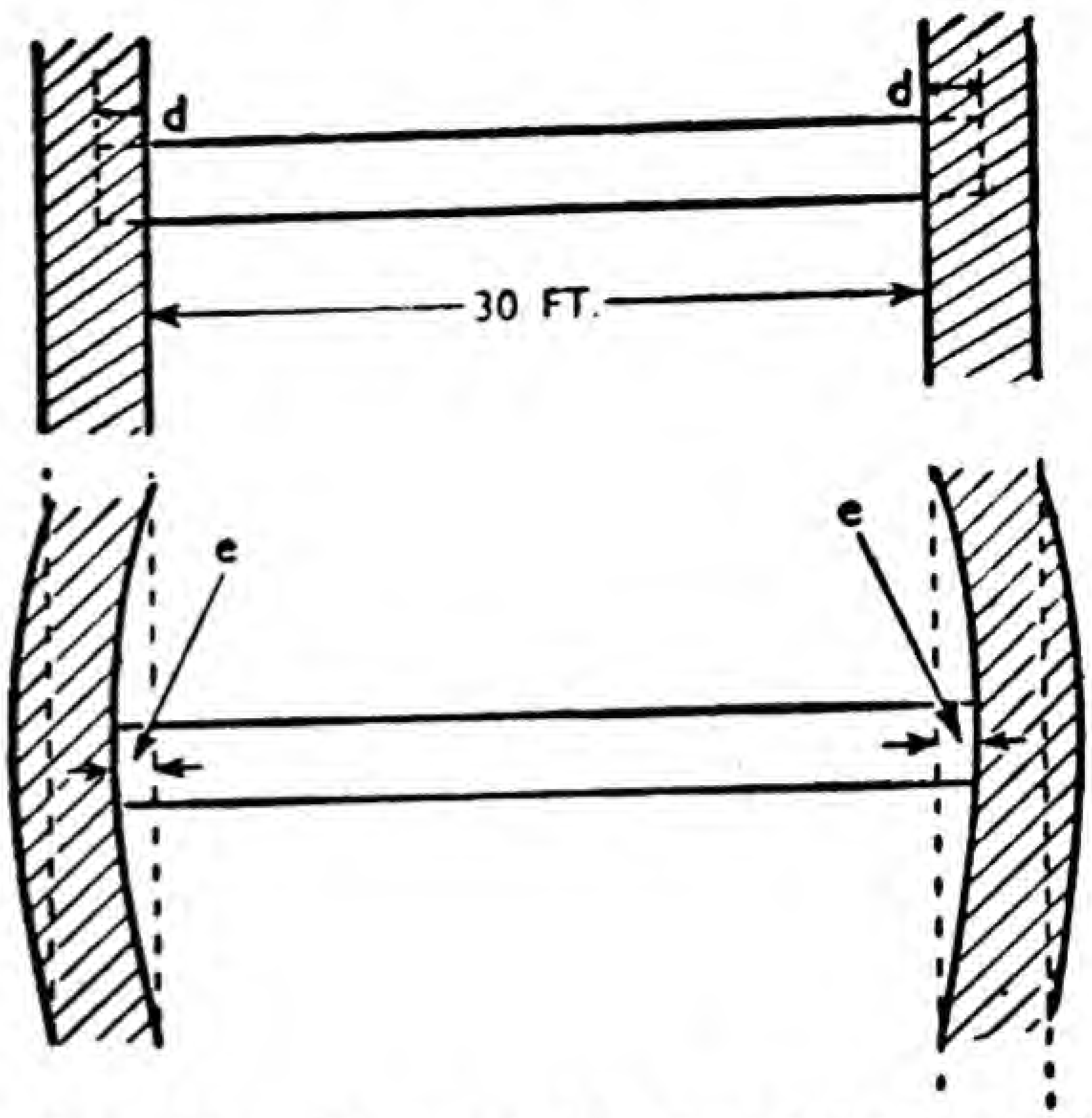


Fig. 33. If unyielding resistance is offered to the expansion of a steel beam under a rise of temperature, a stress is developed in the beam. Walls which yield by a distance e offer somewhat less resistance and the temperature stress in the beam is consequently less than the maximum.

For the sake of clarity, the beam is supposed to be exactly 30 ft. long, although it would, in practice, require to be somewhat longer.

It must first be determined what the expansion of the beam would be if the walls exercised no constraint and the beam were quite free to expand. In such a case no stress would be produced in the beam.

Total expansion $= 2d =$ length of beam × rise in temperature
× coefficient of expansion
$= 30$ ft. $\times$ 150 deg. F.
$\times\ 0{\cdot}0000063$ deg. F.
$= 0{\cdot}0284$ ft. $= 0{\cdot}341$ in.

If the walls were perfectly rigid, and did not yield in the least, this

movement would be completely prevented. Such a prevention of movement would be equivalent to applying a compressive force, and causing a contraction of the length of the beam equal to the value of the free expansion worked out above.

Contraction on 30 ft. = 0·0284 ft.

Therefore, strain $= \dfrac{0 \cdot 0284}{30}$

$= 0 \cdot 000947.$

Knowing the value of Young's modulus for steel, it is now possible to find the compressive stress produced in the beam if the walls were perfectly rigid.

$$\frac{\text{Stress}}{\text{Strain}} = E.$$

Therefore, stress = strain × E

$= 0 \cdot 000947 \times 30 \times 10^6$

$= 28{,}410$ lb. per sq. in.

Force exerted = Stress × cross-sectional area

$= 28{,}410$ lb. per sq. in. × 16·18 sq. in.

$= 459{,}674$ lb. = 205·2 tons.

If the walls each yield through a small distance e at each end, the total movement which is prevented is less than $2d$ by the length $2e$. In this instance $e = \frac{1}{10}$ in.

Expansion prevented $= 2d - 2e$

$= 0 \cdot 341 - 0 \cdot 20$

$= 0 \cdot 141$ in.

$$\text{Strain} = \frac{0 \cdot 141}{30 \times 12} = 0 \cdot 000392.$$

Stress $= 0 \cdot 000392 \times 30 \times 10^6$

$= 11{,}760$ lb. per sq. in.

Force on the walls $= 11{,}760 \times 16 \cdot 18 = 190{,}277$ lb.

$= 84 \cdot 9$ tons.

It is obvious that the normal type of brick wall would be unable to withstand a force of this magnitude and the actual movement of the ends of the beam would consequently be much greater than $\frac{1}{10}$ in., probably causing cracks in the walls. A similar problem arises in a long range of steam piping.

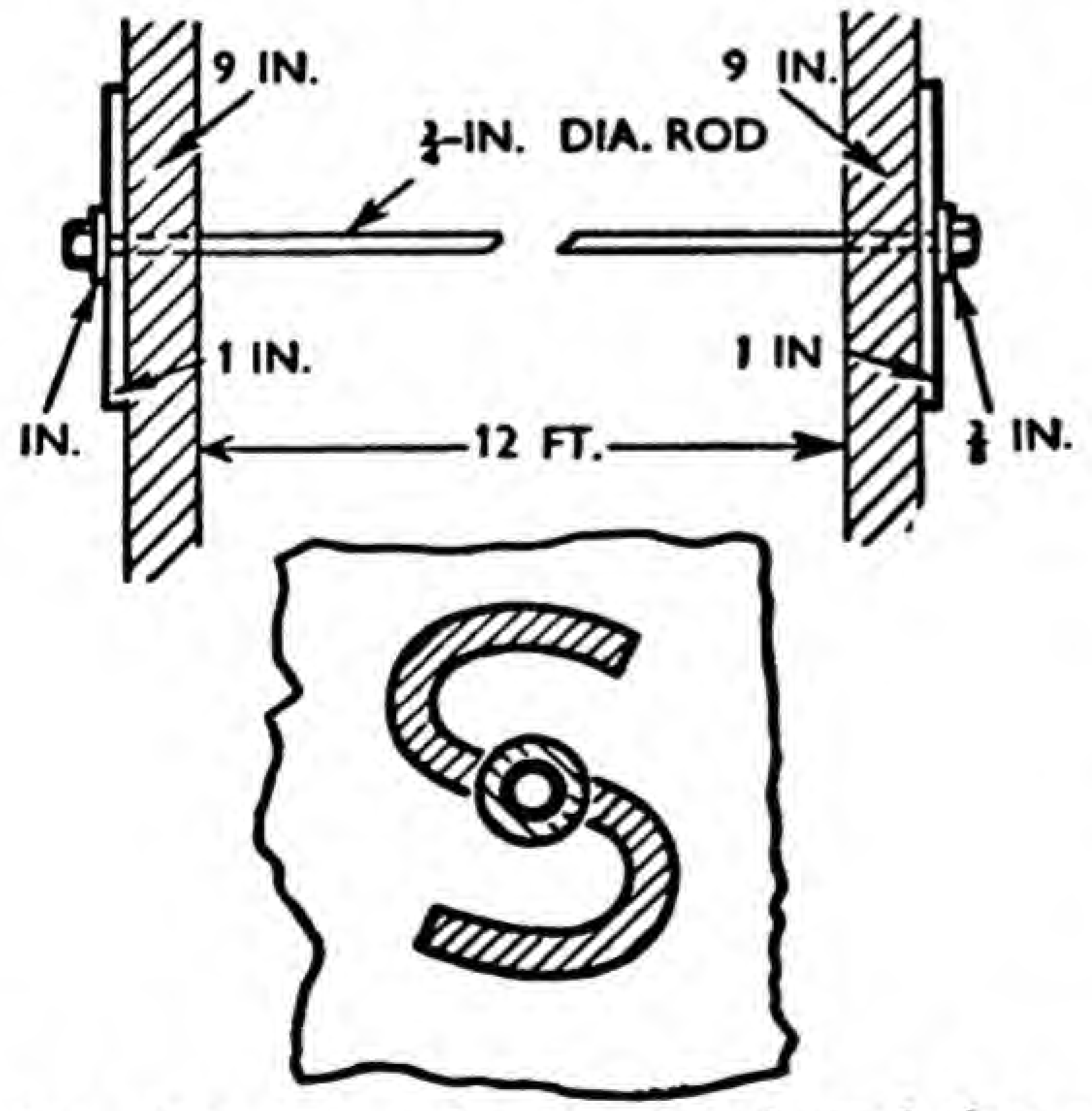


Fig. 34. The force developed by a resistance to temperature deformations can be used to hold weakened walls in place. A bar, tightened up when hot and then allowed to cool, exerts a restraining force inwards, preventing the collapse of the walls.

Pull of Stay Bar

The contraction of a steel bar after it has been raised to a high temperature is used to support and strengthen cottage walls. On the outside of the walls a plate, often in the form of a letter S, carries a rod passing through the building. When this rod has been heated, the nuts on both ends are screwed tight, and when the rod cools it attempts to contract to its original length, and so exerts an inward pull on the walls.

To what temperature would it be necessary to raise the rod shown in Fig. 34 if a pull of 2 tons is required after the rod has cooled to 60 deg. F.? Assume that each wall yields $\frac{1}{80}$ in.

Let the unknown rise in temperature be denoted by the symbol T, since its numerical value is not yet known. The length of the steel

rod, after it has been heated and the nuts tightened on the washers, is 12 ft. + 18 in. + $2\frac{3}{4}$ in. = 164·75 in.

The contraction of the steel rod, when the temperature is lowered, is then :—

164·75 × 0·0000063 × T, or 0·00104T in., where T represents the number of degrees (Fahrenheit) of fall in temperature from the upper value to 60 deg. F.

If the walls did not yield at all, all of the resultant contraction would be prevented, and the deformation per unit length would be :—

$$\frac{164{\cdot}75\text{ in.} \times 0{\cdot}0000063T}{164{\cdot}75\text{ in.}} = 0{\cdot}0000063T,$$

and the stress would become :—

Strain × E = 0·0000063T × 30 × 10^6 = 189T lb. per sq. in.

The force exerted by the bar would be :—

Stress × Area = 189T × area of $\frac{3}{4}$-in. diameter bar
= 189T lb. per sq. in. × 0·442 sq. in.
= 83·54T lb., or 0·0373T tons

But it is required that the force exerted should be 2 tons.

Therefore :— 0·0373T tons must equal 2 tons.

$$\therefore T = \frac{2}{0{\cdot}0373} = 53{\cdot}6 \text{ deg. F.}$$

This is the difference in temperature. Therefore, it is necessary to raise the temperature of the rod to 60 deg. F. + 53·6 deg. F., or to 113·6 deg. F.

Effect of Yielding

Each wall, however, yields by $\frac{1}{50}$ in., and thus the effective strain, or change in length per unit of length, is :—

$$\frac{0{\cdot}00104T - 2 \times 0{\cdot}02}{164{\cdot}75} = 0{\cdot}0000063T - 0{\cdot}00243$$

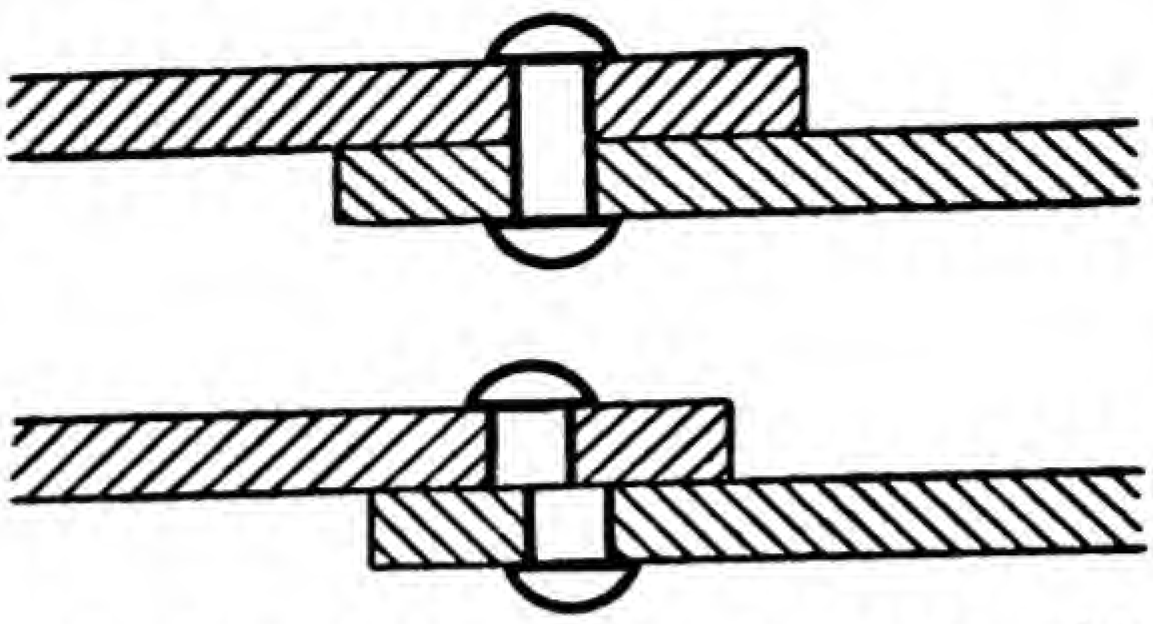


Fig. 35. One of the ways in which a rivet may fail is by a shear or sliding across one section. This is known as a single shear failure.

But, Stress = Strain × E
= (0·0000063T − 0·00243) × 30 × 10^6
= 189T − 72,837

Total load on the bar = Stress × cross-sectional area
= (189T − 72,837) 0·442
= (83·54T − 32,194) lb.
= (0·0373T − 14·37) tons.

But this total load must be equal to 2 tons.

0·0373T = 2 + 14·37.

T = 439 deg. F. rise in temperature above 60 deg. F.

In other words the whole of the bar would have to be raised to a temperature of 499 deg. F. so that, on cooling, it would exert a force of 2 tons. The effect of even a very slight yield of the walls is thus seen to be quite considerable.

Effect of Shear

So far, this section has dealt only with the internal effects of direct stress (tension or compression). Earlier in the chapter, however, the shearing effect of external loads was discussed. The internal effect of this shear on the cross-section of the material must now be examined. It was found that, for direct loading, the direct load divided by the cross-sectional area gives a result which is the load per unit of area. This is called direct stress (tension or compression). In the same way,

the shearing force (from a shearing-force diagram) divided by the cross-sectional area which is tending to slip or shear, gives a value which is the average shearing stress.

Riveted Joints

A simple illustration of shearing stress is provided by a riveted joint. In Fig. 35, a riveted joint is shown before and after failure. It is obvious that the area of the rivet which has sheared is the circular area whose diameter is the nominal size of the rivet.

If the rivet goes through three plates, and two of them pull in one direction while the third pulls in the other, two cross-sectional areas must be sheared through before the rivet fails (Fig. 36). The value or effectiveness of a rivet in such a case, so far as shear is concerned, is theoretically twice its value when sheared on one cross-section only. In the circumstances shown in Fig. 36, the rivet is said to be in double shear.

Suppose the rivet of Fig. 36 is $\frac{3}{4}$ in. in diameter. Its cross-sectional area is then 0·442 sq. in. The effective cross-sectional area in double shear may be taken as twice this, though in certain types of

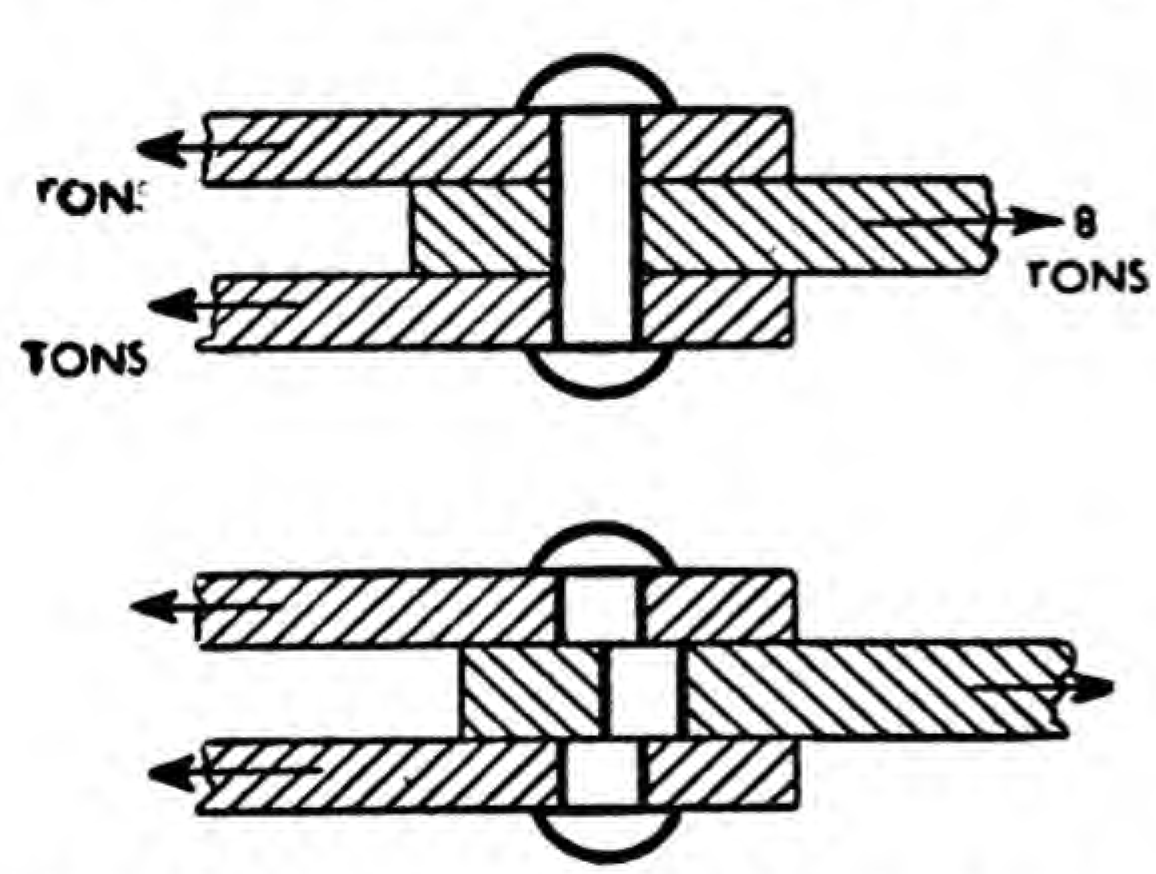


Fig. 36. When there are two sections or planes of shear which must fail before the rivet gives way, the break is known as a double-shear failure.

engineering, the full value of twice the single area is not used.

The shearing stress which would be developed in the rivet, if the rivet did not fail, would be :—

$$\frac{\text{Load}}{\text{Area}} = \frac{\text{Shearing force}}{\text{Sheared area}} = \frac{8 \text{ tons}}{2 \times 0{\cdot}442 \text{ sq. in.}}$$

$$= 9{\cdot}05 \text{ tons per sq. in.,}$$

which is greater than the safe working shearing stress for steel.

Riveted joints, which are so widely used in all types of construction, offer a valuable illustration of the way in which various types of stress must be considered in the design of one component part. The saying that a chain is as strong as its weakest link is well known, and the structural engineer tries in all his construction, to make sure that each part of the structure is equally strong. If failure should take place it should be like the collapse of the 'wonderful one-hoss shay,'

'*All at once, and nothing first,*
Just as bubbles do when they burst.'

Fig. 37 shows four possible ways in which a riveted joint may fail. All of these types of failure must be prevented. It is not enough to put in rivets which are strong in shear if the load tears the plate while leaving the rivets intact. How may these failures be prevented?

(*a*) If the joint is not to fail by the shearing of the rivets, the number supplied must be adequate. This requirement must be found by calculation.

(*b*) If the joint is not to fail by a tearing of the plate, the area of the plate resisting the pull must be sufficiently large.

(*c*) If the plate is not to tear in front of the rivet, the rivet must be sufficiently far from

the edge of the plate. This distance cannot readily be calculated, but it has been found that if the centre of the rivet is at a distance equal to one-and-a-half times the diameter of the rivet from the edge of the plate, the joint will not fail in this way.

(*d*) If the metal in front of the rivet is not to crush, the plate must be of adequate thickness, or there must be a sufficient number of rivets to reduce the crushing stress to a safe value.

Safe Load on Joint

It can be seen from the above that the three conditions represented in (*a*), (*b*) and (*c*) must be satisfied by calculating the strength of the joint in three different ways. The least of these three strengths is the maximum load which can be imposed on the joint with safety.

In Fig. 38 a tie bar, 3 in. wide and $\frac{3}{8}$ in. thick, forms part of a roof truss, and is attached to the main rafter through a gusset plate, which is also $\frac{3}{8}$ in. thick. The problem is to find the safe load which can be applied to the tie bar.

Strength of Plate. The strength of the plate in its original, unperforated state, would be equal to :—

Safe stress × area = 8 tons per sq. in. × (3 × $\frac{3}{8}$) sq. in.
= 9 tons.

From Fig. 38(a), however, it can be seen that the area of plate carrying load at a point where a rivet hole has been perforated is much less than the full area.

Area left after perforating = (3 in. − rivet hole diameter) $\frac{3}{8}$ in.
= $(3 - \frac{13}{16})\,\frac{3}{8}$ = 0·82 sq. in.

Safe load on the perforated plate = safe stress × area left after perforating

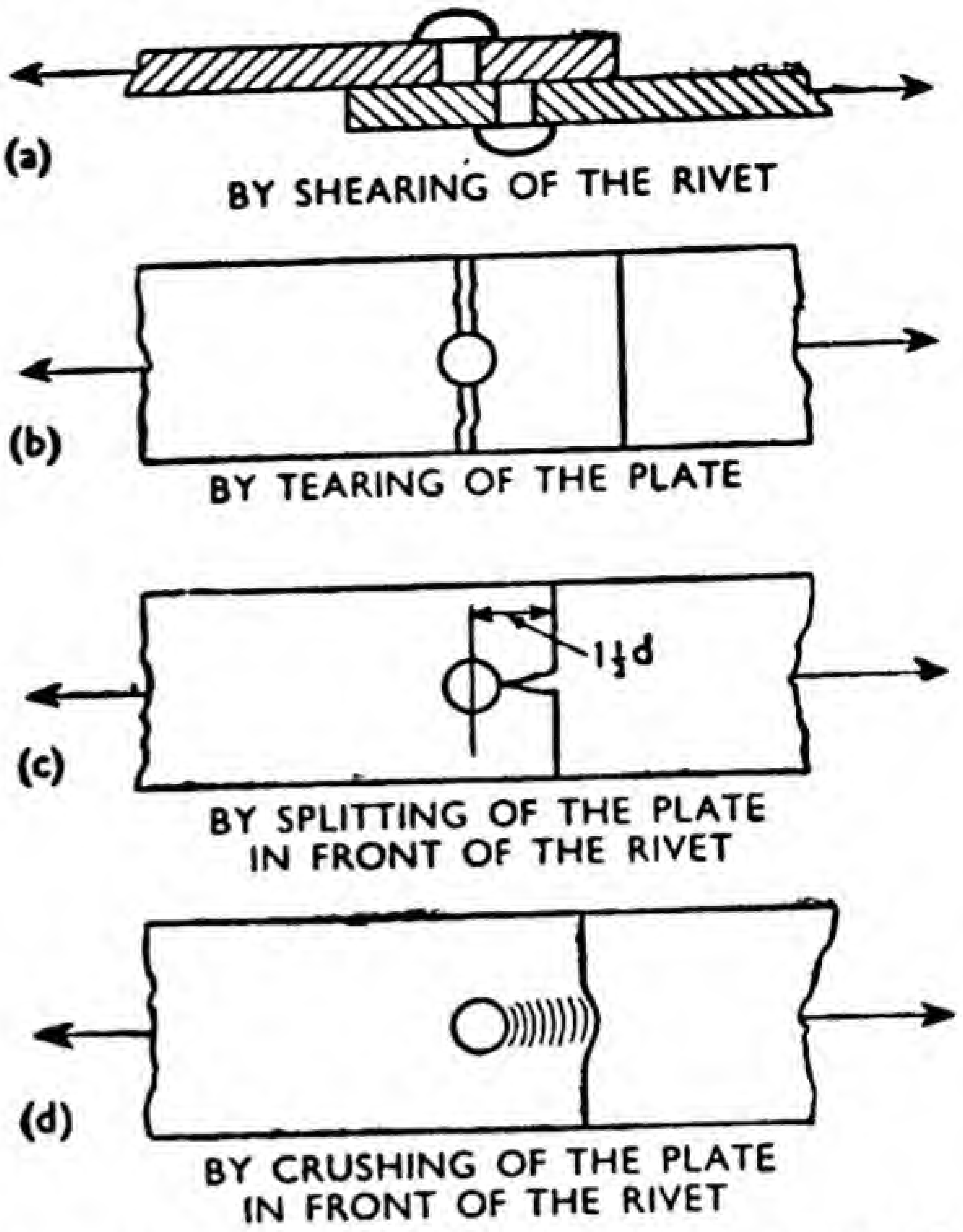


Fig. 37. The weakest part of a riveted joint governs the load which the joint may carry. The plate, as well as the rivet, may fail. The aim of a good design is to make all four types of failure equally unlikely.

= 8 tons per sq. in. × 0·82 sq. in.
= 6·56 tons.

If the load is kept below this figure, the type of failure shown in Fig. 37(b) will not occur.

Shearing Strength of Rivets. The shearing strength of one rivet is :—

Safe shearing stress × cross-sectional area of rivet
= 6 tons per sq. in. × 0·442 sq. in.
= 2·65 tons.

The safe load on the joint, if the rivets are not to risk shear failure, is then 3 × 2·65 tons, because there are three rivets. The strength of the joint, therefore, is 7·95 tons.

Bearing Strength of Rivets. From Fig. 38(b) it can be seen that the pushing front which a rivet exerts on the metal of the joined plate has an area which is rectangular in

shape, one dimension being the thickness of the plate, and the other dimension being the diameter of the rivet.

The safe load which can be exerted in bearing on the plate

= safe bearing stress × bearing area

= (12 tons per sq. in. × $\frac{3}{8}$ × $\frac{7}{8}$ sq. in.) (3 rivets)

= 10·13 tons.

Final Safe Strength of Joint. The joint has been shown to be capable of carrying loads up to 6·56, 7·95 and 10·13 tons. It is obvious that the load must be restricted to 6·56 tons. The strength in shear and bearing of the rivets cannot be fully utilized.

Efficiency of the Joint. It will be noticed that, although the flat tie bar is capable of carrying a 9-ton load, the joint can support only 6·56 tons. If it were possible to make this joint capable of carrying 9 tons, its efficiency would be 100 per cent. Its present efficiency is thus considerably less, and is equal to

$$\frac{6 \cdot 56}{9 \cdot 00} \times 100 \text{ per cent} = 72 \cdot 9 \text{ per cent.}$$

Instead of determining the strength of the joint in three different ways, as was done above, it is convenient to begin by finding what is known as the value of a rivet in the particular joint under consideration. This preliminary step eliminates one of the determinations in the last example.

Value of a Rivet. The value of a rivet is the greatest load which it can safely carry in that particular joint. For example, in designing the joint in a plate measuring 12 in. × $\frac{3}{8}$ in. (Fig. 39) the value of one $\frac{7}{8}$-in. rivet in double shear must be found :—

Safe shearing stress × two cross-sectional areas of rivet

= 6 tons per sq. in. × 2 × 0·60 sq. in.

= 7·2 tons.

It is then found that the value of a $\frac{7}{8}$-in. rivet when it is bearing on the $\frac{3}{8}$-in. plate is,

safe bearing stress × bearing area (Fig. 39)

= 12 tons per sq. in. × $\frac{3}{8}$ in. × $\frac{7}{8}$ in.

= 7·87 tons.

The value of one rivet for this joint, therefore, is the smaller of these values, 7·2 tons.

The next thing to determine is the number of rows of rivets which might be used in the joint. Rivets

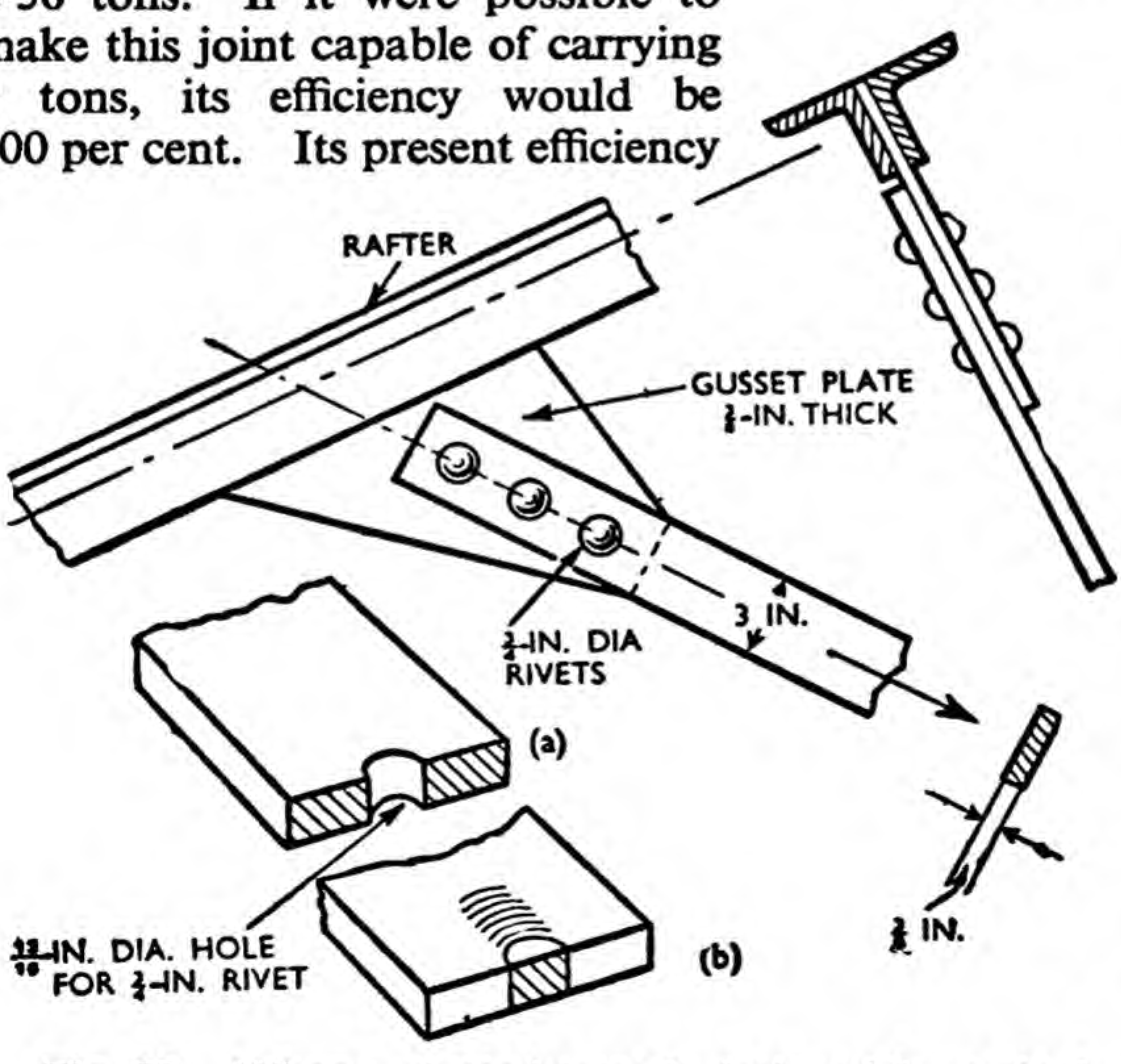


Fig. 38. When a riveted joint is designed for a steel structure, the aim is to make every part of the joint equally strong. The gusset plate is fastened to the rafters by a row of rivets in double shear and the 3-in. tie bar is riveted to the gusset plate. Both these joints should be liable to fail at the same load.

must be kept as close to the edge as is consistent with the condition that tearing must not occur. By this means the edges of the plates are held tightly. Again, rivets must not come closer together than three times their diameter, measuring from centre to centre of the rivets. Taking these two considerations into account it can be seen that there may be either three or four rows. Fig. 39 shows possible arrangements, together with lines along which the plate may fail.

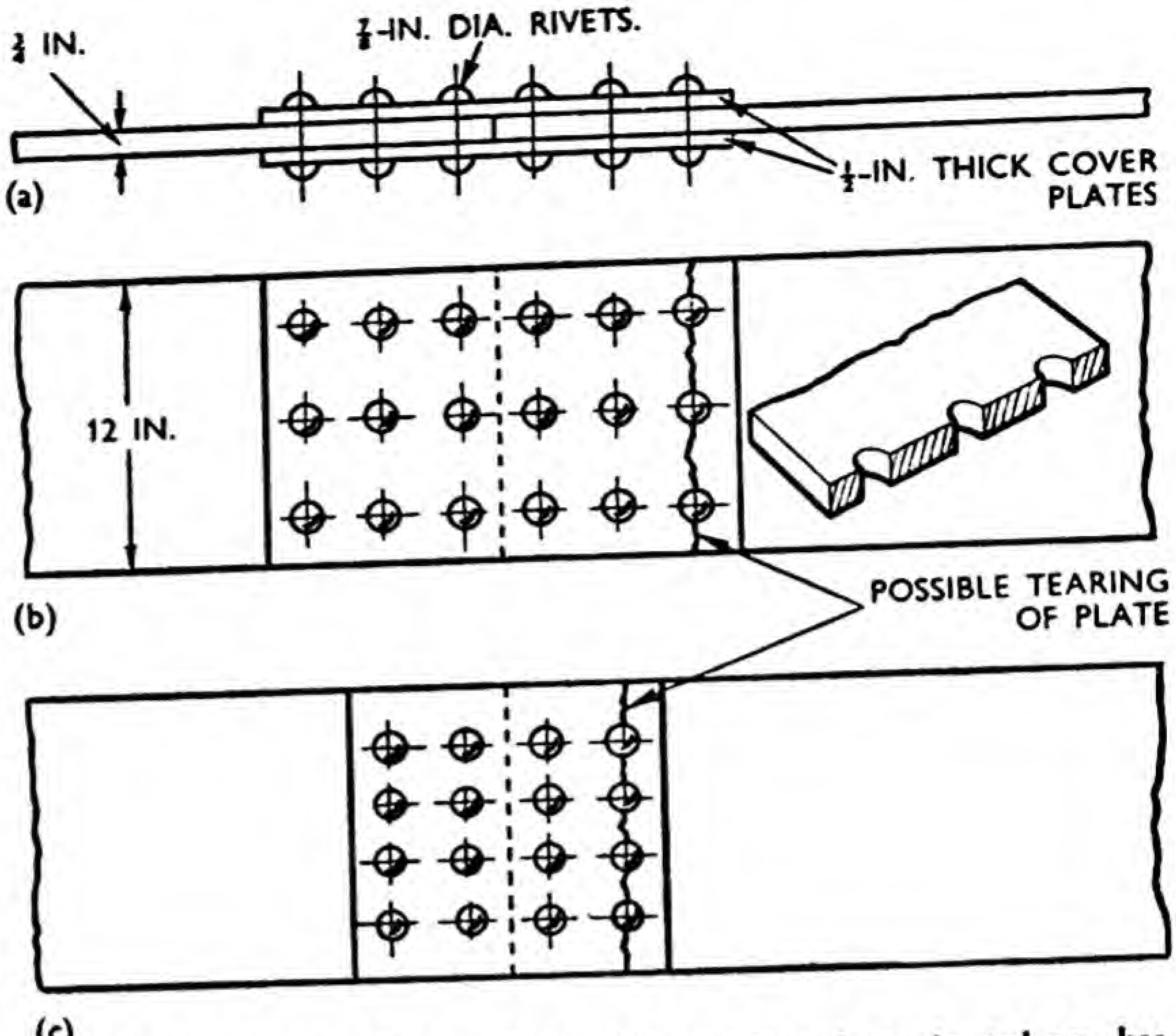


Fig. 39. Here, the load in the main plate is taken by the cover plates, passed across the junction and transferred to the plate on the other side.

Three Rows. When three rivet holes, $\frac{15}{16}$ in. in diameter, have been punched out of the plate, the area of steel left to carry the tensile load is shown shaded in the small diagram.

Cross-sectional area $= (12 - 3 \times \frac{15}{16})\frac{3}{4} = 6{\cdot}89$ sq. in.

Strength of plate = Safe stress $\times$ area $= 8 \times 6{\cdot}89 = 55{\cdot}12$ tons.

The value of one rivet for this joint is 7·2 tons. The number of rivets required, therefore, is $\frac{55{\cdot}12}{7{\cdot}2} = 7{\cdot}7$, or 8 rivets. The rivets are in rows of three, so 9 are required.

Four Rows. When four rivet holes have been punched out, the plate may tear as shown in Fig. 39(c). The strength of the plate then, is

$(12 - 4 \times \frac{15}{16})\,\frac{3}{4} \times 8 = 49{\cdot}50$ tons.

The number of rivets required $= \frac{49{\cdot}50}{7{\cdot}2} = 6{\cdot}9$, or 7 rivets. The rivets are in four rows, so 8 are required.

It is to be noted that the number of rivets calculated must be put in on both sides of the joint.

Efficiency. The unpunched plate will carry a load of :—

$12 \times \frac{3}{4} \times 8 = 72$ tons.

The efficiency of the joint with three rows, therefore, is :—

$\frac{55{\cdot}12}{72} \times 100$ per cent $= 76{\cdot}6$ per cent.

Similarly the efficiency of the joint with four rows of rivets is :—

$\frac{49{\cdot}50}{72} \times 100$ per cent $= 68{\cdot}75$ per cent.

Arranging the Rivets

These efficiencies are not very high for such an important joint in a tie bar. Fig. 40 shows a method of arranging the rivets so that the efficiency is increased. If the plate tears at section *X*, as is shown in the

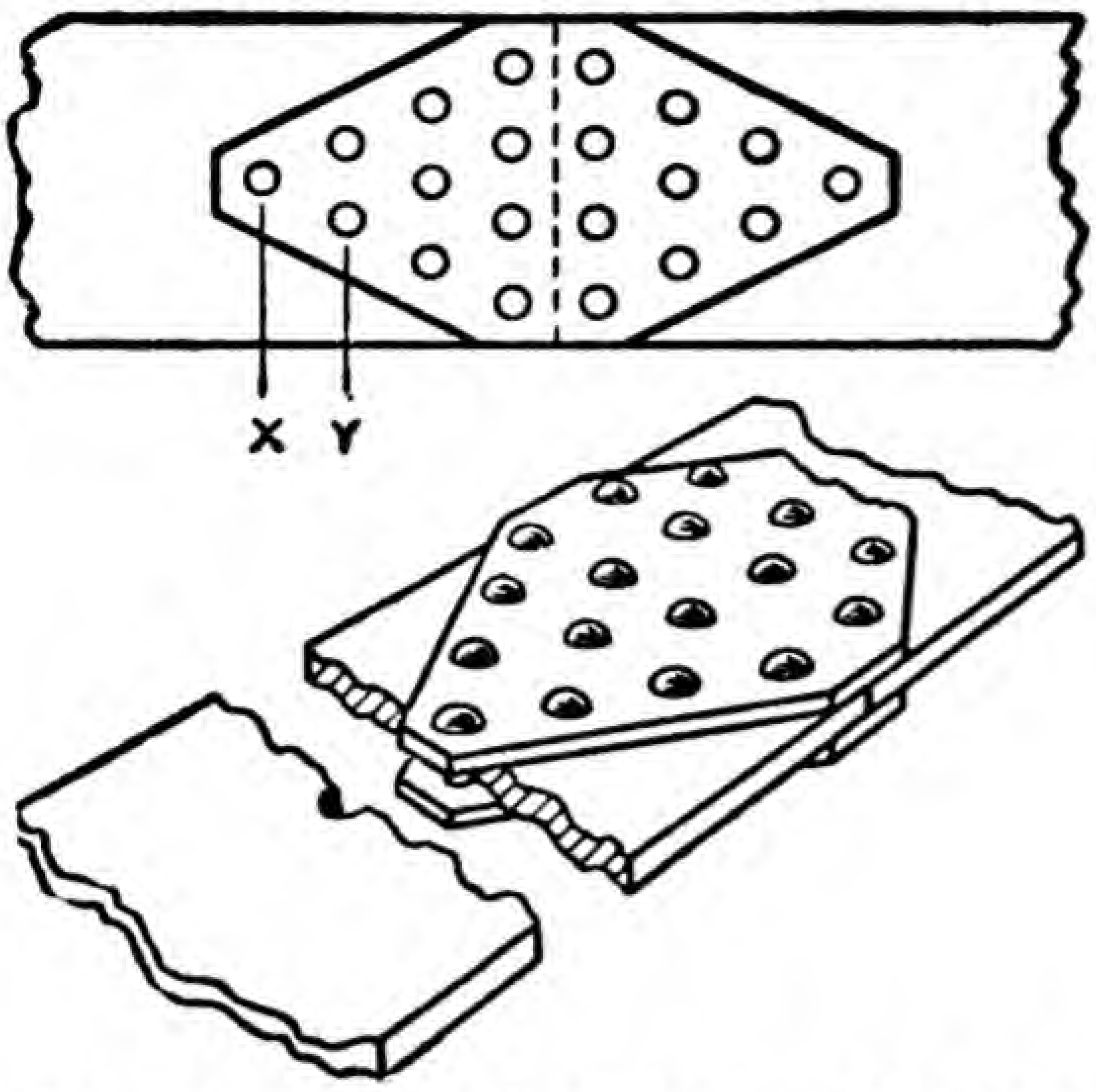


Fig. 40. By drilling rows of holes to take rivets, the engineer weakens the plate in which he is making a joint and makes more likely a failure of the type shown in Fig. 37(*b*). This arrangement of rivets results in the least possible weakening of the plate.

lower view, there is only one rivet hole punched from the plate.

The strength of the plate, therefore, is :—

$(12 - \frac{15}{16})\frac{3}{4} \times 8 = 66{\cdot}38$ tons, which is considerably higher than when the rivets are arranged as in Fig. 39.

The plate might tear where there are two rivet holes taken out, at *Y*, but it must be remembered that it would be necessary, in addition, for the single rivet at *X* to shear before the joint failed completely. The value of a single rivet in this joint is 7·2 tons, so, for the final strength of the joint if the plate tears at *Y*, there is :—

$$(12 - 2)\tfrac{3}{4} \times 8 + 7{\cdot}2 = 67{\cdot}2 \text{ tons},$$

which is stronger than before. The joint is thus more likely to tear at *X*.

The number of rivets required now is :—

$$\frac{66{\cdot}38}{7{\cdot}2} = 9{\cdot}2, \text{ or } 10 \text{ rivets.}$$

Efficiency of the Joint. The efficiency is higher than before, as can be seen by the calculation :—

$$\frac{66{\cdot}38}{72} = 92{\cdot}2 \text{ per cent.}$$

Measuring Shearing Strain

When direct stress and strain were considered, it was found that the ratio between these values is constant for any particular material. This ratio was known as a modulus or modulus of elasticity, and was used as a tool in subsequent calculations.

In considering shearing stress in rivets, nothing has been said about shearing strain, but, in order to consider shear and its effects in more detail, the way in which shearing strain is measured must be determined.

When a shearing stress is applied to a block of material, it tends to cause the various layers or fibres of the material to slide or flow over each other. In any ordinary material, if this sliding exceeds a limiting amount, the material fails in shear, as does the rivet in Figs. 35 and 36. Shearing strain can be shown, however, in model form by the use of a book which is thick in relation to the width of its pages. Fig. 41 shows the shearing of such

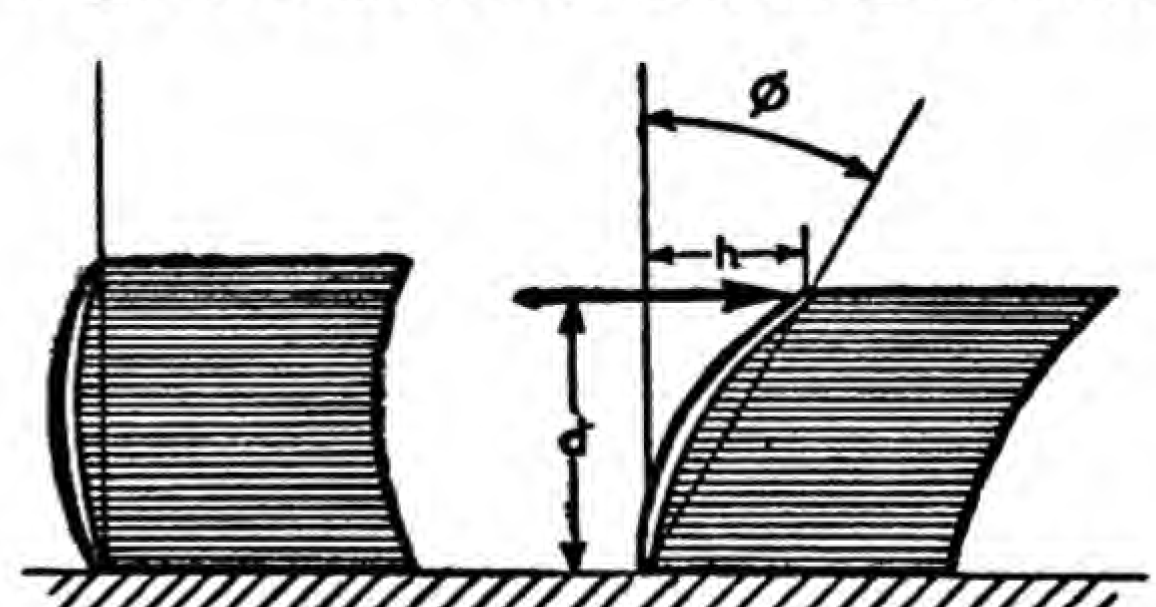


Fig. 41. Direct strain is defined as the longitudinal deformation per unit of length. Shearing strain is the lateral deformation per unit of length or $\frac{h}{d}$. As this value is always very small it can be looked upon as a gradient or angle ø.

a book, each page sliding on the one below it, until the book assumes the shape shown in the second diagram. The strain is the amount of sliding per unit of depth. This can be recorded as $\frac{h}{d}$ or merely as the angle ⌀ (phi), which is normally extremely small.

It is found that a simple relationship exists between shearing stress and shearing strain similar to the relationship existing between direct stress and linear strain. The two can be put together to afford a direct comparison :—

$$\frac{\text{Direct stress}}{\text{Direct strain}} = \text{Modulus of elasticity.}$$

$$\frac{\text{Shearing stress}}{\text{Shearing strain}} = \text{Modulus of rigidity.}$$

For steel, this new modulus is about two-fifths of the modulus of elasticity, or about 12×10^6 lb. per sq. in.

This stage is a convenient one at which to revise and summarize, briefly, what has already been studied in this chapter, in order that a mental picture may be built up in which the more advanced work which follows takes its place.

The effects of the external loading on a structure were first studied, and it was found that these loads tended to cause shearing or slipping of one part on another, and bending of one part relative to another. The variation of the tendency to shear or bend was shown by shearing-force and bending-moment diagrams.

Secondly, it was found that in order to determine the internal effects of such shearing forces and bending moments, it was necessary to know something about the cross-sections of the member or beam on

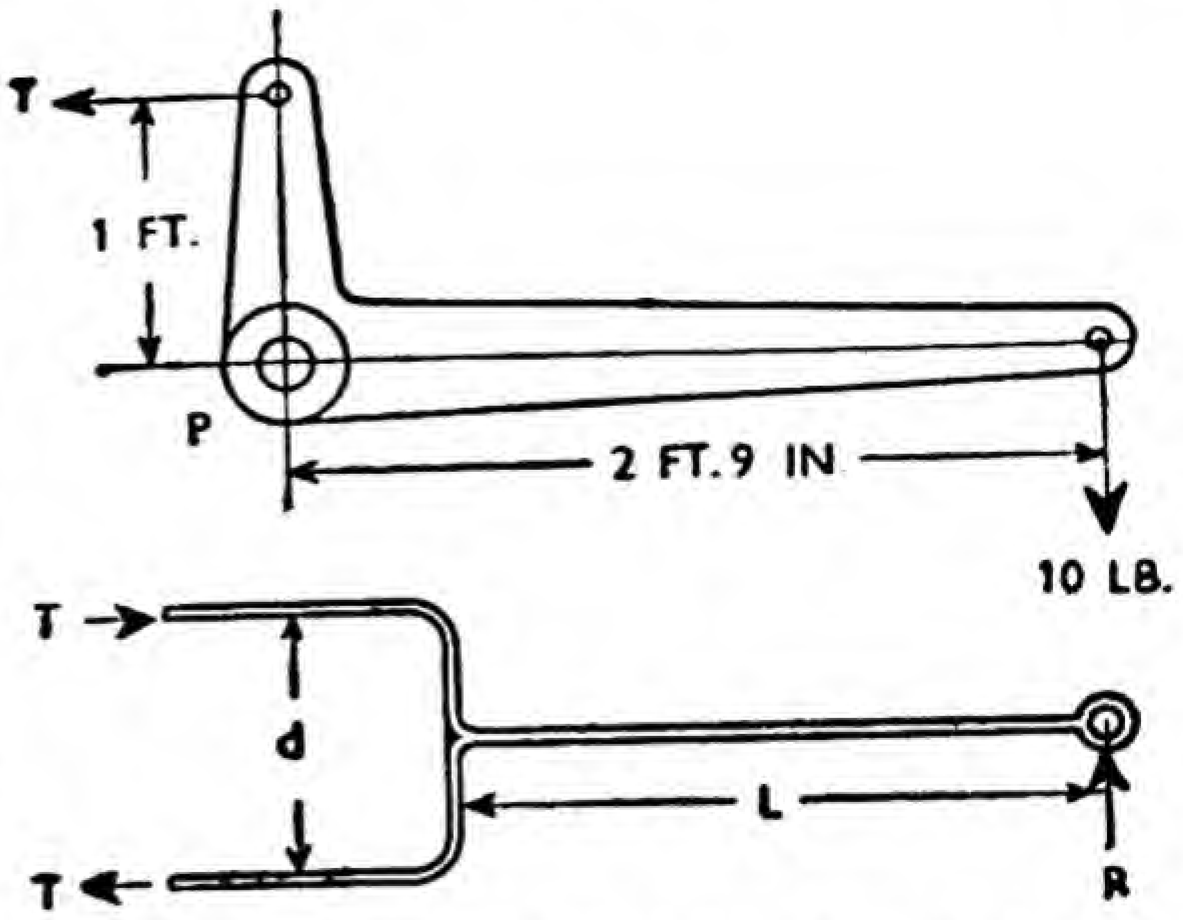


Fig. 42. The first diagram illustrates how one force at the end of each arm can hold the bell-crank lever in equilibrium about its hinge. In the second diagram, the moment of a force *R* is being resisted by a couple formed of two forces *T*.

which the forces and moments act. The two important properties of a cross-section were found to be the area in sq. in. and the moment of inertia in in.^{4}. Of these, the area has been used in solving problems concerning direct stress and linear strain, and also when considering shearing stress. The next part of the chapter will show how the moment of inertia can be employed in finding the values of stresses caused by bending and shear.

Determination of Stress

The most important use to which the moment of inertia can be put is in the determination of the stresses occurring in a beam under the influence of bending moments. In order to show how these stresses are called into play it is necessary to go back to the principle of moments (Chapter 2). The bell-crank lever, shown in Fig. 42, carries a weight of 10 lb. at the end of its longer arm. The force *T* which must be exerted at the end of the shorter arm, in order to balance this load, must be greater

than 10 lb. in the ratio of $\frac{2 \text{ ft. } 9 \text{ in.}}{1 \text{ ft.}}$ which is equal to 2·75. Therefore, T is 27·5 lb., so that :—

$T \times 12$ in. $= 10$ lb. $\times 33$ in.

Let this illustration be extended a step further to the second diagram in Fig. 42. This represents a two-pronged toasting fork. Suppose that the end of the handle is pushed up by a force R. In order to resist the turning effect of this force on the fork, the two prongs must be held, one in each hand, and a resisting couple applied, one hand pushing and the other hand pulling with a force T. Then :—$R \times L = T \times d$. This is merely a variation of the bell-crank lever of the first part of Fig. 42.

When a beam bends under the influence of external loads, the top of the beam is compressed and the bottom part is stretched. If the load exceeds the capacity of the beam, the crushing and tensile stresses will cause failure of the beam, as is shown in Fig. 43. In the second half of Fig. 43 a beam is shown under normal working loads, there being a tensile force on the bottom half, and a compressive force on the top half at whatever section the beam is cut. The cut portion shown is in equilibrium under the action of the forces R and T, just as was the toasting fork. Once again :—$R \times L = T \times d$. At some point in the depth of the beam there is neither tension nor compression. At this neutral position, or neutral axis, the change over from tension to compression takes place.

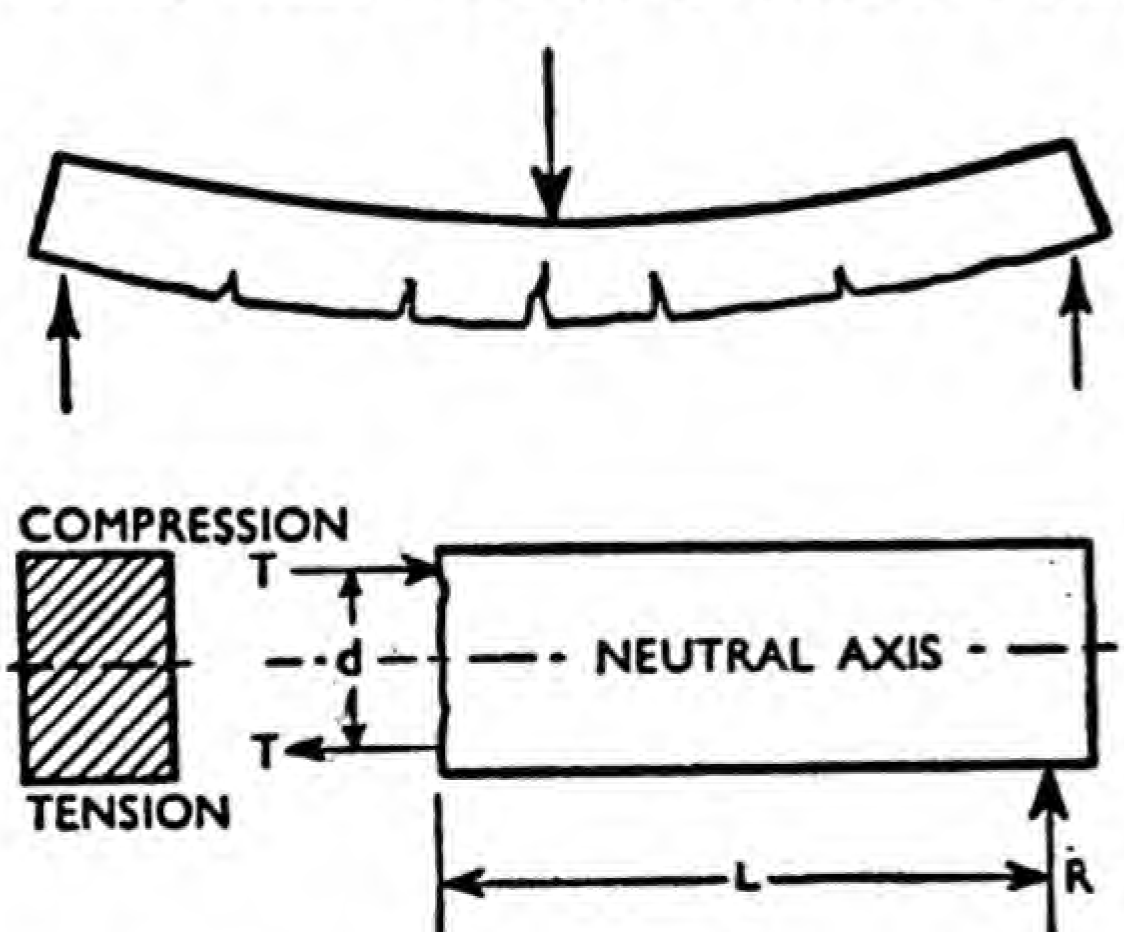


Fig. 43. This diagram shows how the toasting-fork effect of Fig. 42 can illustrate the mechanism by which a beam resists bending. Compressive and tensile forces are set up within the material of the beam.

Of course, it can be realized that in a beam of the type shown in Fig. 43, there are not merely two forces T in action, but a number of forces, varying from a maximum at the top and bottom to zero at the neutral axis. The simple resistance couple $T \times d$ then becomes something more complex.

The Flexure Formula

In Fig. 44, the maximum force has been changed, for convenience, into a stress multiplied by an area. The halves of the beam above and below the neutral axis have both been divided into eight equal areas, each called A. As the neutral axis is approached, the force on each area A becomes smaller in proportion as the distance from the outside edge increases, the forces on the eight areas being :—

$1fA$; $\frac{7}{8}fA$; $\frac{3}{4}fA$; $\frac{5}{8}fA$; $\frac{1}{2}fA$; $\frac{3}{8}fA$; $\frac{1}{4}fA$; $\frac{1}{8}fA$.

Now there are eight different couples, all acting together, and forming the same type of resistance couple, or resistance moment which was defined as $T \times d$ in Figs. 42 and 43. The bending moment $R \times L$, is thus equal to the sum of all these eight resistance couples, as follows :—

$$M = fA(2y) + \tfrac{7}{8}fA(2 \times \tfrac{7}{8}y) + \tfrac{3}{4}fA(2 \times \tfrac{3}{4}y) + \tfrac{5}{8}fA(2 \times \tfrac{5}{8}y)$$

$+ \frac{1}{2}fA(2 \times \frac{1}{2}y) + \frac{3}{8}fA(2\times \frac{3}{8}y) + \frac{1}{4}fA(2 \times \frac{1}{4}y) + \frac{1}{8}fA(2 \times \frac{1}{8}y)$

$= 2fAy(1^2 + \frac{7}{8}^2 + \frac{3}{4}^2 + \frac{5}{8}^2 + \frac{1}{2}^2 + \frac{3}{8}^2 + \frac{1}{4}^2 + \frac{1}{8}^2).$

$= 2A(3\frac{3}{16})yf.$

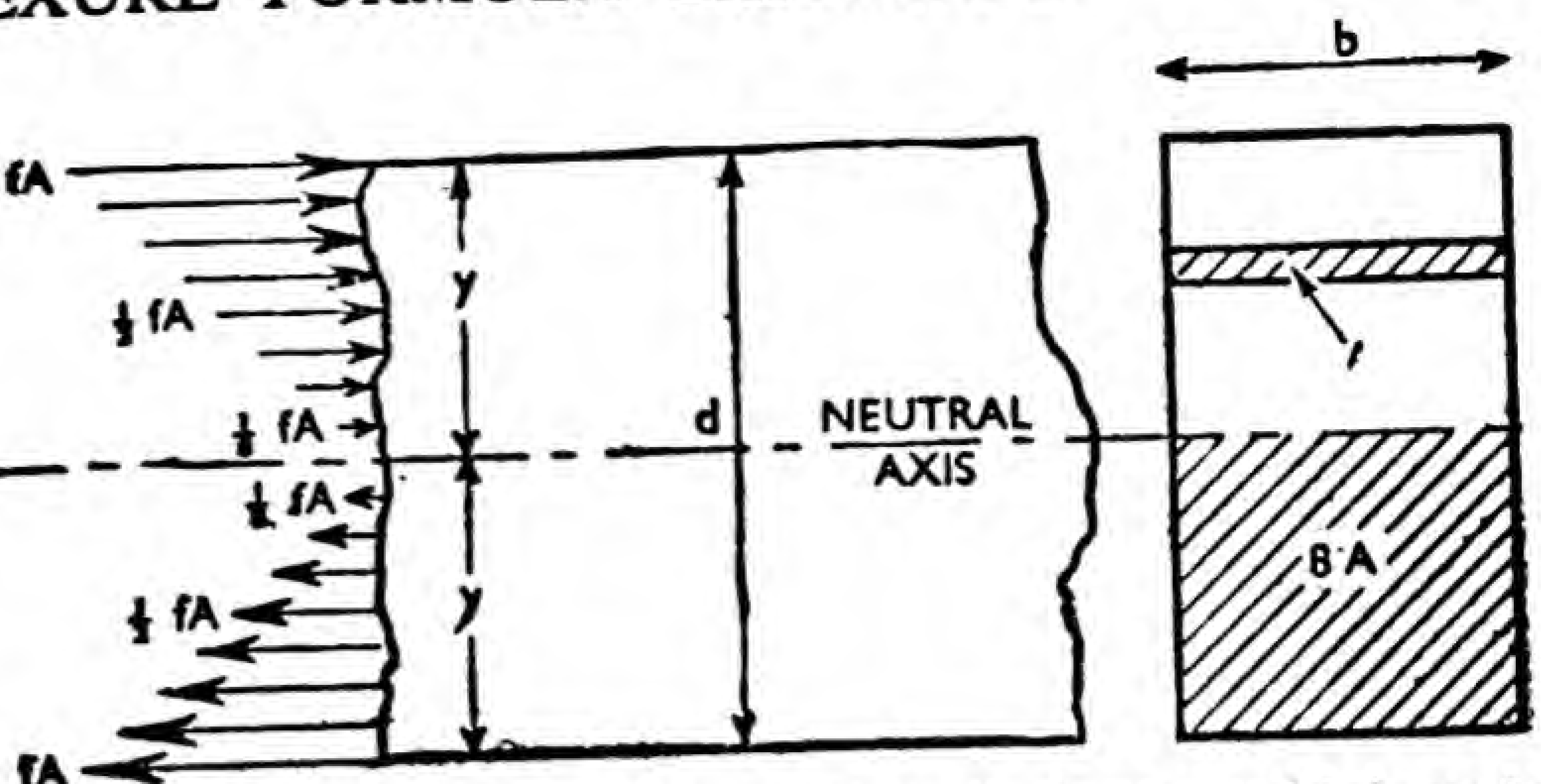


Fig. 44. The compressive and tensile forces which resist bending are not single forces as shown, in a simplified form, in Fig. 43. They consist, rather, of an intensity of resisting stress which acts across the whole cross-section of the beam and varies from a maximum at the upper and lower faces to zero at the neutral axis. In a normal beam, the stress above the neutral axis is usually compressive, and that below, tensile.

The maximum stress f can now be found by writing this equation as :—

$$f = \frac{M}{2A(3\frac{3}{16})y}.$$

It will be remembered, however, that the moment of inertia of an area is the second moment of the area about the given axis. Thus the moment of inertia of any of the sixteen small areas (A) about the neutral axis is :—

$A \times$ (distance of area from axis)^2.

The moment of inertia of the whole beam cross-section will thus be the sum of the sixteen moments of inertia just defined :—

$$I \text{ about } NA = 2\,[A\,\{y^2 + (\tfrac{7}{8}y)^2 + (\tfrac{3}{4}y)^2 + (\tfrac{5}{8}y)^2 + (\tfrac{1}{2}y)^2 + (\tfrac{3}{8}y)^2 + (\tfrac{1}{4}y)^2 + (\tfrac{1}{8}y)^2\}\,] = 2A(3\tfrac{3}{16})y^2.$$

Looking back at the equation for M, above, it can be seen that the value $2A(3\frac{3}{16})y$ is merely $\frac{I}{y}$. The maximum stress on the section, caused by bending, therefore, is :—

$$f = \frac{M}{\frac{I}{y}} = \frac{My}{I}.$$

Important Expression

This is a most important expression and is called the flexure formula. Putting it into words, it can be said that the bending stress caused at a distance y from the neutral axis by a bending moment M, is the bending moment multiplied by y and divided by the moment of inertia of the whole cross-section.

It has been noticed that mathematicians say that the moment of inertia of a rectangle is $\frac{bd^3}{12}$. In taking only sixteen small areas to represent the large cross-section, an approximation has been made. If $y = \frac{1}{2}$ depth, and $A = \frac{bd}{16}$, then $2A(3\frac{3}{16})y^2$ becomes $6{\cdot}375 \times \frac{bd}{16} \times \frac{d^2}{4} = \frac{bd^3}{10}$. If more small areas of A had been taken, it would have been closer to the correct value of $\frac{bd^3}{12}$, which is found by the use of the integral calculus.

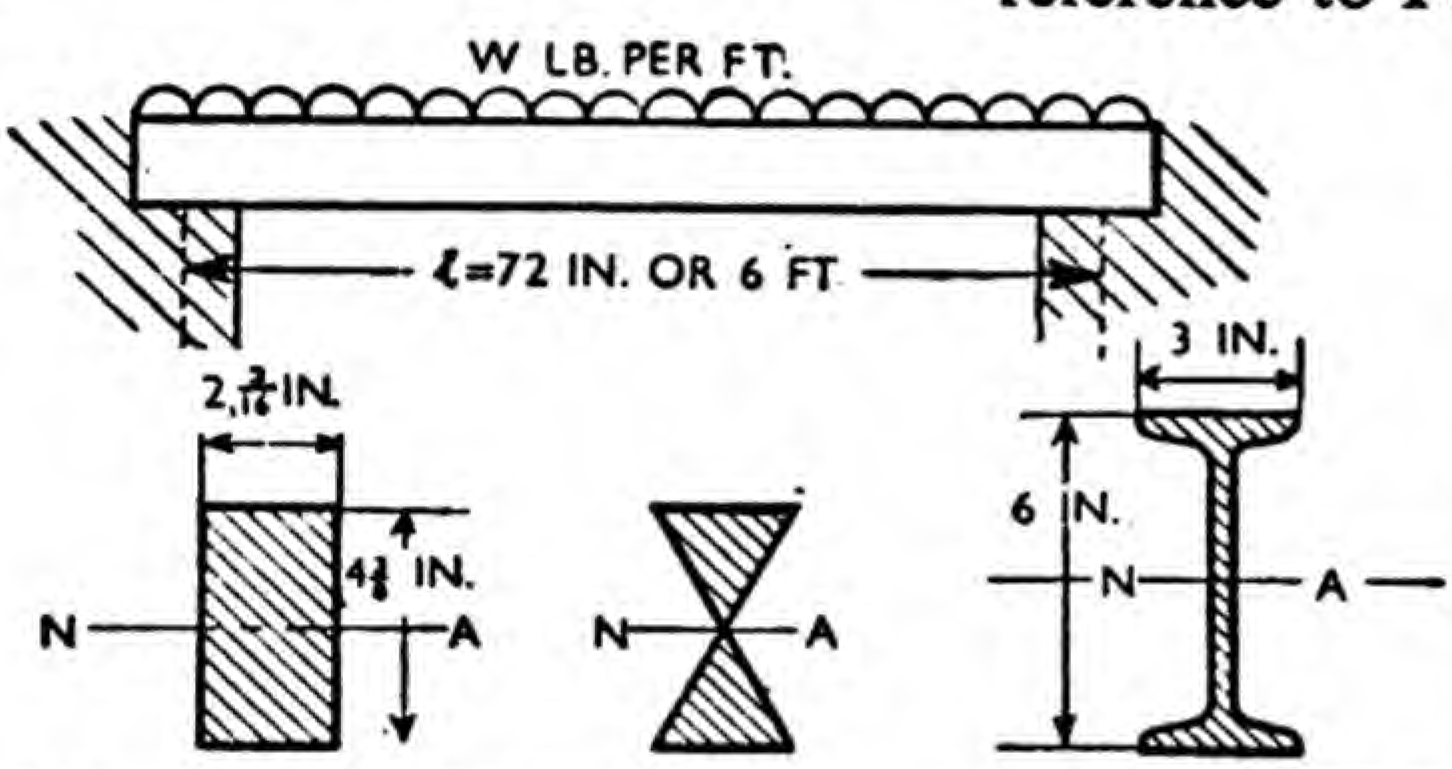
In using the flexure formula, it is not usual to be interested in any stress but the maximum, and thus y is usually made as large as possible, the greatest distance from the neutral axis to the upper or lower edge of the beam.

The neutral axis, which passes through the centre of gravity of the section, may not always be at half

the depth as it was in the problem of Fig. 44. When this occurs, the value of maximum y measured towards the tension side of the beam will be different from the value of y measured towards the compression side of the beam. The maximum values of compressive and tensile stress, therefore, are not equal.

Section Modulus

It has been seen above that $f = \frac{My}{I}$. This is often written as



I	15·3 IN.^{4}	11·5 IN.^{4}	21·0 IN.^{4}
Z	7·0 IN.^{3}	5·2 IN.^{3}	7·0 IN.^{3}
Y	2·2 IN.	2·2 IN.	3·0 IN.
WEIGHT	32·6 LB. PER FT.	16·3 LB. PER FT.	12·0 LB. PER FT.

Fig. 45. If bending stresses are to be kept small, a beam must have as large a moment of inertia as possible. By making the section of the beam in the form of the letter I it is possible to obtain a high moment of inertia without using a heavy beam section.

$M = fz$ where $z = \frac{I \text{ in.}^4}{y \text{ in.}} = \frac{I}{y} \text{ in.}^3$.

The value of z for any beam is known as the section modulus, and is useful as a tool in designing beams, for, since $f = \frac{M}{z}$ all beams with the same section modulus sustain the same maximum stress for a given bending moment.

Fig. 45 shows a steel beam of rectangular section carrying a load of w lb. per ft., uniformly distributed. What load can this beam safely carry in this way, if the safe tensile and compressive stress in the steel is 8 tons per sq. in.? Modern mild steels are often assumed to carry safely 9 or 10 tons per sq. in.

The section modulus required for this beam is :—

$$z = \frac{M \text{ in.-tons}}{8 \text{ tons per in.}^2}$$

where M is the maximum bending moment induced in the beam by a uniformly distributed load. By reference to Fig. 19, it will be seen that the maximum bending moment is $M = \frac{wl^2}{8}$.

This is the first problem in which it is necessary to consider both the external effects of loading (bending moment), and the internal effects within the beam. The flexure formula links the external to the internal, as it contains both M, an external effect, and I or z, a property of the shape of the cross-section.

Let the values which are known be determined and combined in the flexure formula :—

Maximum bending moment (M)

$$= \frac{wl^2}{8} = \frac{w \times 6^2}{8} \text{ ft.-lb.}$$
$$= 54w \text{ in.-lb.}$$

$$\text{Moment of inertia } (I) = \frac{bd^3}{12}$$

$$= \frac{2\frac{3}{16} \times (4\frac{3}{8})^3}{12} = 15{\cdot}3 \text{ in.}^4.$$

Maximum distance from N.A. to

the outer edge $(y) = \frac{d}{2} = 2\frac{3}{16}$ in.

Section modulus $(z) = \frac{I}{y} = \frac{15{\cdot}3 \text{ in.}^4}{2\frac{3}{16} \text{ in.}}$

$= 7{\cdot}0$ in.^{3}.

Writing these values into the flexure formula :—

$M = fz$ or $54w = (8 \times 2{,}240) \times 7$.

$$w = \frac{56 \times 2{,}240}{54}$$

$= 2{,}320$ lb. per ft.

The beam, therefore, can carry a load of 2,320 lb. per ft., including its own weight. Steel weighs 0·283 lb. per cu. in., and the weight per ft. of a beam of the rectangular section shown in Fig. 45 is :—

Cross-sectional area (sq. in.) $\times$ 12 in. $\times$ 0·283 lb. per cu. in.

$= 4\frac{3}{8} \times 2\frac{3}{16} \times 12 \times 0{\cdot}283$

$= 32{\cdot}6$ lb. per ft.

The net weight which may be safely applied to the beam, therefore, is:—

$2{,}320 - 33 = 2{,}287$ lb. per ft.

Development of I Beam

Now, as was noticed in Fig. 44, much of the metal near the neutral axis is very lightly stressed. The safe stress of 8 tons per sq. in. for which this beam is designed occurs only at the outer fibres, top and bottom of the section. It would be more economical, instead of making the beam of a rectangular section, to cut out the steel not taking much stress.

One way of doing this is illustrated in the second small diagram of Fig. 45. Since the stress decreases towards the neutral axis, the metal of the cross-section might be cut away in proportion to the distance from the top and bottom of the beam, and the shape of the cross-section would be reminiscent of an hour-glass. This is a better design than the rectangular shape, for the weight of the beam is halved, while the moment of inertia and section modulus are reduced by only 25 per cent. The aim of the designer is always that of obtaining the strongest and stiffest beam for a given weight, or the lightest beam for a given strength.

However, it is easily seen that a beam of this shape is quite impracticable. It does, however, indicate how a suitable section might be evolved, by eliminating material near the neutral axis, and concentrating it at the top and bottom of the section. If, however, this reduction in weight is not to decrease the moment of inertia, the beam must be made deeper. Using these arguments, engineers finally arrived at the shape of the standard steel beams, which can be seen in use in thousands of buildings and other structures.

The final table shows that a 6-in. $\times$ 3-in. **I** section weighs only 12 lb. per ft. and still has the same section modulus as the rectangular beam. The metal of the cross-section is so well placed that the moment of inertia is considerably increased, and a final advantage is that the deflection of the beam in the centre, when loaded, is not so great as for the rectangular section. This **I** beam, on a span of 6 ft., will carry $2{,}320 - 12 = 2{,}308$ lb. per ft. for the same maximum stress as is developed by the rectangular section in carrying 2,287 lb. per ft.

Unsymmetrical Beam Sections

T beams are somewhat different from **I** beams in that the neutral axis is not in the centre of the depth. The neutral axis always passes through the centroid (or centre of gravity) of the section, and

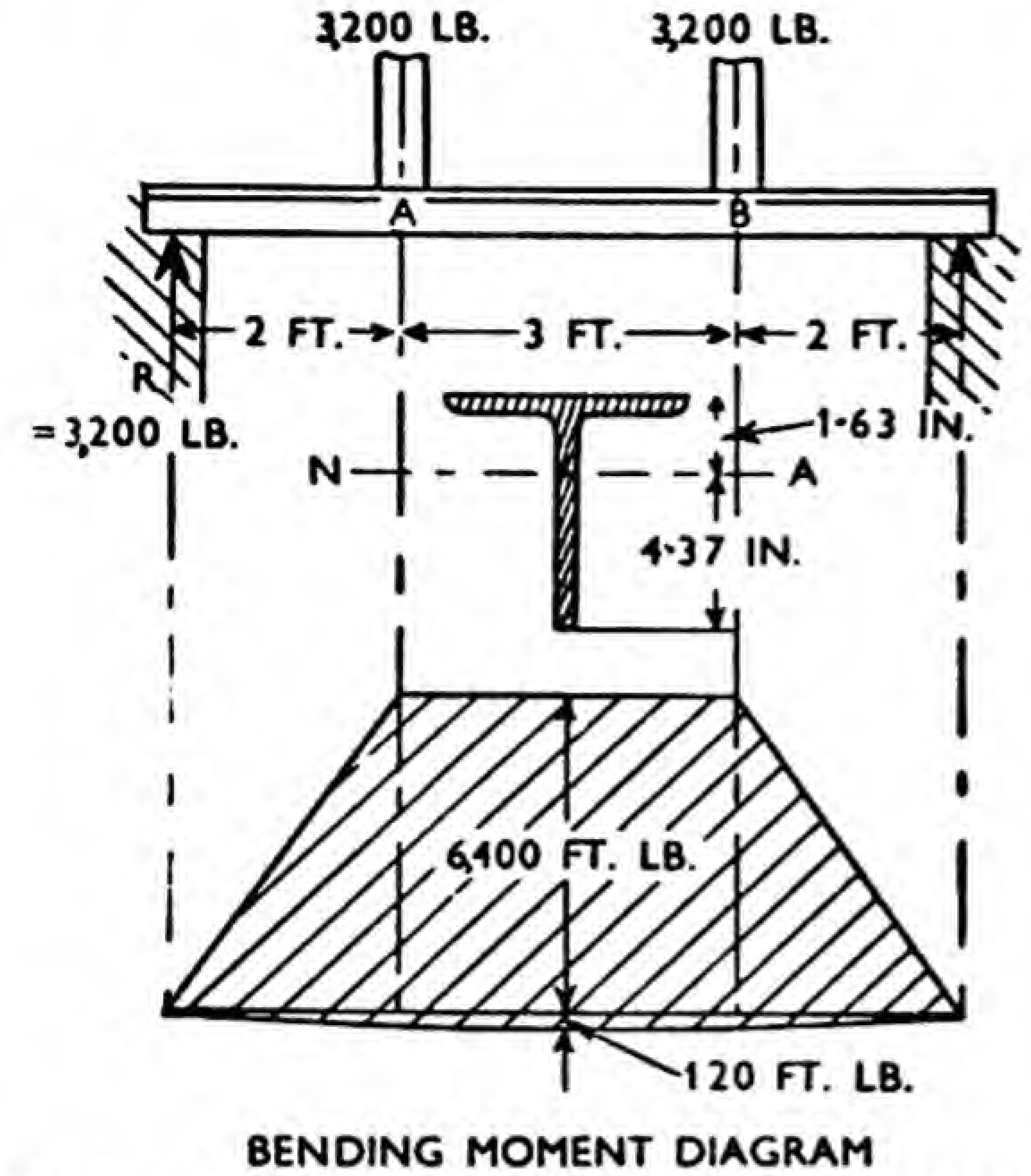


Fig. 46. If a beam cross-section is symmetrical about the centre, the neutral axis is midway from the top or bottom. When the centroid or centre of gravity of the cross-section is above the centre, so is the neutral axis. Two examples of this are the steel T beam and the reinforced concrete beam.

Fig. 46 shows that the neutral axis of a T measuring 6 in. × 6 in. × ½ in. is 1·63 in. below the top.

Suppose this T beam is used to support two loads of 3,200 lb. each, on a span of 7 ft., as is shown in Fig. 46. What is the maximum tensile stress and the maximum compressive stress?

The first thing to do is to draw the bending-moment diagram, in order that the maximum bending moment on the beam may be determined. The loading consists of the two vertical loads, and also of the weight of the beam itself. The bending moments for these two loads may be drawn separately, and the results added together.

Covering up the right-hand portion of the beam beyond *A*, it is seen that the bending moment at *A* is *R* lb. × 2 ft. = 6,400 ft.-lb. This is constant between *A* and *B*. The maximum bending moment due to the weight of the beam itself (19·62 lb. per ft.) is $\frac{wl^2}{8}$. This is $\frac{19{\cdot}62 \times 49}{8} = 120$ ft.-lb. The total maximum bending moment is the sum of these two, or 6,520 ft.-lb. Converting to in.-tons for convenience :—

$$6{,}520 \text{ ft.-lb.} = \frac{6{,}520 \times 12}{2{,}240} \text{ in.-tons}$$

$$= 34{\cdot}8 \text{ in.-tons.}$$

But $f = \frac{My}{I}$, from which the maximum stress can be found. In this instance there are two different values of y, and thus two different values of stress are obtained :—

Tension side $y = 4{\cdot}37$ in. ;

$$f = \frac{34{\cdot}8 \times 4{\cdot}37}{19{\cdot}04} = 8 \text{ tons per sq. in.}$$

Compression side $y = 1{\cdot}63$ in. ;

$$f = \frac{34{\cdot}8 \times 1{\cdot}63}{19{\cdot}04} = 3 \text{ tons per sq. in.}$$

Compound Beam

When a beam is required to carry a heavy load, it may be built up of a

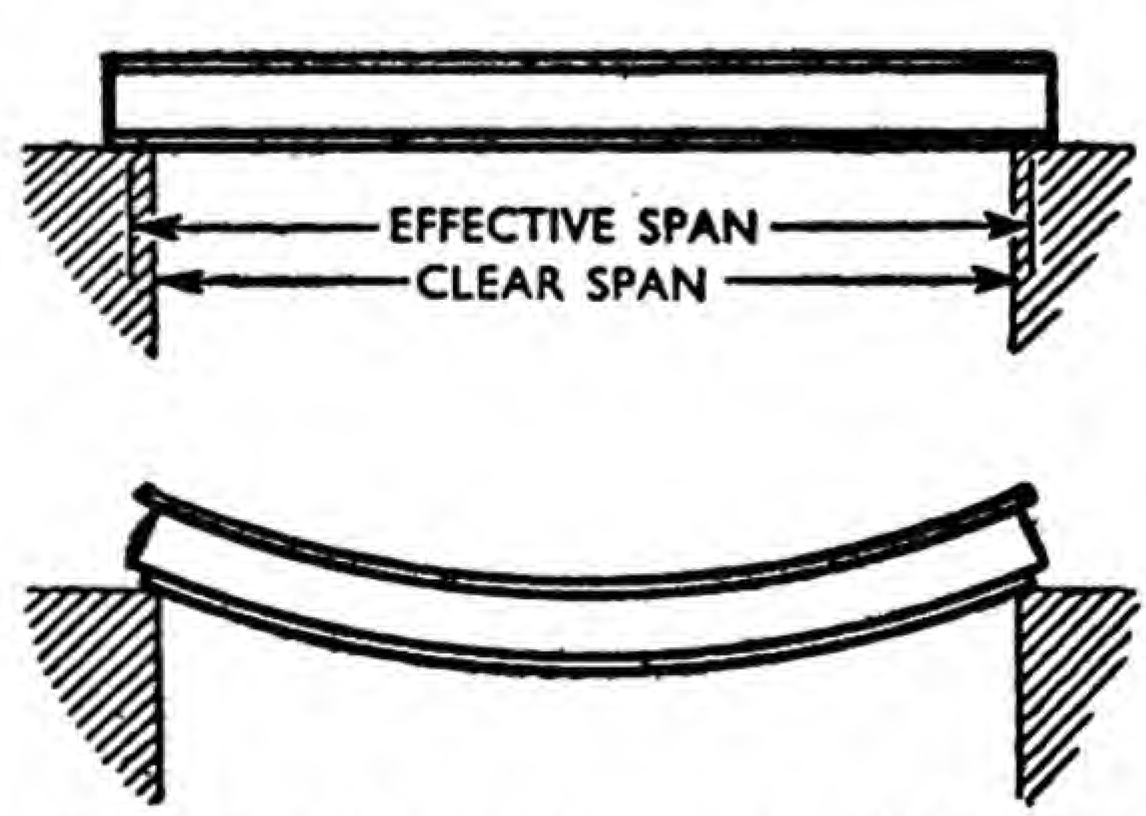


Fig. 47. When a compound beam is bent under a load, a shearing action takes place between the plates and the I beam. If this sliding were not prevented by rivets the plates would slip, as shown in the second figure.

standard **I** beam with plates on each flange, as in Fig. 26. These plates must be riveted to the beam, and it is necessary to find out how many rivets will be required. In Fig. 47 the effect of bending such a beam is shown to an exaggerated scale. The beam, carrying an extra flange plate, top and bottom, is loaded, and as it deflects the plates tend to retain their original length.

The result is that the top plate tries to slide outward relative to the beam and the bottom plate to slide inward. This tendency to slide is a horizontal shear. The function of the rivets is to prevent such sliding, and so stiffen the beam, which then acts together with the plates, as one monolithic whole.

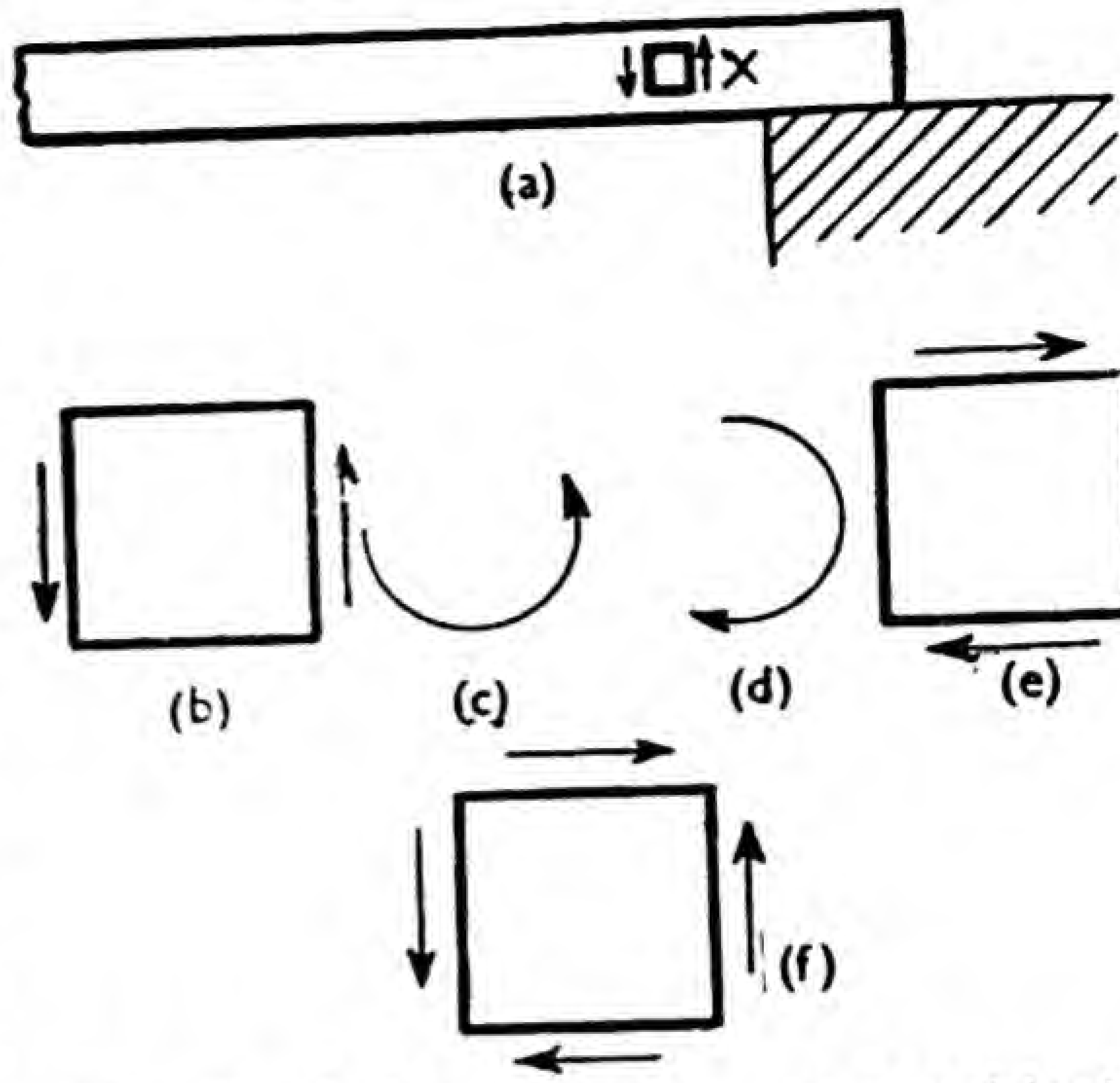


Fig. 48. It is customary, in considering stress effects, to examine the behaviour of an infinitesimally small portion of the material. Here, a very small cube is under the influence of both a vertical and a horizontal shear stress. The cube has been much magnified even in (*a*).

The connexion between vertical shear, which has been studied earlier in the chapter, and horizontal shear, is illustrated in Fig. 48. In Fig. 48(a) there is a beam similar to that shown in Fig. 9.

If a small square portion of the beam is cut out, and enlarged (Fig. 48(b)) it will be seen that it is acted on by a negative shear (Fig. 5). Such shearing forces would cause the little block of material to spin (Fig. 48(c)) and, since it is known that no part of the beam does spin, there must be other forces acting on the portion *X*, in order to keep it in equilibrium. These forces must tend to cause a spin in the opposite direction (Fig. 48(d)), and forces such as are shown in Fig. 48(e) will give this effect. The way in which the vertical and horizontal shears balance each other is shown in Fig. 48(f). In other words, a vertical shearing stress gives rise to a horizontal shearing stress, and it can be shown by a simple calculation that these are, in fact, equal in intensity.

This fact can be used to determine the spacing or pitch of rivets in a compound beam such as is shown in Fig. 49. What load can this beam safely carry on an effective span of 28 ft., and what should be the pitch of the rivets at the end of the beam? The load is to be uniformly distributed, and to include the weight of the beam itself.

The first thing to do is to calculate the moment of inertia of the plated joist :—

Moment of inertia of a British Standard beam measuring 12 in. × 5 in. is 221 in.^4.

Moment of inertia of two plates measuring 9 in. × $\frac{1}{2}$ in., is, according to the methods of Fig. 26 :—

$$2(4\tfrac{1}{2} \times 6\tfrac{1}{4}^2) = 352 \text{ in.}^4.$$

Total moment of inertia

$$= 221 + 352 = 573 \text{ in.}^4.$$

If, however it is imagined that

the beam is cut through at a point such as X in Fig. 49(c), the section will look like Fig. 49(a). Two rivet holes have been cut out from this section, and the gross moment of inertia must be reduced by the moment of inertia of these two blank areas :—

Moment of inertia of rivet holes
$= \text{Area} \times (6 \text{ in.})^2$
$= (1{\cdot}03 \times \frac{13}{16})\ 36 = 61 \text{ in.}^4.$

Net moment of inertia of the section, taking into account the loss of area due to the presence of rivet holes is :—
$573 - 61 = 512 \text{ in.}^4.$

Finding Bending Moment

The section modulus is $\frac{I}{y}$, where y is the distance from the neutral axis to the extreme fibres, top or bottom. Here, y is $6\frac{1}{2}$ in., and the section modulus is $\frac{512}{6\frac{1}{2}} = 79 \text{ in.}^3$.

The bending moment which the beam will carry is found from the expression $M = fz$, where f is the allowable safe stress, which is 8 tons per in.^{2}. Thus :—

$$M = 8 \text{ tons per in.}^2 \times 79 \text{ in.}^3$$
$$= 632 \text{ in.-tons.}$$

Since the load is uniformly distributed along the length of the beam, the maximum bending moment (Fig. 19) is $\frac{wl^2}{8}$.

$$M = \frac{w \times 28^2}{8} = 98\ w \text{ ft.-tons}$$
(w is in tons per ft.)
$$= 1{,}176\ w \text{ in.-tons.}$$

The moment of resistance which is found to be 632 in.-tons must, of course, be equal to the bending moment, if the beam is not to fail; therefore, $632 = 1{,}176\,w$, from which $w = 0{\cdot}54$ tons per ft.

In order to determine the pitch of rivets required, vertical and horizontal shear must be considered. In Fig. 45 it was shown that the most important parts of a beam section, in resisting bending stresses, are the top and bottom areas, in other words the flanges. It was shown how these areas could be concentrated at the top and bottom, as in an I beam. The thin portion connecting these flanges is known as the web, and its activity is to keep the flanges apart so that they can function in resisting bending moment. In other words, the web resists the shearing forces on the beam.

In the beam of Fig. 49, the web is 12 in. deep, and the total loading

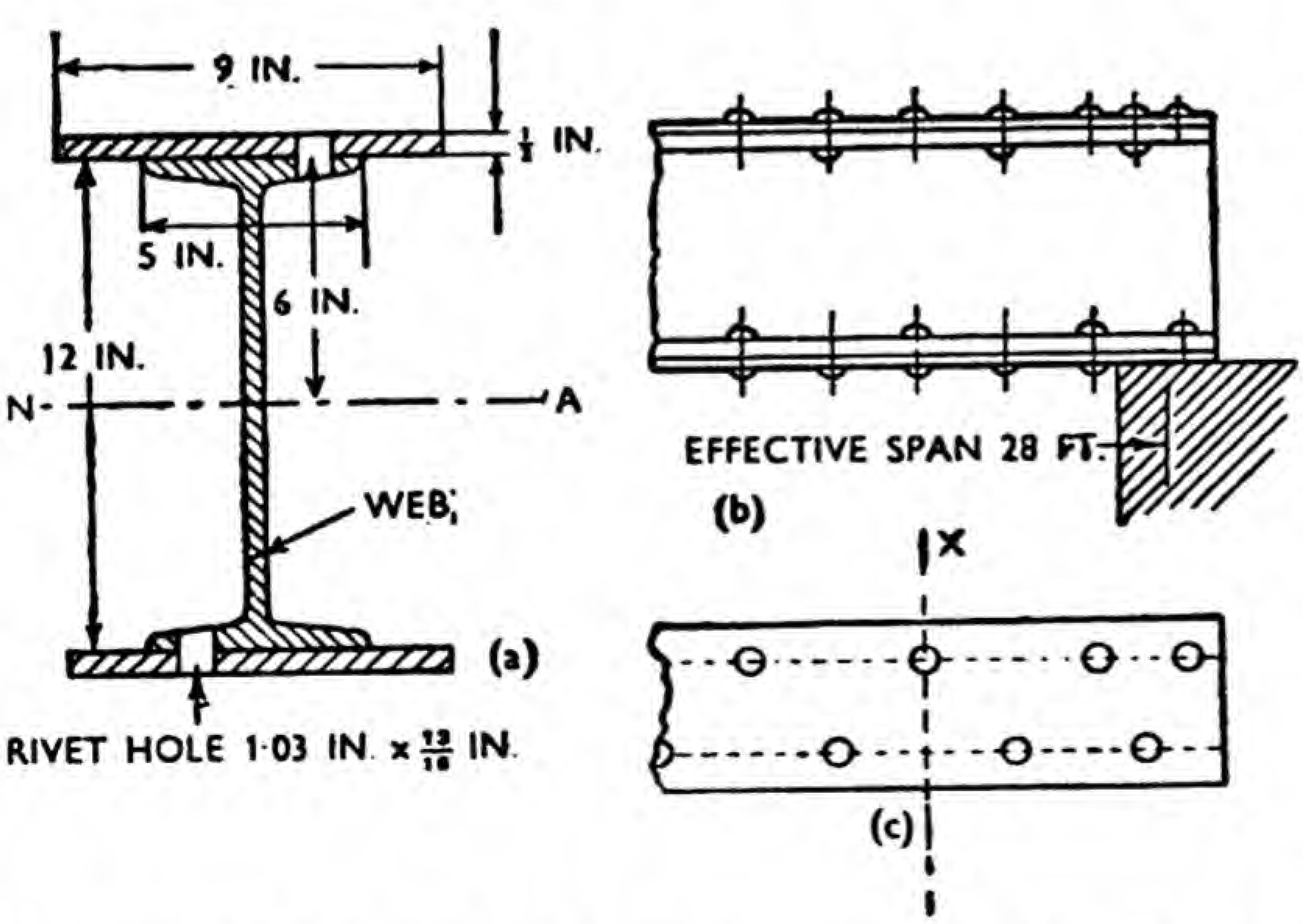


Fig. 49. When a rolled I section is not of a sufficiently high moment of inertia to resist the bending imposed on it, plates may be riveted on the upper and lower flanges. The additional cross-sectional area thus added at some distance from the neutral axis, considerably increases the section modulus.

on the beam is 15 tons. The maximum shearing force occurs at the ends, and is equal to half the load, or $7\frac{1}{2}$ tons. The total vertical shear on the web, therefore, is $7\frac{1}{2}$ tons per ft. of depth, since the depth happens to be 1 ft.

According to Fig. 48 this vertical shear calls into play a horizontal shear of an equal intensity, which tries to make the top and bottom plates slide on the beam flanges (Fig. 47). If this sliding is to be safely prevented, there must be enough rivets between the flange plate and the beam to resist a shearing force of $7\frac{1}{2}$ tons per ft. A $\frac{3}{4}$-in. rivet in single shear has a rivet value of 2·65 tons, so the number of rivets per ft. required to fasten the top plate, and the bottom plate, to the beam is $\frac{7\frac{1}{2}}{2 \cdot 65} =$ 2·83 rivets per ft. If these rivets were all in one row they would be $\frac{12}{2 \cdot 83} = 4\frac{1}{4}$ in. apart. Since they are in two rows, the pitch on each row could be $8\frac{1}{2}$ in. In fact, the pitch would not be made so large, 6 in. probably being used.

Deflection of Beams

In designing a beam such as that of Fig. 49, it would be necessary to calculate the deflection of the beam at the centre for, although the beam may be sufficiently strong, it may bend too much under the load. This point must be carefully watched if the beam is carrying a plaster ceiling or other covering that is likely to crack. The usual allowance in buildings is a deflection of $\frac{1}{325}$ part of the span. Sometimes a big deflection is quite unimportant as long as the beam is sufficiently strong.

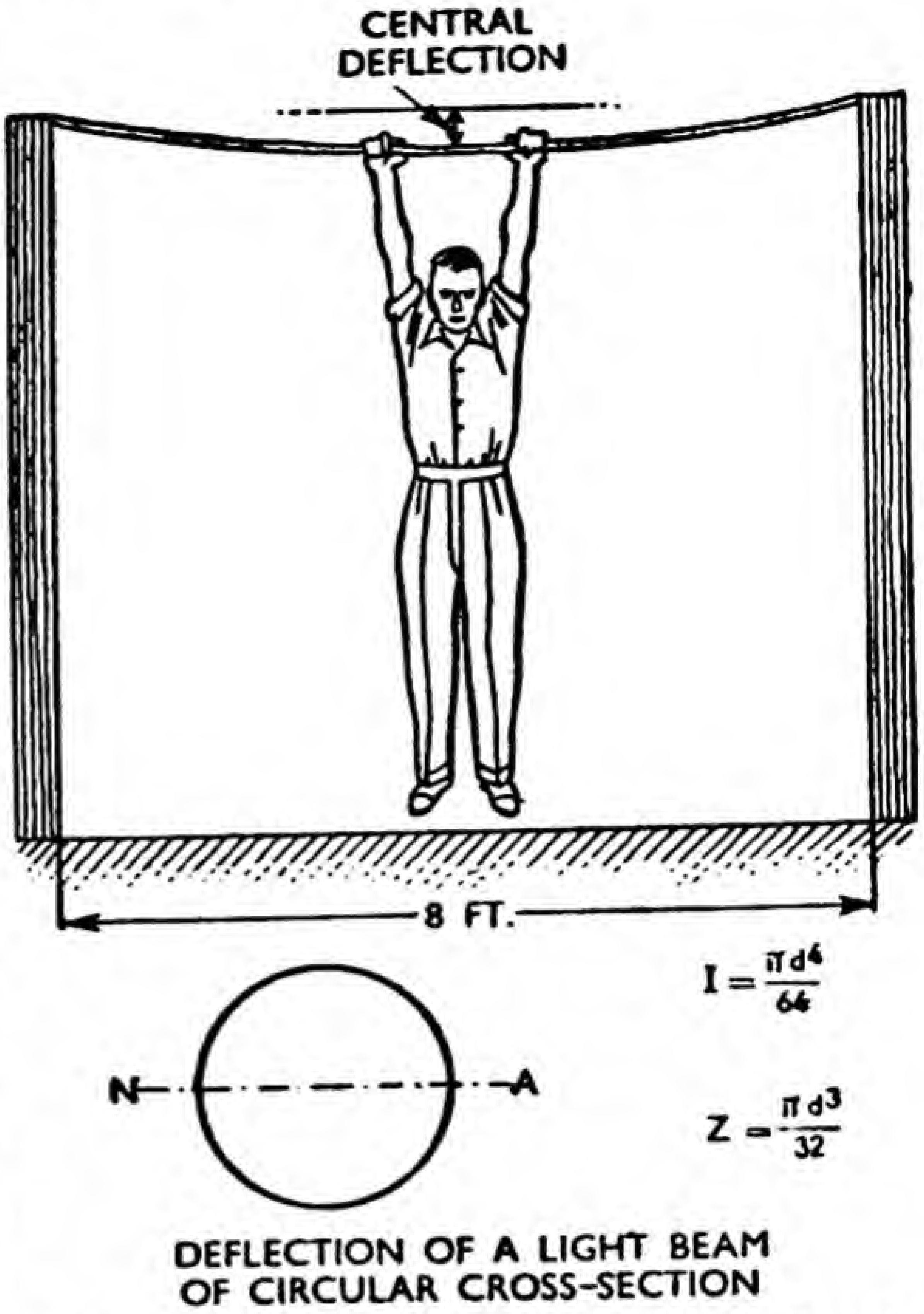


Fig. 50. The amount of deflection shown by a bridge girder or by a steel beam carrying, for example, a plaster ceiling, may be very important. The engineer calculates the deflection of a beam from a knowledge of the load and the moment of inertia of the section.

A man weighing 11 stones hangs by his hands in the centre of a steel bar in a gymnasium. What should be the diameter of the bar? What is its deflection at the centre? (Fig. 50).

From Fig. 19 the maximum bending moment is $\frac{Wl}{4}$. From Fig. 23 the moment of inertia is $\frac{\pi d^4}{64}$, and it is known that for steel the safe stress is 8 tons per in.^{2} or sometimes 9 tons per in.^{2}. It is first necessary to calculate the numerical values of the various quantities, and then apply them to the flexure formula :—

$$M = \frac{(11 \times 14) \times 8}{4} = 308 \text{ ft.-lb.}$$
$$= 1 \cdot 65 \text{ in.-tons.}$$

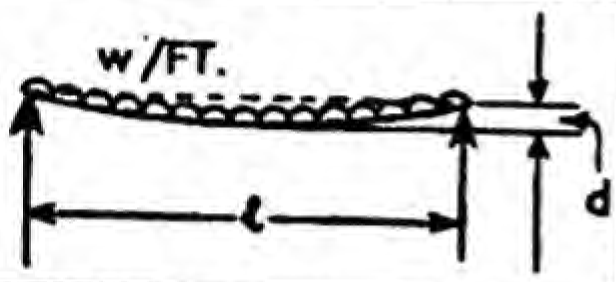
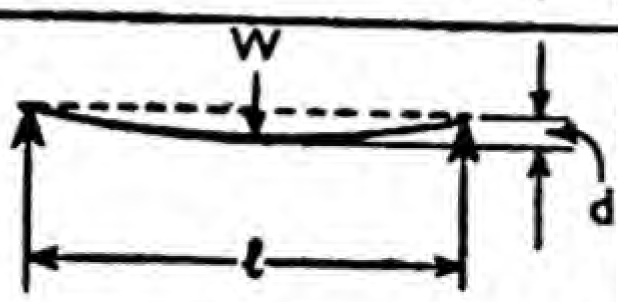
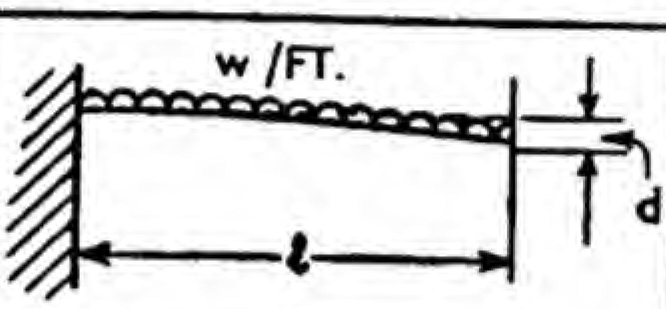
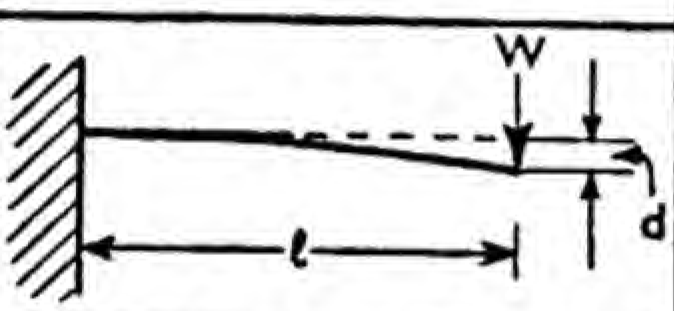
VALUES OF MAXIMUM DEFLECTION	
LOADING	d
	$\frac{5wl^4}{384EI}$
	$\frac{Wl^3}{48EI}$
	$\frac{wl^4}{8EI}$
	$\frac{Wl^3}{3EI}$

Fig. 51. It is useful to have easy reference to the values of the maximum deflections caused by loadings frequently encountered in buildings. It is obvious that deflection varies inversely as the moment of inertia and (for a given total load) directly as the cube of the span.

$z = \frac{I}{y} = \frac{\pi d^4}{64} \times \frac{2}{d} = \frac{\pi d^3}{32}$ (d is diameter in inches).

Flexure formula :–$M = fz$;

$$1{\cdot}65 = 8 \times \frac{\pi d^3}{32}$$

$$\therefore d^3 = 2{\cdot}1 \text{ in.}^3$$

$$d = 1{\cdot}28 \text{ in., or approximately } 1\tfrac{1}{4} \text{ in. diameter.}$$

The deflection is dependent on loading, span, and moment of inertia. As the first two increase, the deflection increases, and as the third increases, the stiffness of the beam increases and the deflection decreases. The mathematical expressions for deflection under simple loading conditions are given in Fig. 51, and only the numerical values of the various terms need to be inserted.

Using lb. and in. as the units, the deflection of the steel bar will be :—

$$\frac{Wl^3}{48EI} = \frac{154 \text{ lb.} \times 96^3 \text{ in.}^3}{48 \times 30 \times 10^6 \text{ lb. per in.}^2 \times 0{\cdot}12 \text{ in.}^4}$$
$$= 0{\cdot}79 \text{ in.}$$

Welded Joints

The joining of two pieces of metal by heating so that they fuse or melt into each other has been practised from time immemorial by blacksmiths. In modern times advances have, of course, been made. Now, the heat is applied just where it is required by means of a gas flame or an electric arc, and, in addition, extra metal is run into the joint. The parent metal, in other words the original plates which are to be joined, and the weld metal, which is the extra metal that is to be melted into the joint, fuse together to give a strong connexion between the two plates.

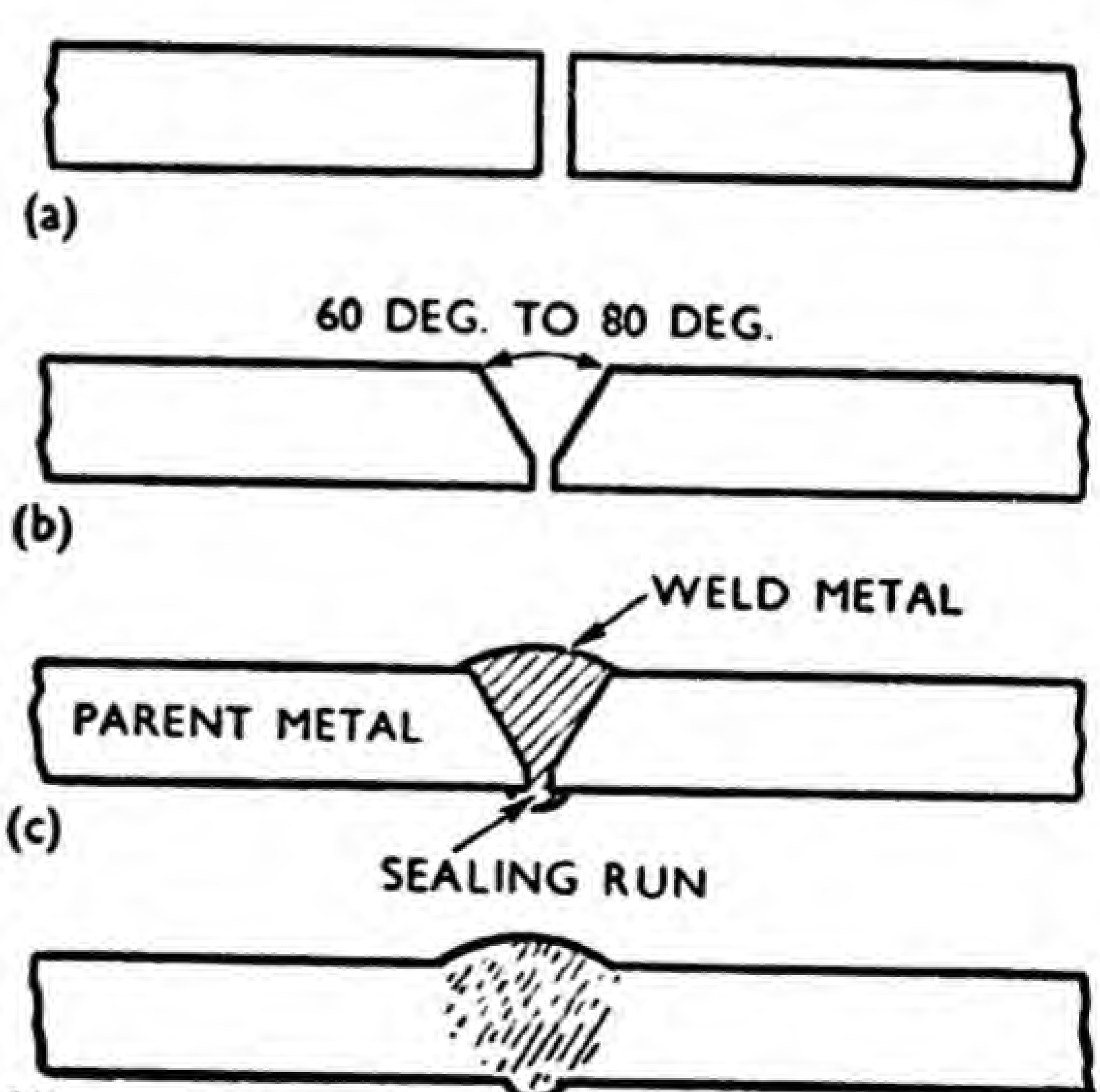


Fig. 52. Welding is an important method of structural jointing. Plates which are to be joined in a butt weld are chamfered at the ends to form a V notch. The molten weld metal is then run in so that parent and weld metal fuse into a homogeneous whole.

In electric-arc welding there are two types of welded joints. Fig. 52 shows a butt weld in which the plates are put end to end and welded together in that position. It is obvious that it would be almost impossible to run in weld metal if the plates were cut square as shown in Fig. 52(a). It is usual, therefore, to shape the ends to form a groove (Fig. 52(b)) or notch which can be filled with weld metal. The weld metal is used in the form of thin rods or electrodes through which the electric current passes, and at the end of which the arc is formed. The electrodes become shorter as they melt and run into the notched joint.

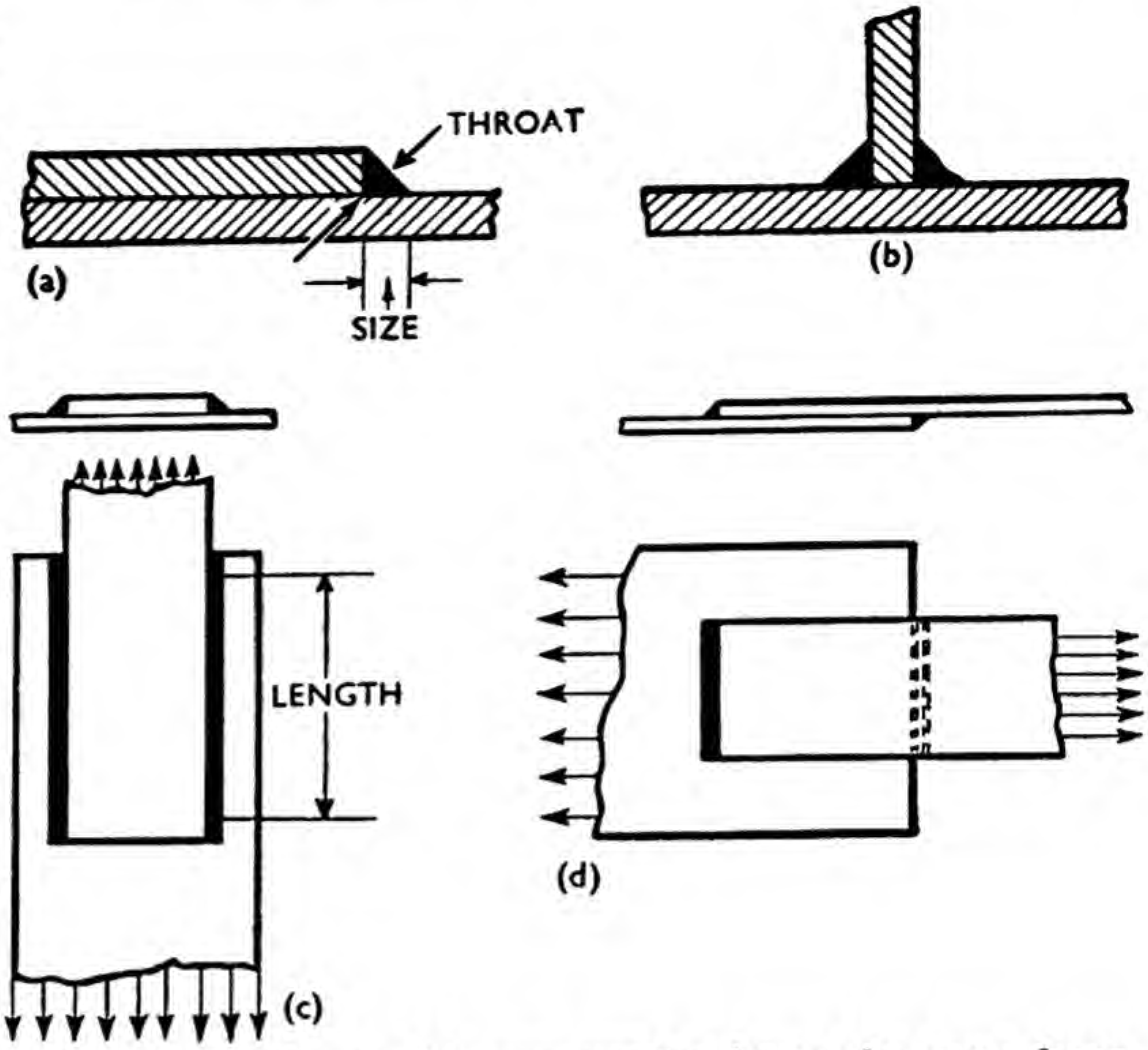


Fig. 53. Fillet welds are used where the two faces of the material are at right angles. The weld metal forms a triangular fillet which fuses with both faces of the parent metal. Plates may be joined at right angles (*b*) or, in lap joints (*a*), (*c*) and (*d*), by means of end or side fillets. The effective throat thickness of a fillet is 0·7 times its size.

Fig. 52(c) shows the conventional representation of a butt weld, but, in fact, the weld metal and the parent metal are much more closely fused together and Fig. 52(d) gives a much better impression of how the plates are joined.

It is not possible to design this type of joint or make calculations. The metal should be merely run in under the best conditions and by skilled workmen. The joint would then be as strong as the plate, with an efficiency of 100 per cent.

Fillet Welds

The second type of welded joint made by the electric-arc process is the fillet weld. This is used where one plate is laid on another, either lying flat, or at right angles to each other. The weld metal is then run in to form a small triangular fillet, as shown in Figs. 53(a) and (b).

If lap joints are considered when one plate is lapped or laid on the other, the fillet welds can be arranged in two ways. They may either run in the same direction as the pull on the plates (Fig. 53(c)) when they are called side fillets, or they may run at right angles to the direction of pull on the plate, when they are called end fillets.

The smallest thickness of a fillet is called the throat thickness, and since fillets are supposed to be made with the sloping face at 45 deg. to the plate, this throat thickness can be considered to be 0·7 of the size (Fig. 53(a)). The

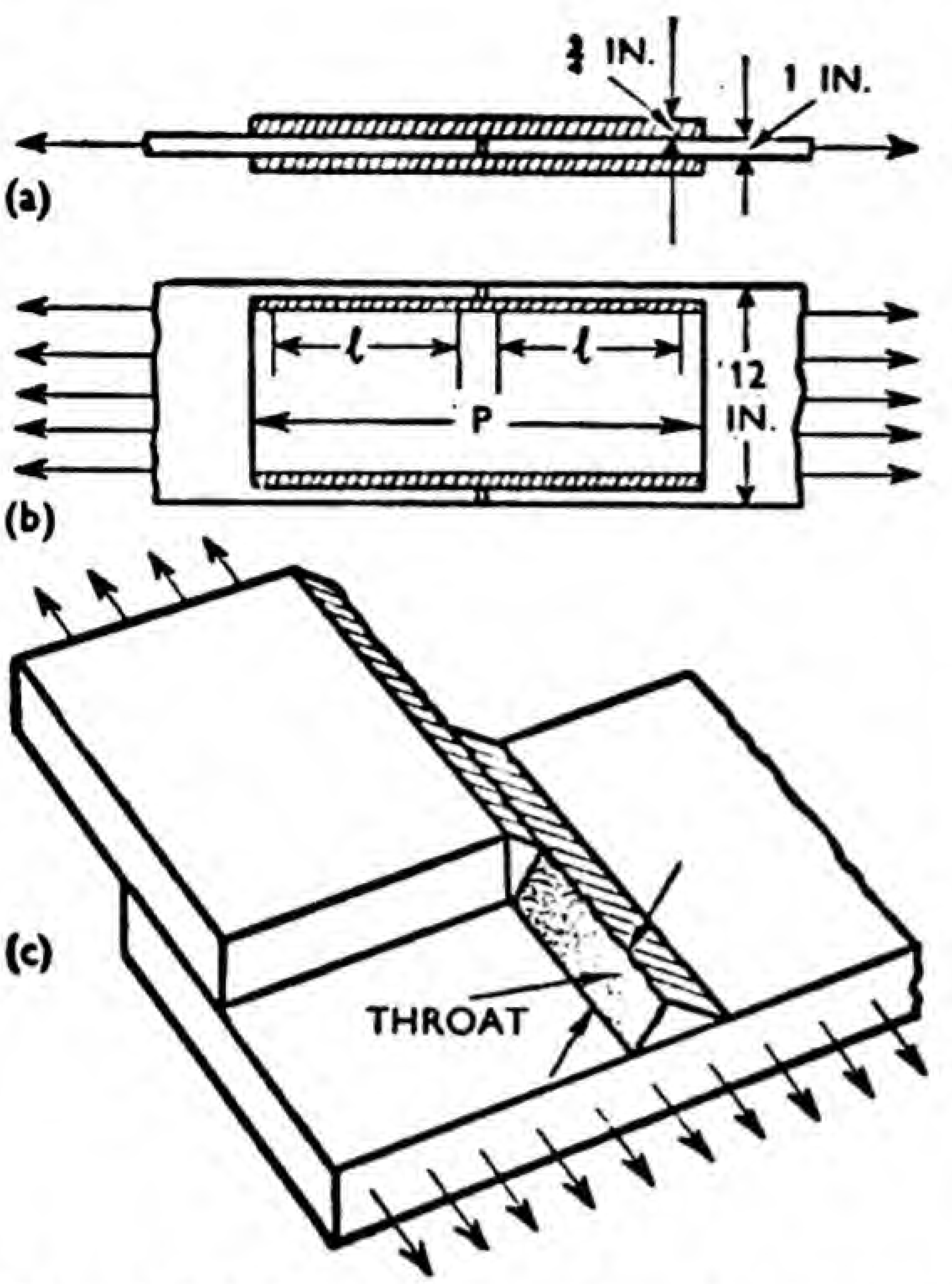


Fig. 54 (*a*) and (*b*). Double-covered butt joints may be made by welding cover plates to the main tie bar by side fillet welds. (c) Shows how such a weld might fail by shearing along its length on the narrowest (throat) area. A safe working stress, much less than the ultimate stress, is used in practice.

length of the weld to be used in calculations (Fig. 53(c)) is less than the overall length by twice the size, because a short length at each end must be considered ineffective.

When designing fillet welds, the problem usually consists of determining the size of the weld when its length is known, or of finding the length when the size has been fixed.

Covered Butt Joint

Suppose, for example, that it is necessary to make a joint in a steel plate measuring 12 in. × 1 in. It is proposed to use double cover plates which are each ¾ in. thick, so the size of the welds cannot be more than ¾ in. (Fig. 54).

Each plate is fastened to the covers by four welds, each of an effective length l, two above and two below. If one of these welds fails it is assumed to shear, as shown in Fig. 54(c). The sheared area is found by multiplying the length by the throat thickness.

Strength of Weld

The load which the original plate may carry at 9 tons per sq. in. is 12 sq. in. × 9 tons per sq. in. = 108 tons. Each of the four welds must thus safely carry a load of 27 tons.

The throat thickness of a ¾-in. weld is ¾ × 0·7 = 0·515 in.

The safe allowable stress on a side fillet is 5 tons per sq. in.

The load which each weld can carry, therefore, is :—

$l \times 0{\cdot}515 \times 5 = 2{\cdot}575l.$

This must, of course, be equal to 27 tons, from which relationship the effective length l is found to be

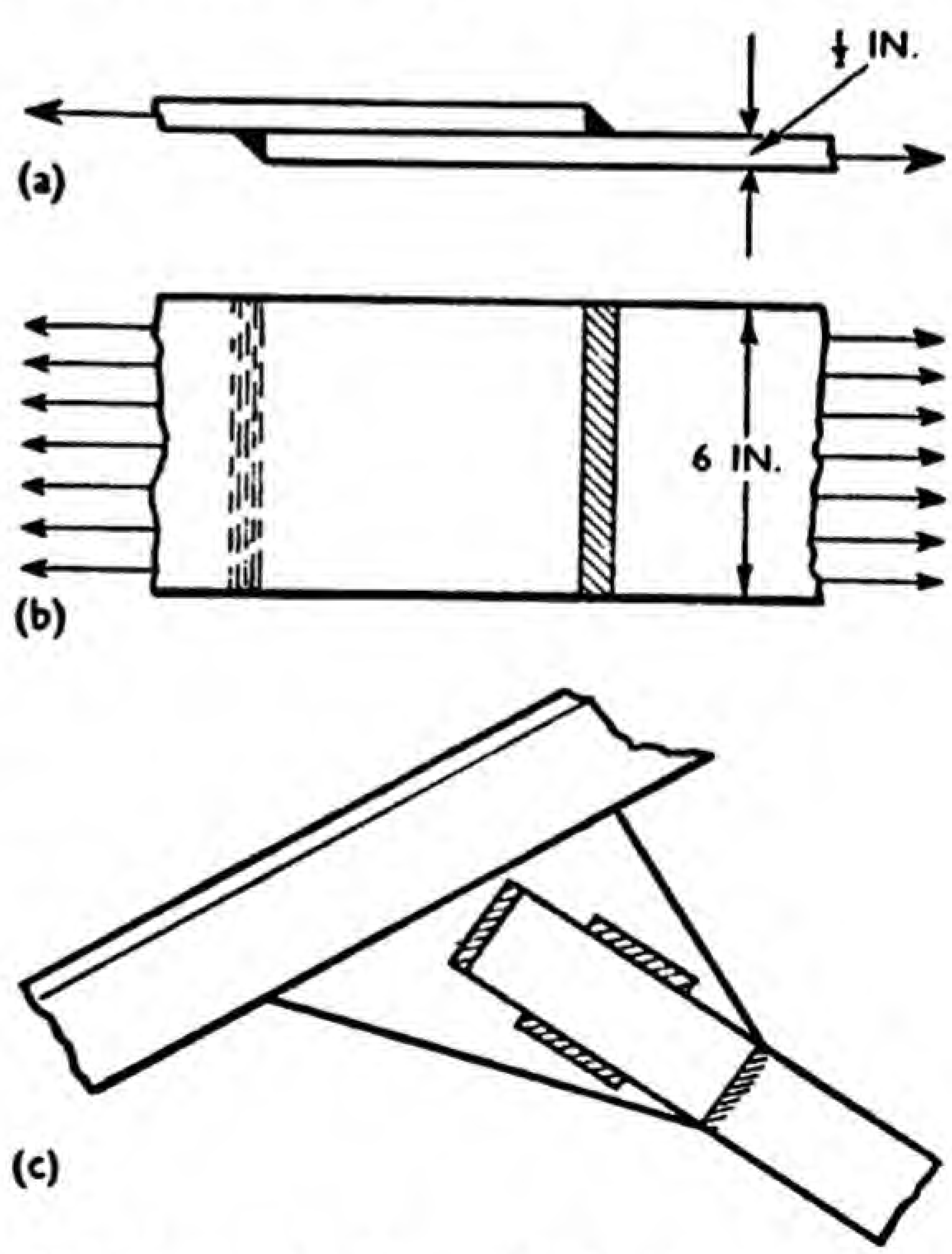


Fig. 55. Fillet welds lying at right angles to the direction of the load on a lap joint, are known as end fillets. The design of a lap joint end fillets is shown here. If these fillets do not provide a sufficient total length, they may be reinforced by additional side fillet welds.

10·5 in. The total length P, of the cover plate must be :—

$$2 \times l + 4 \times \tfrac{3}{4} = 24 \text{ in.}$$

The 6-in. plate shown in Fig. 55 is to be joined by a lap joint with two end fillets. What load may the plate safely carry?

The effective length of each weld is less than 6 in. by twice the size of the weld. The effective length is, thus, $6 - 2 \times \tfrac{1}{2} = 5$ in.

The area on which the calculations may be based is the effective length multiplied by the throat thickness, remembering that there are two welds.

Cross-sectional area of welds $= 2(5 \times \tfrac{1}{2} \times 0{\cdot}7) = 3{\cdot}5$ sq. in.

The allowable stress on end fillets in lap joints is 7 tons per sq. in., and the welds together will thus safely carry $7 \times 3{\cdot}5 = 24{\cdot}5$ tons.

The original plate, at 9 tons per sq. in., would safely carry a load of $6 \times \tfrac{1}{2} \times 9 = 27$ tons. The two welds together are thus not so strong as the original plate. However, in such an instance, it is often possible to strengthen the welding by the addition of side welds. For example, the roof truss joint of

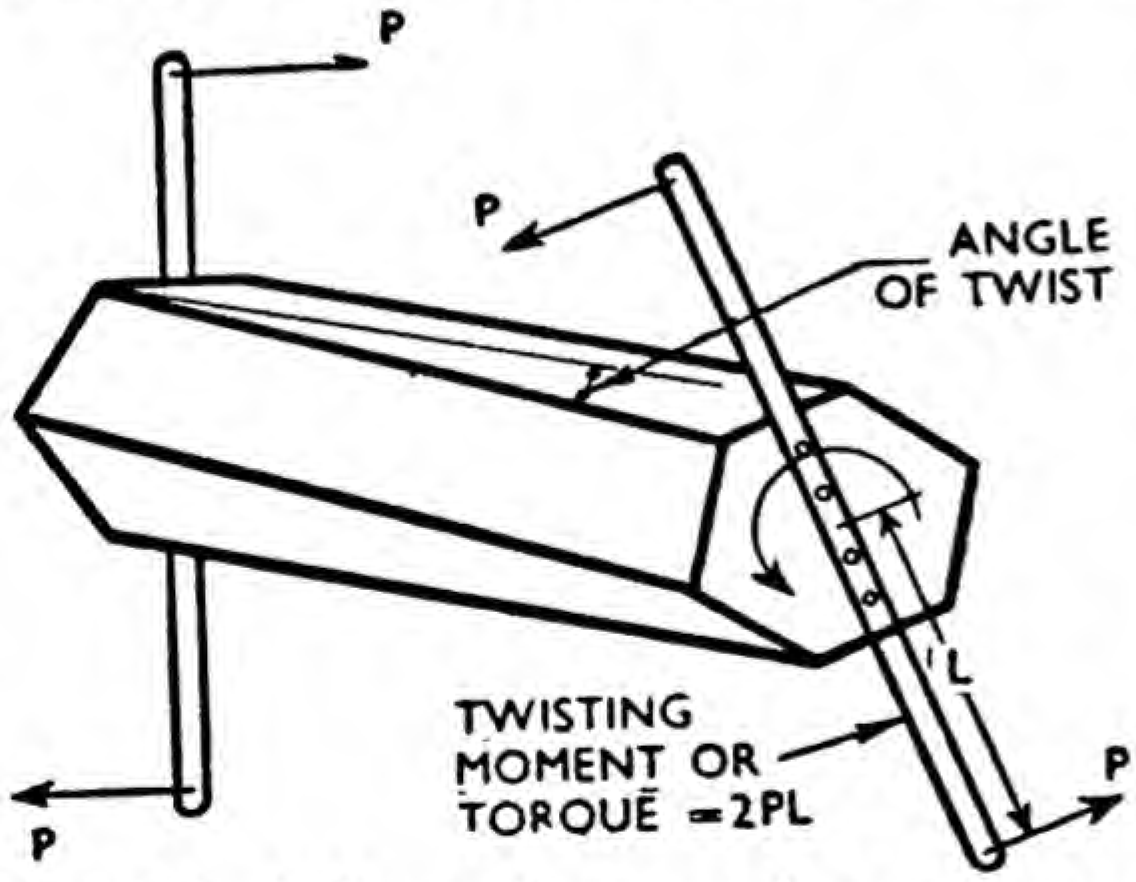


Fig. 56. A long hexagonal pencil may readily be twisted through a small angle, and the distortion of such a bar under torsion can be inspected. This chapter deals only with torsion of circular shafts. Problems involving torsion of non-circular sections are beyond the scope of this book.

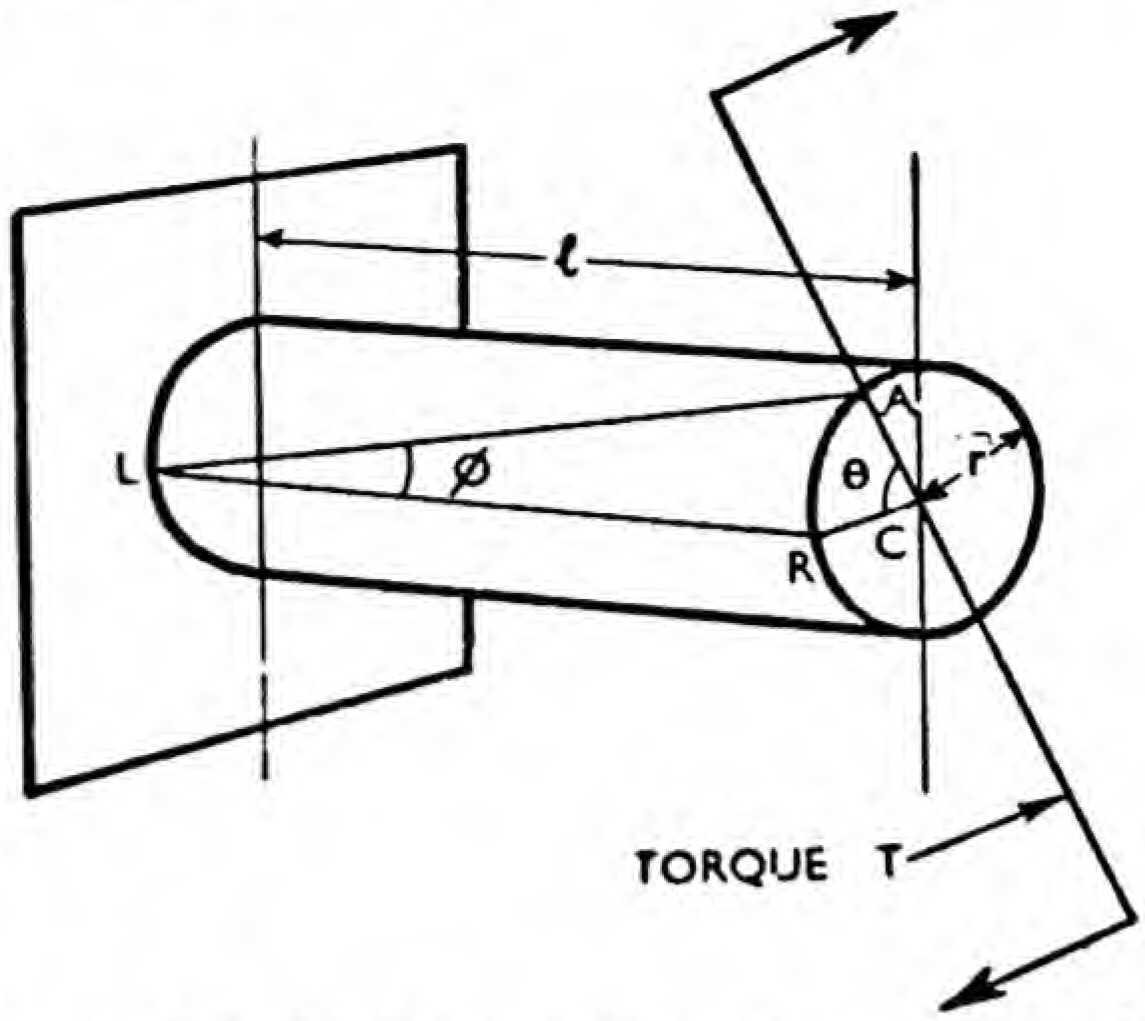


Fig. 57. When a circular bar is held rigidly at one end and twisted at the other end by a torque, the angle of twist produced bears a definite relationship to the stresses developed in the bar. The study of torsion is important in the design of shafts for transmitting power.

Fig. 38 can be made stronger than the original tie bar by the use of end and side fillets, as shown in Fig. 55.

It has now been shown how the moment of inertia of a section can be used to design beams, and to calculate the tensile and compressive stresses in them. Simple shear stress as exemplified in riveted and welded joints has also been studied.

Torsion of Circular Shafts

Turn now to another type of loading which produces a shear stress of a more complex character. This loading is known as torsion or twisting. Hold a pencil by the ends, and twist the hands in opposite directions. It can then be seen that each portion of the pencil tries to rotate relative to the next. If a long hexagonal pencil is given a torsion loading in this way, the distortion of the corners of the pencil can be easily noted (Fig. 56). The torsion of any section other than circular, however, involves

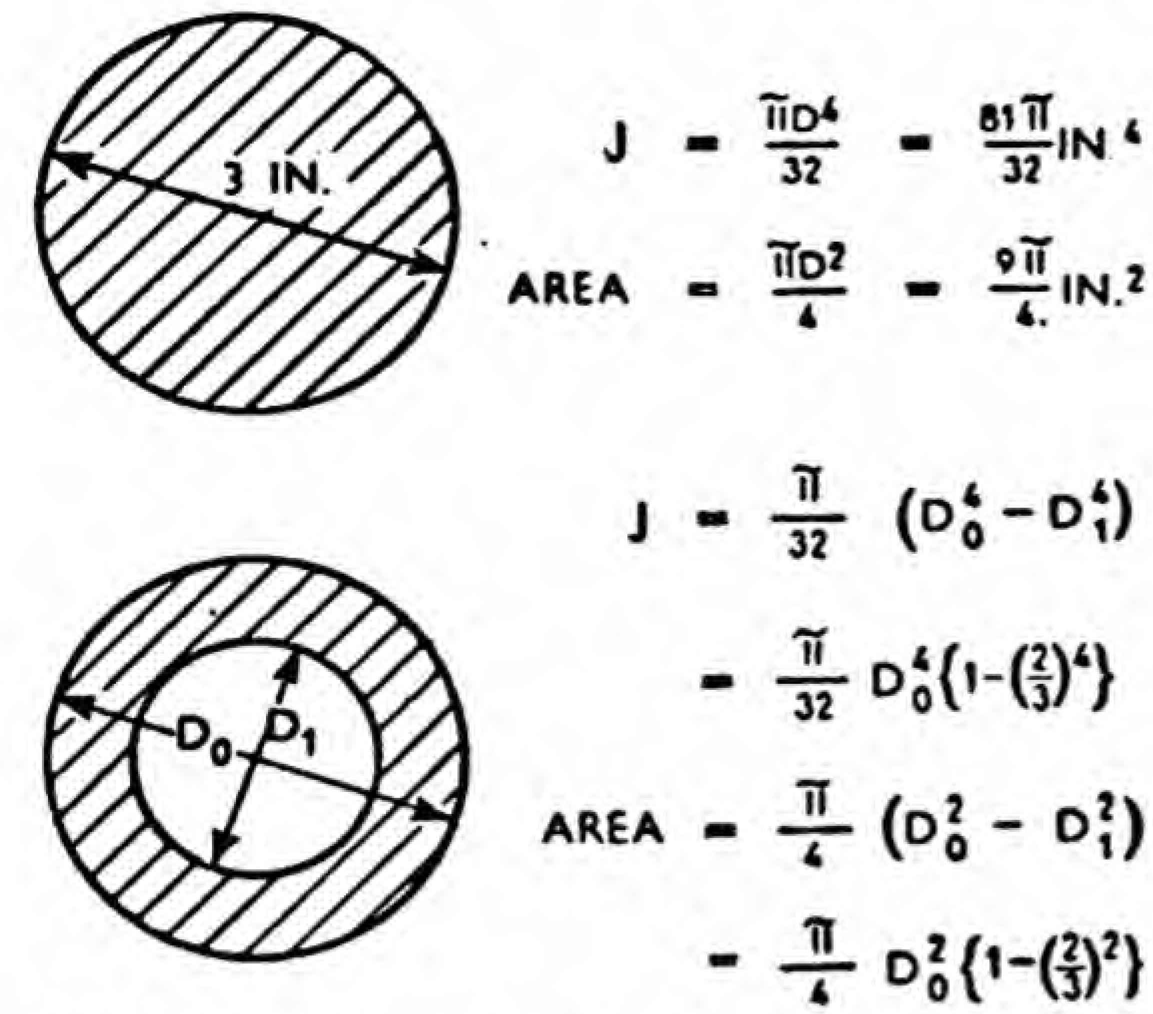


Fig. 58. It is quite possible to remove material from a shaft without reducing very appreciably its resistance to torsion. Massive material is not of much value unless it is distributed where it is most effective.

more advanced problems than can be dealt with here, and only circular sections will be considered.

Fig. 57 shows a shaft, held firmly at one end and subjected to a torque at the other end. A line *LR*, drawn on the shaft, will, when a torque *T* is applied, twist round to the position *LA*. The angle through which the end of the shaft rotates is θ, and the angle through which the line *LR* is twisted is $\varnothing$. Both these angles are normally extremely small, and since they both subtend or measure off the length *RA*, the angles may be considered inversely proportional to the lengths l and r respectively. In other words :—

$$\frac{\varnothing}{\theta} = \frac{r}{l}.$$

$\varnothing$ is the shear strain (Fig. 41), and is thus equal to stress over modulus, or, in symbols :— $\frac{q}{N}$.

Therefore, $q = \varnothing N$.

But, $\varnothing = \frac{\theta r}{l}$.

Therefore, $q = \frac{\theta r N}{l}$, or $\theta = \frac{ql}{rN}$.

So if the angle through which the twisted end of the shaft rotates is known, the maximum shear stress produced can be found. Alternatively, if the shear stress produced is known, the angle through which the shaft will turn can be found. This twisting of the shaft under a torque is analogous to the deflection of a beam under a bending moment. Both are distortions due to induced stress.

Further analogies between a beam subjected to bending and a shaft subjected to torque are shown in Table II. It can be seen that the flexure formula $\left(f = \frac{My}{I}\right)$ is analo-

TABLE II

Comparison of Formulæ and Symbols for Bending and Torsion

Bending	*Symbol*	Torsion	*Symbol*
Bending Moment	M	Torque	T
Distance from N.A. to top or bottom of beam	y	Radius of Shaft	r
Moment of Inertia	I	Polar Moment of Inertia	J
Maximum Tensile or Compressive Stress	$f = \frac{My}{I}$	Maximum Shear Stress	$q = \frac{Tr}{J}$
Modulus of Elasticity	E	Modulus of Rigidity	N

gous to the formula for determining shear stress $\left(q = \frac{Tr}{J}\right)$.

A steel shaft is 3 in. in diameter, and transmits a torque of 2,190 ft.-lb. What is the maximum shear stress in the shaft, and how much does the shaft twist between two pulleys 10 ft. apart?

$T = 2{,}190 \times 12$ in.-lb.

J for a solid circular shaft

$= \frac{\pi D^4}{32}$ (Fig. 23)

$\therefore \frac{J}{r} = \frac{\pi D^3}{16}$

and $q = \frac{Tr}{J} = \frac{2{,}190 \times 12 \times 16}{\pi \times 3^3}$

$= 4{,}950$ lb. per sq. in.

and θ (angle of twist) $= \frac{ql}{rN} =$

$\frac{4{,}950 \times 120}{1{\cdot}5 \times 12 \times 10^6} = 0{\cdot}033$ or $\frac{1}{30}$ radians.

Thus a horizontal line, such as RC in Fig. 57, rotates to a new position AC at a slope of 1 in 30. The angles of twist in Fig. 57 are, of course, exaggerated.

If this shaft is to be replaced by a hollow shaft, with an inside diameter of two-thirds the outside diameter, and the same maximum shear stress is to be permitted, what is the saving in weight?

Fig. 58 shows the hollow shaft together with the solid one which it is to replace. Fig. 45 shows that beams carrying the same bending moment while developing the same maximum stress, have the same value of section modulus. Similarly, shafts which carry the same twisting moment (torque) while developing the same maximum shearing stress must have the same value of $\frac{J}{r}$.

$\frac{J}{r}$ for solid shaft $= \frac{J}{r}$ for hollow shaft.

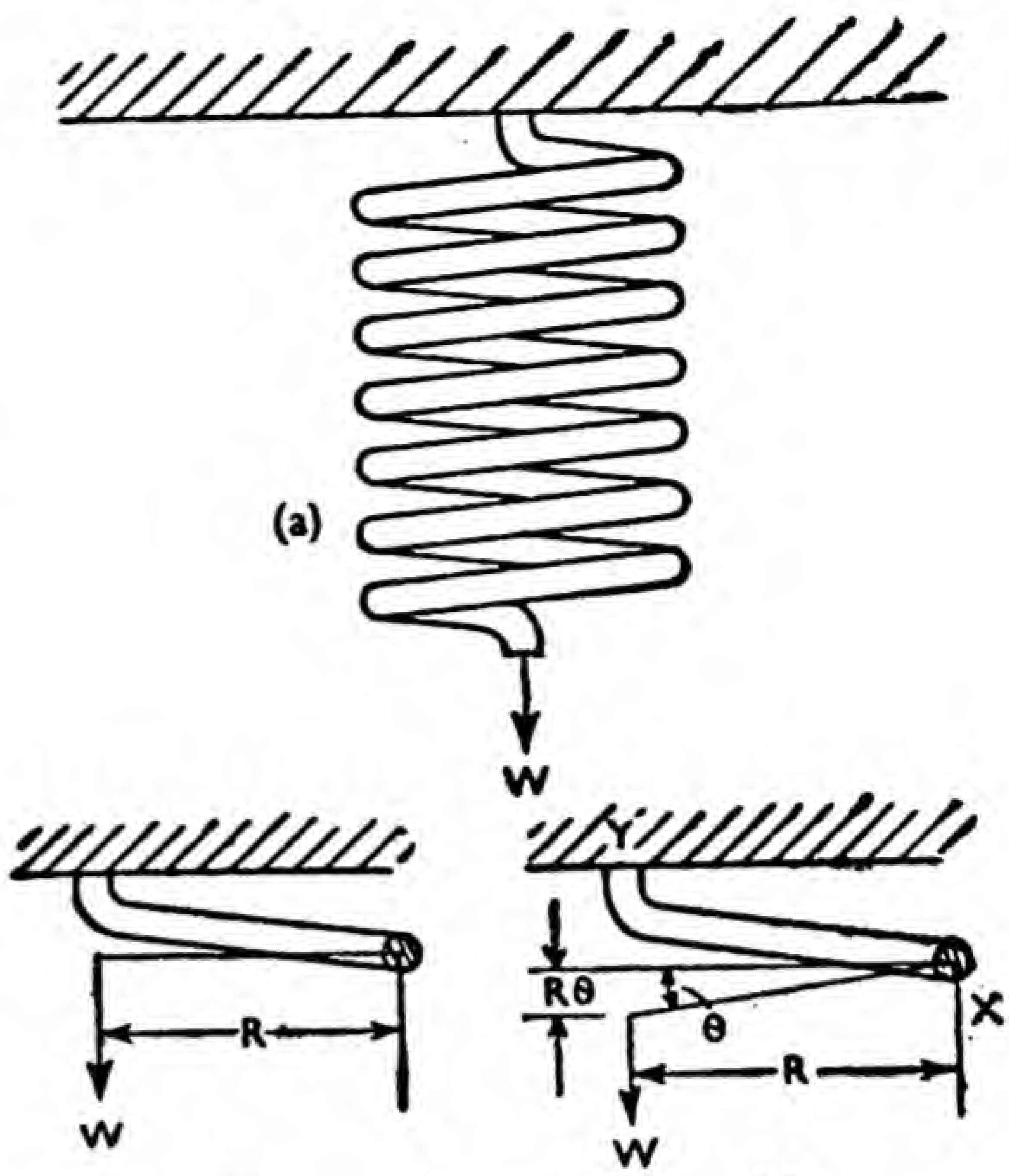


Fig. 59. A twisted straight bar may have a spring-like effect and such a bar is often used as a door-closer (see Fig. 61). If the bar is coiled, however, a much longer and more powerful spring can be put into a much smaller space. The cross-section is still subjected to a torque.

$$\frac{\pi D^3}{16} = \frac{\pi D_o^4}{32}\left(1 - (\tfrac{2}{3})^4\right)\frac{2}{D_o}$$

$$27 = D_o^3\,(1 - \tfrac{16}{81})$$

$$D_o^3 = \frac{81 \times 27}{65}$$

$$D_o = 3{\cdot}2 \text{ in.}$$

The percentage saving in weight is the same as the percentage decrease in cross-sectional area. The original area was $7{\cdot}07$ sq. in. (Fig. 58). The new area is $\frac{\pi}{4} \times 3{\cdot}2^2 \times (1 - \frac{4}{9}) = 4{\cdot}47$ sq. in. The saving in weight is thus :—

$$\frac{7{\cdot}07 - 4{\cdot}47}{7{\cdot}07} \times 100 = 37 \text{ per cent.}$$

Close-Coiled Springs

The spring balance, a well-known household instrument, is an example of how the effects produced by torsion can be put to

immediate practical use. When the coils of a spring lie close together and are almost at right angles to its length, the spring is known as a close-coiled helical spring. Fig. 59 shows such a spring, slightly open to show its construction, and supporting a weight W.

Let the first quarter of the first coil be considered, and imagine it to be cut as shown in Fig. 59(b). It will then be seen that all the spring below the cut section is merely a means of transferring the load W to that section. Since W, however, is at a distance R, which is the radius of the coil, from the section, it has a torsional effect on the section. As has been previously seen, a torque of this kind produces a definite rotation of the end X, and the amount of this rotation depends on the length of the bar affected, viz., from X to Y.

Suppose the angle of twist is θ, then the load W, which can be imagined to be hanging on a stiff arm of length R, will drop vertically through a height $R\theta$ (Fig. 59(c)). Now suppose another quarter coil is added, the length affected will be double that previously examined, but W will still be at a distance R from the cut section. The same torque WR acts, but on a longer length of rod, thus producing a larger angle of twist θ, and, therefore, a larger deflection ($R\theta$) of W.

If still more of the rod is used, a still greater deflection of W is obtained. Finally, by using a long length, coiled up for convenience, there is, in fact, a long rod with a torque WR at one end. Such a long coiled rod is known as a spring, an instrument which is sensitive to changes in torque, or changes in W, since R is constant. In lengthening or shortening under increase or decrease of load, the spring balance is merely registering changing angles of twist under varying torques.

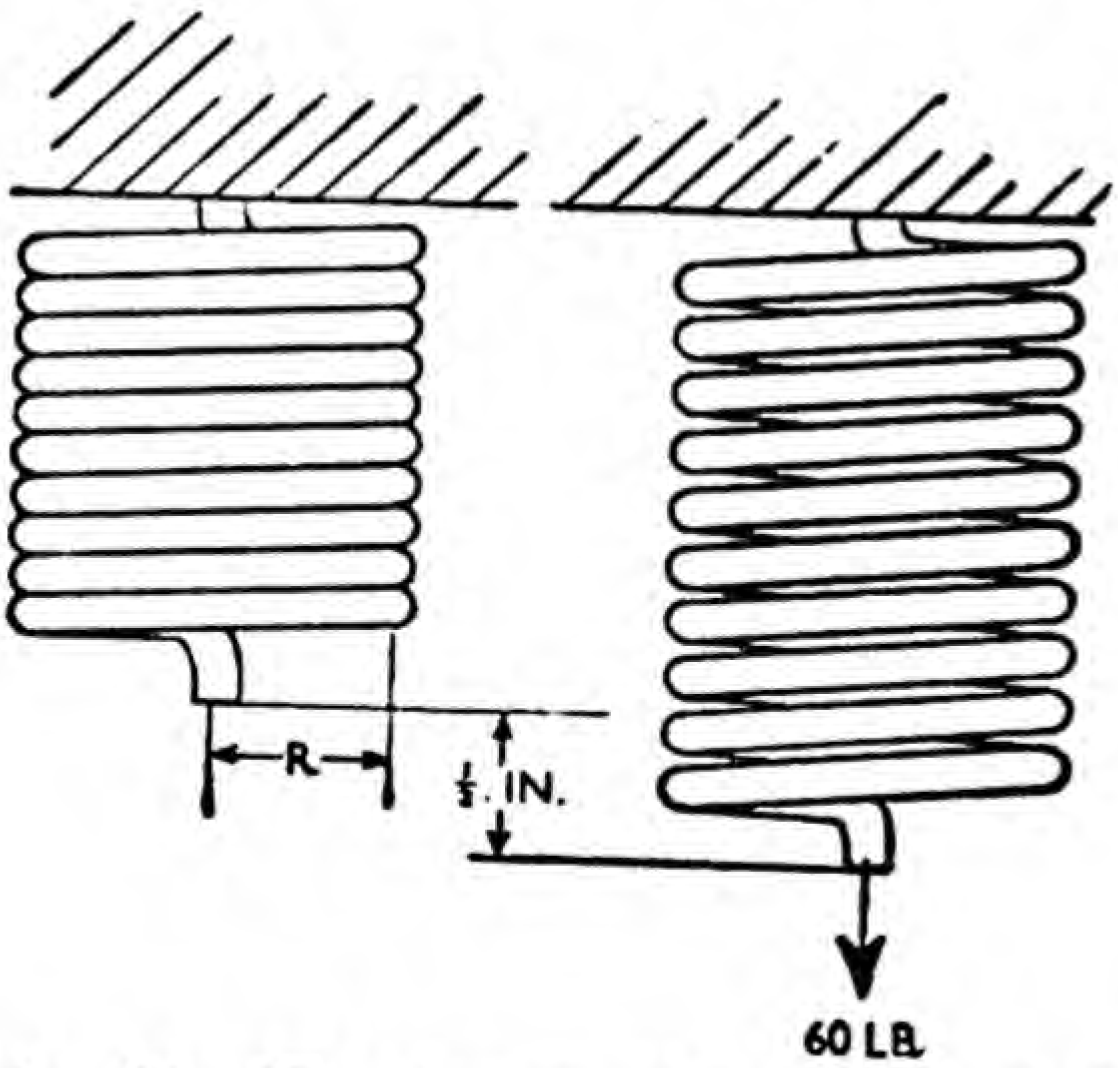


Fig. 60. The design of a close-coiled spring depends on the deflection required under a given load (stiffness), the radius of the coil (R), and the polar moment of inertia of the coiled bar (Fig. 23). The maximum load must be such that the elastic limit of the steel is not exceeded.

Design a close-coiled helical spring (Fig. 60) to carry a maximum load of 60 lb. with an extension of ½ in. The diameter of the steel wire available is 0·3 in. ($r = 0{\cdot}15$ in.), and the allowable shearing stress is 12,000 lb. per sq. in. (q).

It is known that $q = \dfrac{Tr}{J}$,

where $J = \dfrac{\pi d^4}{32} = 0{\cdot}000795$ in.^{4}.

Thus the torque $T = \dfrac{qJ}{r}$

$$= \frac{12{,}000 \times 0{\cdot}000795}{0{\cdot}15} = 63{\cdot}7 \text{ in.-lb.}$$

But in a spring the torque =

$WR = 60R = 63{\cdot}7$ in.-lb.

Therefore, the required mean radius of the coil is $R = 1{\cdot}06$ in.

Now, each coil is nearly a circle, and has a length $2\pi R$. If there are

n coils, then the total length of the rod or wire forming the spring is $2\pi Rn$, and this length can be used in the other relationships that have already been studied.

It is known from previous work that $\theta = \dfrac{ql}{rN}$.

The deflection required is $\frac{1}{2}$ in., which should be equal to $R\theta$.

Therefore, $\dfrac{qlR}{rN} = \frac{1}{2}$ in.

Since the material is steel, N, which is the modulus of rigidity, $= 12 \times 10^6$ lb. per sq. in., and the total length of the spring is, $2\pi Rn = 2{\cdot}12\pi n$ in.

Thus $\frac{1}{2}$ in. $= \dfrac{qlR}{rN}$

$$= \frac{12{,}000 \times 2{\cdot}12\,\pi n \times 1{\cdot}06}{0{\cdot}15 \times 12 \times 10^6}.$$

From which the number of coils required, $n = 10{\cdot}6$ coils.

Fig. 60 shows this coiled spring in the open and closed positions.

The effect of torsional shearing stress is also used in everyday life in the simple automatic door-closer illustrated in Fig. 61. A thin rod is clamped at one end to the door, and at the other end to the wall. The rod is very nearly vertical, and when the door is opened, the rod is subjected to an angle of twist, θ, equal to the angle of the open door.

Inducing a Torque

This angle of twist induces a torque, which has the effect of pushing the door back to its original position when the pull on the handle is released.

In Fig. 61, the handle is 2 ft. 3 in. from the 4-ft. long vertical rod. The rod is of steel, and is $\frac{3}{16}$ in. in diameter. Find the pull on the handle when the door is opened to 45 deg., or $\dfrac{\pi}{4}$ radians.

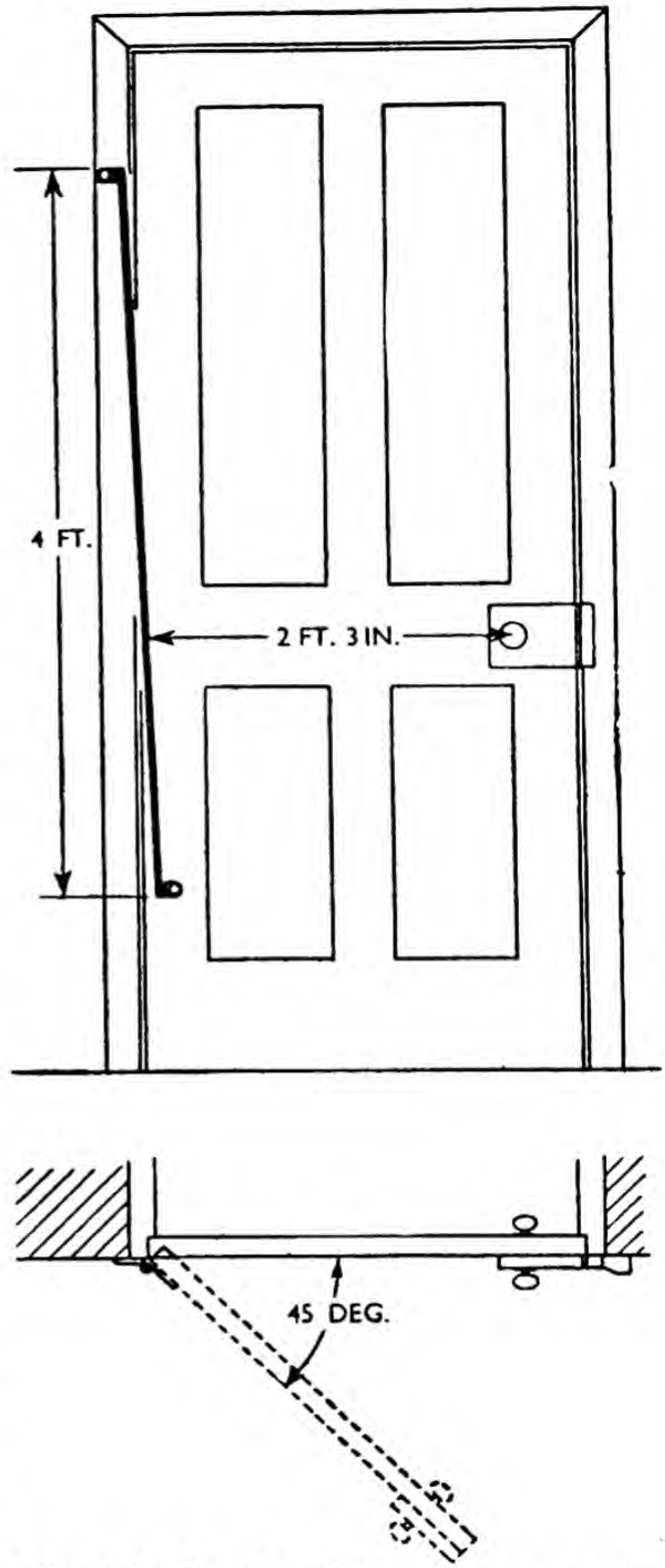


Fig. 61. A rod subjected to a torque has elastic energy stored within it by virtue of the angle through which it is twisted. When the torque is removed, this energy is released and can be used in a simple door-closing device. A close-coiled helical spring also consists of a rod subjected to a torque.

As in the previous problem, it is necessary to know the value of J for a $\frac{3}{16}$-in. diameter rod.

$$J = \frac{\pi d^4}{32} = \frac{\pi \times (\frac{3}{16})^4}{32}$$
$$= 0{\cdot}000121 \text{ in.}^4$$
N(steel) $= 12 \times 10^6$ lb. per sq in.

$l = 48$ in. $\quad \theta = \frac{\pi}{4}$ radians.

Now it is known that
$$\theta = \frac{ql}{rN} \text{ and } q = \frac{Tr}{J}.$$
Substituting for the value of q in the first expression,
$$\theta = \frac{Tl}{JN} \text{ or } T = \frac{\theta JN}{l}$$
$$\therefore T = \frac{\frac{\pi}{4} \times 0{\cdot}000121 \times 12 \times 10^6}{48}$$
$$= 23{\cdot}8 \text{ in.-lb.}$$
This torque or twisting moment, like all moments or couples, is the product of a force and a distance. In this instance, the force of interest is the pull required on the door handle. The distance of the door handle from the centre of the twisted rod is 27 in., so there is :—

Pull on handle $\times$ 27 in. = 23·8 in.-lb.

$$\text{Pull} = \frac{23{\cdot}8}{27} = 0{\cdot}88 \text{ lb.},$$
or approximately 14 oz.

Door springs of this kind usually remain in a permanently twisted condition even when the door is shut, in order that the door should be held closed by a small force. Suppose that the spring keeps the door closed with a force of 1½ lb., then in order to open the door to 45 deg. a total pull on the handle of 1 lb. 8 oz. plus 14 oz., or 2 lb. 6 oz. is required. Of course, the assumption is that the hinges are frictionless, which would be nearly correct if they were well oiled.

Scope of Chapter

This chapter has attempted to show how external effects of loading can finally be related to internal effects within the various materials used in constructing buildings, furniture and common objects of everyday life. The effect of the external loads in producing shearing forces, bending moments and torques can be determined without any knowledge of the type of structure supporting these forces.

However, when considering the shape and size of the component parts which are resisting the external forces, something about the properties of the various sections used must be known. It was found that the two chief properties required were cross-sectional area and moment of inertia, which includes the polar moment of inertia, J.

The flexure formula $\left(f = \frac{My}{I}\right)$, and the torsion formula $\left(q = \frac{Tr}{J}\right)$, link together the external and the internal effects, and connect external loading, internal stress, shape of cross-section and nature of material. M and T refer to the external loading, f and q to the internal stress, and the type of material, while I, y, J and r refer to shape and size of cross-section.

By using these connecting formulæ it is possible to estimate the intensity of stress produced at any point in a structure by a given set of external loads. Whether such stresses are safe and within the elastic limit depends on the material of which the component parts of the structure are made, and on the properties of that material. It is thus essential to study the properties of the most commonly used materials of construction, as is done in Chapter 9.

CHAPTER 9

PROPERTIES OF MATERIALS

COMPOSITION OF STEEL. CARBON STEEL. ALLOY STEELS. HEAT TREATMENT OF MATERIALS. TESTING. LIMIT OF PROPORTIONALITY. PROPERTIES OF STEEL. NON-FERROUS METALS. TIMBER. BUILDING STONE. PRESERVATION OF STONE. CONCRETE. PLASTICS.

OF all metals, steel is by far the most widely used, and the engineer must know much about the properties of the many varieties of it now available. Other materials of importance are non-ferrous metals and alloys, concrete, timber, plastics and building stone.

Steel is a material consisting chiefly of iron, combined with a small proportion of carbon, and often with additional and varying proportions of other metals. The manufacture of steel can be mentioned only briefly, but consists of three stages :—

(1) the manufacture of impure iron from iron ore,
(2) the total or partial removal of the impurities in the iron, and
(3) the addition of the required amounts of carbon and other materials.

(1) is carried out in a blast furnace, and (2) and (3) in either a Bessemer converter, or an open-hearth furnace.

Wrought Iron

First consider those steels which consist almost entirely of iron and carbon, with not more than 1 per cent content of other metals. When there is no carbon content, the metal is iron. The pure metal is never extracted commercially, but an impure form, which is soft and malleable, is used for wrought-iron work such as is seen in ornamental gates. Before the methods of steel manufacture had been well established, iron was used for building quite large structures. The Eiffel Tower, for example, is constructed of wrought iron.

Composition of Steel

With the addition of a small amount of carbon to the iron, a remarkable change takes place. The material, which may now be called steel, is much stronger and more ductile than wrought iron, and its sphere of use is much extended.

As more and more carbon is added, the steel becomes still stronger and harder, but at the same time more brittle ; it cannot be safely used for purposes where heavy loads or impact loads occur. There would then be danger of the material suddenly breaking. Such high-carbon steels are, however, very useful where cutting edges are required, or resistance to wear must be provided.

The maximum carbon content which is ever used in steel is not much over $1\frac{1}{4}$ per cent, but this amount is relatively unusual, and most of the steel made contains only a few tenths of 1 per cent of carbon.

In steel all of the carbon is

chemically combined with the iron. When the carbon content is over 1·6 per cent, the carbon begins to separate out in the form of particles or flakes of graphite. The material then becomes very brittle compared with steel and will melt at a much lower temperature, in fact, it is now what we usually call cast iron. Ordinary foundry iron contains about 3½ per cent of carbon, most of which is graphite. It can be readily cast into almost any required shape, but its strength in tension is much lower than that of mild steel and it offers much less resistance to shock. Other constituents, particularly silicon, modify its properties very considerably.

There are a number of types of cast iron now in use, containing nickel or chromium, or both, which have a tensile strength much nearer to that of steel, and these are used for such castings as motor-car and aero-engine cylinders, where lightness and strength are important combined with resistance to high temperatures.

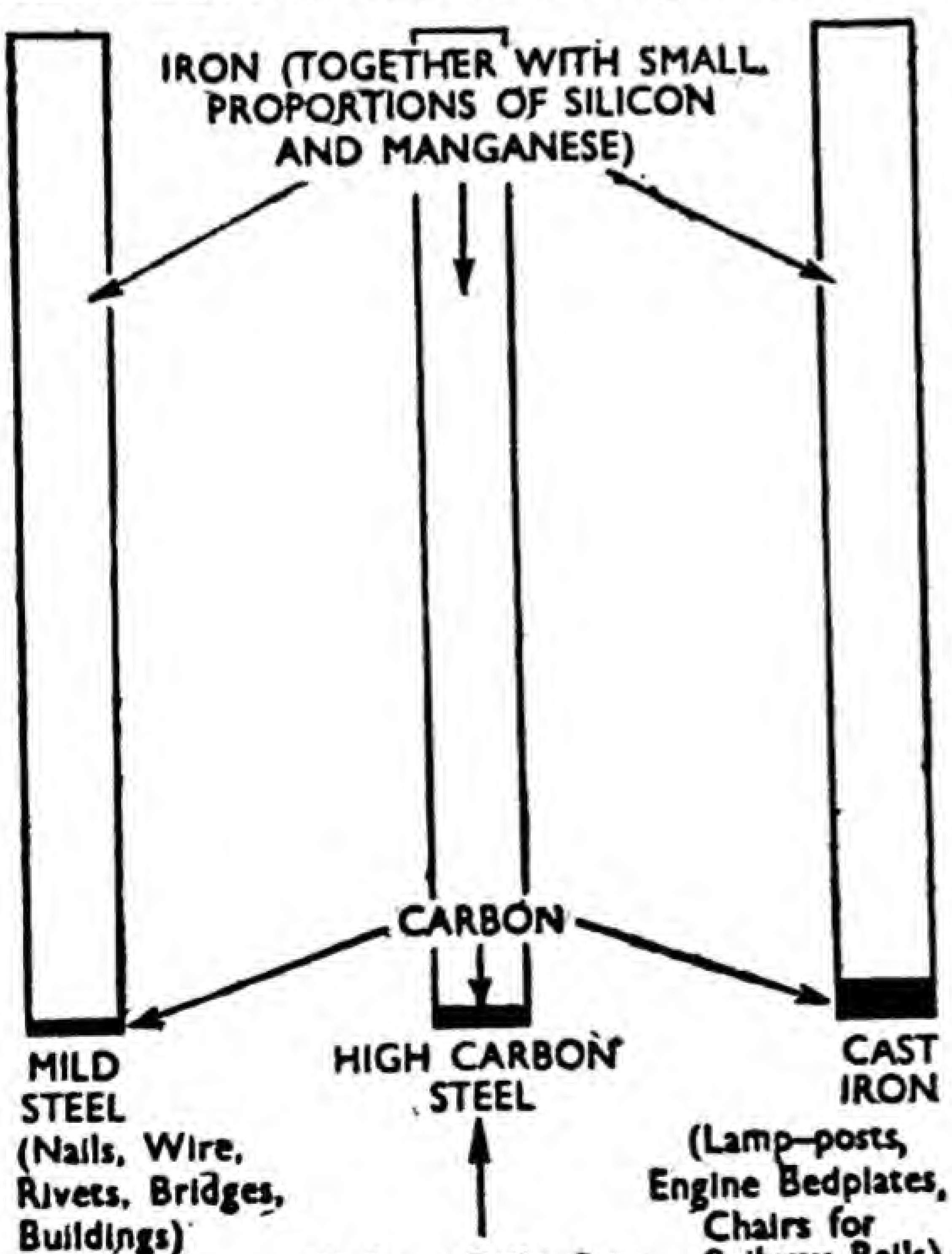


Fig. 1. Steel consists chiefly of iron with a small, but very important, inclusion of carbon. This figure shows, approximately, the proportions of iron to carbon for various steels.

Proportion of Carbon

The proportion of carbon in steel need only be varied very minutely in order to effect a considerable change in the properties of the material. Fig. 1 shows the amount of carbon in mild steel, high-carbon steel and cast iron, respectively. Although the properties of any one of these materials are very different from those of the other two, the change in carbon content is relatively small.

It has already been noted that an increase in strength, obtained by an increase in carbon content in carbon steels, is accompanied by increased brittleness. Also, apart from other considerations, an increase in carbon content does not improve the strength of carbon steel sufficiently to meet the multifarious uses for which it is required in industry. Thirdly, the heat treatment of straight carbon steels presents serious problems. The occurrence of these difficulties has led to many experiments being made to find a way of simultaneously improving the desirable properties of steel, i.e., strength and ductility. These experiments have resulted in the production of alloy steels, in which other metals are added, or alloyed to the iron and carbon.

The chief metals now used to improve the qualities of steel are manganese, nickel, chromium and tungsten. Other elements used in alloy steels are molybdenum, vanadium, silicon and cobalt. These are

generally used in combination with one or more of the others, and with correct heat treatment improve the strength, ductility and resistance to shock very considerably.

The first three diagrams of Fig. 2 show alloy steels, in which one of the alloying elements predominates, but where iron still forms the major proportion of the steel. The content of the alloying material, added to the steel, has a very noticeable effect on its properties. Manganese makes the steel capable of being hardened by blows, and this type of steel is used for railway points, excavator buckets and other objects subjected to repeated shock.

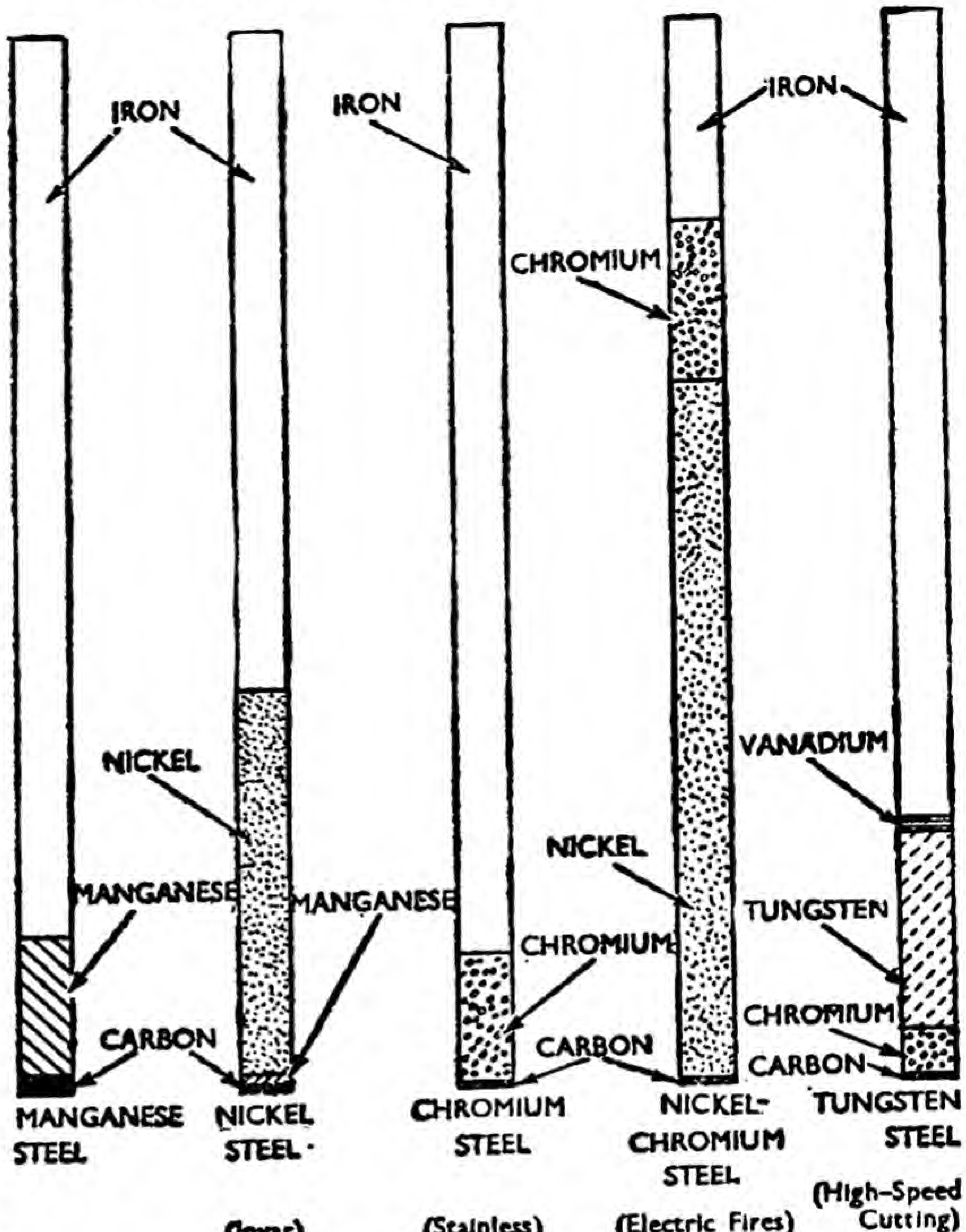


Fig. 2. Alloy steels sometimes contain so much of the alloying material and so little iron that it is doubtful whether the material is a steel in the sense used in Fig. 1. This figure gives the approximate proportions of materials used in common alloys.

Steel for measuring-tapes and parts of clocks, which must not change in length during a change of temperature, are made of Invar, steel containing 36 per cent of nickel. This steel has a coefficient of expansion which is practically zero.

Finally, a high chromium content in the steel results in resistance to corrosion (stainless steel).

Alloying Elements

Smaller amounts of alloying elements are, however, often more profitably employed in combination. Nickel and chromium, for example, may be used together in nickel-chrome steels. Such alloys possess several of the valuable properties of both the nickel and the chromium steels. The nickel-chrome steel which is illustrated in Fig. 2, shows how the steel may contain more of the alloying elements than it contains of iron.

In high-strength steels for structural and other purposes the alloying elements are present in only very small amounts, but even an apparently insignificant quantity may have a considerable effect on the properties of the steel. The

tungsten steel shown in Fig. 2 contains some chromium and a small percentage of vanadium, which, however, improves the qualities of the steel when used for cutting at high speeds.

Heat Treatment

The reason for the treatment of steel by heating is that, at certain temperatures, such as 730 deg. C., changes in the internal constitution of the steel take place. By suitable adjustment of the temperature to which steel is heated, and the rate at which it is subsequently cooled,

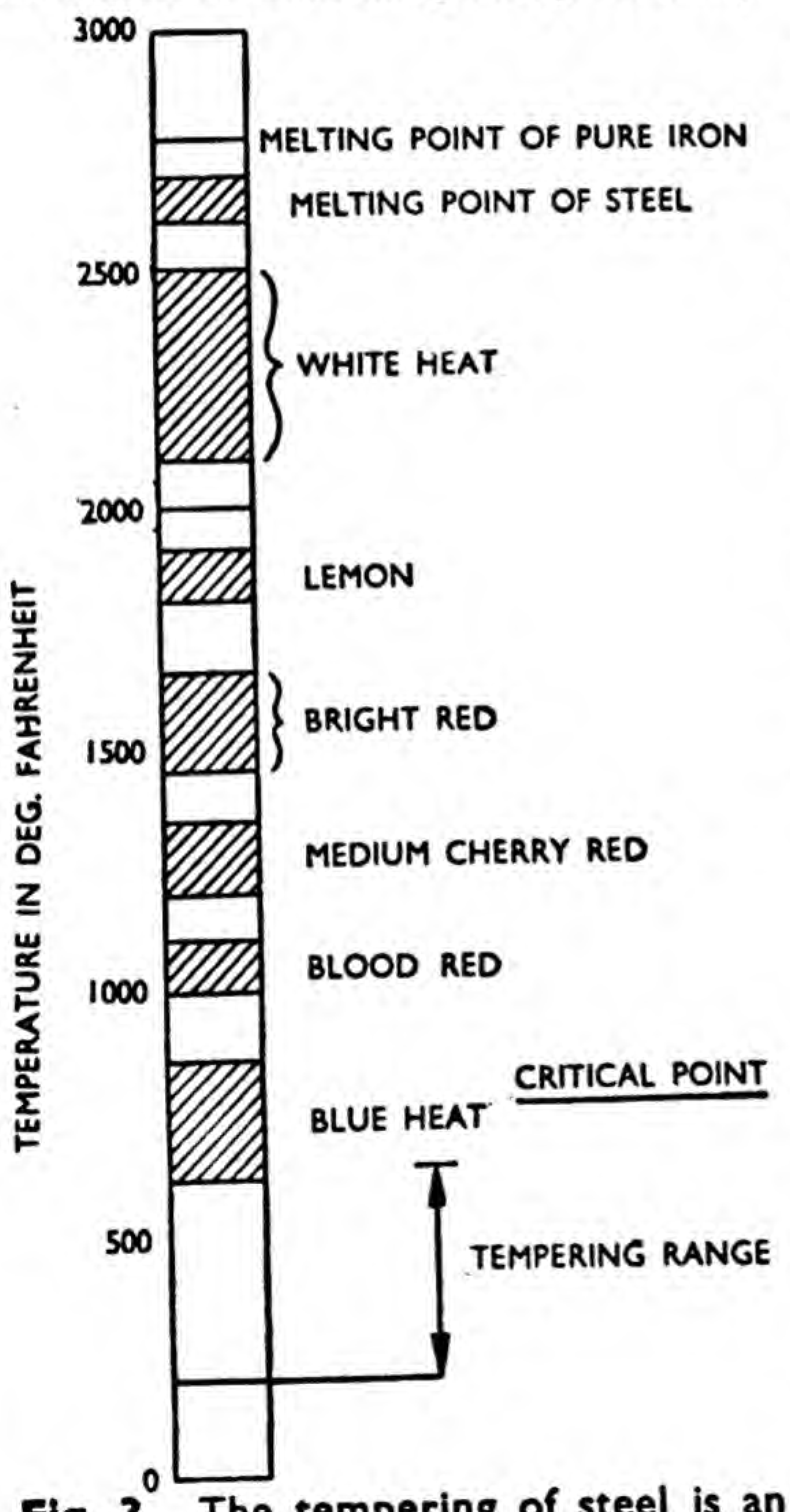


Fig. 3. The tempering of steel is an art which has long been known and practised by blacksmiths. The choice of the correct temperature at which quenching should take place is made by the craftsman by watching the colour of the metal.

appropriate changes in the properties of the steel may be made. Heat treatment is especially required for alloy steels in order to develop the full effect of the alloying elements.

The steel is first heated to a specified temperature, according to its composition, and then cooled from that temperature by a sudden immersion or quenching in water or oil. This treatment results in a steel which is very hard, but also very brittle. Such quenching is, however, only the first step in the process of heat treatment, and steel cannot profitably be used in the quenched condition.

By subsequent heating to a much lower temperature, the brittleness is reduced, although the hardness obtained by quenching is substantially retained. This second process is known as tempering. The colours shown by steel at different temperatures are described in Fig. 3. Such colours are a useful guide to skilled workers, but accurate measurement of temperatures by pyrometers is necessary for the best results.

Other Methods

This rapid cooling, followed by tempering, is not the only method of heat treatment employed. The process of heating to a relatively low temperature, soaking at that temperature, and then slow cooling, is known as annealing. This slow cooling has the opposite effect to that of quenching in that it softens the metal and refines the grain. Heating for long periods at a high temperature has, however, an opposite effect, since it increases the grain size and makes the steel brittle. Intermediate rates of cooling, or,

in other words normalizing, are also employed for special purposes.

One of the advantages of the use of alloy steels is that the presence of the alloying elements modifies the necessity for very sudden cooling in order to obtain a hard material. The sudden shock of plunging red-hot carbon steel into water may easily result in the cracking of the forging which is being treated. With alloy steels the cooling may be slower, and some alloy steels harden merely on cooling in air. Such slow cooling makes the hardening safer and more thorough, for the effect of the cooling has time to penetrate the material.

Fig. 4 shows how the composition of the steel affects the heat-treatment temperatures. The choice of correct temperatures depends on a thorough knowledge of the technicalities of metallurgy.

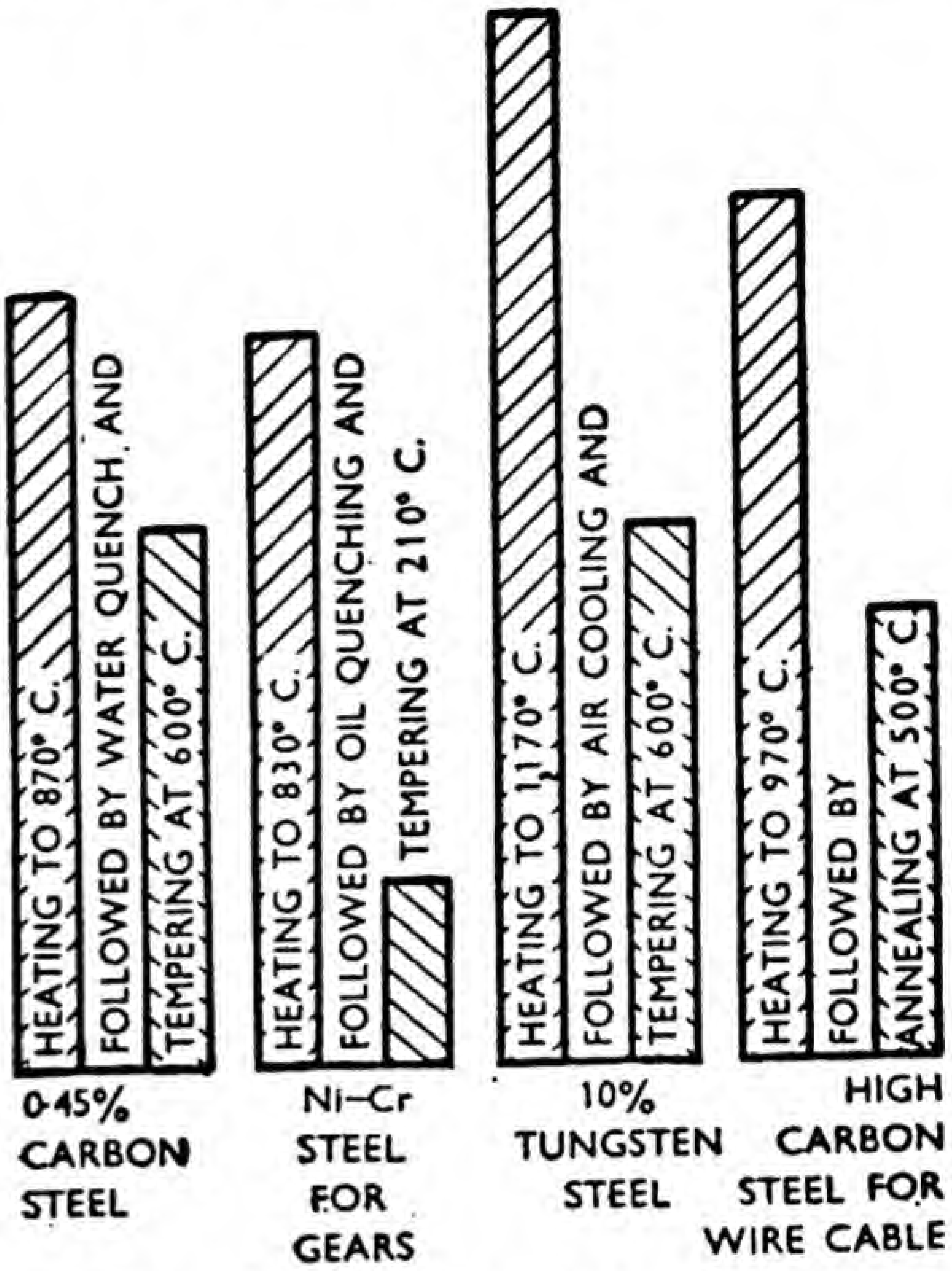


Fig. 4. Heat treatment of modern steels requires great care, and a more extensive knowledge is demanded of the steel-worker than was possessed by the village blacksmith. Small changes in the temperatures and techniques employed may cause considerable differences in the finished products.

Standard Tests

A corollary to the study of the properties of materials is the development of suitable tests for satisfying the engineer that the materials he is using have been manufactured to a standard satisfactory for the purpose he has in mind. The type and size of sample specimens, and the conditions under which these tests are carried out are, so far as is possible, minutely described, so that the results obtained in one part of the country may be reasonably compared with the results obtained elsewhere.

The chief range of specifications with which the properties of materials should comply, in this country, is prepared by the *British Standards Institution* (*B.S.I.*) and issued as *British Standard Specifications* (*B.S.S.*). A list of a few of these is given at the end of the chapter. In the United States, the leading body is the *American Society for Testing Materials* (*A.S.T.M.*).

The most important property of steel from the structural point of view is its strength, or ability to resist load. The strength of any material, however, cannot adequately be defined unless the type of stress it is expected to resist is stated. The strength of a material in tension may be very different from its strength in shear, and its strength under a static load may not be a sufficient criterion its ability to resist an impact or repeated load.

The static tensile test, when the material is gradually stretched

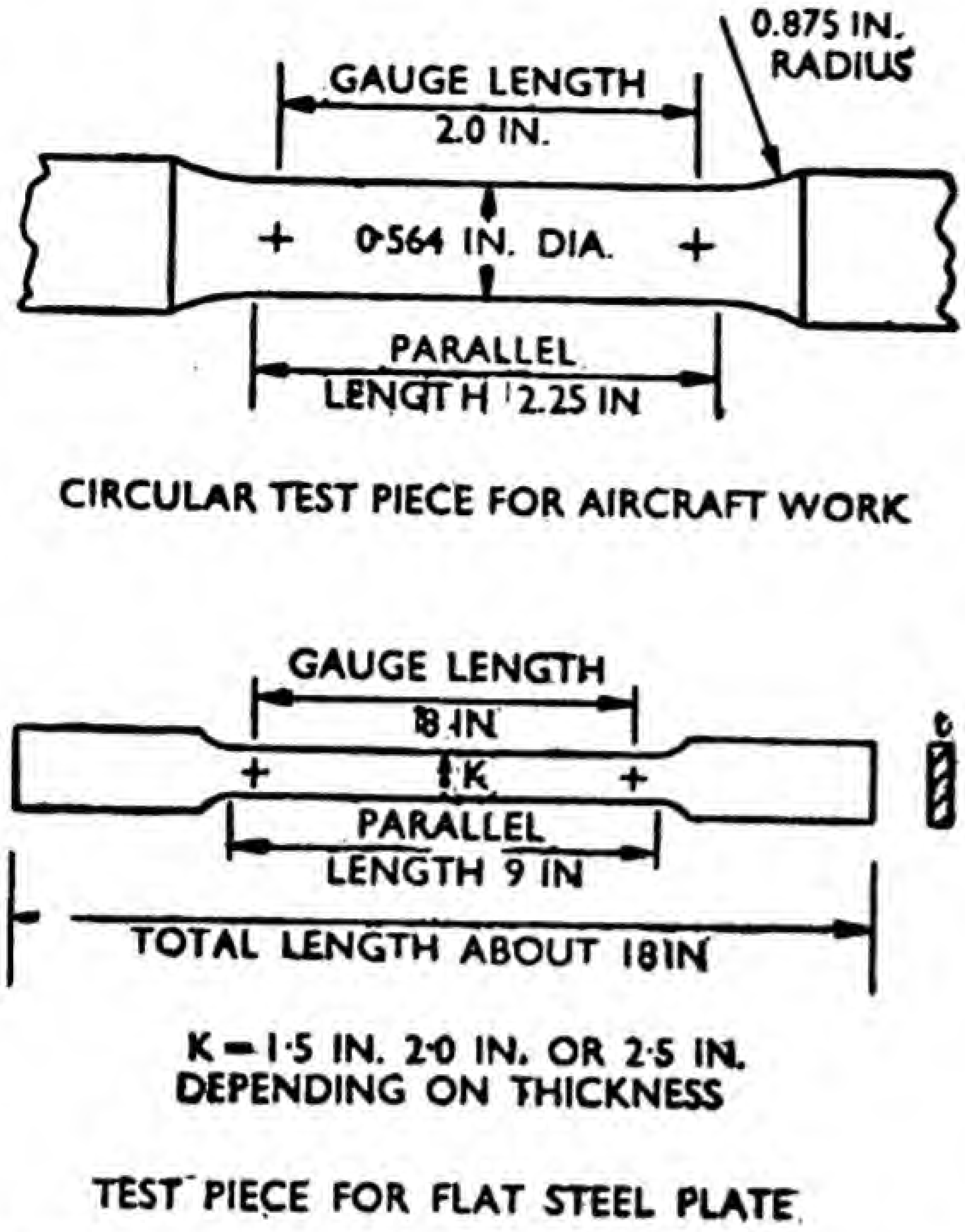


Fig. 5. It is important that the results of tests made in one part of the country can be compared with results obtained elsewhere. In order to achieve this condition, tests are always made on standard test pieces described in the various *British Standard Specifications.*

until it breaks, is, however, one of the most important of the tests to which steels and other materials are subjected, and this is described below as an example of how tests are used to determine whether a material is satisfactory.

Standard Specimens

It has been found that the type of test specimen used affects the results obtained, and it is important that a standard shape of specimen should be employed in the test, so that one steel may be compared with another. Fig. 5 shows two of the standard test pieces specified by the B.S.I.

During the static tensile test, several important points are noted. At first, as the load is increased, only a sensitive measuring device can record any change in length. The steel is very stiff at low loadings. The strain produced is proportional to the stress applied during this stage, and the value of the stress-to-strain ratio is known as the modulus of elasticity. The higher this modulus, the stiffer the material (Fig. 6).

At the limit of proportionality (*A*), the strain begins to increase at a faster rate than the stress, and a critical point in the test then occurs when the bar suddenly stretches, even when no extra load is added. The stress at which this occurs is known as the yield point (*B*).

After the yield point, the stretching increases rapidly, and at a certain maximum stress the bar can carry no more load (*C*). The portion of the graph between *C* and *D* is not of great practical interest, and occurs so rapidly that a curve, such as Fig. 6, can best be completed by automatic apparatus, allowing the figure to be drawn by the testing machine itself.

The steel in a structure, of course, is of very little practical use after it has been loaded beyond the yield

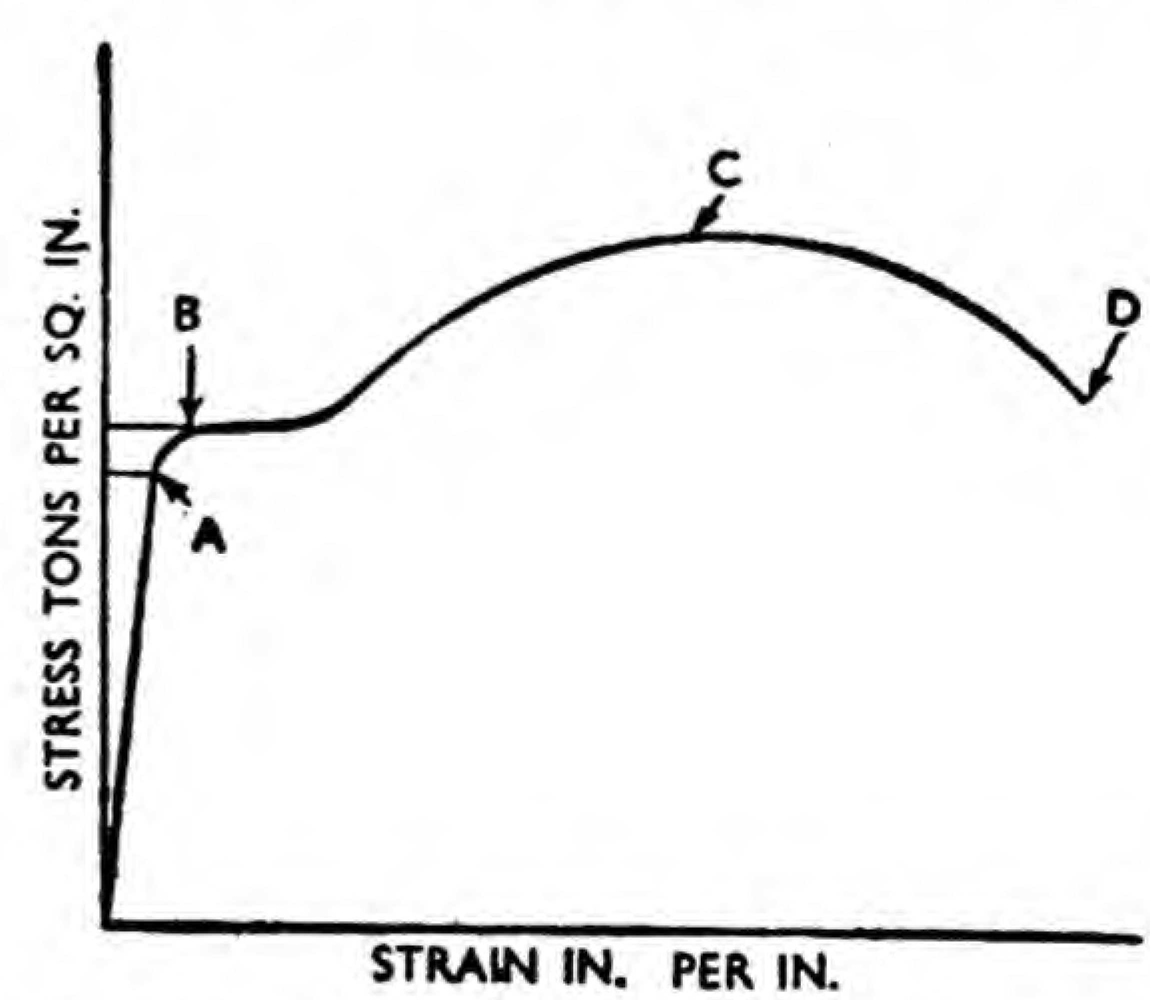


Fig. 6. The stress-strain diagram for steel has a characteristic shape. At first, the strain is extremely small and proportional to the stress applied; then there is a sudden yielding of the material and, finally, a very rapid extension until the breaking stress is reached.

point, since the removal of the load leaves the material permanently extended. In practice, the factor of safety by which the maximum stress is divided results in a working stress well below it. However, although the yield point may be considered as the maximum usable strength of the material, it is important that the maximum stress should also be determined, for the range of stress between yield and maximum gives a measure of the margin of safety offered by the steel to sudden and unexpected loads.

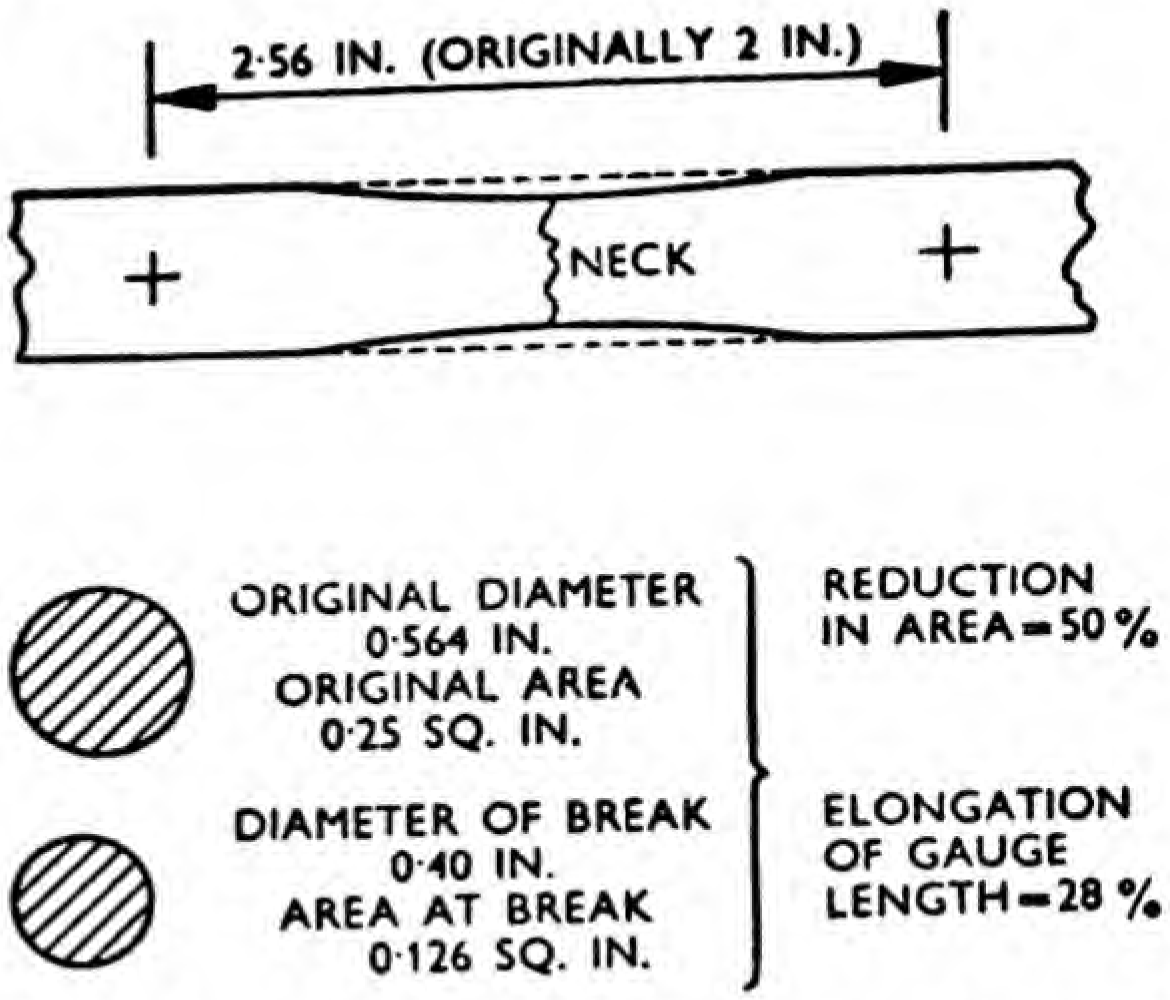


Fig. 7. The ductility of a structural material can be expressed in two ways —by the percentage elongation of a tensile specimen, and by the percentage reduction in cross-sectional area. This diagram shows how a ductile specimen forms a neck at the point of fracture.

Ductility and Hardness

But strength is not the only property required of steel. It must also possess ductility or the ability to undergo considerable deformation while carrying a high stress. The deformation which occurs at a high stress is represented by the horizontal distance between *B* and *D* or between *C* and *D* (Fig. 6). Between *C* and *D* the elongation is very localized, and occurs at the point where the bar finally breaks, the point at which the break occurs being reduced in area from the size of the original section.

The ductility of the test piece is measured by the percentage elongation of a specified length of the bar lying on both sides of the point of fracture. The greater the percentage elongation before the specimen breaks, the higher is the ductility of the steel. A secondary check on the ductility is given by the percentage reduction in area at the point of fracture. Fig. 7 shows typical results for the fracture of a steel test specimen.

Another important test is that for hardness. The results obtained in this test may be used in two ways. Firstly, it may be necessary for the steel to possess a particular hardness for the use to which it is to be put. Secondly, hardness tests of successive consignments of the same type of steel allow of a rapid check on the consistency of manufacture. Consistent hardness figures indicate consistent strength and heat treatment.

Hardness is understood to mean the resistance which the metal offers to scratching, indentation or abrasion, and this property is usually measured by indenting the material by means of a heavy load, and measuring the size of the impression produced. The larger the impression, the softer is the material.

Hardness Numbers

A hardness number is used for comparing one material with another. This number is obtained by dividing the load in kilograms by the area of the surface of the indentation in square millimetres. The hardness number, therefore, really represents the stress in kg.

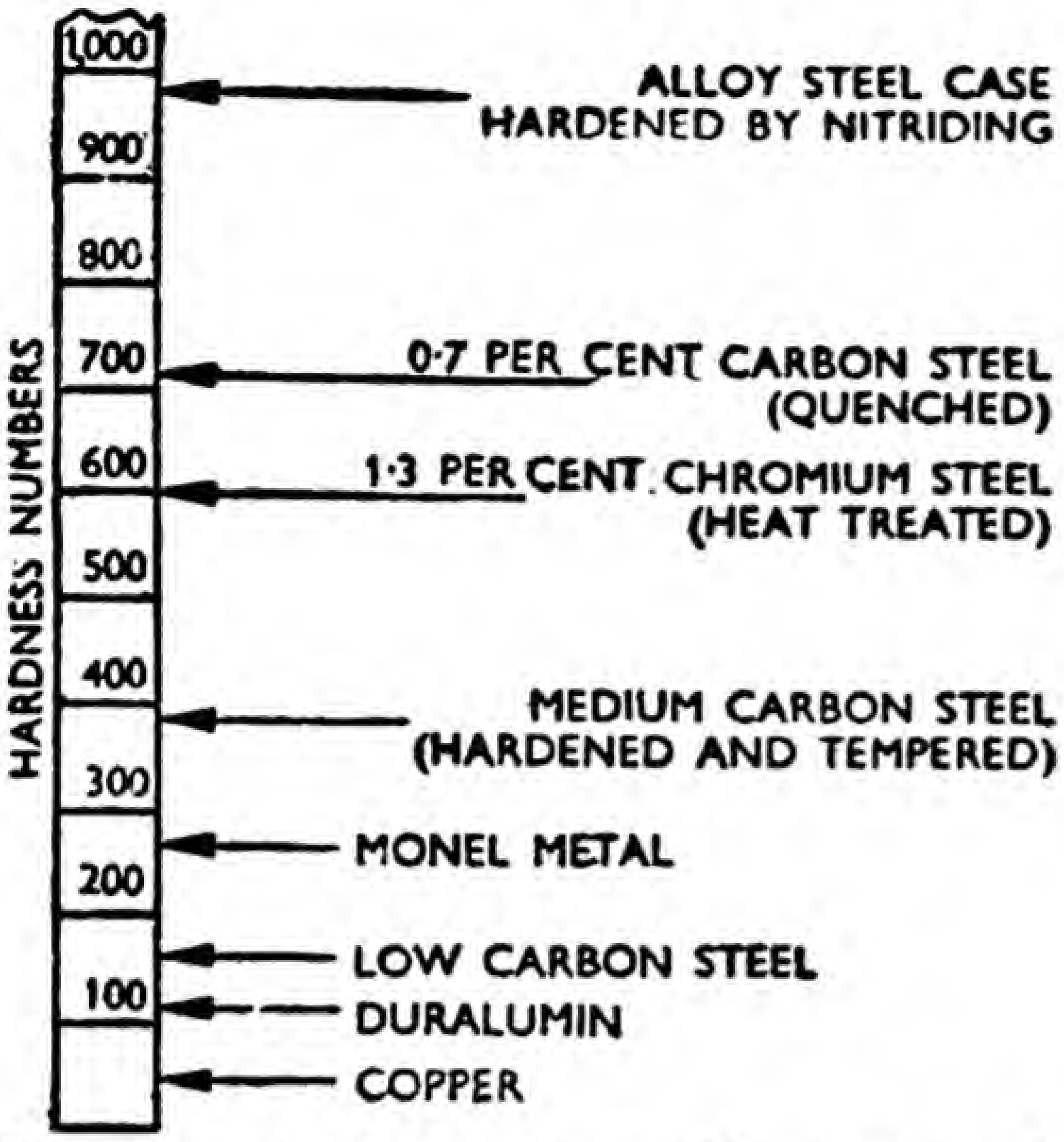


Fig. 8. Hardness is usually measured by the size of an impression made in the material by a hard steel ball or a pyramidal diamond pressed in under a standard load. The hardness number is obtained by dividing the load by the superficial area of the indentation.

per sq. mm. which the material can withstand locally without further indentation. Fig. 8 shows the relative hardness of various materials.

Indenting Methods

The two chief methods of applying the indenting load are by means of a steel ball, the Brinell method, and by means of a very small diamond, shaped to the form of a pyramid, the Vickers diamond method.

The diamond method, which is also used in the Rockwell and Firth hardness testers, has several advantages over the steel-ball method. The machine used makes the application and release of the load less dependent on the human element; the mark is almost invisible to the naked eye; finished articles may be tested without disfigurement. In both methods, the diameter or diagonal of the impression is measured by a microscope, and reference to a set of tables then gives the required hardness number (Figs. 8 and 9).

Fig. 10 summarizes briefly the various conclusions of this rapid survey of the properties of steel. The first three diagrams show properties of carbon steels, with

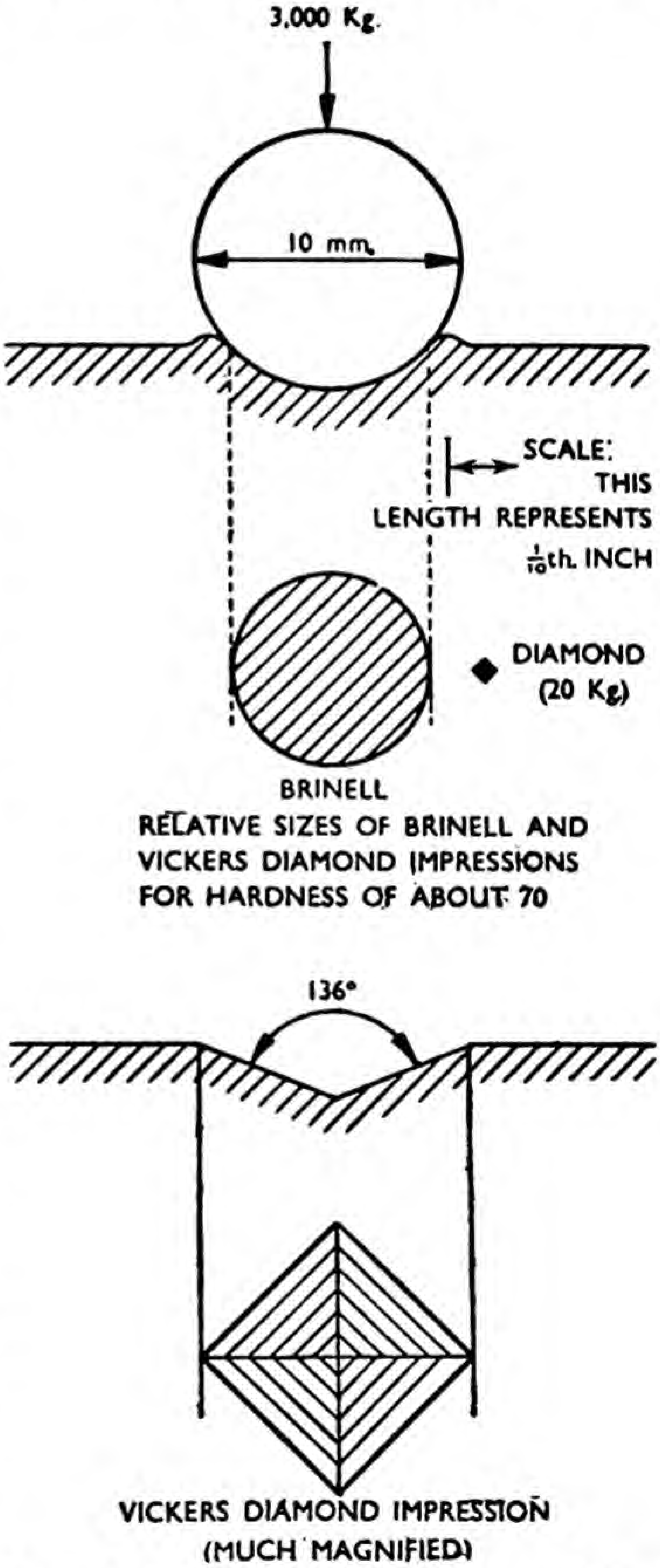


Fig. 9. Hardness testing with a pyramidal diamond has the advantage over testing with a steel ball, that the impressions are very much smaller. The hardness of finished articles can be determined without disfigurement. This diagram shows the relative sizes of the two indentations.

increasing carbon contents. The final three diagrams show similar values for alloy steels. The following main points appear :—

(1) In carbon steels, as carbon content increases, the strength and hardness increase and the ductility decreases.

(2) By the use of alloy steels, much greater strengths can be achieved than by the use of straight carbon steels.

(3) Reduction of ductility with increase of strength is not so pronounced in alloy steels as in straight carbon steels.

(4) There is a close relation between strength and hardness. The relation is, in fact, as follows :—

Strength of carbon steels in tons per sq. in. = 0·23 × hardness number.

Strength of alloy steels in tons per sq. in = 0·21 × hardness number.

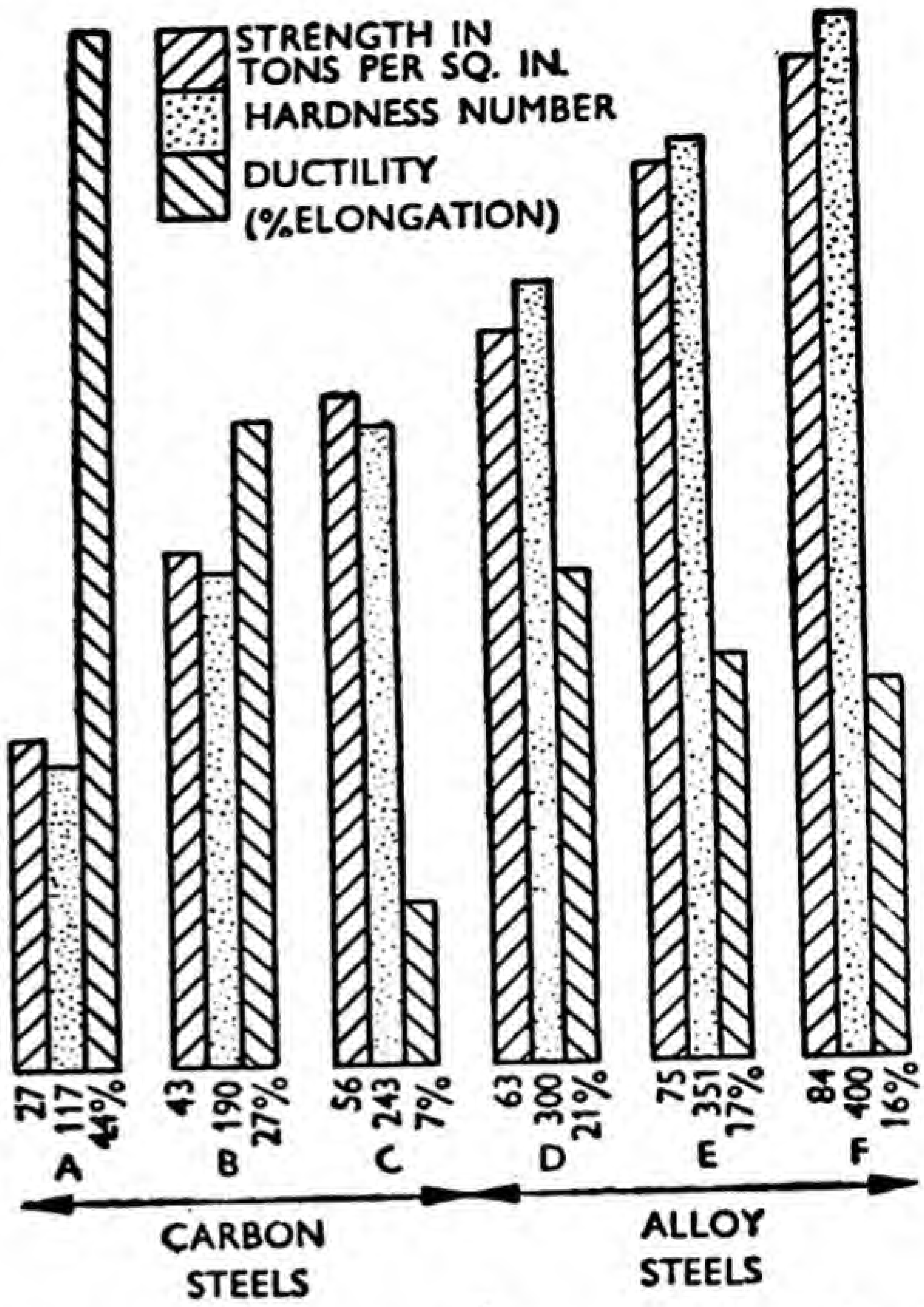


Fig. 10. These diagrams show that strength and hardness in steels may be said to go hand in hand. The value of the hardness may be used as a measure of the strength, and so, indirectly, of the consistency of the heat treatment.

Non-Ferrous Metals

The outstanding importance of iron and steel has led to the broad classification of metals as ferrous and non-ferrous. The methods of testing non-ferrous metals are similar to those used for steel, and need not be discussed further. Space allows of only a brief reference to some of the more important of these metals and alloys.

Copper is used in greater quantity than any other non-ferrous metal, and is the base of numerous important alloys. Aluminium, which resembles copper in its resistance to corrosion and in its high conductivity for heat and electricity, is also widely employed. An alloy of these two important metals is known as duralumin, and is used in the aircraft industry because of its light weight and high strength.

Some of the more important non-ferrous metals and their alloys are shown in Fig. 11. This diagram helps to show, in broad outline, how the non-ferrous metals may be combined, but offers only a simplified picture of the real position. The chief alloys shown may contain, in addition to the main constituents, small proportions of a large selection of other metals, and may also possess a wide range of properties, according to the proportions of the main constituents.

Special Alloys

Of the non-ferrous metals in Fig. 11, note may be made of magnesium, which forms many light and easily machined alloys

whose importance has increased as the methods of mass production have been extended in scope. In mass-production manufacture, alloys are also required for diecasting, viz., casting in permanent metal moulds. Large numbers of similar articles may thus be produced, and the diecasting alloys used are composed of zinc, copper and aluminium.

In diecasting, the moulds are made of cast iron or steel and articles can be cast exactly to size. The dies can be used continuously, whereas a sand mould can be used only for a single casting.

Non-ferrous metals are more widely used in the pure, or almost pure state than is iron, and, in addition, scores of special alloys are regularly produced for special purposes.

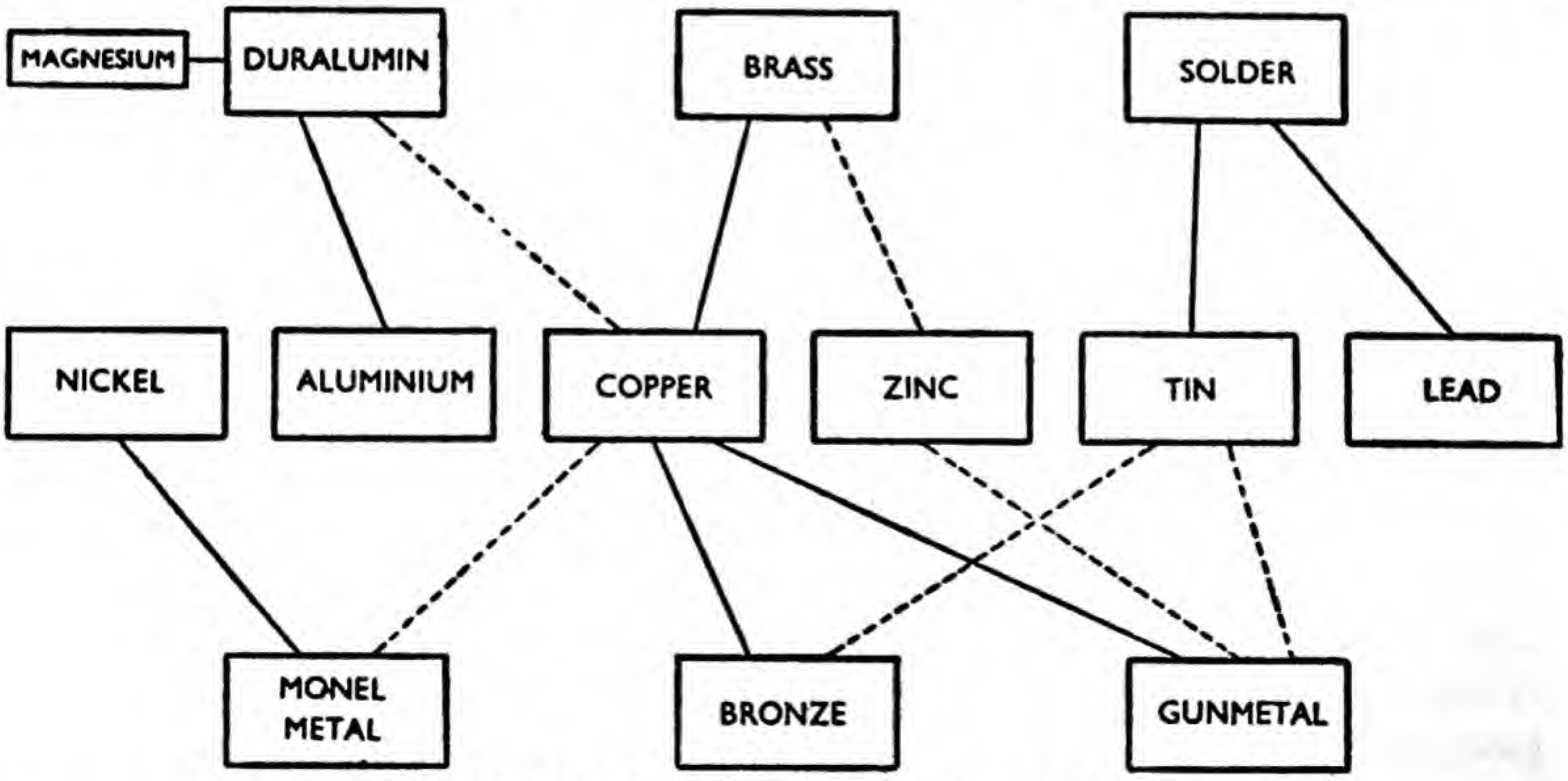
Use of Timber

Although the place occupied by timber as a principal structural material for many centuries has now been usurped by metals, timber is still required in large quantities for many types of building and engineering construction.

Timbers can be divided into (*a*) softwoods and (*b*) hardwoods. The softwoods are obtained from coniferous trees, and the hardwoods from broad-leaved trees. Generally speaking, softwoods are lighter and softer than hardwoods, although there are exceptions to this rule. Pitch pine, for example, is as heavy as the hardwood teak ; the weight of mahogany is less than that of the softwood larch (Fig. 12).

Methods of Seasoning

The term seasoning refers to the removal of the sap, and the consequent reduction of the moisture content of the timber to about 15 per cent. This can be done either by natural drying, or by artificial drying. The latter is the method which is now used to an increasing extent. It is rapid, and continuous control can be exercised. The wood is steam-heated in



SOME METALS AND THEIR ALLOYS

Fig. 11. The non-ferrous metallic elements may be combined to form many different alloys. From bronze, well known to the ancients, to the lightweight present-day alloys containing aluminium and magnesium, they cover a wide field of usefulness and a wide range of properties.

special kilns. Seasoning should be carried out sufficiently gradually to prevent cracks or checks appearing at the surface of the timber.

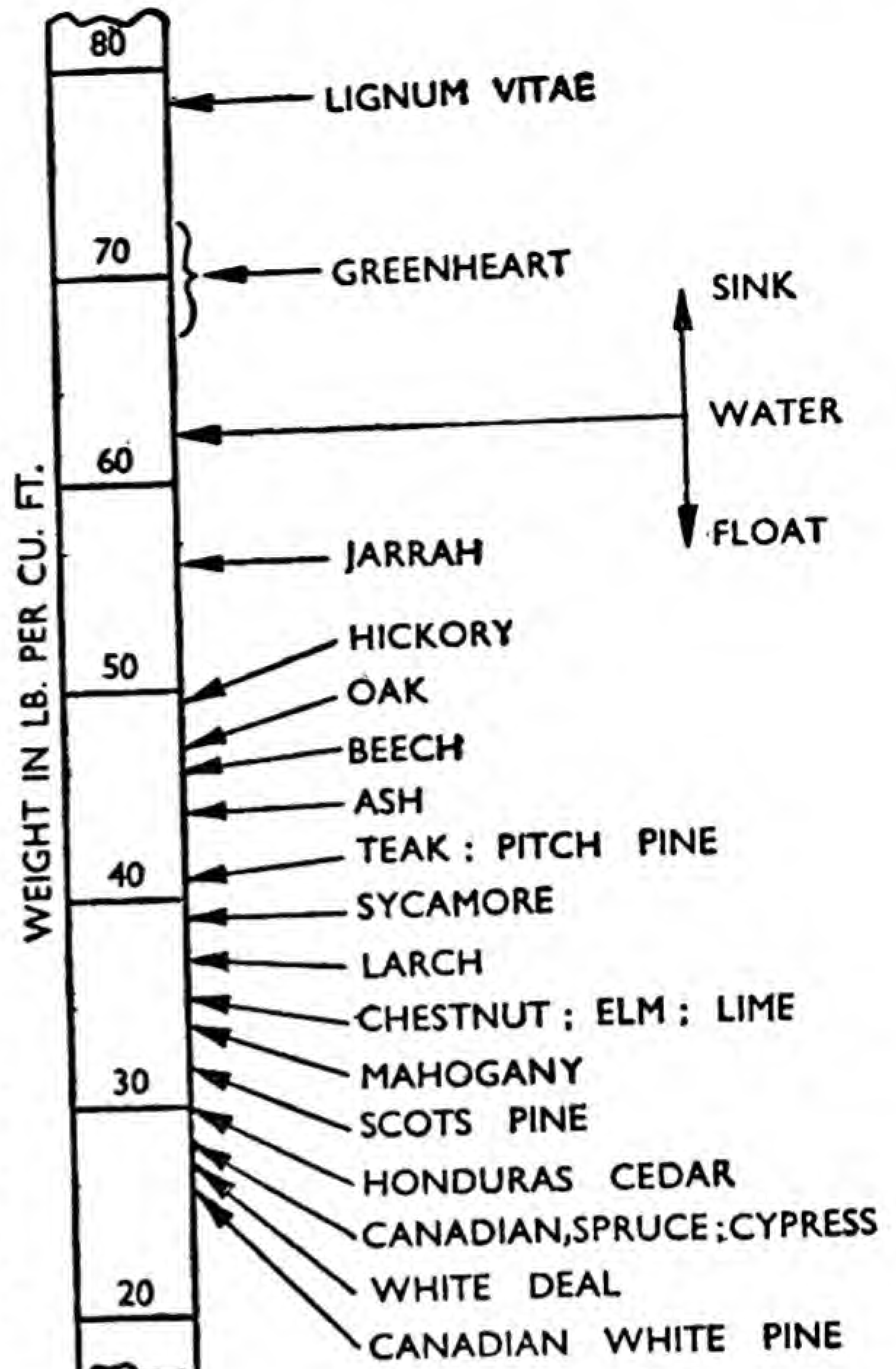


Fig. 12. Most woods are lighter than water, but one or two of the hardest and most durable cannot float. The timbers used in construction generally weigh between 25 and 50 lb. per cu. ft. with the hardwoods in the heavier class, and the coniferous softwoods in the lighter range.

Prevention of Decay

Timber is liable to decay, disease and attacks by insects ; it must be treated with a preservative if used in unfavourable circumstances. The most usual preservative is creosote, forced into the timber under high pressure, but chemical compounds, such as copper sulphate, are also employed. Thorough ventilation of the timber when in place helps to prevent the attacks of diseases such as dry rot.

The most generally used building timbers are softwoods, and the most useful softwood is obtained from the Scots pine. This timber, like others, is known by several names, the chief of these being red deal and yellow deal. It withstands a compressive stress, parallel to the grain, of over 5,500 lb. per sq. in., and a shear stress of over 1,100 lb. per sq. in. The modulus of elasticity is $1{\cdot}7 \times 10^6$ lb. per sq. in.

The hardwoods, such as oak, ash and beech, each have their own properties and uses, and a great variety is available for cabinet-making.

Building Stones

Building stones are subjected chiefly to compressive stress, attacks of the weather and of the polluted atmosphere of cities. Their flexural strength, or strength in bending, may also be of importance where a stone spans an opening. Therefore, strength and durability are both valuable properties, but durability, which can be judged in some measure by chemical composition and by the known performance of the stone, is probably more important.

The builder can develop the natural resistance of a stone by careful work. Sandstones and limestones, for example, should always be laid carefully on their natural bedding planes. If supervision is not adequate to ensure this, decay may set in rapidly. Also the design of the building should be such as to ensure that no stone is subjected to excessive wetting, either from drips, or by the results of faulty drainage.

The hardness of a stone generally depends upon the hardness of the cementing material. Very often,

stones composed of soft grains, well bonded, are harder than stones containing hard minerals but with poor bonding material. Some stones harden after they are quarried. This is due to the evaporation of the quarry water which deposits minerals in the pores of the stones.

Chemical Action

Apart from the effect of structural defects in the stone, the chief cause of decay is chemical action. Rain, polluted atmosphere, and the solution of chemicals from mortar, all cause acids and dissolved salts to be carried into the stone. Corrosion by acids, sulphuric and carbonic, and splitting pressure due to the crystallization of salts or the freezing of water in the pores of the stone, lead to disintegration.

Building stones may be broadly grouped into three classes:—granites, limestones, sandstones. Table I shows one or two of the main properties of these stones, according to different authorities.

Granites were formed by the cooling of masses of molten rock, and are found in Britain, chiefly in Cornwall, Leicestershire, Westmorland and Aberdeenshire. Their colour varies according to the mineral constituents, and is generally grey or red. These rocks are very durable, and are useful for heavy engineering work.

Limestones are composed chiefly of lime formed from the shells and fossil deposits of living organisms. These stones were deposited in layers under water, and different types vary considerably in texture, colour and stratification. Sometimes, as in Bath stone and Portland stone, the direction and thickness of the various layers or beds are almost indistinguishable. Marbles are limestones which have been changed in character by heat.

Sandstones are composed of

TABLE I—PROPERTIES OF BUILDING STONES

Property	Type of Building Stone		
	Granite	Limestone	Sandstone
Chief Constituents	Quartz Felspar Mica	Calcium Carbonate or Magnesium Carbonate	Small grains of Quartz (Silica)
Weight lb. per cu. ft.	160 to 170	130 to 160	130 to 160
Crushing Strength tons per sq. ft.	1,150 to 1,290	300 to 2,600	130 to 900
Bending Strength (Modulus of Rupture) lb. per sq. in.	2,300 to 3,900	1,200 to 4,700	360 to 1,300
Porosity per cent	Up to 1½ per cent	½ per cent to 14 per cent	5 per cent to 28 per cent
Coefficient of Expansion per deg. F.	·000004 to ·000009	·000004 to ·000006	·000006 to ·000009

TABLE II—REPRESENTATIVE BRITISH BUILDING STONES

Geological Type or Period	Representative Stone	Chemical Constitution	Weight lb. per cu. ft.	Strength tons per sq. ft.	Example of Use
Igneous Rocks	De Lank Granite (Cornwall)	Biotite —Muscovite	163	1,170	Smeaton's Eddystone Lighthouse (1756)
	Rubislaw Granite (Aberdeen)	Muscovite —Biotite	162	1,288	Old Waterloo Bridge (1817)
Carboniferous Limestone Series	Hopton Wood Stone	Limestone 98 per cent $CaCO_3$	158	806	Imperial Institute (1881)
	Prudham Freestone	Sandstone 86 per cent SiO_2	142	455	Central Station, Newcastle (1850)
Millstone Grit Series	Darley Dale Stone	Sandstone 96 per cent SiO_2	150	550	St. George's Hall, Liverpool (1842)
	Kenton Stone	Sandstone 93 per cent SiO_2	140	—	Town Hall, Newcastle (1856)
Coal Measures	Forest of Dean Stone	Sandstone 81 per cent SiO_2	149	569	Old Bailey, London
	Pennant Stone	Sandstone 82 per cent SiO_2	172	1,001	Buildings in Cardiff and Bristol
Permian	Huddlestone Stone	Dolomite 54 per cent $CaCO_3$ 41 per cent $MgCO_3$	138	278	King's College Chapel, Cambridge
	Bolsover Moor Stone	Dolomite 51 per cent $CaCO_3$ 41 per cent $MgCO_3$	152	484	Part of Houses of Parliament
Trias	Storeton Stone	Sandstone 96 per cent SiO_2	138	—	Town Hall Birkenhead (1833)
	Hollington Stone	Sandstone 87 per cent SiO_2	135	289	Hereford Cathedral
Jurassic	Doulting Stone	Limestone 96 per cent $CaCO_3$	150	212	Wells Cathedral
	Portland Stone	Limestone 96 per cent $CaCO_3$	137	240	St. Paul's Cathedral
Cretaceous	Kentish Rag Stone	Limestone 91 per cent $CaCO_3$	167	—	Rochester Castle (1078)

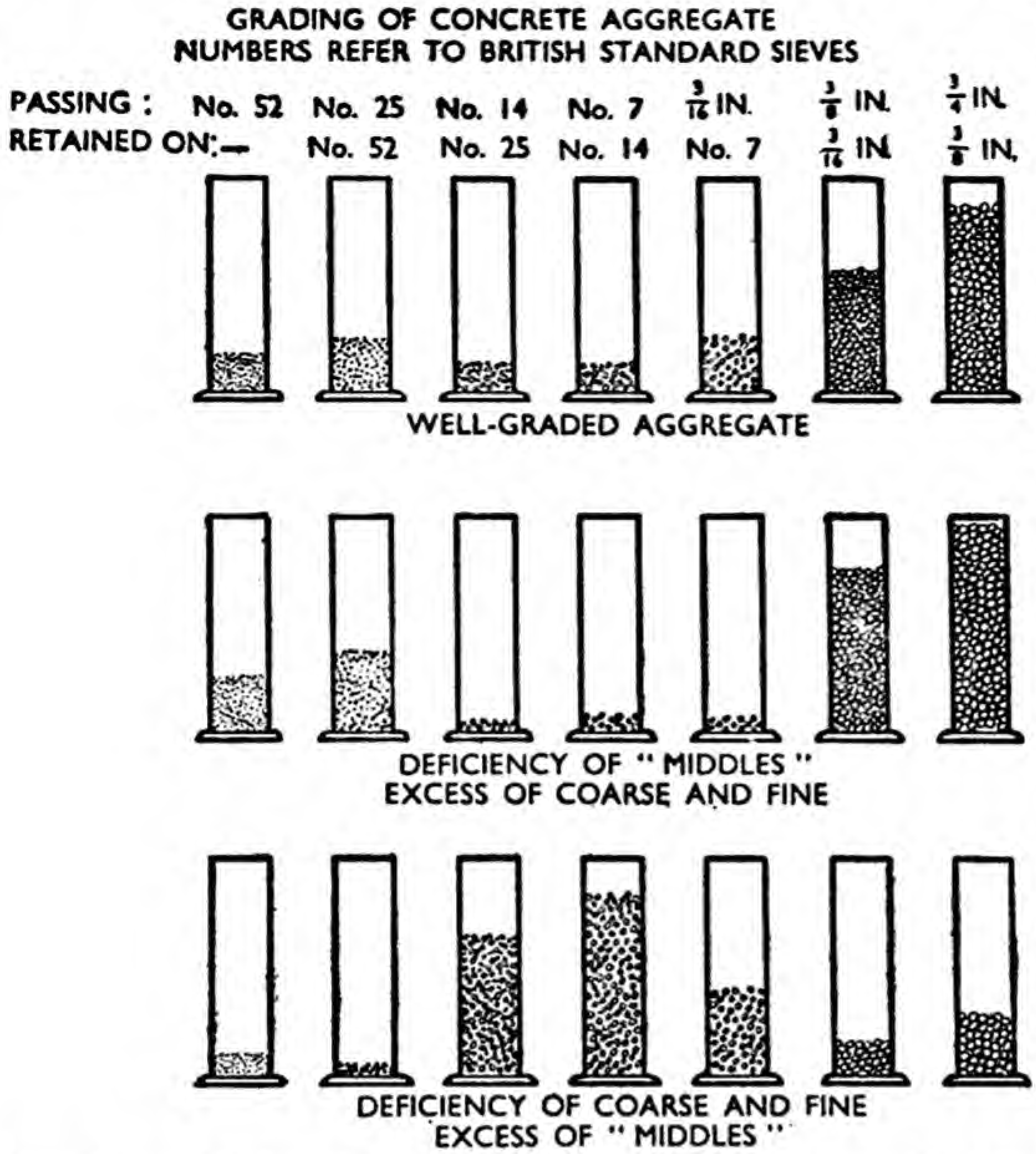


Fig. 13. If the various constituents of concrete are to form a workable and efficient mix, the sand and stone must be 'graded' in suitable proportions. In the ideal mix *all* possible sizes of stone and sand should be present from the largest to the finest. The first diagram shows a suitable grading.

grains of quartz, or silica, cemented together into a hard rock. The grains may be sharp or rounded, and the cementing material may be durable or soft. Sandstones, like limestones, were formed by deposition under water. Table II gives examples of typical rocks dating from various geological periods.

Preserving Stone

For the preservation of building stone, the aim is, essentially, to restrict the entry of water carrying destructive acids and salts. Various substances are used for this purpose, but the discovery of the ideal preservative is not easy. It must penetrate deeply, must not be washed away by rain, and must not completely fill the pores of the stone.

Preservation can be carried out either by consolidating the stone by a deposition of silica within the pores, or by the exclusion of water by means of a water-repelling substance. Consolidation is carried out by treating the stone with potassium or sodium silicate, or with shellac, but the best method is probably the application of an alcoholic solution of silicon ester. Various oils, resins and fats have been used in the waterproofing process, and paraffin wax dissolved in coal-tar naphtha gives satisfactory results if properly applied. The stone should be clean before any application is made. Cleaning by steam jets is the most efficient method.

Concrete

Although concrete is one of the most widely employed materials in building, it is one which is probably the least understood. The fact that it contains only one manufactured constituent, Portland cement, and consists chiefly of natural stone, sand and water, makes it seem too commonplace to require much understanding or study. However, if the best of the properties of concrete are to be developed, much

care must be taken over its manufacture.

Concrete is an artificial stone and, like a natural stone, it must possess the properties of strength and durability. In addition, it must be sufficiently fluid when freshly mixed to flow easily into the mould prepared for it.

When concrete is made, the dry materials are all mixed together before the water is added, but in order to understand how the properties of concrete are developed, it must be visualized as the mixture of two materials only. These materials are (*a*) mixed aggregate, stone and sand, and (*b*) cement paste (Portland cement and water).

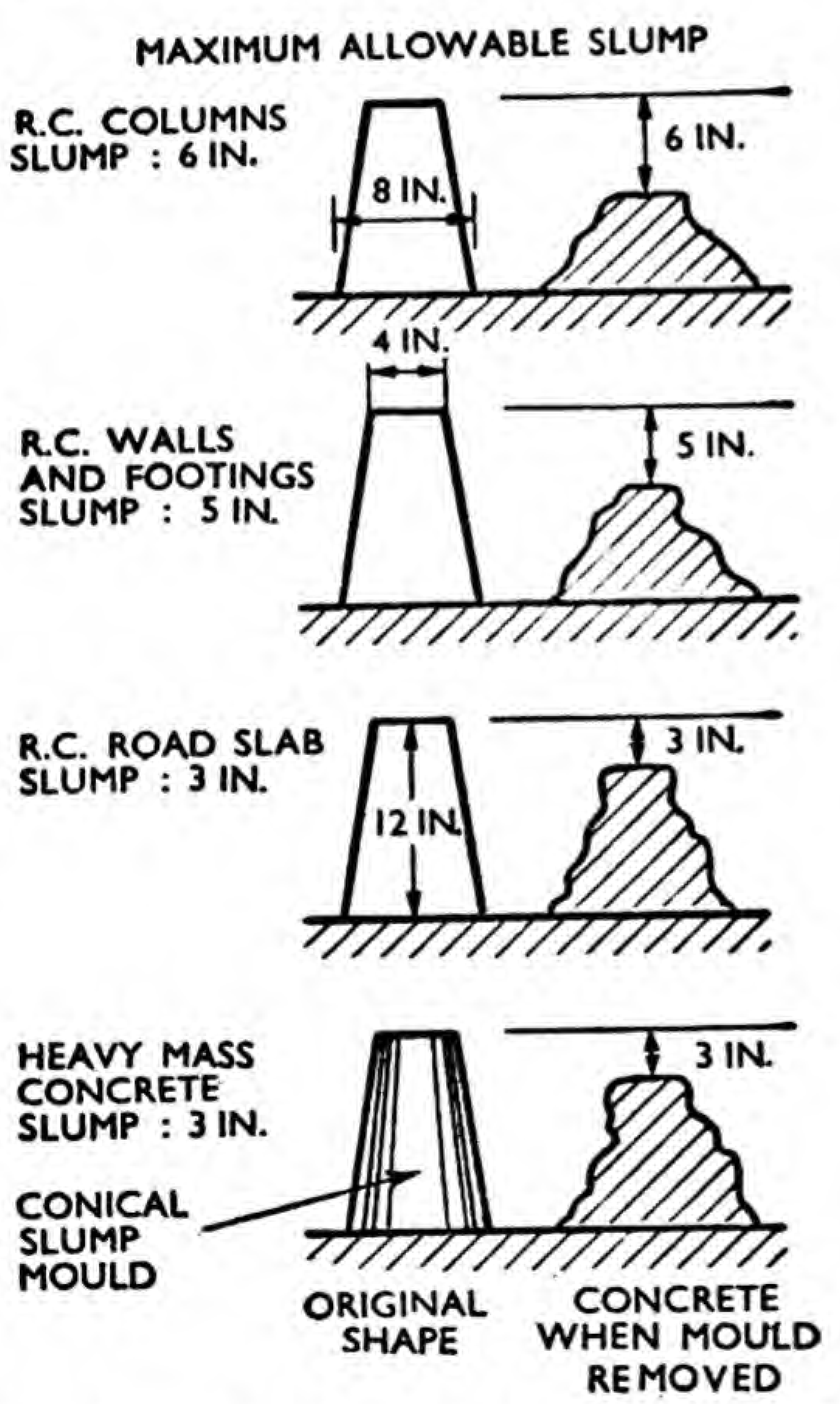


Fig. 14. The slump test checks the workability of concrete. Where there are reinforcing bars, the concrete must flow more readily than in mass work. These sketches show slumps commonly used in practice.

The inert aggregate must be so constituted that all sizes of material are present, from the largest to the smallest. If there is no important size missing, the material is said to be well graded. Fig. 13 shows the amounts of material of different sizes sieved out from mixed aggregates, and compares well-graded and badly graded materials.

Fig. 15. The less water which can be used in concrete, the stronger and more resistant is the resultant material. Water content is usually defined by stating the water-cement ratio.

The cement paste has an important bearing on the strength of the concrete.

An excess of water results in a weak paste and, consequently, in a weak concrete. The water-cement ratio is extremely important and should be kept as low as is consistent with the given conditions of the job in hand.

One method of designing a suitable concrete mix is as follows : (*a*) The strength required of the concrete is known, and an appropriate water-cement ratio is chosen from the results of previous experience. (*b*) It is then decided in what proportions a cement paste of this quality and a well-graded aggregate should be mixed in order to give a concrete of the required consistency, or workability. The consistency of concrete is measured by the slump test, in which a pile

of concrete, cast in a conical mould, is allowed to slump down as the cone is removed (Fig. 14).

Figs. 14 and 15 show, diagrammatically, the water-cement ratios and slumps commonly required for different types of construction.

Strength of Concrete

Concrete is very strong in compression. If suitable material is used, and suitable methods employed in mixing, concrete may carry a working stress of from 750 to 1,000 lb. per sq. in. Concrete is, however, very weak in tension, and in portions of a structure where tensile stresses are developed, steel must be inserted to carry the tensile load.

Although only recently coming prominently into public notice, plastics are not a recent development. A plastic can be defined as a synthetic material formed into its finished shape while in a plastic condition, and the first, celluloid, was discovered in 1868. It was followed by casein and Bakelite before the outbreak of the First World War in 1914. In the nineteen - thirties and nineteen - forties, however, experiment in and use of many new types of plastics made them well-known materials, and they have certain important mechanical and structural applications.

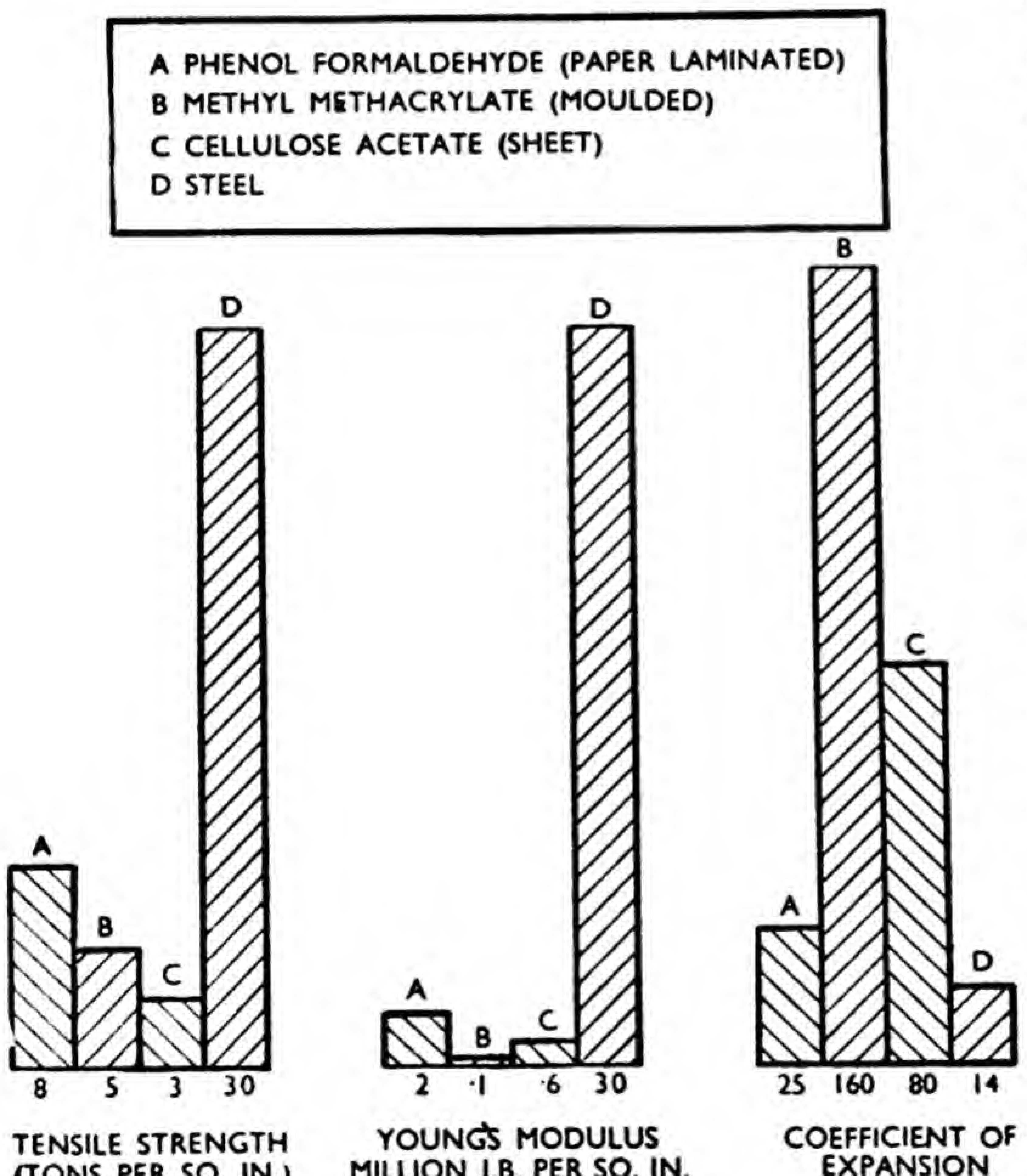


Fig. 16. Three plastics, chosen at random, are here compared with steel with regard to strength, modulus of elasticity and coefficient of expansion. These particular plastics are not so strong as steel, and they change in length more than steel does when heated. Other plastics and other varieties of these plastics may, of course, have quite different properties.

In the manufacture of a plastic, the raw materials used are usually two or more of the following: coal, natural gas, wood alcohol, skimmed milk, alum and cotton linters. The result of suitable manufacture is the production of a large number of important new materials which may be moulded, cast and extruded into many useful forms.

At some point in the manufacture of the finished article, there is an application of heat, which either causes the material to become

TABLE III—SOME USEFUL BRITISH STANDARD SPECIFICATIONS

B.S.S. dealing with the properties of :—	B.S.S. Number
Steel for Bridges and General Construction	15
High-Tensile Steel for Bridges	548
Steel for Shipbuilding	13
Spring Steel for General Engineering Purposes	970
Steel Arches for Mines	227
Duralumin Bars	477
High-Tensile Aluminium Bronze Castings	1,073
Brass Sheets, Strip and Foil	265–7
Phosphor-Bronze Castings for Gears ..	421
Cast-Iron Drain Pipes	437
Timber for Building Construction ..	1,018
Portland Cement	12
Natural Aggregate for Concrete.. ..	882
Common Bricks	657
Synthetic Resin Bonded Fabric Sheet..	972

plastic, to solidify on cooling, or causes changes in the material, freezing it into its new shape.

Just as the valuable properties of concrete are best exploited when concrete is reinforced with steel, plastics are often combined with other materials in order to develop their best qualities. Such plastics as are made from resins, usually contain fillers such as wood flour or asbestos fibre, in order to decrease brittleness or to give other valuable properties to the material. Other additions take the form of sheets of paper or cloth, which are soaked in the plastic material and squeezed into laminated blocks.

When the properties of plastics are considered, it is found that there is one factor which does not enter into any discussion of other types of structural material used under normal conditions. This factor is change of temperature. Rise of temperature, for example, does not affect the properties of steel until the temperature becomes so high as to be encountered only in unusual circumstances. At such high temperatures, metals may creep, or gradually change shape under loading. Plastics, however, creep at normal temperatures, and due consideration must be taken of this fact when deciding on shapes and loadings of plastic materials (Fig. 16).

The production of numerous ferrous and non-ferrous alloys, the development of plastics and the increasing use of the powerful and adaptable tool of heat treatment, all indicate present-day tendencies.

Future Possibilities

In the future, the functions and forms of structures and machines will not be limited, as they were for centuries, by the unalterable properties of a few available materials. Rather will the nature and properties of the material to be used be dictated by and adjusted for the required conditions (Table III). There are many indications that the middle of the twentieth century is the beginning of a period of close control over the nature and properties of materials. The principles of mechanics, outlined in this book, indicate the conditions which do exist. We can say then, that it may be expected that, in the future, such conditions will be recognized, met and resisted in a much more precise fashion than ever before.

CHAPTER 10

MACHINE PRINCIPLES AND DESIGN

PRINCIPLE OF WORK. LIFTING MACHINES. DIFFERENTIAL WHEEL AND AXLE. VELOCITY RATIO. MECHANICAL ADVANTAGE. EFFICIENCY. GEARS AND TRAINS OF GEARS. COMPOUND TRAINS. INTERNAL GEARS. BEVEL GEARS. WORM GEARS. HELICAL GEARS. EPICYCLIC GEAR TRAINS. SCREW CUTTING. BELT DRIVES. HOW A BELT TRANSMITS POWER. LEATHER AND RUBBER BELTS. PISTONS AND CYLINDERS. CALCULATING HORSE-POWER. DIFFERENCE BETWEEN INDICATED AND BRAKE HORSE-POWER. MECHANICAL EFFICIENCY.

IN an age of mechanization with the almost infinite variety of mechanical aids to life, from the alarm clock that wakes us up in the morning to the switch that puts out the light when we go to bed, it should scarcely be necessary to define what a machine is. Yet each machine, no matter how complex, is in reality composed of a number of elements; the lever, pulley, gearwheel, screw, piston and cylinder, etc. In the language of mechanics, each of these simple elements is called a machine, and when we can understand the laws governing the simple machines, it is not difficult to comprehend the most intricate mechanism of which the simple elements are a part.

First Law of Machines

The first law of machines is that a machine does not generate work or power, it merely transforms the energy that is put into it into a more useful or convenient form. Thus, in one of the earliest machines, the waterwheel, the energy is in the falling water, and the wheel takes up some of this energy and gives it out in a more convenient form, as rotational energy of the shaft. The greater the amount of water that is admitted to the wheel, the greater is the energy that may be converted to work at the shaft. This is expressed in the principle of work, which, for a simple machine, may be stated as follows :—

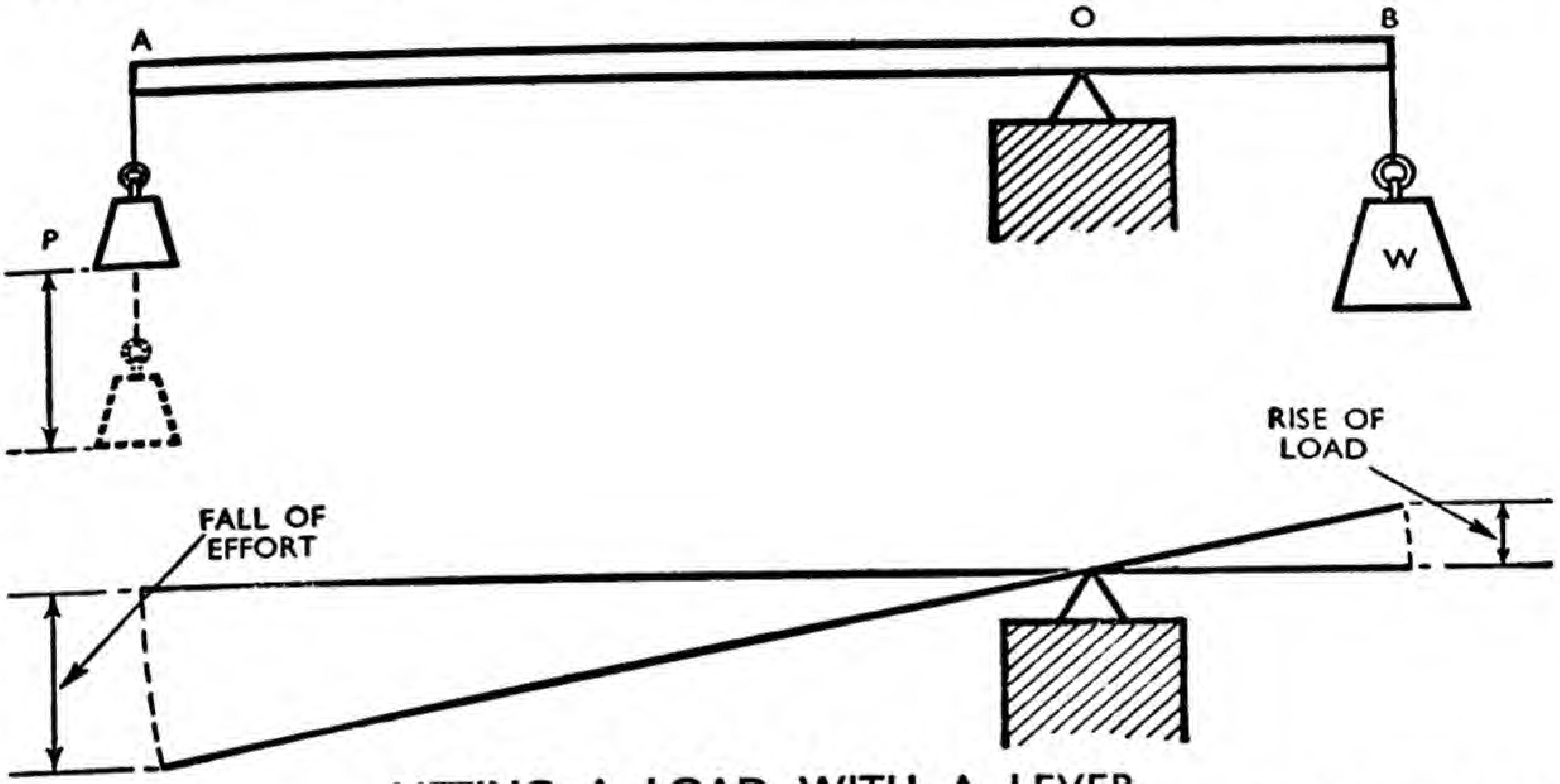
Input side		*Output side*
Work done by the effort	=	Work done against the resistance.

Force and Speed

A machine is used when we wish to exert a large force by the application of a small one, in order to obtain purchase or mechanical advantage, or when we wish to obtain an increase in speed or to secure operation at a distance. At first sight it might appear that by using a machine to exert a large force by the application of a small effort, we were getting an increase of work on the output side, but it will always be found that the effort has to move a proportionately greater distance, and thus to put at least as much energy into the machine as is obtained from it.

Conversely, when an increase in speed by the use of a machine is obtained, there is a corresponding reduction in the force exerted for a given amount of work done. This dependence of force upon speed or distance moved, in accordance with the principle of work, i.e., that the work put in is equal to the work got out, is the basis of the popular saying, 'What is gained in power (force) is lost in speed.'

A familiar example nowadays is the gearbox of a motor car, or the three-speed hub of a bicycle, which, by the provision of three alternative speeds, enables a suitable gear to be selected to suit the resistance that has to be overcome. Thus, when we approach a hill, we change down, that is, select a lower gear and thus secure a greater mechanical advantage. The steeper the hill, that is to say, the greater the resistance, the lower is the gear engaged, and the lower the speed. However, it is to be noted that the power input by the engine or the cyclist remains the same.



LIFTING A LOAD WITH A LEVER

Fig. 1. One of the simplest machines is the lever. With its aid a heavy load *W* can be raised by a small effort *P*. Ratio of W to *P* depends upon the leverage *AO*/*OB*, and in raising the load, work done on both sides is equal because the smaller effort moves through the greater distance.

The Lever

Now let us apply this principle to a simple machine, the lever, and explain some of the terms used in greater detail. Fig. 1 shows a lever by means of which an effort P applied at A is used to raise a load W suspended at B. The leverage will depend upon the ratio of the distances of the effort and the load from the fulcrum O. Thus, if AO is three times the length of OB, the leverage will be 3 to 1 and, neglecting friction, an effort P at A will raise a load W, which is equal to $3P$, at B. In other words, the mechanical advantage is 3, where we define mechanical advantage as follows:—

$$\text{Mechanical advantage} = \frac{\text{Load, or Resistance}}{\text{Effort}}$$

But it will also be obvious from Fig. 1 that if AO is three times

the length of OB, the fall of the effort will be three times the rise of the load. This ratio of the distance moved by the effort to the distance moved by the load in a given time is called the velocity ratio. Thus :—

Velocity ratio

$$= \frac{\text{Distance moved by effort}}{\text{Distance moved by load, or resistance}}$$

In this example, the velocity ratio will be 3, so that if the fall of the effort is s ft., the rise of the load will be $\frac{s}{3}$ ft. Now, by applying the principle of work, we get an equation as follows :—

Input side		*Output side*
$P \times$ distance moved by P	$=$	$W \times$ distance moved by W
$P \times s$ ft.-lb.	$=$	$3P \times \frac{s}{3}$ ft.-lb.
Ps	$=$	Ps

From the fact that in this example the mechanical advantage and the velocity ratio both have the value 3, it might be inferred that the mechanical advantage is always equal to the velocity ratio. This is not so, except when, as in this example, friction is neglected. It will be apparent that if the effort P has first to overcome the friction at the pivot or fulcrum at O before it can start to move the load W, the load it can raise will be less than $3P$. Also, a small effort will be required to overcome the friction even when the load W is zero, and under these conditions the mechanical advantage will be zero.

Velocity Ratio a Constant

On the other hand, the velocity ratio is unaffected by the friction of the pivot and depends entirely upon the relative lengths of AO and OB. This enables us to make a clear distinction between the two quantities, which applies to all machines.

The velocity ratio can always be calculated from the dimensions or geometry of the machine, and is a fixed value for that machine, being unaffected by the friction or the load. The mechanical advantage, on the other hand, can only be determined by an actual test, and will vary with the friction and the load. The relationship between them can, perhaps, best be visualized by making use of the idea of the ideal load, that is, the load which the machine would raise if it were perfect and there were no friction, so that :—

Ideal load

$$= \text{Effort} \times \text{Velocity ratio.}$$

The effect of friction is always to make the actual load less than the ideal load. The less the friction, the more closely does the actual load approach the ideal load, and the more closely does the value of the mechanical advantage approach the value of the velocity ratio.

At first sighı, it might be thought that the principle of work would not be strictly true for the actual machine in which the load is less than the ideal value owing to friction. A glance at the following equation, however, shows that the work done against friction must be included in the work done on the output side, just as the fixed charges must appear with the profits on a financial balance sheet.

Input side		*Output side*
Work done by the effort	$=$	Work lost in friction
	$+$	Useful work done on the load.

This not only shows us that the principle of work applies equally well to the case where friction is included, but also that if we know the amount of work lost in friction, we can estimate the useful work done on the load, and hence the mechanical advantage. We shall consider this further in the sections dealing with the efficiency and testing of machines.

The Wheel and Axle

The disadvantage of the lever as a lifting machine is that only a small lift is possible, and the load must be supported whilst the fulcrum is changed to a new position if a further lift is required. This disadvantage is overcome when a wheel or drum is used to raise the load, and Fig. 2 shows a simple device of this kind. It is called the wheel and axle. The load is raised by means of a rope or chain which is wound on to the drum or axle, as the effort causes the axle to rotate by unwinding the effort rope from the wheel.

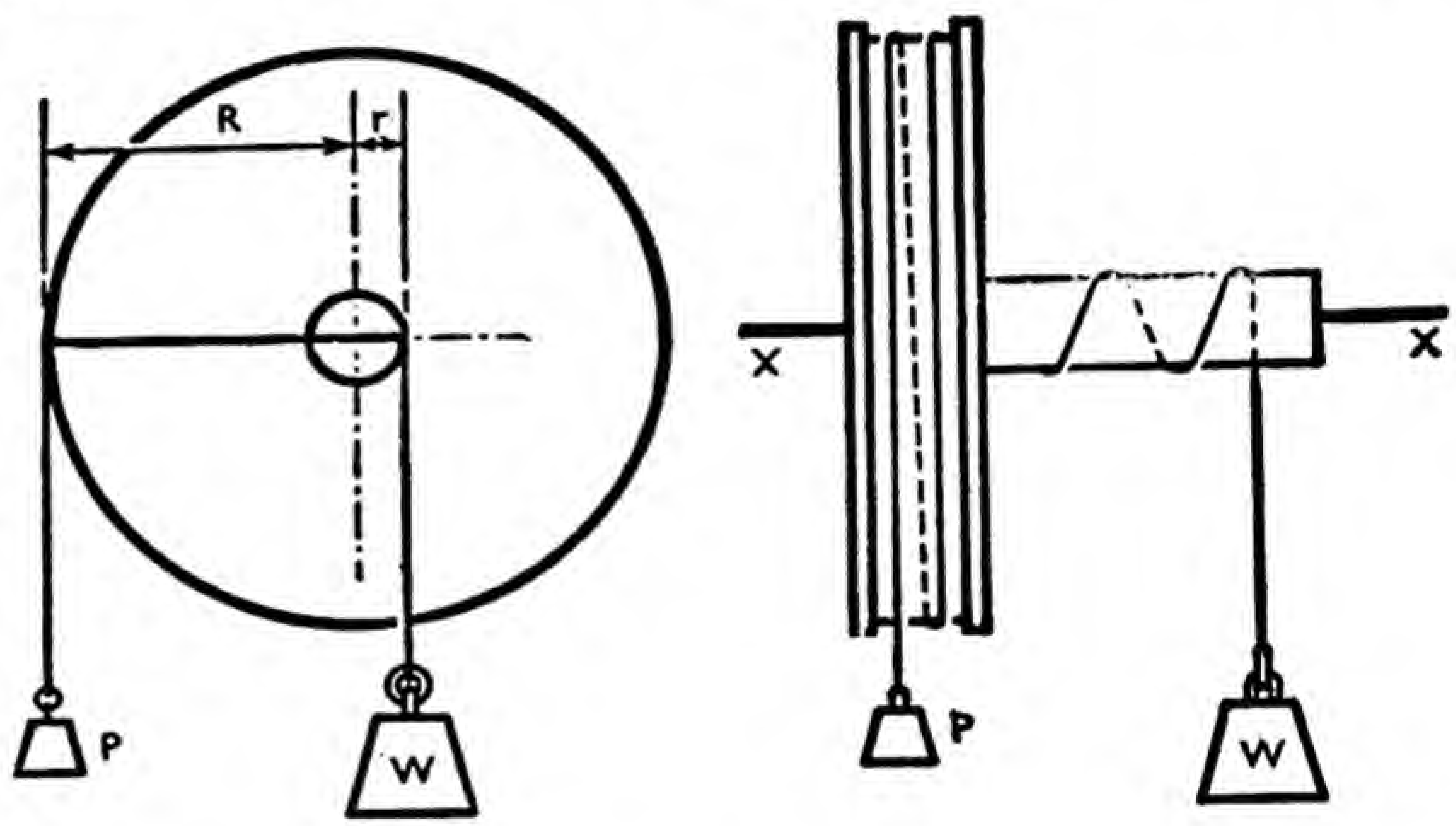


Fig. 2. The wheel and axle shown enable the load to be raised continuously through any desired distance. As effort P unwinds the cord from the wheel, load W is raised by means of a rope which winds on to the axle. The axis XX corresponds to the fulcrum and the radii R and r to the arms of the lever.

Balance of Effort and Load

From the end view at the left of Fig. 2, it can be seen that the effort P and the load W balance about the axis of the axle XX in the same way that with the lever they balance about the fulcrum. The arms of the lever are represented by R and r, the radii of the wheel and the axle respectively. The velocity ratio of the device is, therefore, $\frac{R}{r}$. Alternatively, to determine the velocity ratio, we can consider the distances through which the effort falls and the load rises in, for example, one revolution of the wheel and axle. The effort will fall a distance equal to the circumference of the wheel, that is $2\pi R$, and, similarly, the load will rise a distance $2\pi r$.

Hence :—

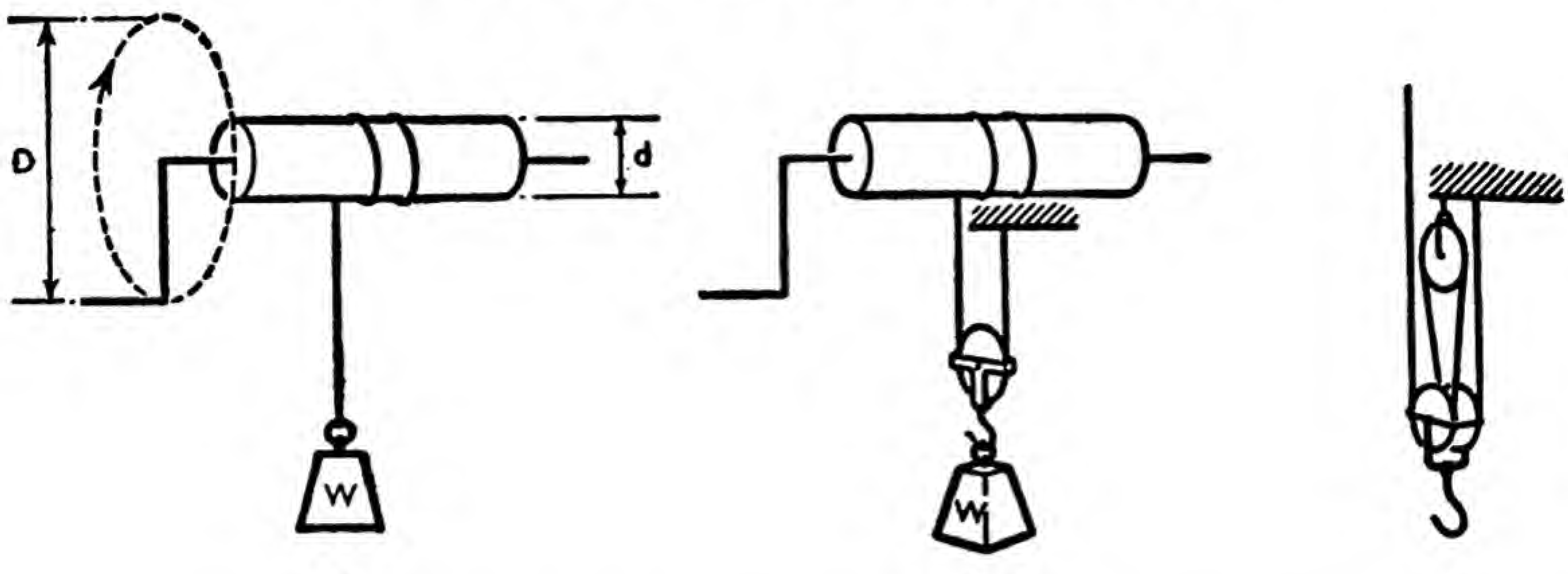
Velocity ratio

$$= \frac{\text{Distance moved by effort}}{\text{Distance moved by load}}$$

$$= \frac{2\pi R}{2\pi r}$$

$$= \frac{R}{r}.$$

For example, if R is 12 in. and r is 2 in., the velocity ratio is 6. Owing to the effects of friction in the bearings, the mechanical advantage will be slightly less than 6, say 5·5. So that an effort of 20 lb. would be able to raise a load of $5{\cdot}5 \times 20 = 110$ lb., that is, approximately a hundredweight.



INTRODUCING A PULLEY DOUBLES VELOCITY RATIO

Fig. 3. Common windlass is a form of wheel and axle in which the wheel is replaced by a crank handle. (*b*) Introducing a pulley doubles the velocity ratio because 2 feet of rope must be wound on the drum for each foot the load is raised.

If the wheel is replaced by a crank handle, we get the common windlass, shown in Fig. 3(a). As in the case of the wheel and axle, the velocity ratio is given by the ratio of the radius, or, as it is more frequently called, the throw of the crank to the radius of the drum. Or, what amounts to the same thing, the ratio of the diameter of the crank circle, shown dotted, to the diameter of the drum.

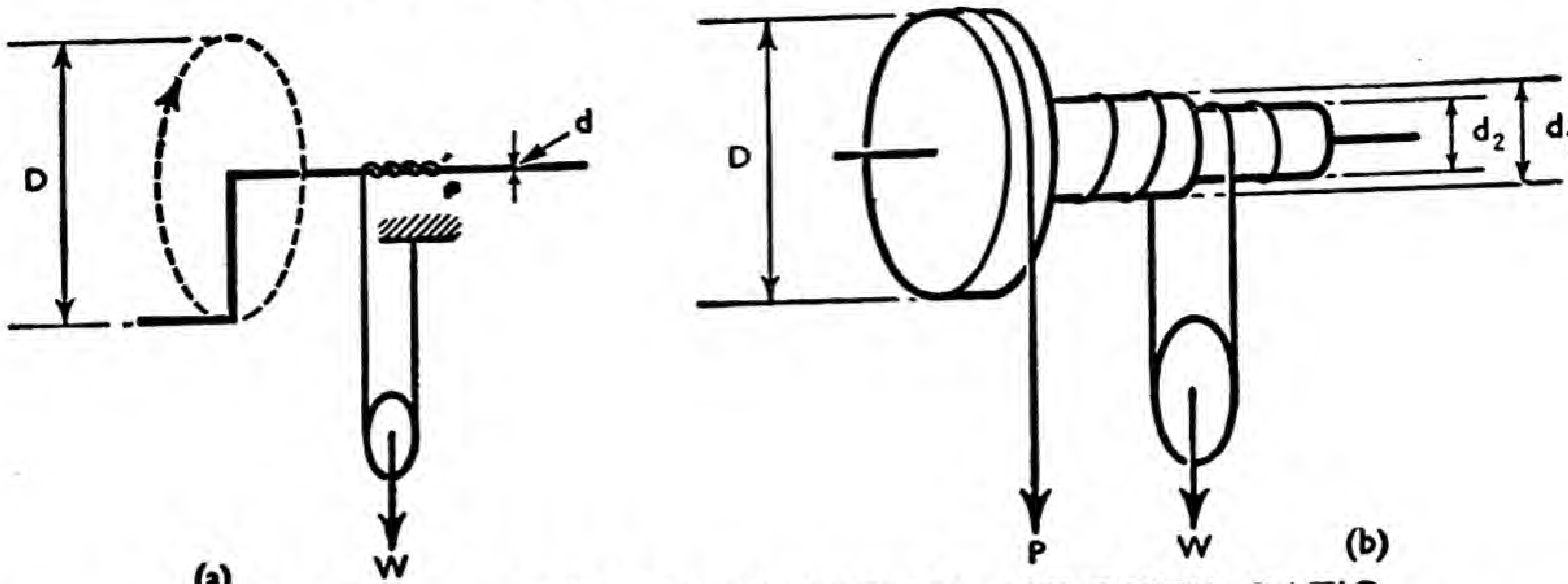
Obtaining Greater Purchase

A greater purchase can be obtained by introducing a pulley block as shown in Fig. 3(b). In this arrangement, one end of the rope is attached to a fixed point beneath the drum, and the pulley to which the load is attached hangs in the loop so formed. It can readily be seen that to raise the load 1 ft., it is now necessary to wind in 2 ft. on to the drum, that is, to shorten the sides of the loop by 1 ft. each. Therefore, this pulley arrangement itself has a velocity ratio of 2, and the total velocity ratio for the windlass shown in Fig. 3(b) is $\frac{D}{d} \times 2$. Using the same figures as for the wheel and axle, the velocity ratio would be $6 \times 2 = 12$, and a man exerting a force of 20 lb. at the crank handle, would be able to raise a load of almost 2 cwt.

Fig. 3(c) shows how, by using two additional pulleys, the load can be raised by a double loop of rope, and the velocity ratio for the pulleys alone will then be 4, and for the windlass as a whole, $6 \times 4 = 24$.

This is a very useful relationship, that when we have two or more simple machines coupled together in this way to give a compound effect, the velocity ratio of the combination is equal to the product of the velocity ratios of the separate components. By the use of several components, the multiple effect can be made very large indeed, but we must not forget that each additional component introduces more friction, and that although we may obtain a very large velocity ratio, the mechanical advantage may be only one half or two-thirds of this figure.

Another way of obtaining a large velocity ratio is shown in Fig. 4(b). This is called the differential wheel and axle, and



DIFFERENTIAL AXLE GIVES HIGHER VELOCITY RATIO

Fig. 4. (*a*) Velocity ratio $2D/d$ for the wheel and axle may be magnified either by increasing D or decreasing d. A small axle, however, would not be strong enough to support the load. (*b*) Difficulty is overcome by means of the differential wheel and axle in which the velocity ratio depends upon the difference in the diameters of the two parts of the axle.

you will notice that the drum or axle is composed of two parts of different diameter. This is where the differential principle comes in. From what has been said already about the simple wheel and axle, it will be obvious that, unless a system of additional pulleys is used, a large velocity ratio cannot be obtained except by making the diameter D of the wheel very large, or by making the diameter d of the axle very small, as shown in Fig. 4(a).

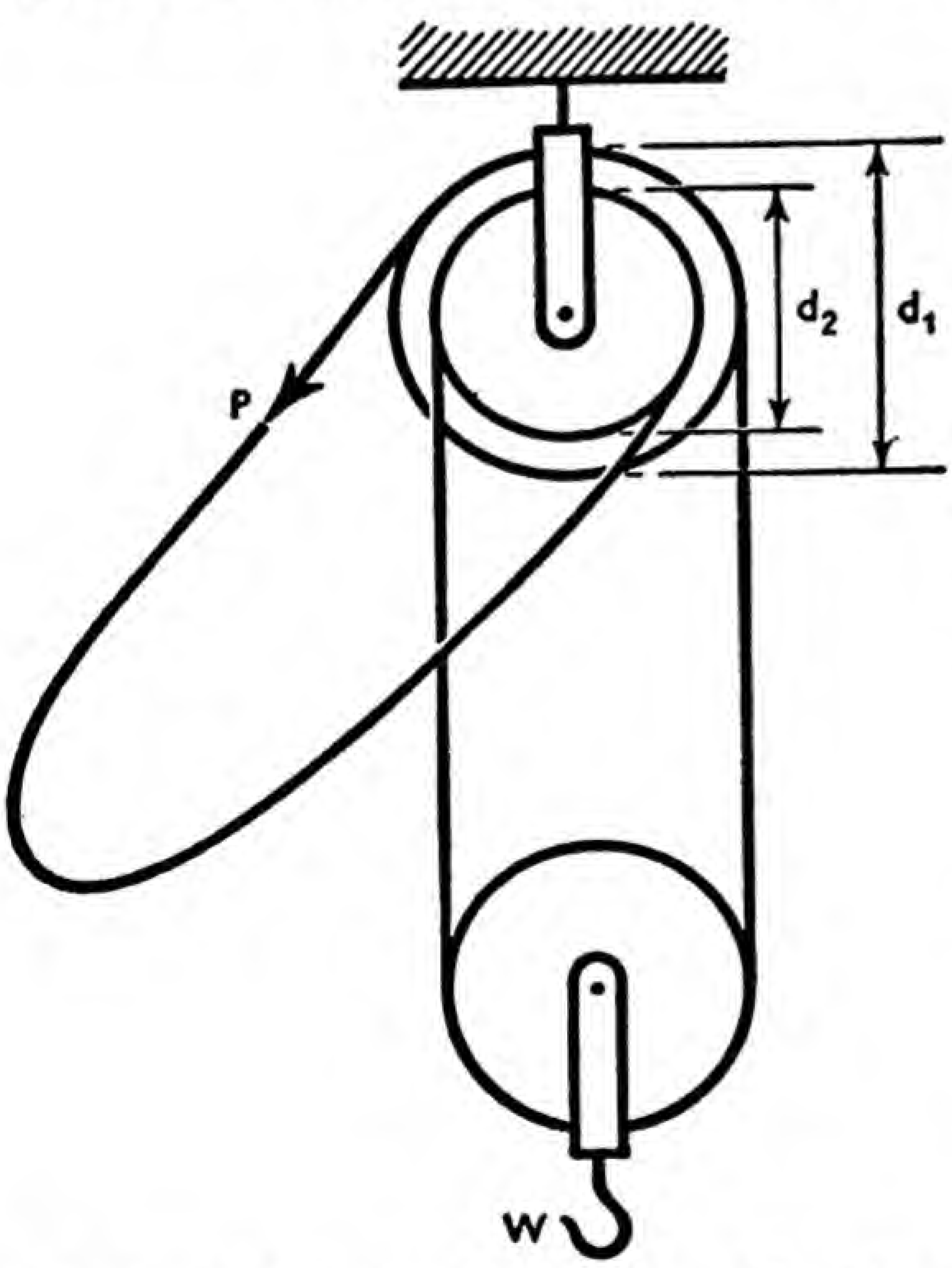


Fig. 5. In Weston's differential pulley block, pulleys of diameters d_1 and d_2 rotate as one. Effort P rotates the pulleys, so shortening the loop by winding rope on to large pulley and unwinding a shorter length from smaller pulley.

Differential Principle

If the axle is made very small, it will not be strong enough to support the load. How does the differential principle get us out of this difficulty? Returning to Fig. 4(b), it will be seen that the axle may be as large as we wish, because the velocity ratio depends upon the difference in diameter of the two portions. In one revolution the rope will wind up on to the large drum by an amount πd_1, equal to the circumference of the large drum. But the rope will also unwind from the small drum by an amount πd_2, equal to the circumference of the small drum, and, therefore, the total length of rope in the loop will be shortened by the amount :—

$$\pi d_1 - \pi d_2 = \pi(d_1 - d_2).$$

The load, as before, will rise a distance equal to one half the

amount by which the rope is shortened, i.e., in one revolution, the distance moved by the load

$$= \frac{\pi(d_1 - d_2)}{2}.$$

The effort, or force, at the crank handle, moves a distance πD. Therefore :—

$$\text{Velocity ratio} = \frac{\pi D}{\frac{\pi(d_1 - d_2)}{2}}$$

$$= \frac{2D}{(d_1 - d_2)}.$$

This differential principle is very useful when a large purchase is required, and Fig. 5 shows how it can be applied to a pulley block. The two pulleys at the top are of different diameter, and are fixed together so that they rotate as one. The endless rope, or chain, passes round all the pulleys, and it will be seen that in raising the load, the rope winds up on to the large pulley and unwinds from the smaller pulley, exactly as with the differential axle. The velocity ratio is :—

$$\frac{2d_1}{(d_1 - d_2)}.$$

This arrangement is known as Weston's differential pulley block, and with it one man can easily lift a load of half a ton or more.

For example :—

$d_1 = 10$ in. $d_2 = 9\frac{3}{4}$ in. so that $(d_1 - d_2) = \frac{1}{4}$ in.

$$\text{Velocity ratio} = \frac{2 \times 10}{\frac{1}{4}} = 80.$$

Tests show that the loss by friction in this case is rarely less than 60 per cent; in fact, this is why the load does not begin to fall when the effort is removed.

Allowing for friction, the mechanical advantage might be, say, 35. Therefore, for a 40-lb. pull, the load raised would be $35 \times 40 =$ 1,400 lb.

Modern pulley blocks, operating on a different principle, have an efficiency of over 85 per cent.

Another method of obtaining

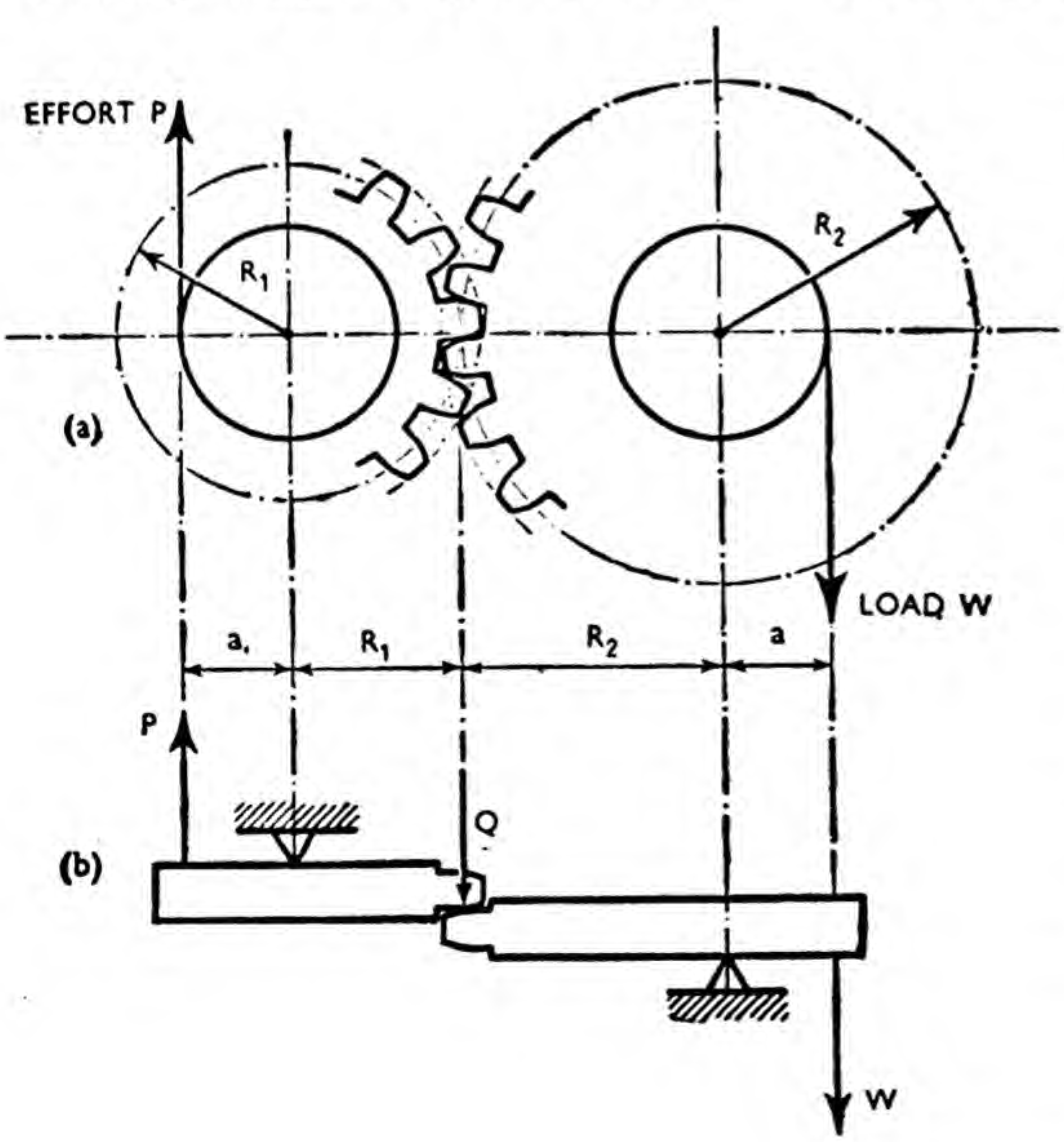


Fig. 6. The action of spur gears may be likened to that of a series of levers in which axes of the wheels are fulcrums. Effort and load exert moments which are balanced by moments of force Q acting between gear teeth. Leverage obtained is proportional to the radii of the gearwheels and, therefore, to numbers of teeth on the two wheels. (*a*) Shows a pair of spur gears by means of which effort P raises load W. (*b*) Shows the equivalent lever system. Mechanical advantage $\frac{W}{P} = \frac{R_2}{R_1} = \frac{N_2}{N_1}$ where N_1 and N_2 are the numbers of teeth.

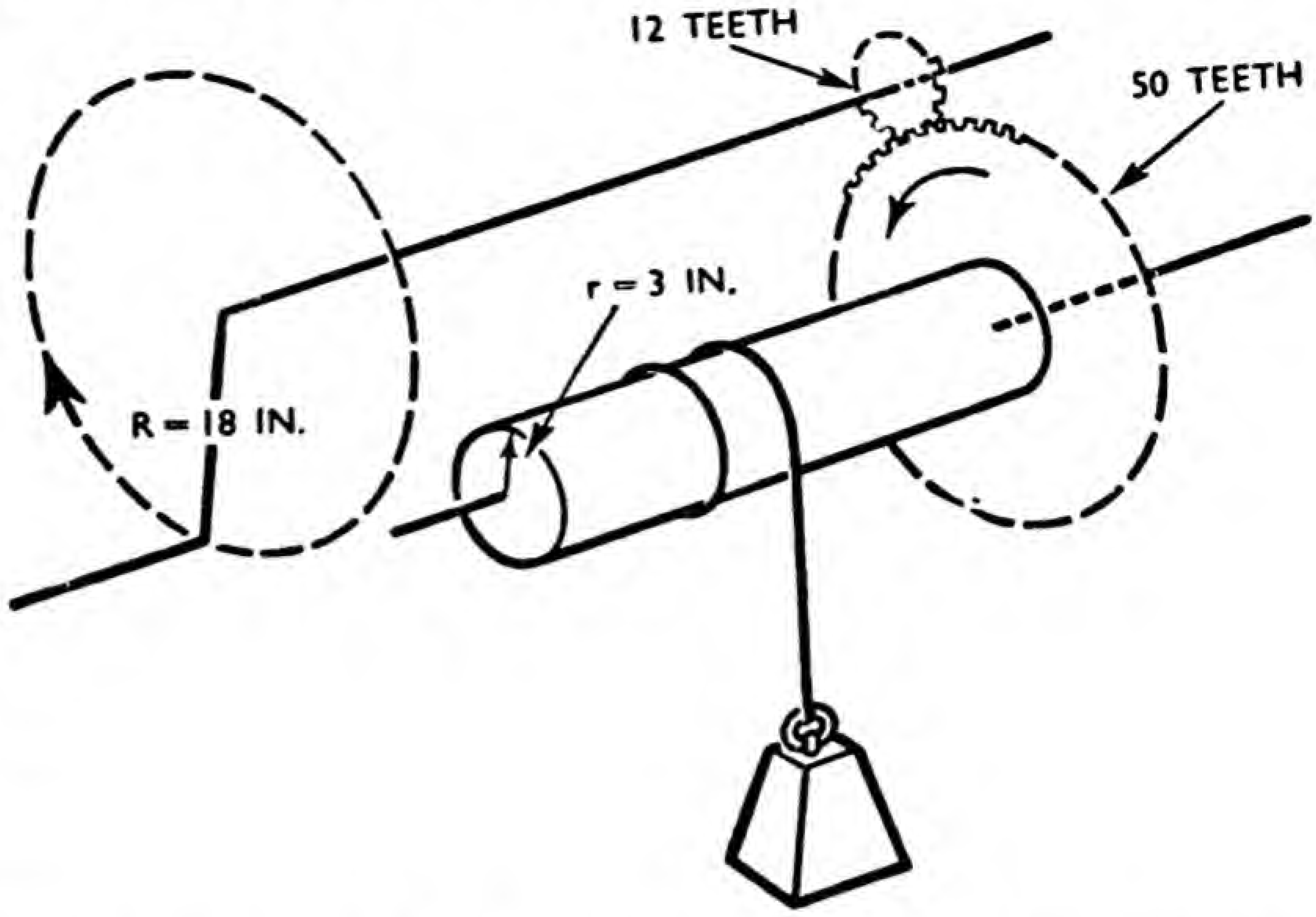


Fig. 7. The geared winch is a simple windlass provided with gears. Without gears the velocity ratio would be $\frac{R}{r}$ or 6. But the pair of gears has a velocity ratio of $\frac{50}{12}$ giving a total velocity ratio of 25. The use of gears increases the velocity ratio and enables a much greater load to be lifted with a given effort. The load, however, will rise more slowly.

mechanical advantage, which is of great importance at the present time, is by means of toothed gearing. As with pulleys, we can use the analogy of the lever to determine the relationship between the forces. This is shown in Fig. 6(a), where a pair of spur gears is shown by means of which an effort P is used to raise a load W, and in Fig. 6(b) is shown the equivalent lever system. By taking moments about the axes of the gearwheels, it can be seen that the effort P will exert a force Q at the point of contact of the teeth, and that this force Q will then balance the load. Neglecting friction at the pivots and between the teeth, the relationship between the forces will be as follows :—

$$P \times a = Q \times R_1,$$

$$\text{and, } W \times a = Q \times R_2.$$

$$\therefore \text{Mechanical advantage} = \frac{W}{P} = \frac{R_2}{R_1}.$$

More frequently when dealing with gearwheels we consider the number of teeth on each wheel rather than the radii of the wheels. So that :—

$$\text{Mechanical advantage} = \frac{W}{P} = \frac{N_2}{N_1},$$

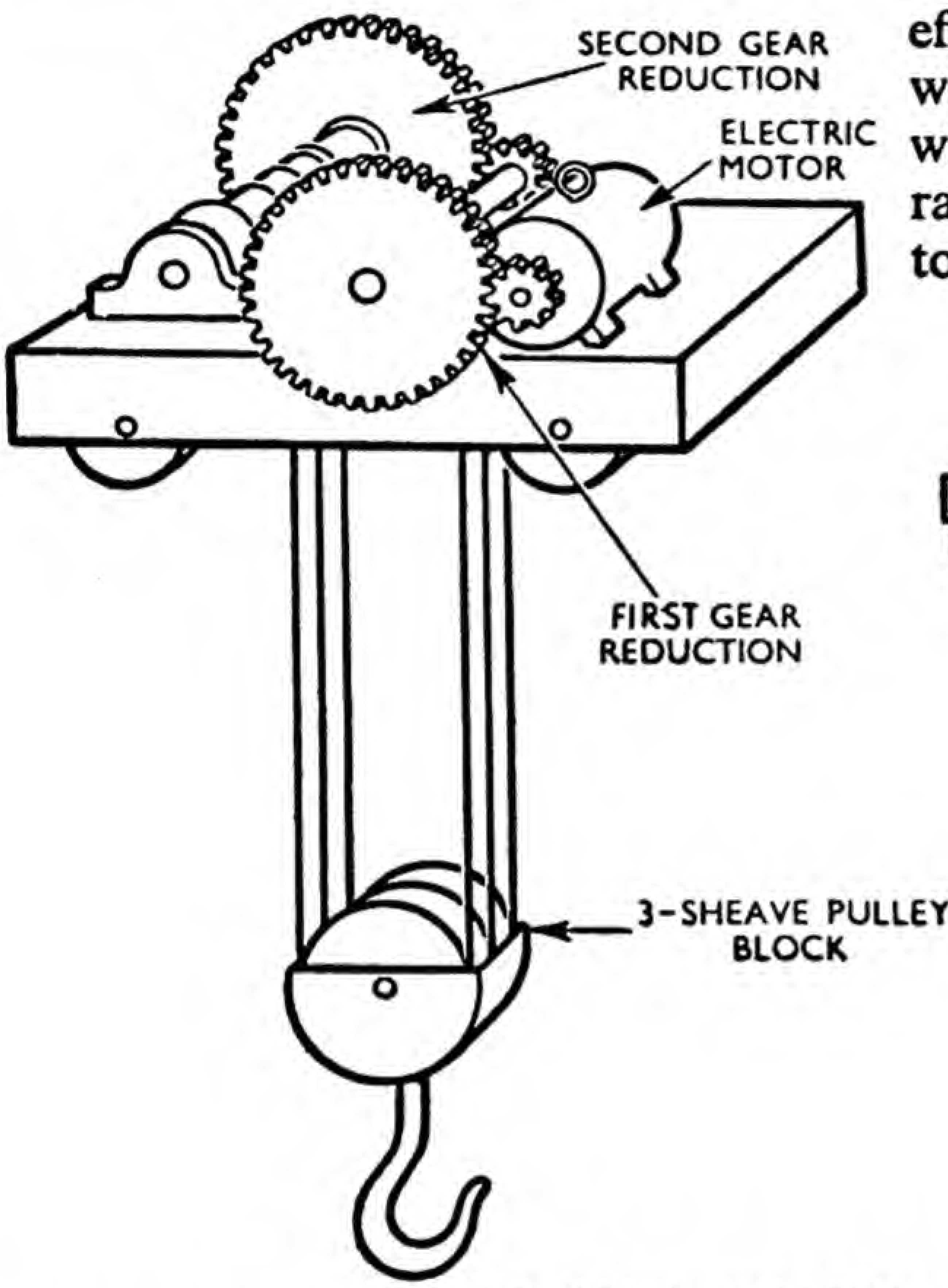
where N_1 and N_2 are the numbers of teeth on the wheels of radius R_1 and R_2 respectively. It will be obvious that if two wheels are to gear together, the teeth must be of the same size, and the number of teeth will, therefore, be proportional to the circumference. Thus, a 40-tooth wheel will have twice the circumference and, therefore, twice the radius of a 20-tooth wheel, and these wheels meshing together would give a velocity ratio of $\frac{40}{20}$, viz., 2.

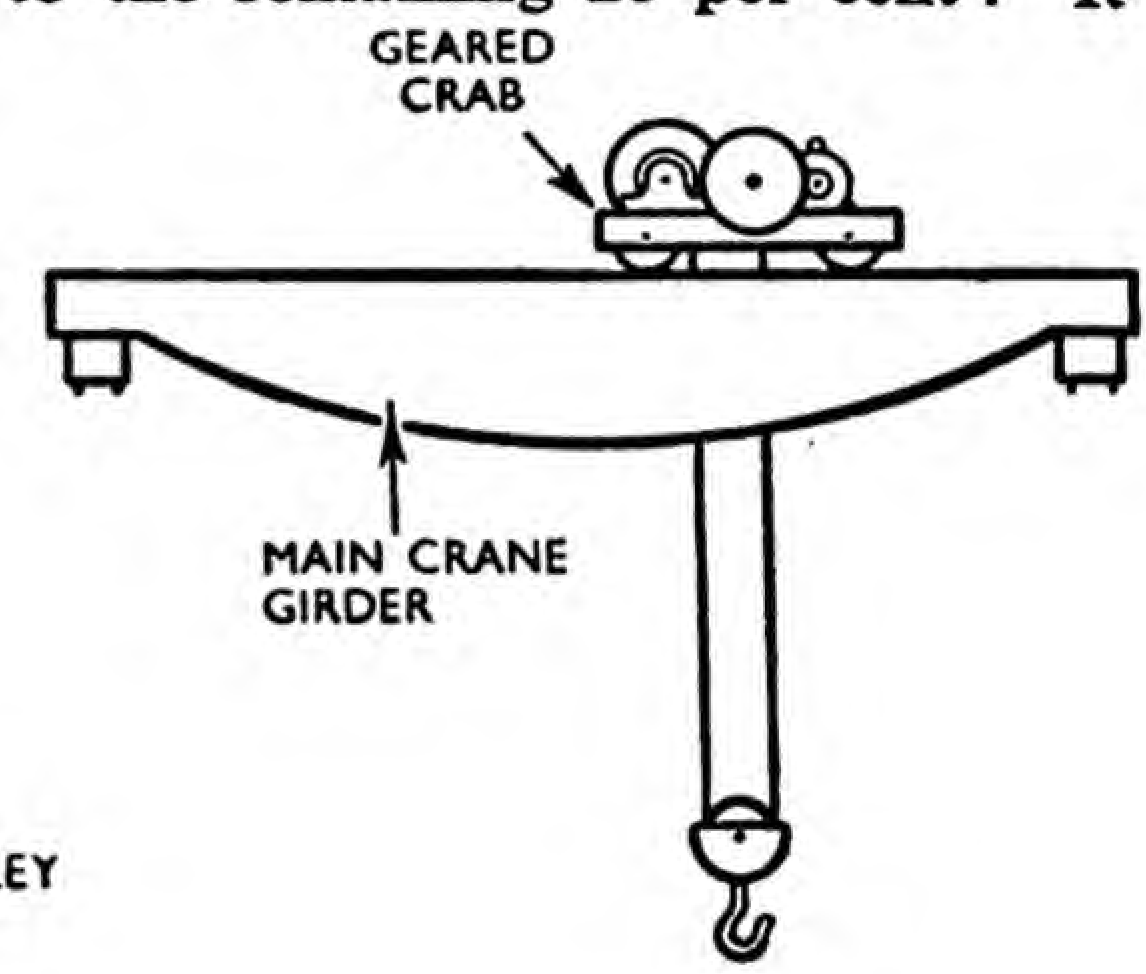
It will be noticed that it is not the actual number of teeth that matters, but the ratio of the numbers of teeth on the two wheels. Thus, we could select many combinations such as 24 and 12, 60 and 30, 94 and 47, to give the velocity ratio of 2. The loads on the teeth are, however, affected by the choice made.

Use of Gears

The winch is very frequently provided with gears of this kind so as to give a stronger pull, and a typical geared winch or crab is shown in Fig. 7. Taking the crank handle and drum first, as in the simple windlass, this will

have a velocity ratio of $\frac{18}{3}$, which is 6. The pair of gears will have a velocity ratio of $\frac{50}{12}$, and hence the total velocity ratio will be $6 \times \frac{50}{12} = 25$. There will be friction in the gears, of course,





GEARS AND PULLEYS IN COMBINATION

Fig. 8. In an overhead crane, the lifting mechanism is mounted on a movable trolley or crab. Its powerful action is obtained by a combination of gears and pulleys. In the example shown above, the electric motor drives the rope drum through two sets of gears, and the velocity ratio of the drum gears is increased six-fold by the three-sheave pulley block.

and allowing for this, the mechanical advantage might be 20. The use of the gears enables us to raise a load almost four times as great as we could without them, but the load will rise more slowly (Fig. 8).

Efficiency of Machine

When we say that a person or an organization is efficient, we mean that they obtain the maximum effect with the minimum of effort. Much the same interpretation also applies to a machine, but we are a little more precise in the definition, and say :—

Efficiency of a machine

$$= \frac{\text{Useful work done on the load}}{\text{Work done by the effort}}.$$

Thus, if a winch has an efficiency of 0·8, that is, if it is 80 per cent efficient, then 80 per cent of the work put in at the crank handle will reappear as work done in raising the load. What happens to the remaining 20 per cent? It is dissipated in overcoming friction.

We have seen already that whether levers or ropes and pulleys or gearwheels are used, there will be friction wherever the parts slide or turn on each other. In overcoming these frictional resistances, work is done, inefficient work, so increasing the work done by the effort for a given amount of useful work done in raising the

load. You will notice that we use the term useful work here, and in the definition of efficiency, to distinguish it from the total work done on the output side, which, by the principle of work must, of course, be equal to the work done by the effort.

Using the figures just given for the winch, we have :—

$$\text{Work done by the effort} = \text{Useful work done on the load} + \text{Work done in overcoming friction.}$$

$$100 \text{ per cent} = 80 \text{ per cent} + 20 \text{ per cent.}$$

Arranged in this way it is easy to see that a reduction in the friction to, say, 10 per cent, would increase the efficiency to 90 per cent, but even if it were possible to eliminate the friction altogether, the maximum efficiency could not exceed unity, that is, 100 per cent.

It is important to notice that the efficiency is not measured by the size of the load in relation to the effort applied ; this is measured by the mechanical advantage. But we can measure the efficiency by comparing the size of the actual load with the size of the ideal load, the load which the same effort would raise if there were no friction.

Let us take another example. Two machines A and B can each be used to raise a load by the application of an effort of 50 lb. A has a velocity ratio of 5 and raises a load of 200 lb. B has a velocity ratio of 8 and raises a load of 300 lb. Which is the more efficient ?

$$\text{Ideal load for } A = \text{Effort} \times \text{velocity ratio}$$
$$= 50 \times 5$$
$$= 250 \text{ lb.}$$

$$\text{Efficiency of } A = \frac{\text{Actual load}}{\text{Ideal load}}$$
$$= \frac{200}{250}$$
$$= 0{\cdot}8 = 80 \text{ per cent.}$$

$$\text{Ideal load for } B = 50 \times 8$$
$$= 400 \text{ lb.}$$

$$\text{Efficiency of } B = \frac{300}{400}$$
$$= 0{\cdot}75 = 75 \text{ per cent.}$$

Thus, A is the more efficient machine although it raises the smaller load. This is because with its lower velocity ratio, it raises the load further, and does more work on the smaller load for the same work done by the effort. Work it out and see ! You should find that the ratio of the useful work done by A to the useful work done by B for the same fall of the effort, say, for example, 1 ft., is as 80 is to 75.

High Ratio, Low Efficiency

You will often find that a machine with a high velocity ratio has a low efficiency. It is one of the penalties that have to be paid for gaining a large purchase. This is because each pulley or gear-wheel that is added to get extra purchase also introduces additional friction. In fact, if we have a simple winch with an efficiency of 80 per cent, and we add a pair of gears which have an efficiency of 90 per cent, then, although the effort to raise the same load will be much smaller, the efficiency will have fallen to 72 per cent.

Let us see how this is obtained. When dealing with velocity ratio we saw that if a number of simple elements were arranged to give a compound effect, the overall velocity ratio was obtained by

multiplying together the velocity ratios of the individual elements. It is just the same with efficiency. The overall efficiency is obtained by multiplying together the efficiencies of the various components. So that in the example above :—

Overall efficiency $= 0{\cdot}8 \times 0{\cdot}9$
$= 0{\cdot}72 = 72$ per cent.

Usually the benefit obtained from the increased mechanical advantage which accompanies the high velocity ratio, more than offsets the lower efficiency. For instance, when driving a motor car and we have to change down to climb a steep hill, we do not worry about the lower efficiency so long as the increased mechanical advantage enables us to climb the hill. On the other hand, we do not remain in low gear longer than is necessary because the lower efficiency will have an adverse effect on the petrol consumption.

The relationship between the mechanical advantage and the velocity ratio is also a useful measure of the efficiency. Earlier, it was shown that in the ideal machine, the mechanical advantage was equal to the velocity ratio, but that in the real machine, the effect of friction was always to make the mechanical advantage less than the velocity ratio. In fact, we can write :—

Mechanical advantage
= Efficiency × Velocity ratio.

or

$$\text{Efficiency} = \frac{\text{Mechanical advantage}}{\text{Velocity ratio}}.$$

This form is very useful when designing a machine to do a particular job, say, for example, to raise a 5-ton load with an effort of 100 lb. It is known from experience what will be the approximate value of the efficiency of the particular arrangement we intend to use ; pulleys or gears, or possibly a combination of the two. Let us imagine it to be 75 per cent. We shall require a mechanical advantage of :—

$$\frac{5 \times 2{,}240}{100} = 112.$$

But Mechanical advantage
= Efficiency × Velocity ratio.

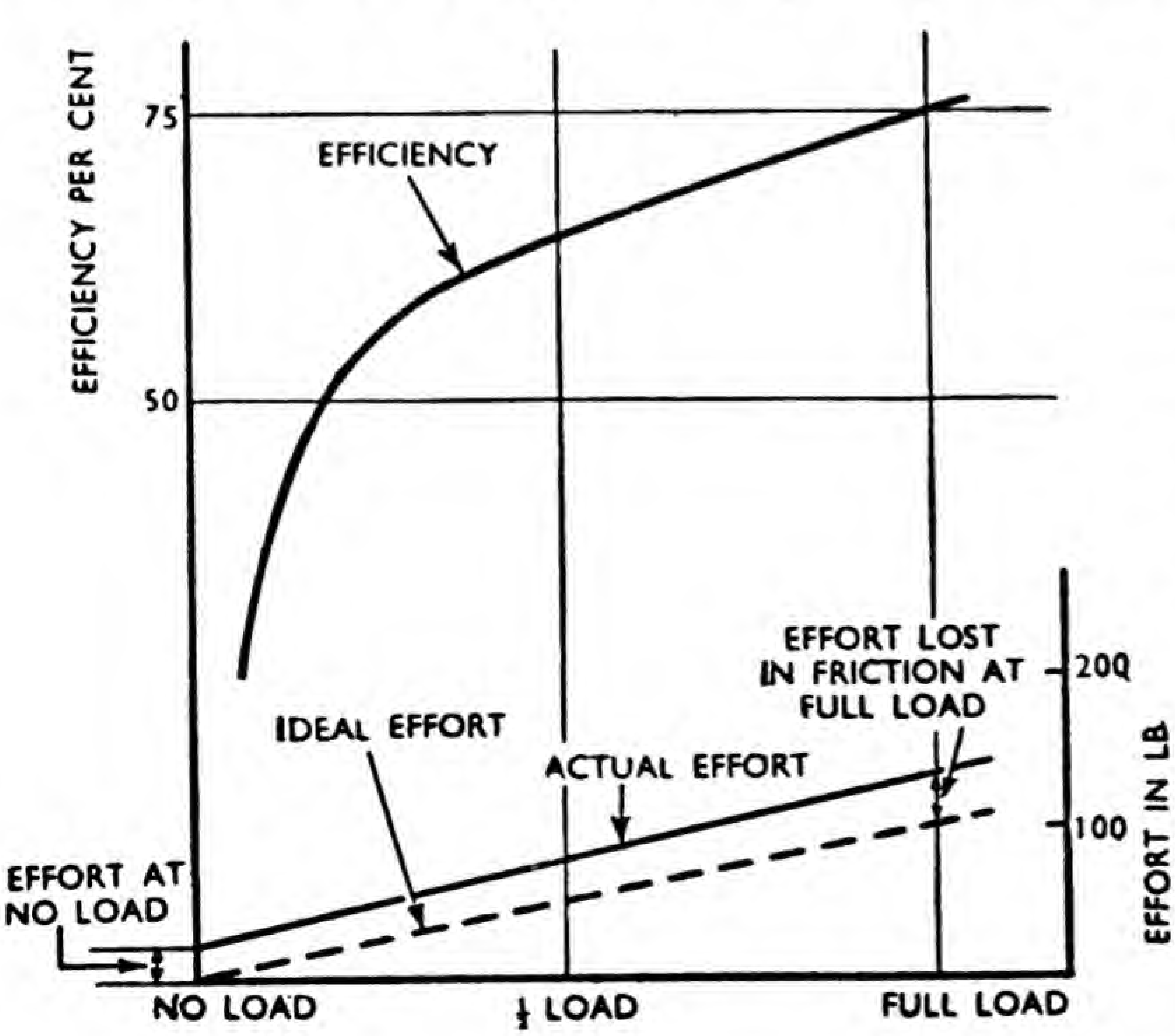
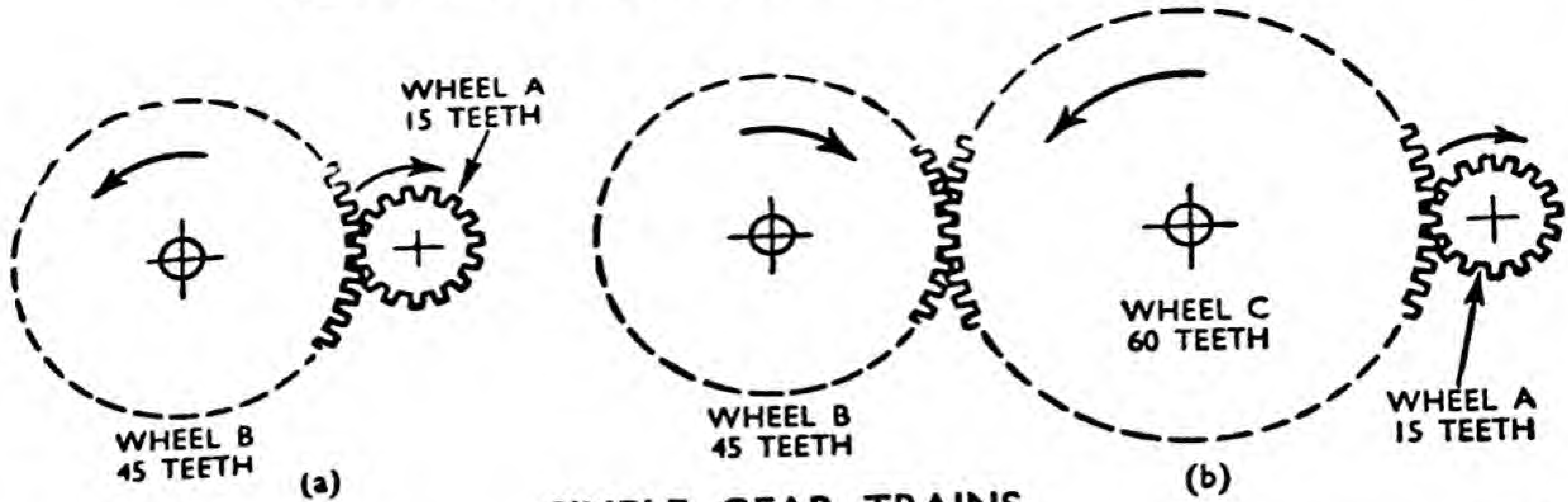


Fig. 9. The efficiency improves as load is increased, and the value of the efficiency at each load can be shown on a graph. An effort is required to overcome friction even at no load, and this is greater in proportion for light loads than for full load, as indicated in the lower diagram. Efficiency is decreased by friction and so will be low for light loads but will increase as full load is approached.



SIMPLE GEAR TRAINS

Fig. 10. A number of gearwheels meshing together directly constitutes a simple train. If a third wheel, C, is interposed between two other wheels, A and B, their relative speeds will be unaffected, but the direction of their rotation will be reversed. Such a wheel is called an idler.

$\therefore$ 112 = 0·75 × Velocity ratio,

$$\text{viz., Velocity ratio} = \frac{112}{0{\cdot}75}$$
$$= 149$$

We can then arrange the geometry of our machine, that is, sizes of pulleys and numbers of teeth on the gearwheels, so as to give a velocity ratio of 149.

Keeping Losses Small

We have spoken here of the efficiency of a machine as if it were a fixed quantity, but the fact must not be overlooked that a high efficiency can be obtained only by keeping the losses small, and that excessive friction due to faulty lubrication or poor adjustment will cause the efficiency of an otherwise good machine to fall. Also, at light loads the frictional resistance will be proportionately greater than at full load, as there is still some friction even when the load is zero, and, therefore, the efficiency will be lower. This explanation will be understood more easily if we spend a little time in studying the graph of Fig. 9. Thus for the geared winch which we have just been considering, the efficiency would be only about 60 per cent at half load, and an effort of about 62½ lb. would be required to raise a 2½-ton load.

We should by now have got a clear mental picture of these three important relationships. The velocity ratio is a fixed quantity for any given machine. The efficiency and the mechanical advantage vary together. When the efficiency is low the mechanical advantage is low, and the higher the efficiency the more closely does the value of the mechanical advantage approach the fixed value of the velocity ratio, but it can never exceed it, because the efficiency can never exceed unity, no matter how perfect the machine.

More about Gears

Gears are so important nowadays, and there is such a wide range of applications, such as engines, machine tools, agricultural machines, spinning and weaving machines, sewing machines, clocks and watches, to mention only a few, that it is necessary that we should know more about them.

A number of toothed wheels meshing together so that rotation of one of them causes the others to revolve is called a train of wheels. If the wheels are arranged as in

Fig. 10, so that each wheel revolves separately on its own axle, it is called a simple train, whereas if two of the wheels are secured to the same shaft so that they rotate as one, as shown in Fig. 11, it is called a compound train. Let us first consider the simple trains of Fig. 10. At (a) we have the initial motion being given to wheel *A*, and, therefore, it is called the driver. Wheel *B* is then driven by wheel *A* and is called the follower. The velocity ratio will depend upon the ratio of the numbers of teeth on the two wheels, and this is usually expressed as follows :—

$$\frac{\text{Speed of follower}}{\text{Speed of driver}} = \frac{\text{Number of teeth on driver}}{\text{Number of teeth on follower}} = \frac{N_a}{N_b}.$$

Note that it is the wheel with the larger number of teeth, in this case wheel *B*, that moves the more slowly.

This was the arrangement we used for the geared winch, where we wanted to gain extra purchase at the expense of speed. If wheel *B* had been the driver and wheel *A* the follower, we should have got an increase in speed with a decrease of purchase. Distinction is made between these two alternatives by speaking of gearing down when there is a reduction of speed, and gearing up when there is an increase in speed. Thus, in the motor car, the very high speed of the engine, which is about 3,000 r.p.m., is geared down to give a suitable speed to the road-wheels, whereas in a sewing machine or a hand-drill, we find that the low speed of the hand wheel is geared up.

One more thing that should be noticed before we leave this

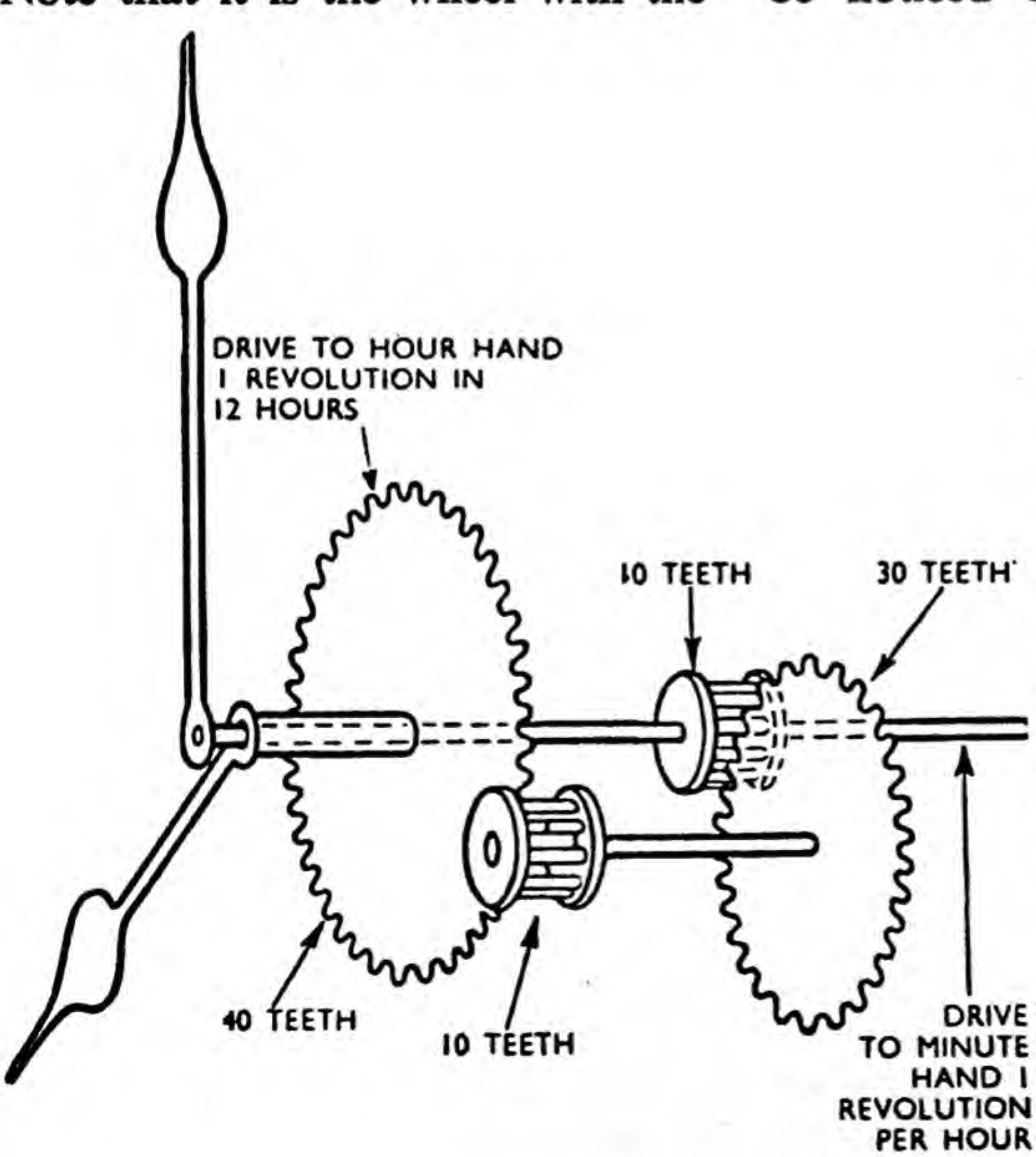
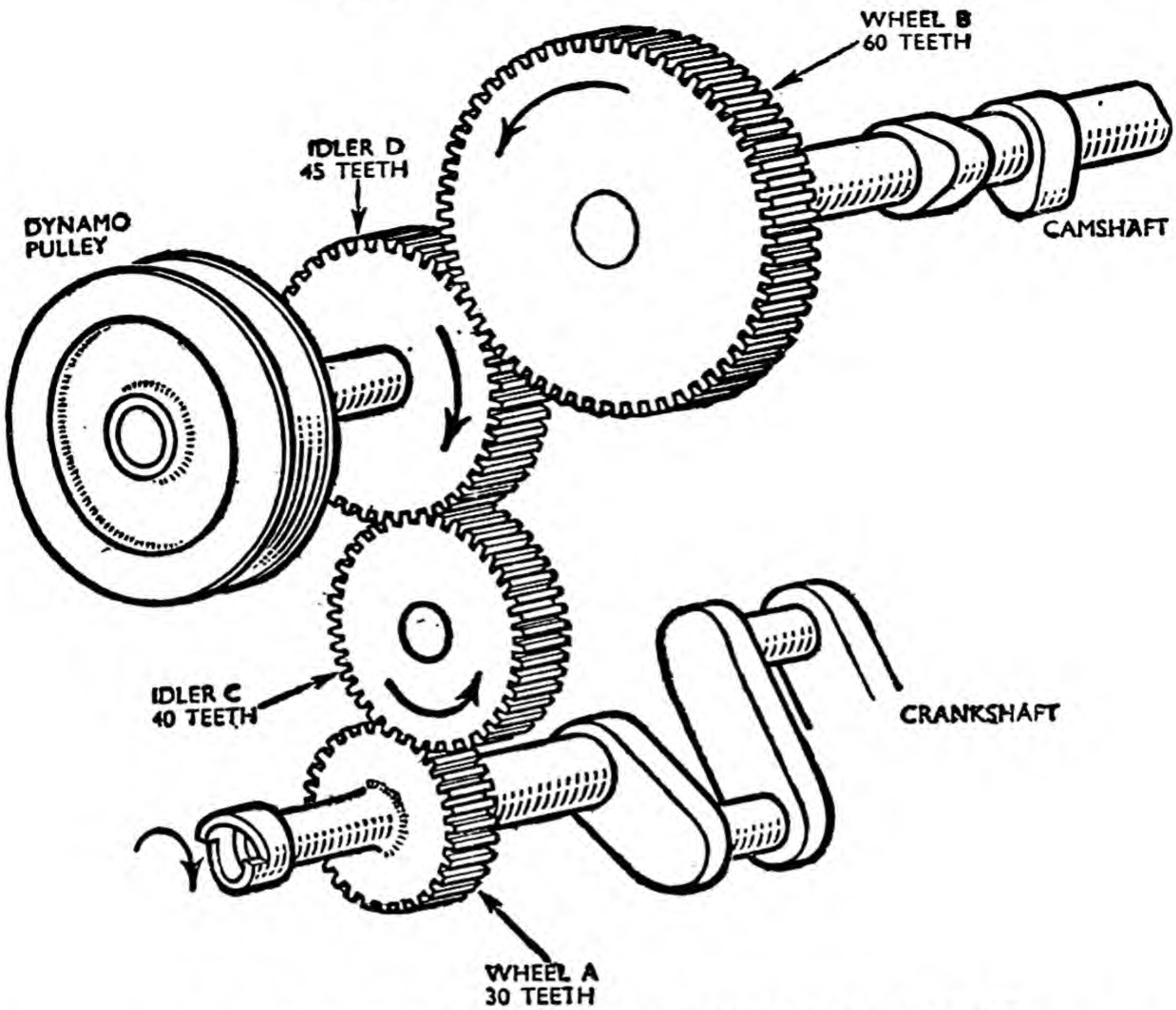


Fig. 11. In a clock, the hour hand is driven by the minute hand, and it is necessary to arrange the gearing so that the hour hand makes one revolution for every twelve revolutions by the minute hand. Also both hands must revolve in the same direction. This is effected by a compound gear train. Secured to the minute hand spindle is a gear-wheel with 10 teeth which meshes with a wheel with 30 teeth. This, therefore, makes one revolution in three hours, and rotates another 10-tooth wheel which in turn meshes with a 40-tooth wheel. Thus, the hour hand rotates once in twelve hours.



THE TIMING GEARS OF A MOTOR-CAR ENGINE

Fig. 12. Gearing must be so arranged that the camshaft revolves at half the crankshaft speed. The gears form a simple train and, therefore, wheel *B* must have twice as many teeth as wheel *A*. The drive to the dynamo is taken from the idler wheel *D*, and as this has 45 teeth, we find that the dynamo pulley will be driven at two-thirds the crankshaft speed.

simplest of simple trains, is that the two wheels *A* and *B* will revolve in opposite directions. For example, if *A* is driven at 60 r.p.m. clockwise, *B* will rotate at 20 r.p.m. counter-clockwise. On the other hand, if *B* is driven at 60 r.p.m. clockwise, *A* will rotate at 180 r.p.m. counter-clockwise.

Role of Idler Wheel

In Fig. 10(b), the same two wheels *A* and *B* are arranged in a simple train, but instead of *A* meshing with *B* directly it meshes with an intermediate wheel *C*, which, in turn meshes with wheel *B*. What are the speed relationships now? First of all consider *A* as the driver, and *C* as the follower, then :—

$$\text{Speed of } C = \text{Speed of } A \times \frac{\text{No. of teeth on } A}{\text{No. of teeth on } C}$$

$$= \text{Speed of } A \times \frac{15}{60}$$

$$= \frac{1}{4} \times \text{Speed of } A.$$

Now consider *C* as the driver and *B* as the follower.

$$\text{Speed of } B = \text{Speed of } C \times \frac{60}{45}$$

$$= \frac{4}{3} \times \text{Speed of } C.$$

But we have just found that the speed of C is $\frac{1}{4}$ of the speed of A, and hence :—

$$\text{Speed of } B = \frac{4}{3} \times \frac{1}{4} \times \text{speed of } A.$$

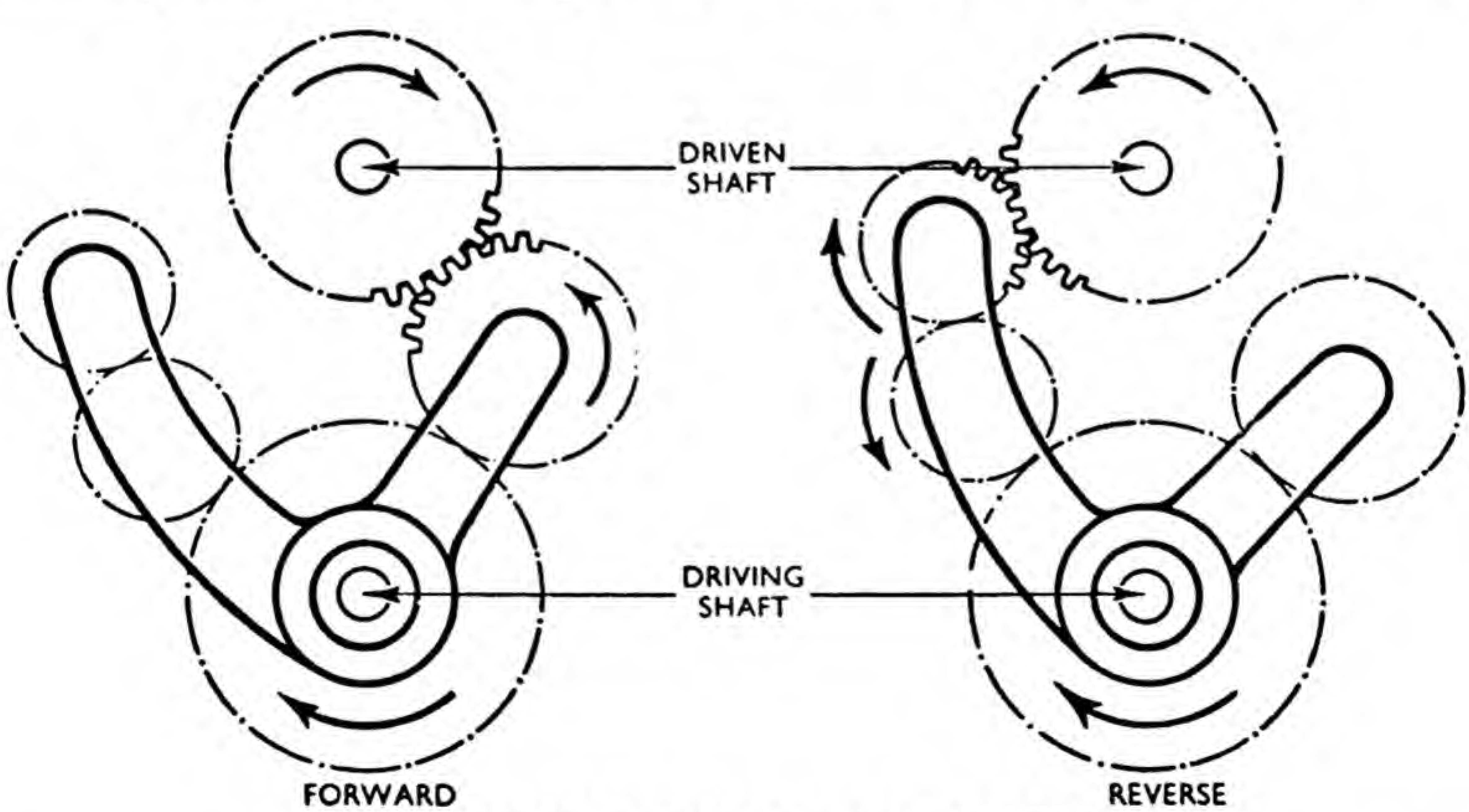
$$= \frac{1}{3} \times \text{speed of } A.$$

You will notice that this is just the same result that was obtained when B was directly driven by A in Fig. 10(a). In fact, as you will find if you work it out for yourself, it makes no difference to the speed of wheel B, whether the intermediate wheel C has 20 teeth, 50 teeth or 100 teeth. Wheel C is, therefore, called an idler wheel or simply an idler, because it serves to pass on the motion without affecting the velocity ratio.

This is a most important relationship, and we can go further and say that however many idlers we interpose in a simple train between the wheels A and B, the speeds will be unaffected. In other words, the speeds of any two wheels in a simple train depend only upon the ratio of the numbers of teeth on the first and last wheels.

Instructive Example

Let us apply this to the simple train shown in Fig. 12, which represents the timing gears of a motor-car engine. Here, two idlers are used to pass on the motion from the crankshaft to the camshaft. The essential requirement is that the camshaft shall make one revolution for every two revolutions of the crankshaft. From what has already been said, you know the answer, which is that the wheel on the camshaft must have twice as many teeth as the wheel on the crankshaft, but we will work it out in full to show the method. We will write N_a, N_b, etc., for the numbers of teeth on wheels A, B, etc., and we will



REVERSING GEAR USING IDLERS

Fig. 13. When a single idler is in use, the shafts rotate in the same direction. When two idlers are put into gear, the shafts rotate in opposite directions. The two sets of idlers are mounted on a movable bracket, so that engagement of one disengages the other.

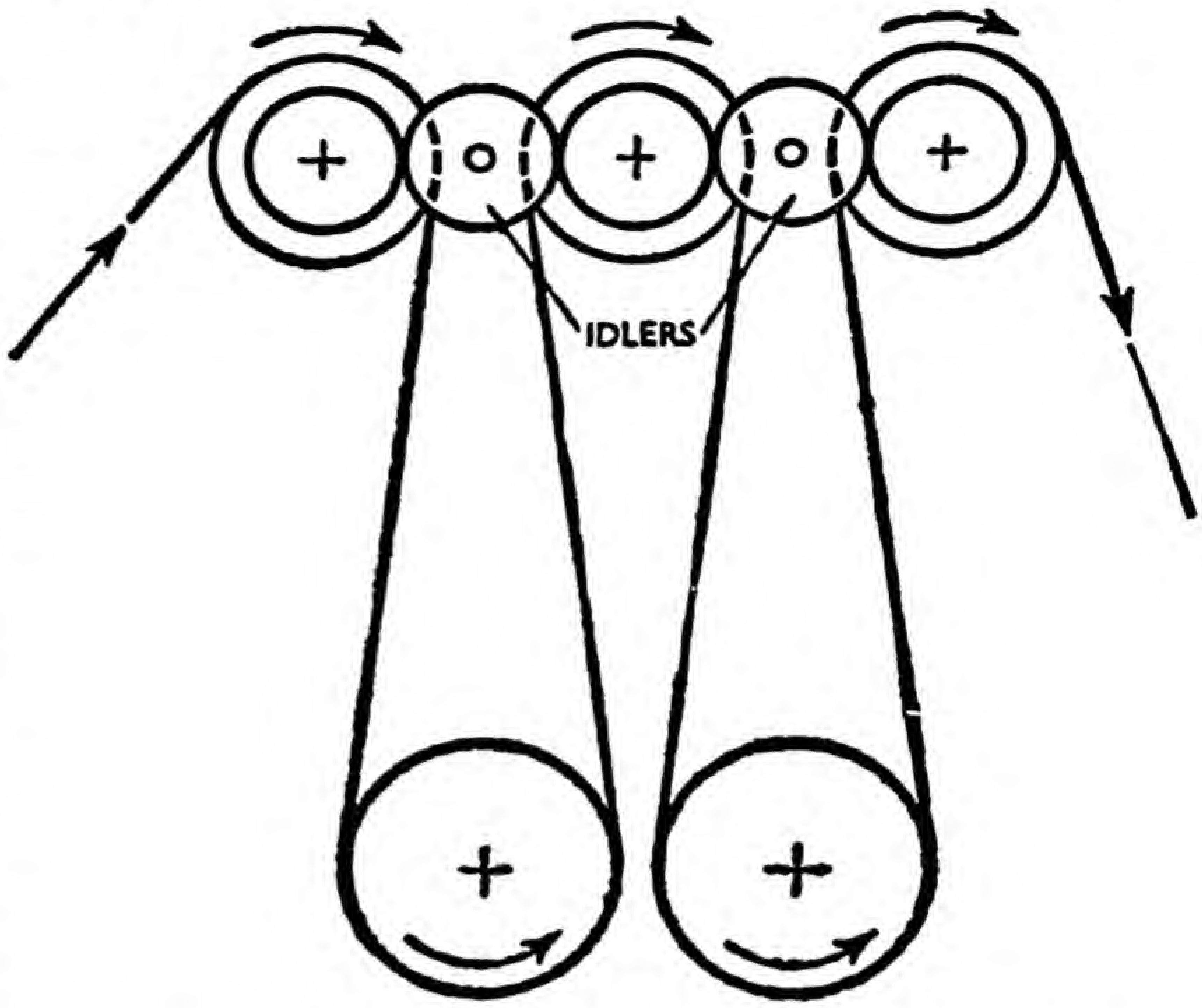


Fig. 14. In both textile and printing machinery, it is most important that various rollers rotate at correct speeds, otherwise cloth or paper will tear if pulled too tightly, or will become slack, so that various printed parts do not register correctly. So, rollers are geared together. They must rotate in the same direction, so idlers are used between various gearwheels. Idlers can be of any convenient size to suit the spacing of the rollers without affecting the relative speeds.

consider each idler in the train in turn; first, as a follower of the wheel preceding it, and secondly, as a driver of the wheel following it.

$$\text{Speed of } B = \text{Speed of } A \times \frac{N_a}{N_c} \times \frac{N_c}{N_d} \times \frac{N_d}{N_b}$$

$$= \text{Speed of } A \times \frac{30}{40} \times \frac{40}{45} \times \frac{45}{60}$$

$$= \frac{30}{60} \times \text{Speed of } A$$

$$= \frac{1}{2} \times \text{Speed of } A.$$

When it is written down in this way it is easy to see that the fractions obtained by considering the drive through the idlers C and D cancel out, and one is left with the ratio $\frac{N_a}{N_b}$.

But look at Fig. 12 again, and you will notice that idler D is not completely idle. It is used to drive the dynamo. Proceeding as before, we can now find the speed of the dynamo.

$$\text{Speed of } D = \text{Speed of } A \times \frac{30}{40} \times \frac{40}{45}$$

$$= \frac{30}{45} \times \text{Speed of } A$$

$$= \frac{2}{3} \times \text{Speed of } A.$$

It is important to notice here that although the number of teeth on D does not matter so far as the speed of B is concerned, it is all important so far as the speed of the dynamo is concerned, and D will go faster or slower according to whether it has fewer or more teeth. This is extremely useful in practice, because the number of teeth on the wheel D can be chosen to suit the speed of the dynamo without affecting the speeds of the crankshaft and camshaft.

Idler wheels are also very useful for changing the direction of rotation. Comparing Figs. 10(a) and 10(b), it will be noticed that when the two wheels mesh directly, they rotate in opposite directions,

but that when they are connected by an idler wheel, they rotate in the same direction. Fig. 12 shows that by introducing a second idler *D*, the direction of rotation is reversed again. The general rule is that when an odd number of idlers is used, the end wheels in the train rotate in the same direction, but when an even number of idlers is used, the end wheels rotate in opposite directions.

A simple reversing gear making use of idlers is shown in Fig. 13. When the single idler is in use as shown at the left, both shafts rotate in the same direction. When this is swung out of the way, and the two idlers are put into gear, the shafts rotate in opposite directions. Another use is in textile and paper machines as shown in Fig. 14. The various rollers must be geared together so as to run at the correct speed, and idlers are used to cause the rollers to rotate in the same direction.

Simple trains are not very helpful when a large velocity ratio is required, as this would mean either that one wheel had to be very large or that the other wheel had to be very small. From what we have just seen, using more than two wheels would not help either, because the additional wheels would merely act as idlers without affecting the velocity ratio. To obtain a large velocity ratio we must arrange a number of simple trains so as to give a compound effect, as shown in Fig. 15. This is known as a compound train.

Double Reduction Gears

Wheel *A* is the driver, and wheel *B*, having more teeth than *A*, will be driven at a slower speed. Wheel *C* is now attached to the same shaft as *B*, and, therefore, runs at the same slow speed. Wheel *C* acts as the driver of wheel *D*, and there is a further reduction in speed due to this second pair. Notice that this is quite different from the simple train in which *C* meshes with *B*, as in this case there would be an

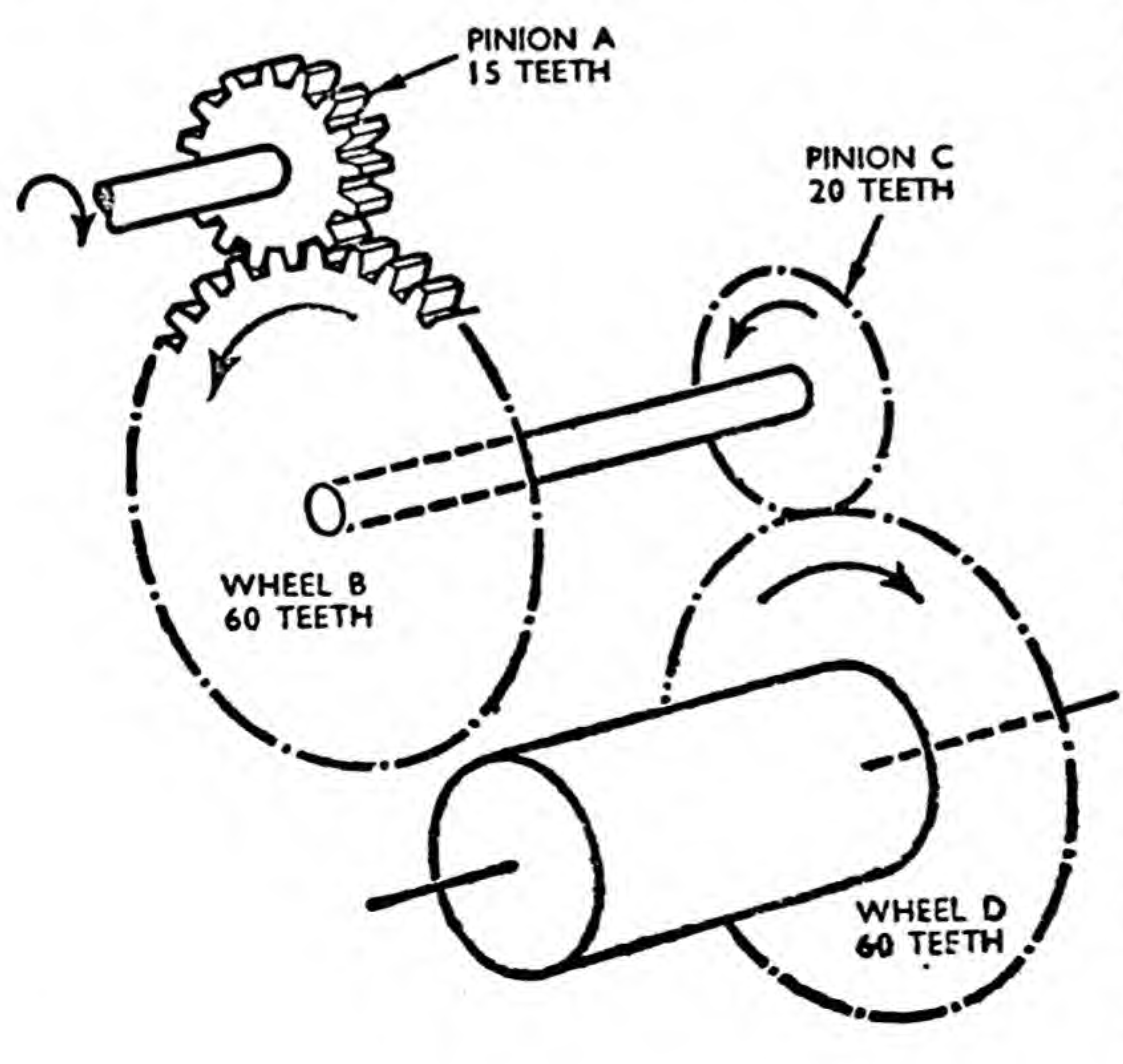
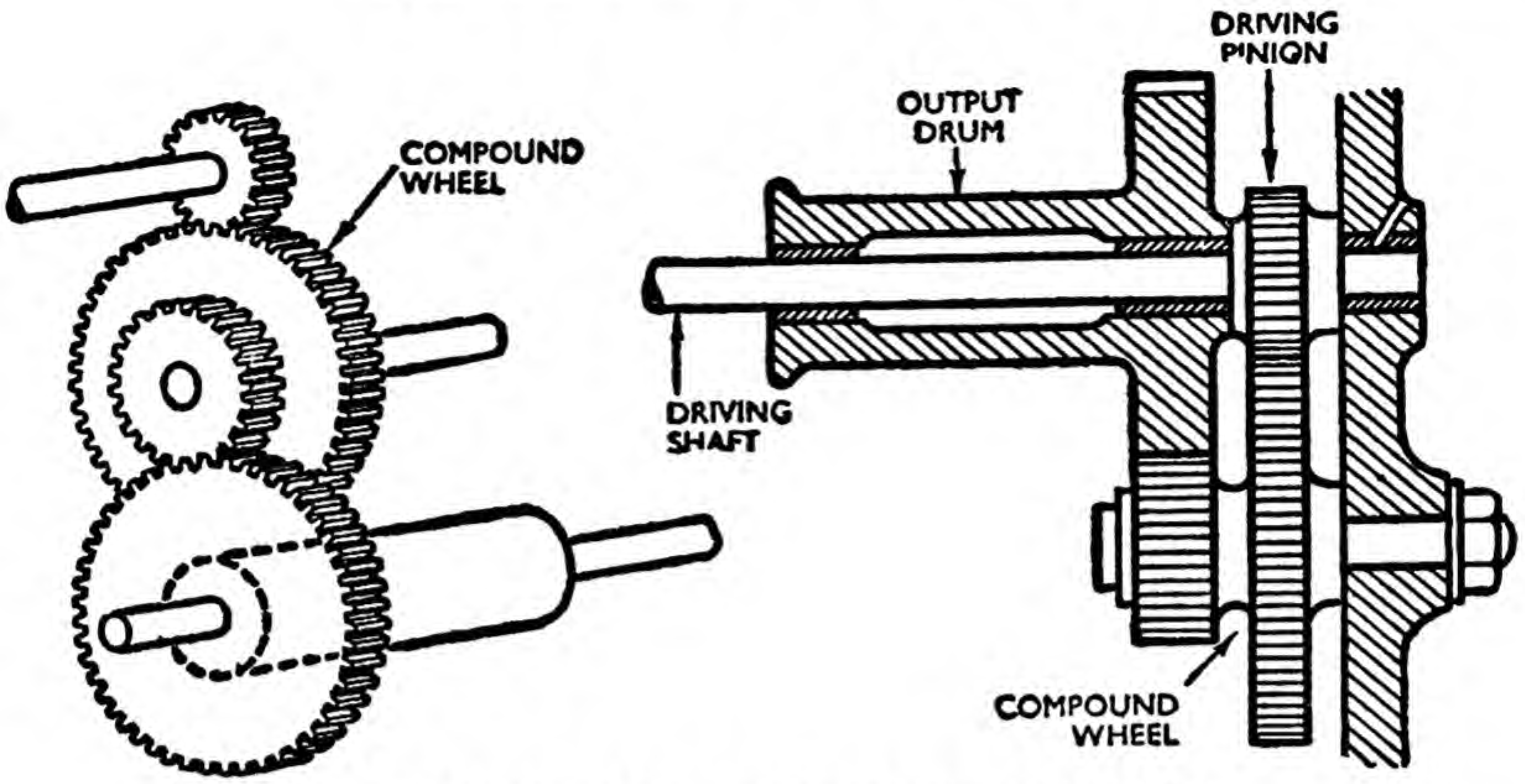


Fig. 15. When it is required to obtain a large velocity ratio with gears, it is convenient to arrange two simple trains so that they form a compound train, as this avoids the use of very large wheels. In such a compound train the speed reduction by one pair is passed on to the second pair as shown. Wheel A is the driver and wheel B, having more teeth than A, is driven at a slower speed. Wheel C attached to the same shaft as B and so running at the same slow speed, acts as driver of wheel D.



SPACE SAVING WITH COMPOUND WHEELS

Fig. 16. Compound wheels avoid the complication of a large number of shafts and bearings. They each consist of a pinion made integral with a gearwheel, and such a wheel can be arranged to rotate on a fixed pin. By suitable choice of sizes of wheels, the compact drive shown on the right is possible. Here the output drum rotates on bearings on the driving shaft.

increase in speed from B to C, to be followed again by a reduction from C to D. In the compound train, the speed reduction by the first pair of wheels A and B, is passed on to the second pair, C and D. Accordingly this is called a double reduction gear. The speed relationships can be written down as follows :—

Speed of A

$$= \text{Speed of } D \times \frac{N_d}{N_c} \times \frac{N_b}{N_a}$$

$$= \text{Speed of } D \times \frac{60}{20} \times \frac{60}{15}$$

$$= \text{Speed of } D \times 3 \times 4$$

$$= 12 \times \text{Speed of } D.$$

This relationship is similar to that which we have had for other machines where a compound effect was obtained. That is, the overall velocity ratio is the product of the velocity ratios of the components. In this case, the overall velocity ratio of 12 is the product of the velocity ratio 4, of the first pair, and the velocity ratio 3, of the second pair. To obtain this same velocity ratio 12, with a simple train using a 15-tooth driver, it would be necessary to use a follower with 180 teeth, that is to say, a wheel three times as big as the largest wheel used in the compound train. By extending the compound train with additional pairs of gears, it is obvious that the velocity ratio can be made as large as is desired.

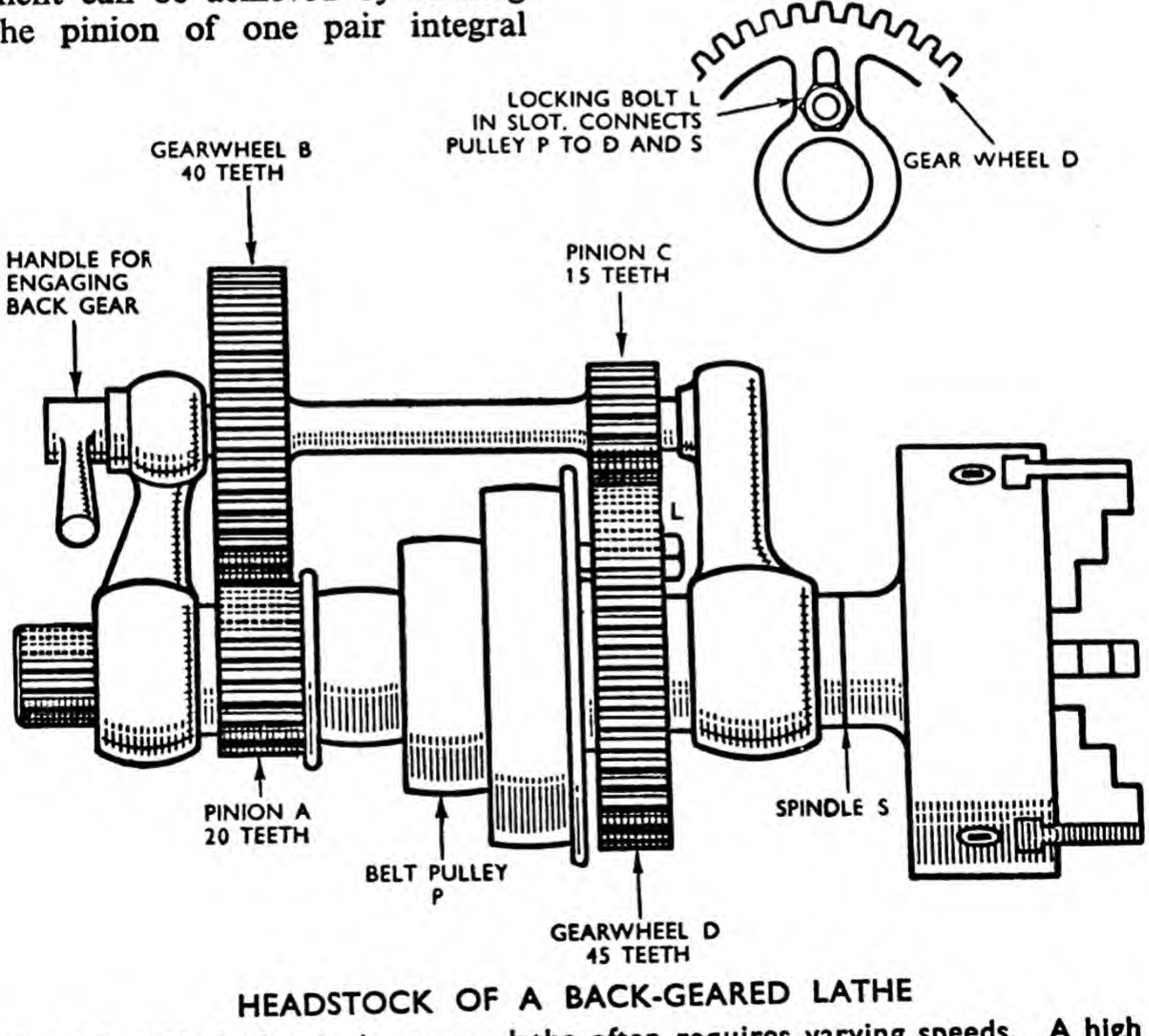
Looking again at the little calculation for the speed relationships, it will be seen that there are fractions $\frac{N_d}{N_c}$ and $\frac{N_b}{N_a}$, but no fraction $\frac{N_c}{N_b}$. This is because wheels B and C are coupled together so as to rotate as one, and, therefore, the ratio of the numbers of teeth on B and C does not affect the speed relationships.

When pairs of wheels are used

in this way, it is usual to call the wheel with the smaller number of teeth, the pinion, and the larger wheel, the gearwheel. Thus, in Fig. 15, wheels *A* and *C* are pinions, and wheels *B* and *D* are gearwheels. When calculating the velocity ratio for a compound reduction train, the numerators of the fractions will be the numbers of the teeth on the gearwheels, and the denominators the numbers of the teeth on the pinions. For a speed-increasing gear, the fractions will be inverted.

When it is desired to economize on space, a very compact arrangement can be achieved by making the pinion of one pair integral with the gearwheel of the preceding pair, as shown in Fig. 16. This has the additional advantage of saving shafts and bearings, because the compound wheels can be mounted so as to rotate freely on fixed pins.

A very interesting example of a compound train is the back gear of a lathe. Here, the requirement is that a high speed is needed for some work, and a lower speed with a greater turning moment for other work, such as heavy roughing cuts on cast iron. Fig. 17 shows how this is done. For high speeds the belt pulley *P* is coupled directly



HEADSTOCK OF A BACK-GEARED LATHE

Fig. 17. Work that is done on a lathe often requires varying speeds. A high speed is obtained by coupling the belt pulley *P* directly with the spindle *S* by means of the locking bolt *L*. For low speeds, the back gear is engaged, which is wheels *B* and *C*. Pulley *P* then turns freely on the spindle *S*, and drives it indirectly through the compound train formed by wheels *A*, *B*, *C* and *D*.

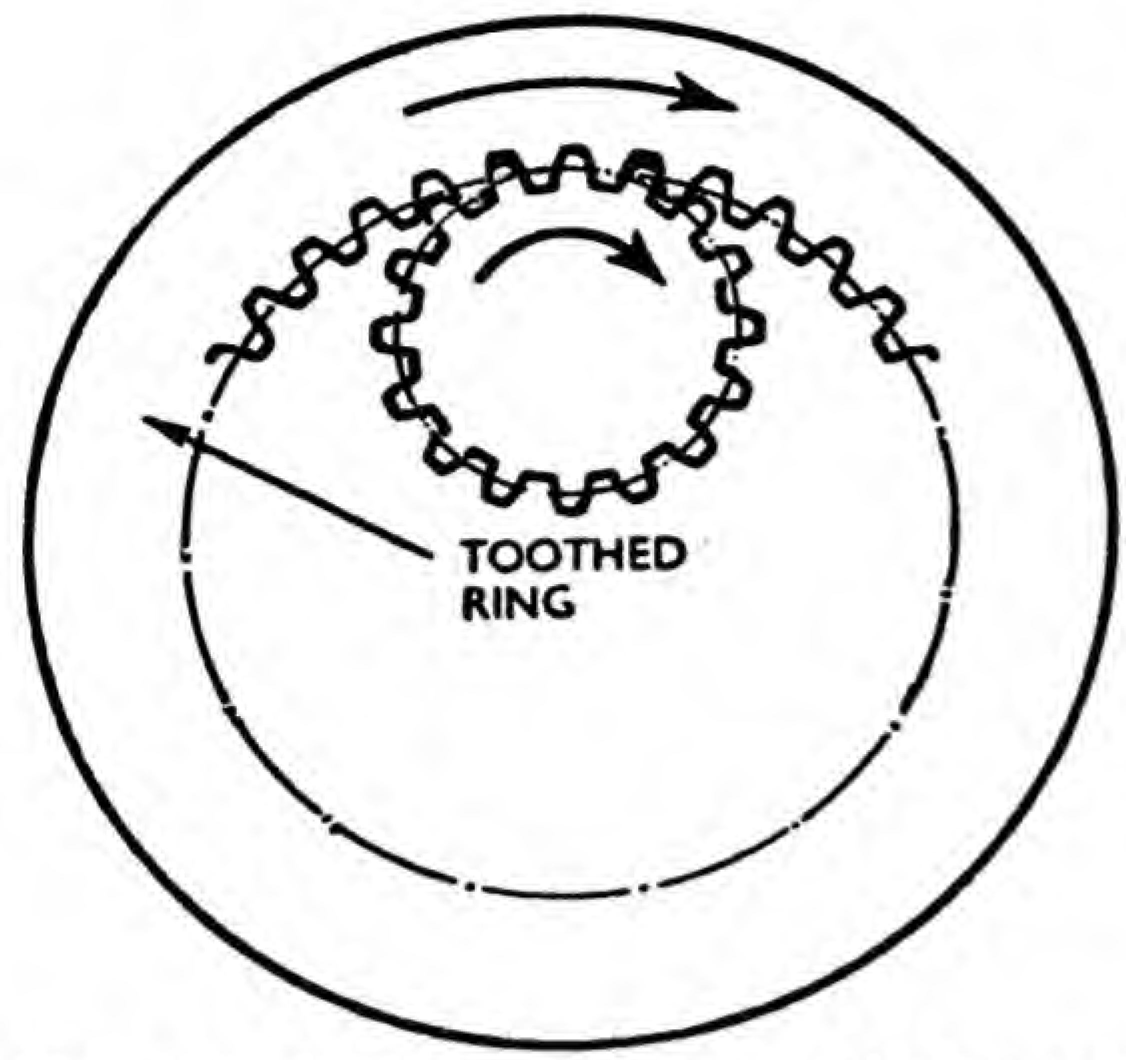


Fig. 18. In internal or annular gears, the follower is a toothed ring and the driver an ordinary pinion. Speed relationship is as with ordinary gears, but wheels rotate in the same direction.

to the spindle S by the locking bolt L so that both rotate as one, and the back gear is thrown out of mesh. For low speeds, the bolt L is withdrawn so that the pulley P cannot rotate the spindle directly, but turns freely upon it. The back gear, consisting of the gearwheel B and the pinion C rigidly mounted on a common shaft, is then brought into engagement. The gearwheel B meshes with the pinion A, which is part of the pulley P and rotates with it, and pinion C meshes with gearwheel D, which is secured to the spindle S.

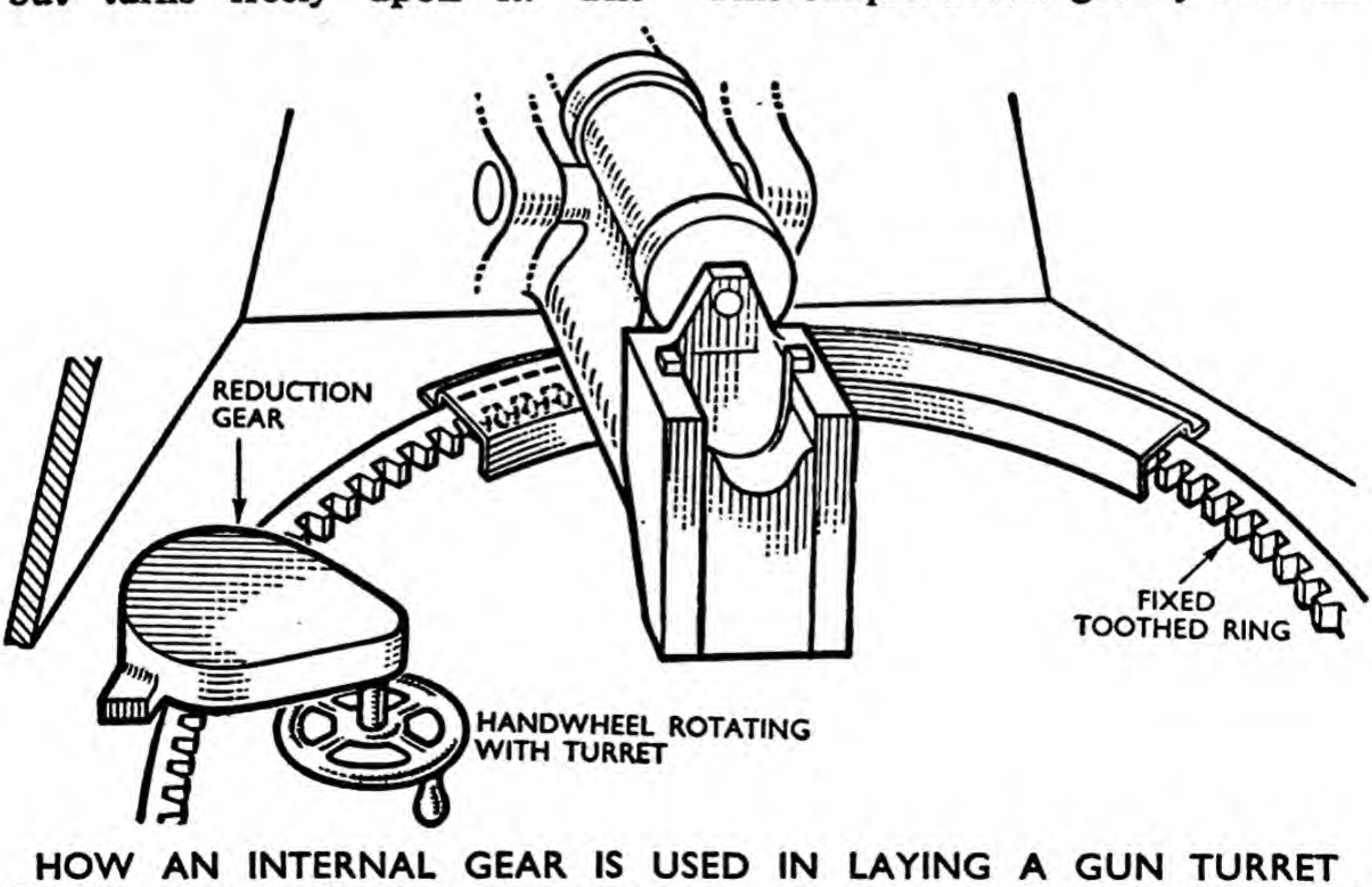
Using the numbers of teeth shown in Fig. 17, we can now follow the operation of the back gear. Pulley P, through pinion A and gearwheel B, drives the shaft at half its own speed. Pinion C and gearwheel D then drive the spindle at one-third the speed of the shaft, viz., at one-sixth the speed of the pulley, or writing as before :—

Speed of spindle

$$= \text{Speed of pulley} \times \frac{20}{40} \times \frac{15}{45}$$

$$= \frac{1}{6} \times \text{Speed of pulley.}$$

This simple device greatly extends



HOW AN INTERNAL GEAR IS USED IN LAYING A GUN TURRET

Fig. 19. Gun turret of a tank is rotated by using an internal gear, as illustrated in the above diagram. In this case, the toothed ring is fixed to the hull of the vehicle, and the pinion, which we see meshes with it, is driven, through a reduction gear, by a handwheel. This handwheel and the reduction gear is fixed to the turret, and so as the turret moves, the handwheel moves with it.

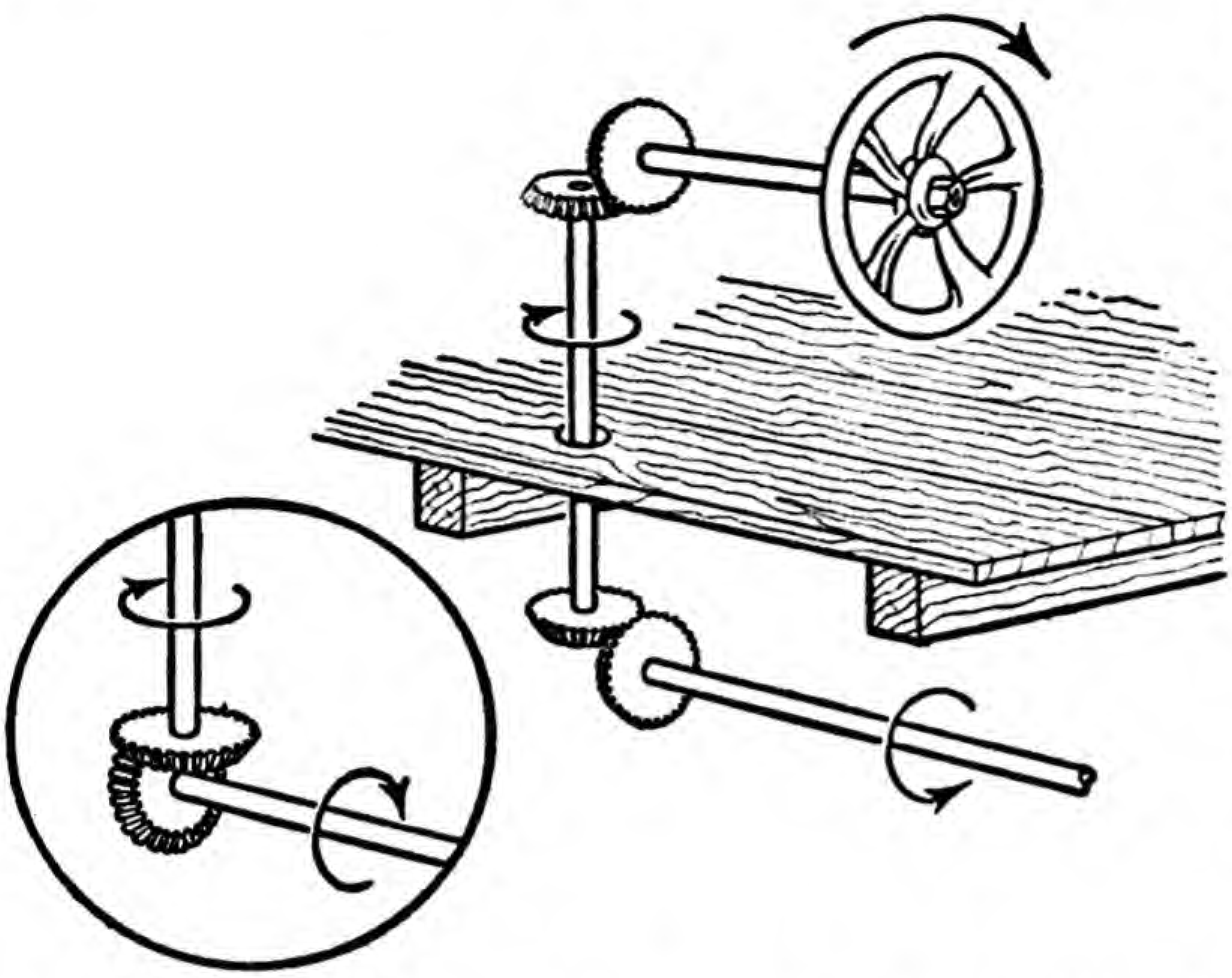
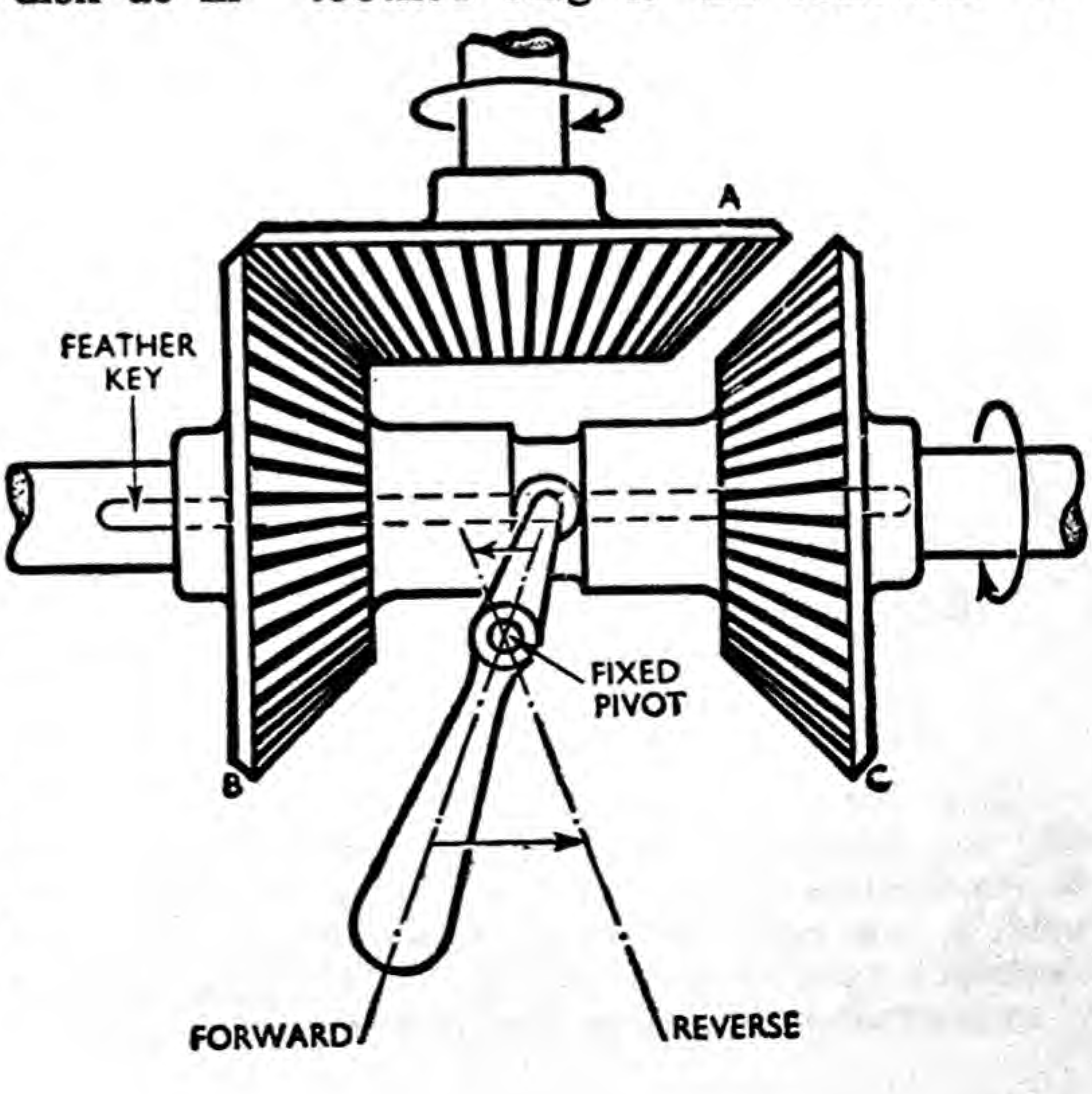


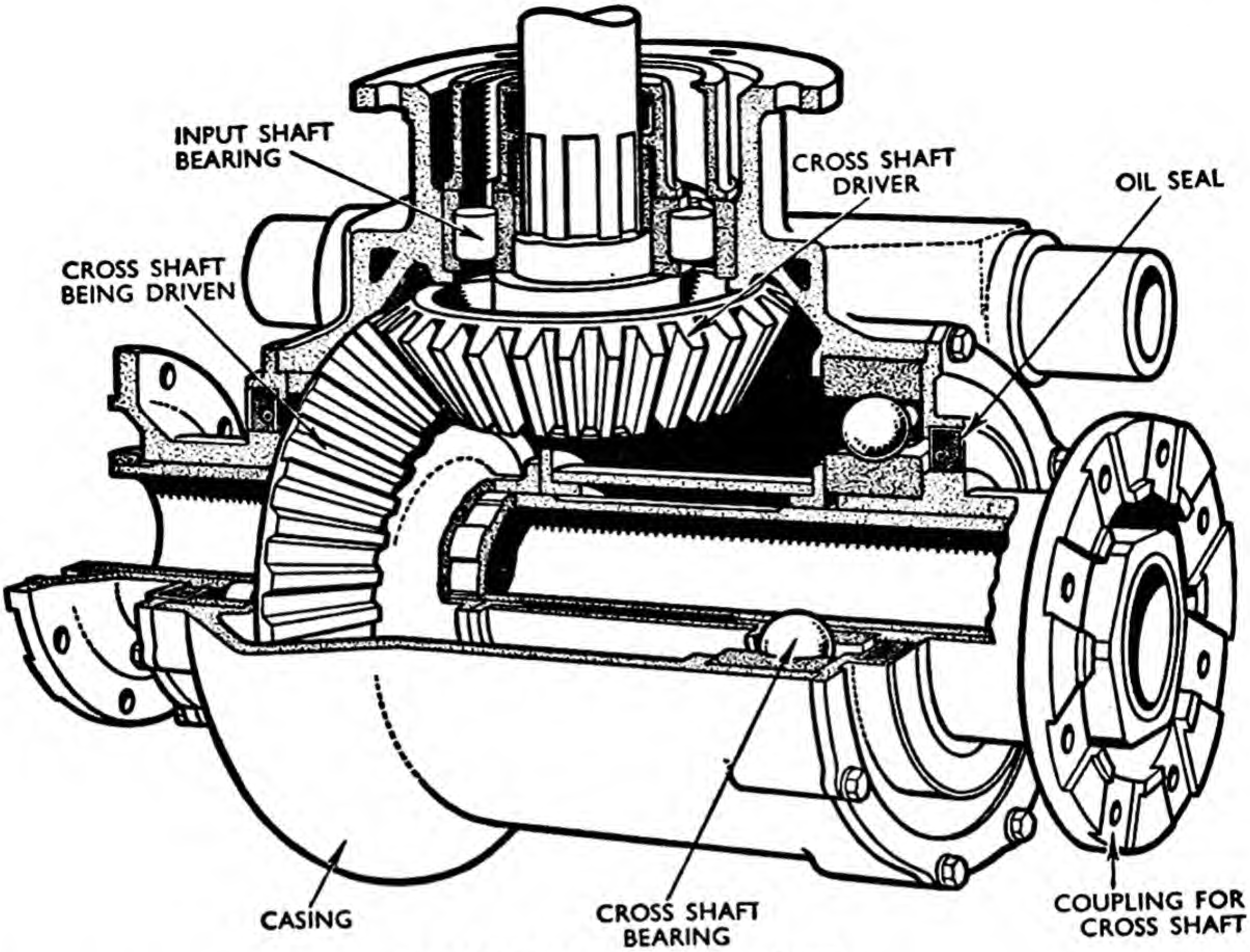
Fig. 20. Bevel gears are used when axes of shafts are inclined to each other or to take a drive round a corner. In the diagram, wheels are the same size and so there is no reduction in speed. Care must be exercised, however, in the arrangement of wheels in order to obtain desired direction of rotation. The inset diagram illustrates how, by changing the position of one bevel wheel, the final shaft may be made to rotate in the reverse direction.

the range of work which can be undertaken.

Internal gears are sometimes convenient for special purposes, although they are not in very general use because they are so much more difficult to manufacture. Fig. 18 shows the principle, and it will be noticed that the teeth on the follower are cut on the inside of a ring instead of on the periphery of a disk as in the other gears we have considered. The driver is an ordinary pinion, and the speed relationship is just the same as for ordinary gears. There is one important difference, and that is that both wheels rotate in the same direction. It will also be obvious that because the pinion must fit inside the internal gear, it is only possible to obtain a speed reduction if the toothed ring is the follower. If

Fig. 21. In jib cranes and other lifting devices, mechanism must be provided for reversing. Diagram on right shows how bevel gears are used to drive the slewing gear in either direction, from the power shaft which rotates always in the same direction. Wheels *B* and *C* are mounted on a feather key so that they are driven by the power shaft but are free to slide along it. Wheel *A* is driven in the forward or reverse direction according as wheel *B* or wheel *C* is made to mesh with it.





POWERFUL BEVEL DRIVE

Fig. 22. Illustrated above is a powerful bevel drive. The teeth are formed on conical surfaces and are tapered. Bevel gears are used here to transmit the drive to the cross shaft, and as they are of different sizes, there will be a speed reduction in the ratio of the number of teeth on the driver to the number on the follower. The drive from the propeller shaft to the back axle of the average motor car will be found to be very similar to this.

an increase in speed is required, the internal gear must be the driver.

An important application is shown in Fig. 19. This shows how the gun turret of a tank is rotated by means of a handwheel which is geared to an internal gear fixed to the roof of the vehicle. In this case, the toothed ring remains stationary and the handwheel moves round with the turret. In large turrets, a motor is geared to the ring in addition to the handwheel, so that, if it is required, the turret can be rotated quickly.

Another important application of internal gears is in epicyclic trains, which are considered in a later section.

Bevel Gears

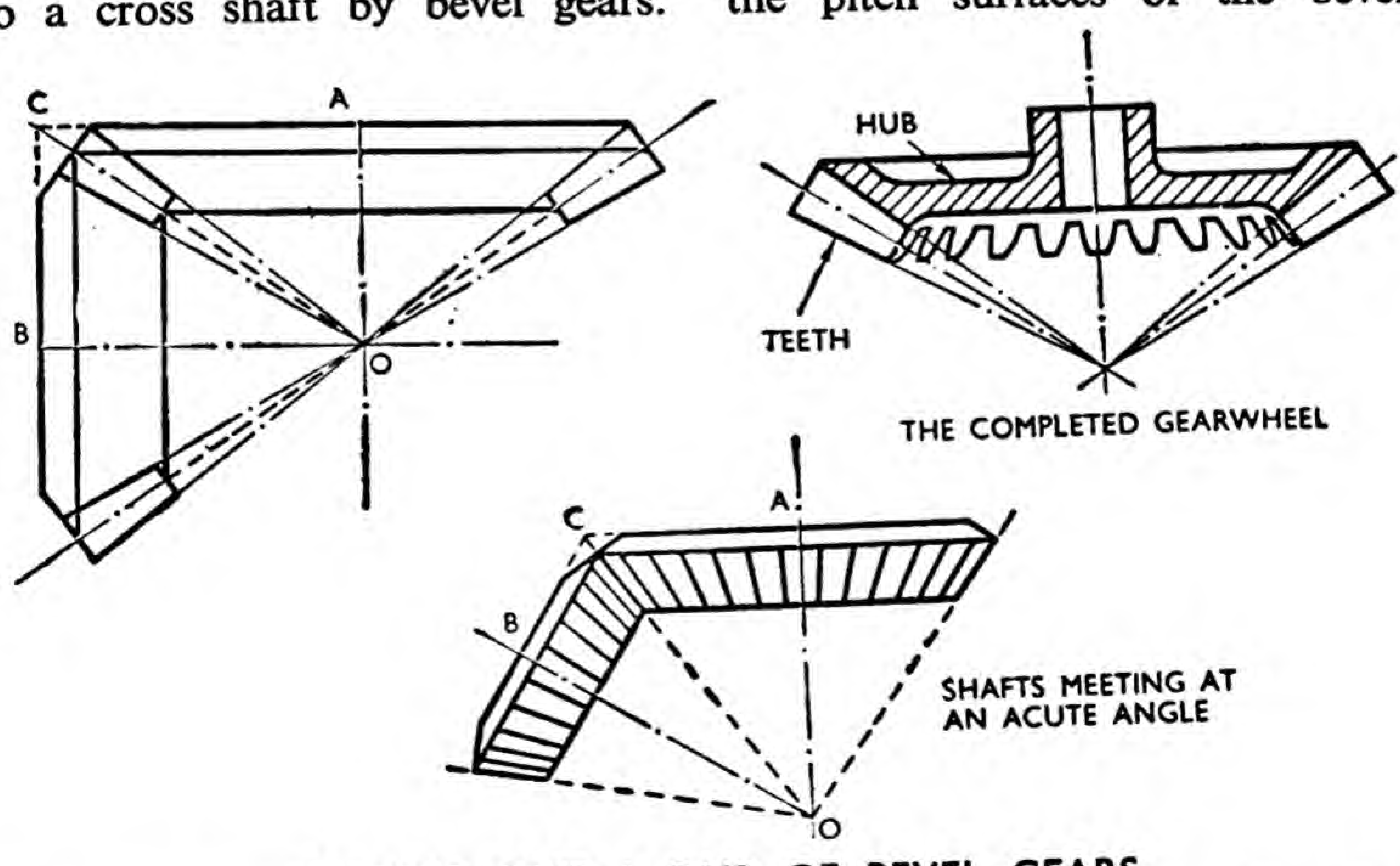
In all the gearwheels we have so far considered, the teeth have been straight and parallel. These are called spur gears and they can only be used to connect shafts which are parallel. Frequently, we want to transmit power between shafts which are inclined to each other, or to take a drive round a corner. This is where bevel gears are useful. A typical arrangement is shown in Fig. 20. Here, all the wheels are the same size, but care must be exercised in arranging the wheels to get the desired

direction of rotation. The small inset diagram shows how, by changing the position of one bevel wheel, the final shaft may be made to rotate clockwise.

This immediately suggests a way of using bevel gears in a reverse gear box, and an arrangement that is very widely used in cranes and hauling machinery is shown in Fig. 21. Two bevel gears *B* and *C* are mounted on a sliding key or feather on the shaft, so that they can slide along it but must rotate with it. When the gears slide to the right, wheel *B* meshes with the gearwheel *A* which is then driven clockwise. When the gears slide to the left, wheel *B* is disengaged and wheel *C* meshes with the other side of gearwheel *A*, which is then driven counter-clockwise.

A better idea of what the teeth of bevel gears are like can be gained from Fig. 22, which shows a drive to a cross shaft by bevel gears. The drive from the propeller shaft to the back axle of the average motor car is very similar. It will be noticed that the teeth are tapered and are formed on conical surfaces. The wheels in this case are also of different diameter, and there will be a speed reduction to the cross shaft in the ratio of the number of teeth on the driver or bevel pinion, to the number of teeth on the follower, just as for ordinary spur gears.

We have mentioned that the teeth of bevel gears are formed on conical surfaces. The method of setting out a pair of bevel gears is shown more clearly in Fig. 23. First find the point of intersection *O* of the axes of the two shafts. Then mark off the point *C* such that the ratio of *AC* to *BC* is equal to the required velocity ratio of the pair of gears. Then the line joining *O* to *C* is the line of contact of the two cones forming the pitch surfaces of the bevel



SETTING OUT A PAIR OF BEVEL GEARS

Fig. 23. Bevel gears must be designed as a pair, and the method of setting out is shown above. *O* is the point of intersection of two shafts which are to be geared together. Point *C* is chosen so that the ratio of *AC* to *AB* is equal to the required velocity ratio of the two gears. *OC* is, then, the line of contact of the two cones forming the pitch surfaces of the bevel wheels.

wheels. It will be seen that the essential requirement is that each cone shall have its apex at the point *O* where the axes of the shafts intersect. In effect, then, the two cones roll on each other as the shafts rotate, and the teeth prevent the cones from slipping.

In practice, of course, only a small portion at the base of the cones is required, as shown by the full lines. The smaller diagram in Fig. 23 shows how the same method is applied when the shafts meet at an acute angle. It will also be obvious from the diagrams that the proportions of the cones will vary for each velocity ratio that is required. This means that any two bevel wheels, even if they have the same size of teeth, will not necessarily mesh together. Bevel wheels must be made up as a pair for a particular velocity ratio, and the ratio cannot be changed except by changing both wheels. This is a disadvantage as compared with spur wheels where any two wheels will mesh together provided they have the same size of teeth, and from a set of wheels a large number of different velocity ratios may be obtained.

Worm Gears

Worm gears provide us with another means of driving shafts at right angles. They are used instead of bevel gears in the back axles of some cars, but their principal use is where a large speed reduction is required, for example, 20 to 1 or more. The principle of the worm gear is that of a screw driving a nut. In one revolution of the screw the nut is moved along by an amount equal to the pitch of the thread. Now if the threads, or parts of

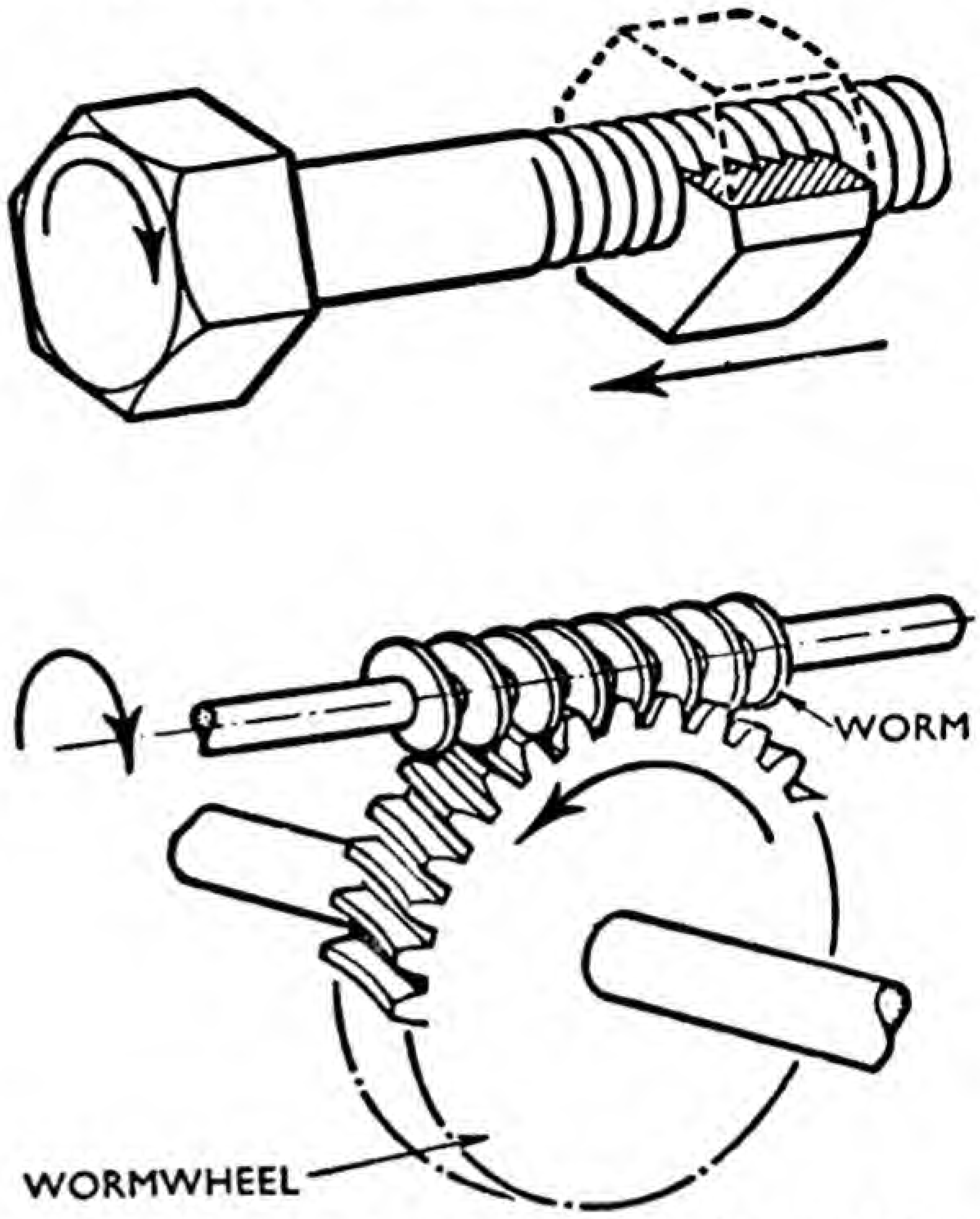


Fig. 24. One revolution of a screw causes the nut to move through a distance equal to the pitch of the thread. The worm and wormwheel behave in a similar manner, the threads being made on the rim of the wheel. The worm revolves once for each tooth on the wormwheel, so giving large speed reduction.

them, are formed on the rim of a wheel instead of in a nut, then the rim of the wheel will move a distance equal to the pitch of the thread for each revolution of the screw (Fig. 24).

Such an arrangement constitutes a worm gear, and the screw is usually called the worm, and the wheel with threads or teeth on its rim, the worm wheel. Its convenience when a large speed reduction is required will now be obvious, for if the worm wheel has 100 teeth, the worm will have to revolve 100 times in order to turn the worm wheel through one complete revolution. Another peculiarity of the large reduction worm gear is that of non-reversibility. The worm will drive the worm wheel, but the worm wheel cannot

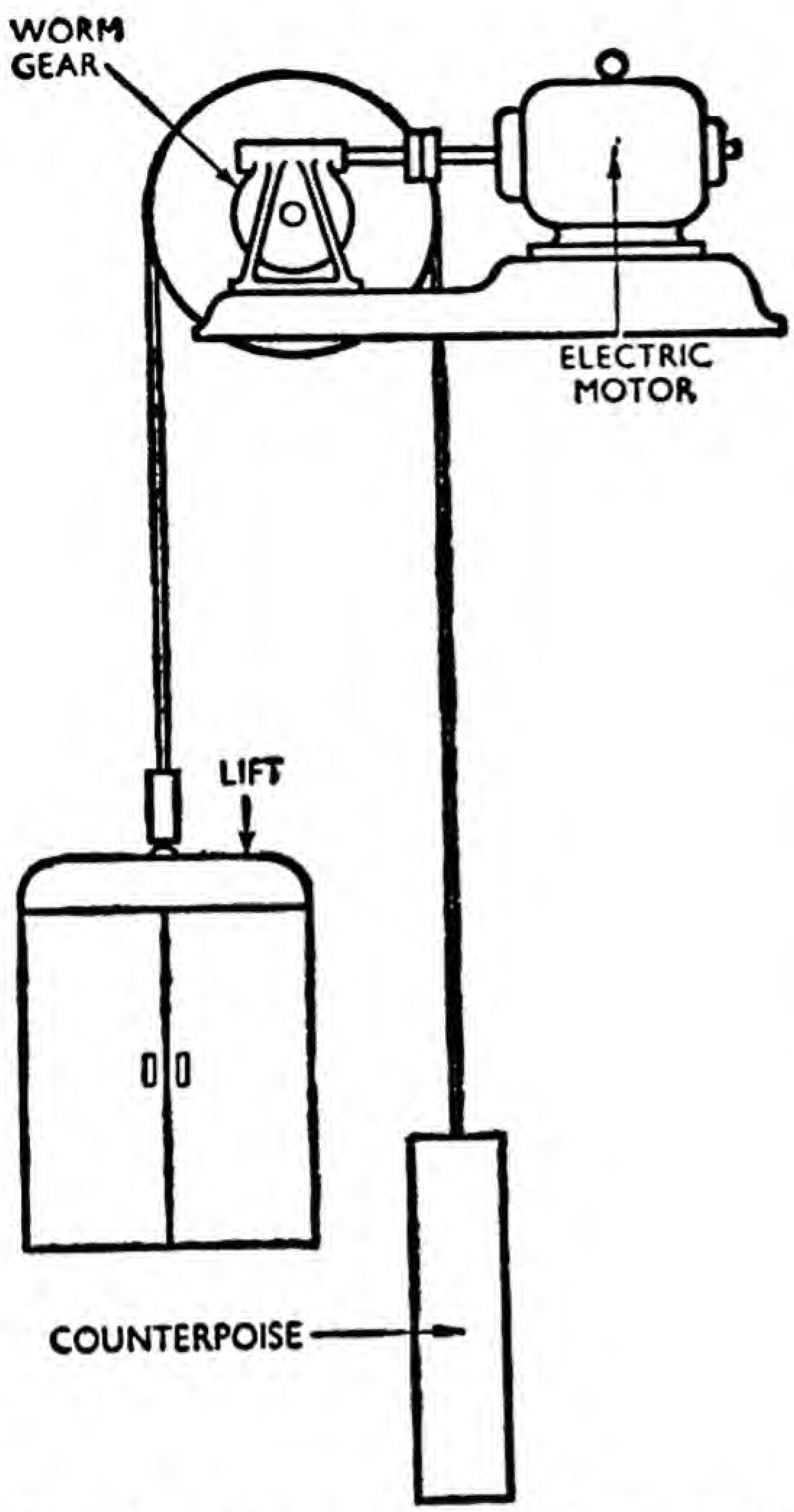


Fig. 25. The property of non-reversibility of the worm gear is utilized in a lift as illustrated above. The worm, driven by a motor, drives the worm-wheel which rotates the cable drum and so raises the lift. But as the worm-wheel cannot drive the worm, the lift will not fall if the motor fails.

drive the worm. The efficiency is, however, very low in this case. This property is frequently a very great advantage as, for instance, in the drive to the lift shown in Fig. 25. The electric motor can raise or lower the lift, but the lift cannot fall of its own accord even if the current fails.

We cannot, however, use a simple worm gear of this type if we require only a small speed reduction, because it is not possible to make a satisfactory worm wheel with less than about 15 teeth, and even this small wheel would give a speed reduction of 15 to 1. To help us out of this difficulty we can use a worm with more than one thread cut on it. This is called a multi-thread worm, or sometimes, a multi-start worm. If the worm has two threads, any one thread will mesh with every alternate tooth on the worm wheel, and one revolution of the worm will move the rim of the worm wheel two teeth. This gives us the general rule for the velocity ratio of worm gears.

Velocity ratio

$$= \frac{\text{No. of teeth on worm wheel}}{\text{No. of threads on worm}}.$$

A four-thread worm meshing

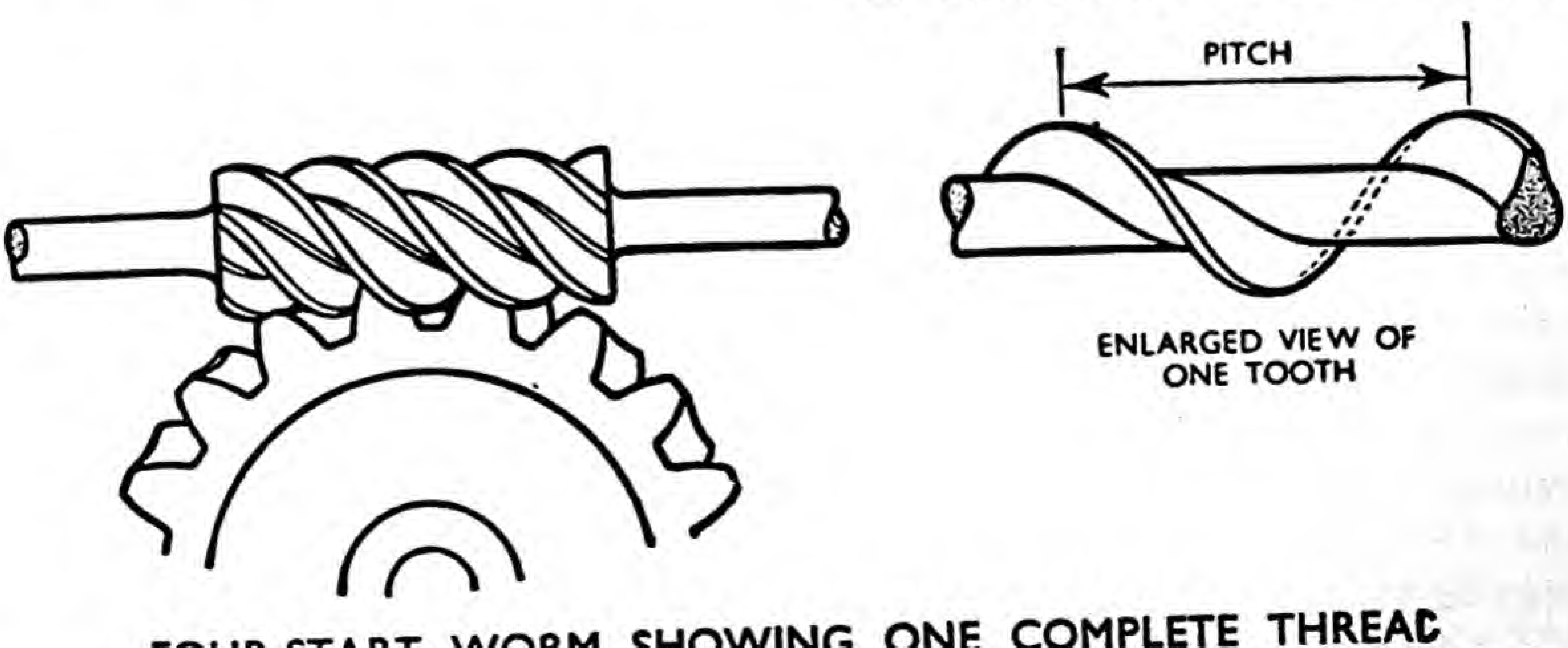
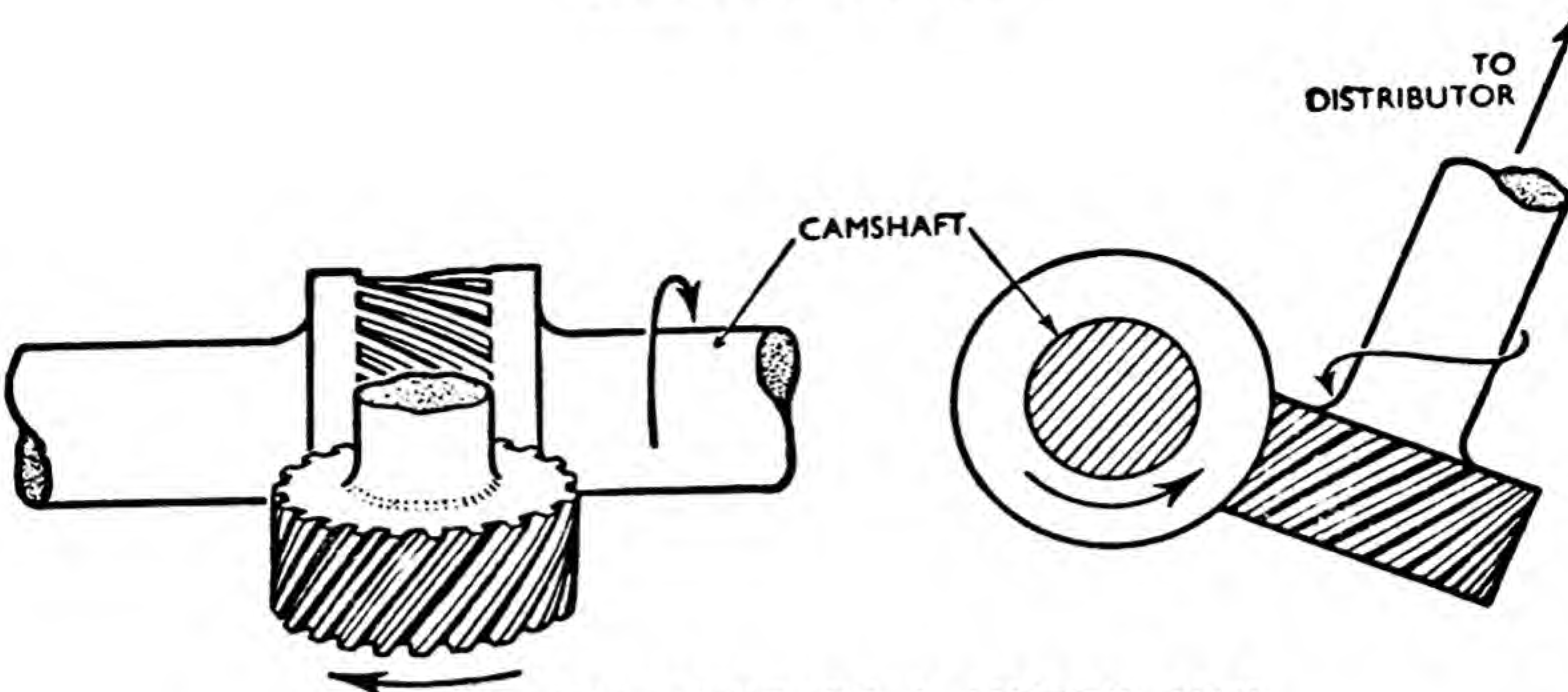


Fig. 26. A multi-start worm may have two or more threads, the twist becoming more pronounced as the number of threads increases. They are used when a worm drive giving a low speed reduction is required, as in the back axle of a motor car.



DISTRIBUTOR DRIVE OF A MOTOR CAR

Fig. 27. Skew gears are similar to multi-start worm gears, but are used when a smaller speed reduction is required. The teeth on both wheels have a very quick pitch. The diagram shows how the distributor on a motor car may be driven from the camshaft by means of skew gears.

with a 16-tooth worm wheel is shown in Fig. 26. This will give a speed reduction of 4 to 1. It will be noticed that as the number of threads increases, the inclination or twist of the threads becomes more pronounced, the worm is said to have a quick pitch, and the teeth on the worm wheel must be inclined to suit. There comes a point when the inclination is sufficiently great for the worm to become reversible, for the worm to be driven by the worm wheel. This is the kind of worm that must be used in a worm-drive back axle of a car, for, although the worm will normally drive the axle, it must be possible for the axle to drive the worm when coasting down hill. The worm drive is usually quieter than the bevel drive, but there is more friction, and, therefore, it is slightly less efficient. A well-designed and well-lubricated worm drive may, however, have an efficiency of 97 per cent.

It is possible to increase the number of threads on the worm to 15 or 20 or even more, but then it begins to look more like a gearwheel with inclined teeth, in fact the worm and worm wheel are then not unlike each other. In this case they are more frequently called skew gears (Fig. 27), and a very familiar application is the distributor drive on a motor-car engine.

Sometimes inclined or helical teeth, similar to the teeth on skew gears, are used on spur gears. This is so as to combine the characteristic quietness of worm gears with the high efficiency of spur gears. These are called single-helical teeth if they are like Fig. 28(a), or double-helical or herringbone teeth if they are as shown at Fig. 28(b). The advantage of double-helical teeth is that the side thrust due to the inclination of the teeth balances out, and there is no end thrust on the shaft or bearings.

Epicyclic Trains

So far, in all the gear trains that have been considered, the axes about which the wheels rotate have remained stationary. In epi-

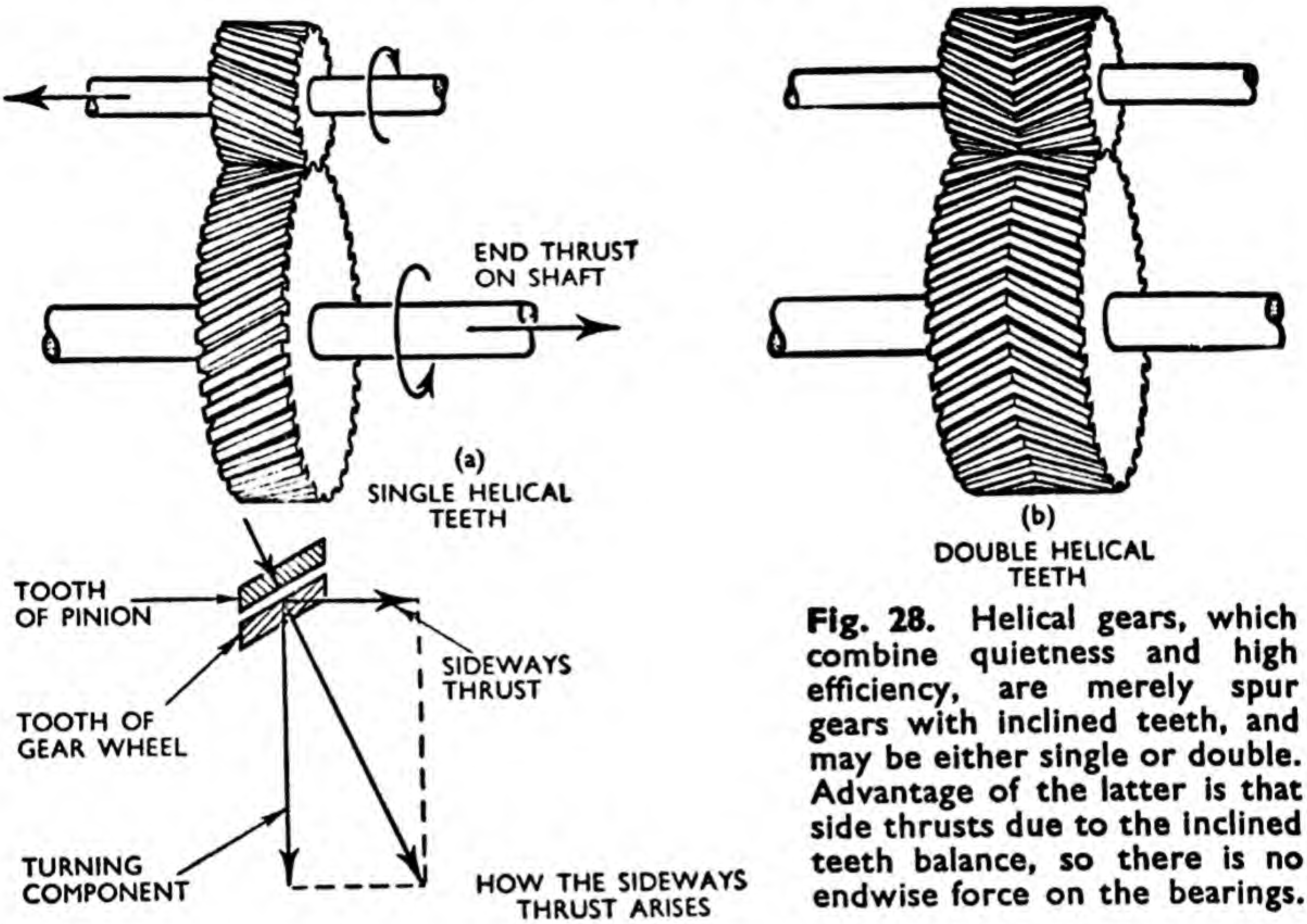


Fig. 28. Helical gears, which combine quietness and high efficiency, are merely spur gears with inclined teeth, and may be either single or double. Advantage of the latter is that side thrusts due to the inclined teeth balance, so there is no endwise force on the bearings.

cyclic trains the axis of one or more of the gears rotates, while the wheel itself may rotate on the axis, in much the same way that the earth rotates about the sun whilst also spinning on its axis. For this reason, epicyclic trains are sometimes called sun and planet gears.

In Fig. 29, wheel *A* is the central sun wheel, wheel *B* is the planet wheel which rotates on a pin mounted on the planet carrier *C*, and the internal gear *D* is the annulus. If the planet carrier is held stationary, the gearwheels form a simple train, so that the sun wheel could rotate the annulus with the planet wheel acting as idler. But what happens if the

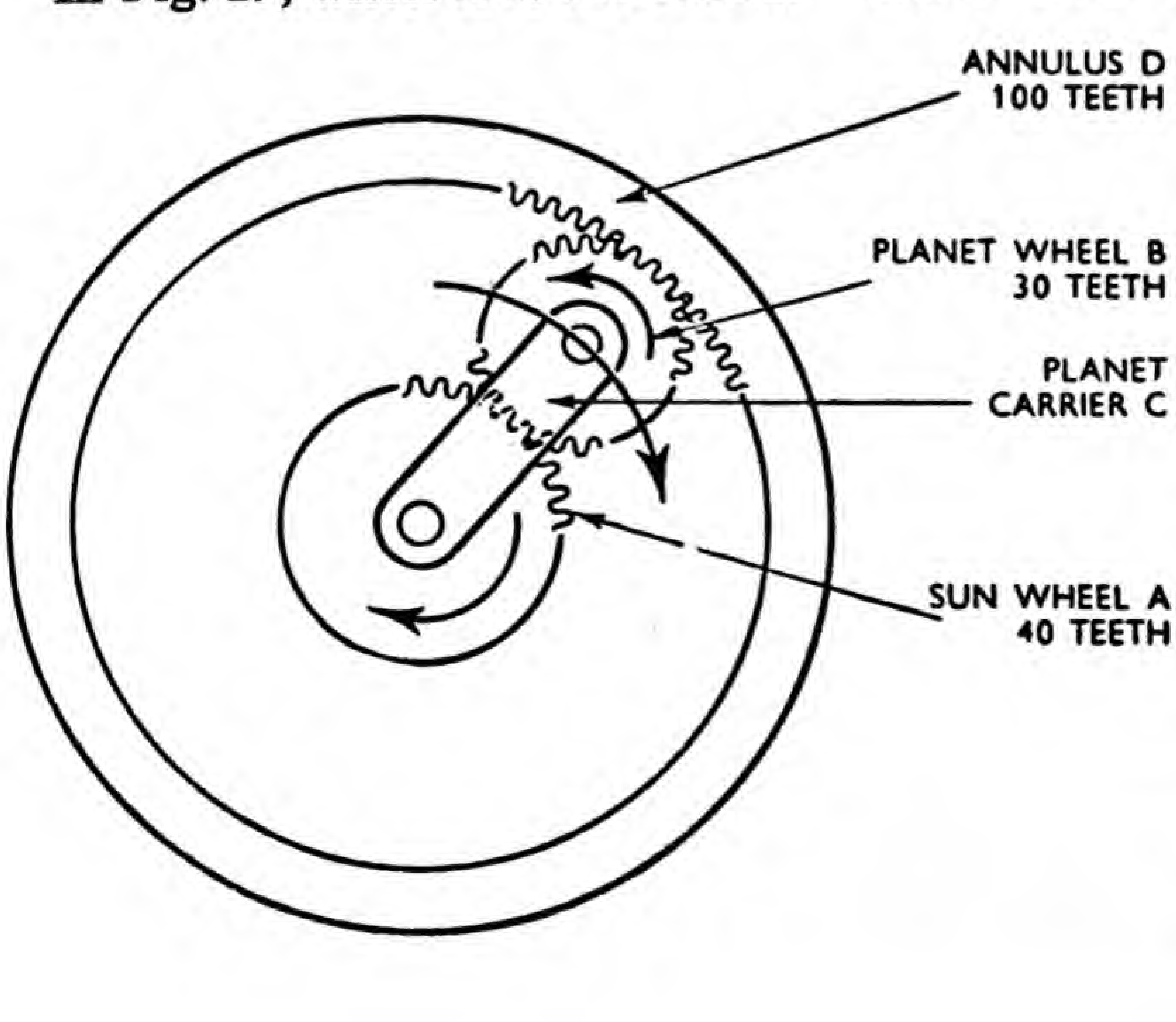


Fig. 29. Characteristic feature of sun and planet gear is that the axis of one of the wheels (*B*) rotates in a circular orbit, while the wheel itself rotates on the axis. *B* is called a planet wheel, and *A*, about which it revolves, a sun wheel. *B* also meshes with internal gear *D*, called the annulus. If *D* is prevented from rotating and *A* is rotated clockwise, *B* will roll round inside the annulus, moving *C* with it at two-sevenths the speed of *A*.

TABLE I. ALL WHEELS MOVING TOGETHER

	Planet Carrier *C*	Annulus *D*	Planet *B*	Sun *A*
1 Revolution of planet carrier, wheels locked	$+1$	$+1$	$+1$	$+1$

TABLE II. PLANET CARRIER FIXED

	Planet carrier *C*	Annulus *D*	Planet *B*	Sun *A*
1 Revolution of annulus, planet carrier fixed	0	-1	$-\frac{100}{30}$	$+\frac{100}{40}$

planet carrier is left free, and the annulus *D* is held stationary? If the sun wheel is rotated, say, for example, clockwise, the planet wheel will be rotated on its axis counter-clockwise, but it will also roll around the inside of the annulus, carrying the planet carrier with it in a clockwise direction. Now, how many revolutions will the sun wheel make for one complete revolution of the planet carrier? That is a difficult question to answer straight away, but it is easy if we do it in two stages.

Overcoming a Difficulty

The difficulty is to know just how much the planet wheel moves with the planet carrier for one revolution of the sun wheel, so let us get over this difficulty by moving the planet carrier a definite amount, one revolution clockwise. If during this process we lock all the wheels together, all three wheels will make one revolution clockwise. We put this down in the first stage of a table, as in Table I.

The plus sign denotes a clockwise rotation. The planet carrier is now in the position we desire, but the annulus is not; it should have remained fixed. Therefore, it will have to be turned back one revolution counter-clockwise while the planet carrier is held stationary. The wheels now form a simple train and we can write down their rotations in the second stage of the table, as in Table II.

The minus sign denotes a counter-clockwise rotation. To obtain the total revolutions, we add together both parts of the table, and we obtain Table III.

This shows that the sun wheel *A* must make $3\frac{1}{2}$ revolutions clockwise for each complete revolution of the planet carrier. Meanwhile the planet itself will have made $2\frac{1}{3}$ revolutions counter-clockwise, rotating on the axis while the axis itself moves clockwise with the planet carrier. With the sun wheel as driver and the planet carrier as follower, this epicyclic train, therefore, would give a

TABLE III. TOTAL REVOLUTIONS

	Planet carrier C	Annulus D	Planet B	Sun A
1 Revolution of planet carrier, wheels locked	$+1$	$+1$	$+1$	$+1$
1 Revolution of annulus, planet carrier fixed	0	-1	$-\frac{100}{30}$	$+\frac{100}{40}$
Total	$+1$	0	$-2\frac{1}{3}$	$+3\frac{1}{2}$

speed reduction of $3\frac{1}{2}$ to 1. Another way of using it would be to use the planet carrier as driver with the annulus as follower, whilst holding the sun wheel stationary. It then gives a speed increase of $1\frac{2}{5}$ to 1. Fill up the table for this case and see if you agree.

A great advantage of epicyclic gears is their compactness and the fact that the input and output shafts can be made concentric. For this reason they are used in the three-speed hubs of bicycles. They can also be used to form compound trains where a large speed reduction is required, as shown in Fig. 30. Here the annulus is fixed for both trains, and forms the casing enclosing the gears. The sun wheel of the first stage is the driver, and the planet carrier of the first stage then drives the sun wheel of the second stage directly. You will notice that each sun wheel has more than one planet wheel. This does not affect the working out of the problem, because all the planets run round the annulus at the same speed, and thus help to distribute the forces to the planet carrier, in this case, a disk.

As a final example of an up-to-date epicyclic train, Fig. 31 shows the speed-reducing gear through

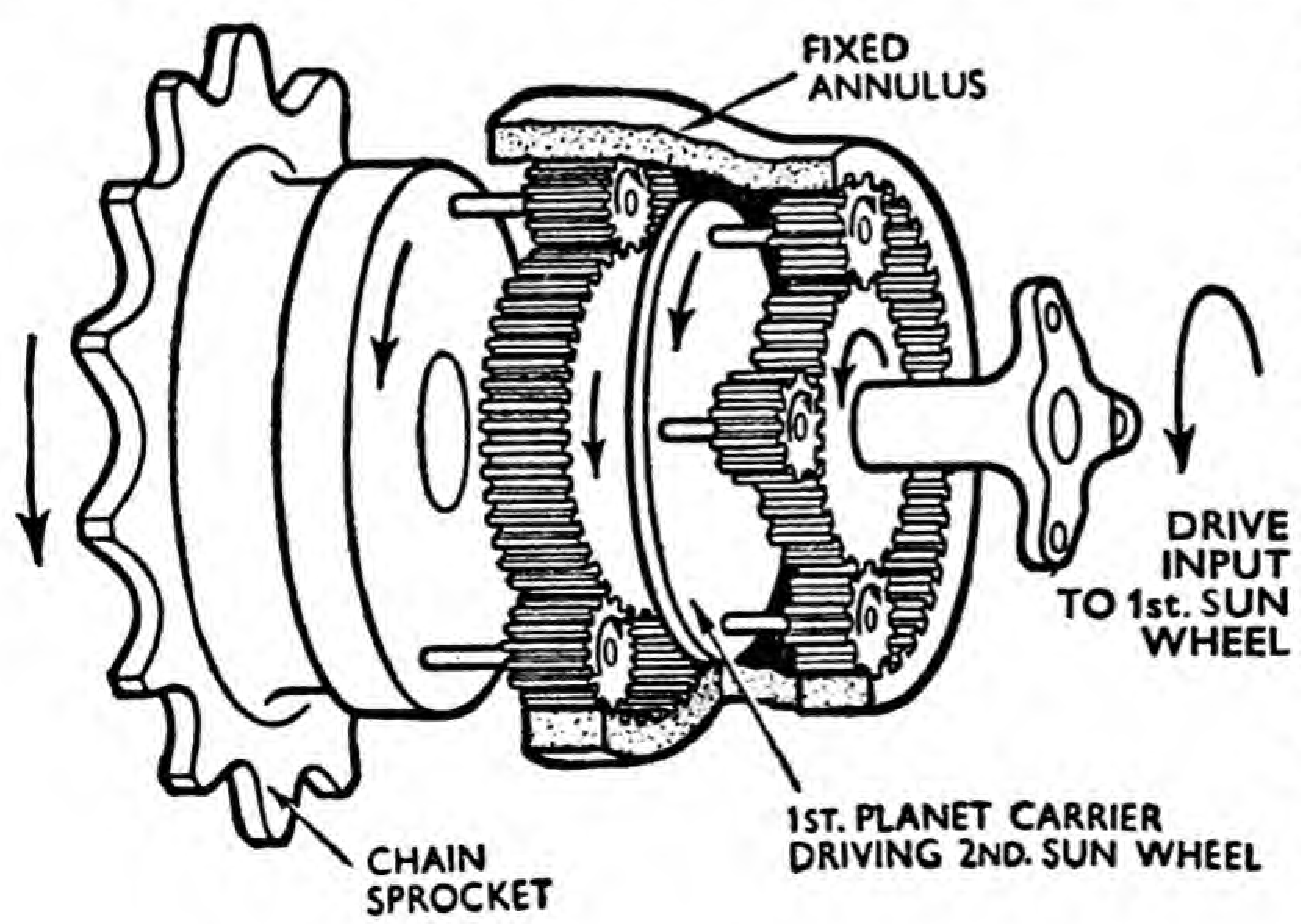


Fig. 30. Two simple epicyclic trains can be combined to form a compound train, and thus to obtain a double reduction effect. In the diagram, the drive is from the small sun wheel on the right to the first planet carrier which drives the second sun wheel at the same slow speed. There is then a further speed reduction to the sprocket, which acts as the planet carrier of the second train.

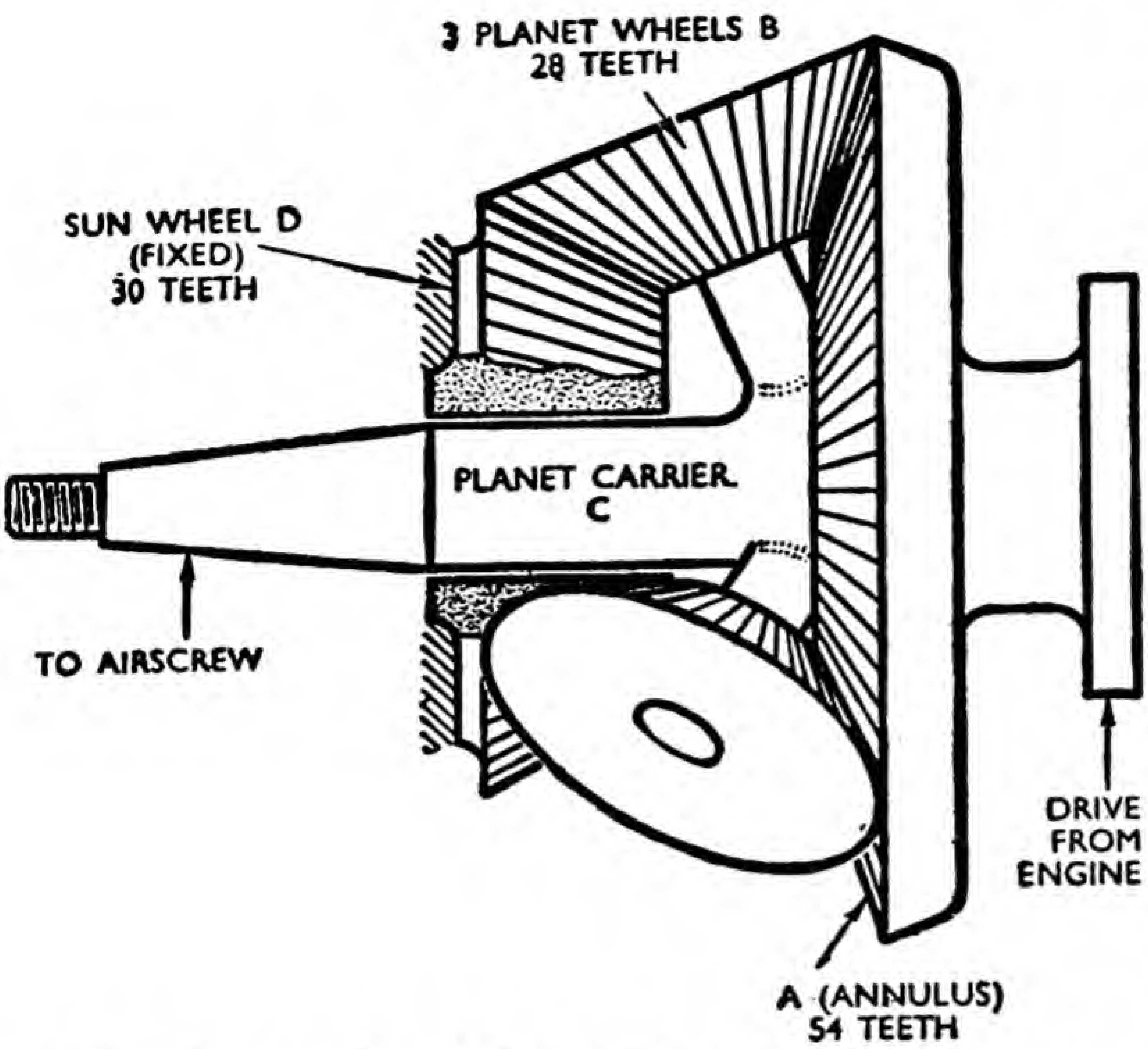


Fig. 31 The high speeds of aircraft engines are not suitable for the operation of the airscrew at its best efficiency. Nowadays, engines usually drive airscrews through a low-ratio reduction gear. This gear must be compact, and it is desirable that input and output should be in line. Here the bevel wheels are used in an epicyclic train. Sun wheel *D* is fixed, and the three planets are driven round it by the large bevel *A*, which is connected to the engine. Planet carrier drives the airscrew at about two-thirds of the engine speed.

which the engine drives the airscrew in a modern aeroplane. This time bevel wheels are used. Wheel *A* is driven by the engine, the three planet wheels *B* rotate with the planet carrier *C* which drives the airscrew, and wheel *D* is fixed. The speed of the airscrew to the speed of the engine is in the ratio of 1 to 1·55. The little table would be as follows :—

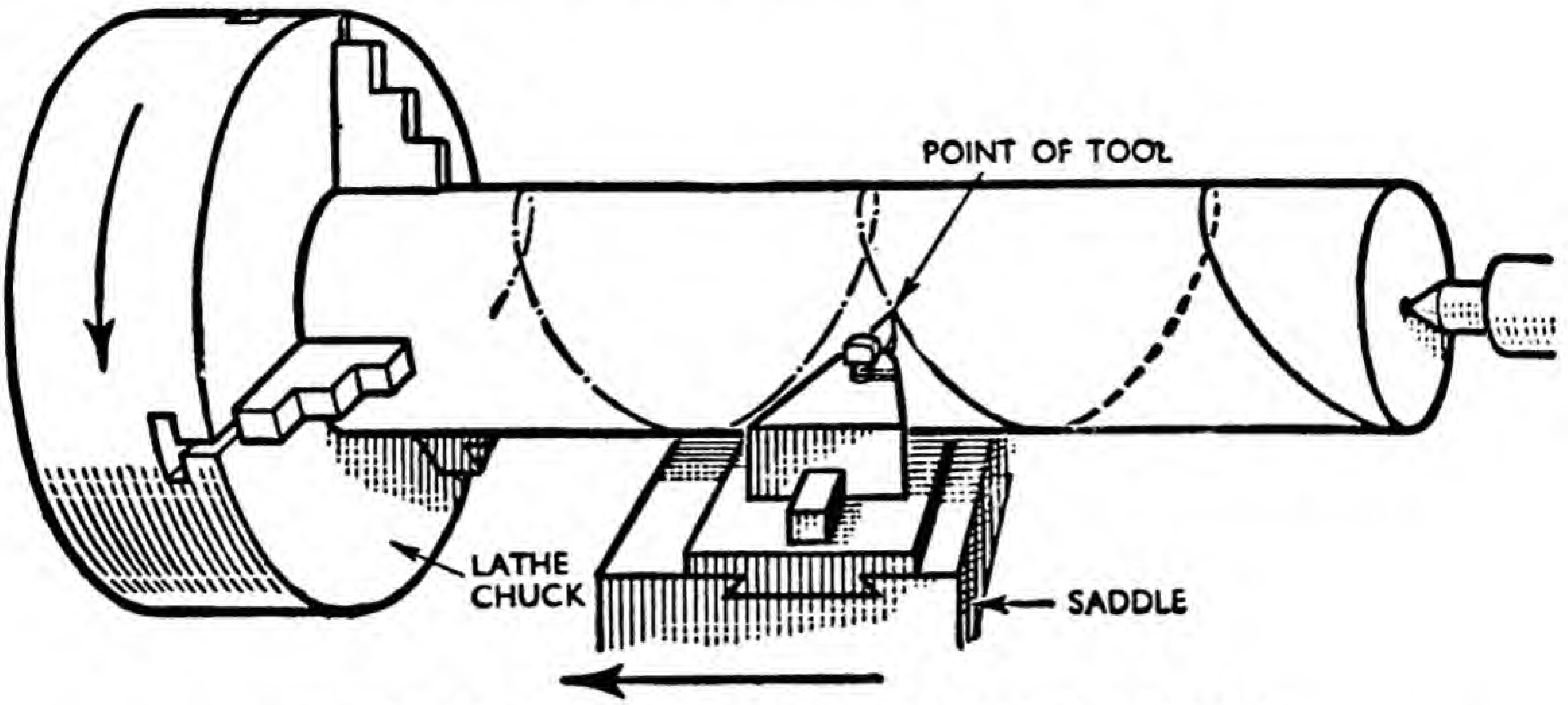
TABLE IV

	D	*C*	*B*	*A*
Locked	+1	+1		+1
C fixed	−1	0		$+\frac{30}{54}$
Total	0	1		$1\frac{5}{9}$

A very natural question at this stage would be, " Well, how are all these complicated shapes of worms and gear teeth made so accurately ? " The answer is, on lathes or milling and shaping machines, or on special grinding machines. These machines differ very much in appearance but there is an underlying principle common to all of them. There are two motions, the motion of the work, the worm or gearwheel, and the motion of the tool or cutter. These two motions are geared together so as to produce any shape required. It is a case of gears producing gears.

Screw Cutting

Now let us take one of the simpler examples, the worm or screw, and see how it is produced. Screw cutting is usually done on a lathe, and the principle will be clear from Fig. 32. Starting with a plain cylindrical bar, this is rotated in a chuck or between the centres of the lathe, and the saddle, carrying the tool or cutter, is caused to slide along the bed of the lathe parallel to the axis of the bar. The combination of these two motions will result in a helical



PRINCIPLE OF THE SCREW-CUTTING LATHE

Fig. 32. To machine a helix, the rotary motion of the work is co-ordinated with the sliding motion of the saddle, which carries the cutting tool. The pitch of the thread or helix is the distance that the saddle slides along the lathe bed for each revolution of the work.

groove being cut in the bar, and the pitch of the helix will be equal to the distance moved by the saddle for one revolution of the work. The groove can be made deeper until a proper thread is formed, by making successive cuts with the tool set further into the work each time.

There are many ways of linking together the two motions of turning and sliding, and a very simple one is shown in Fig. 33. As the spindle rotates with the work, a wire winds up on to it and draws the saddle along. Most screw threads have quite a small pitch, however, only a fraction of an inch, and the method shown would be quite unsuitable for accurate screw threads, although a somewhat similar method has been used for cutting quick-pitch helices, such as the rifling in gun barrels.

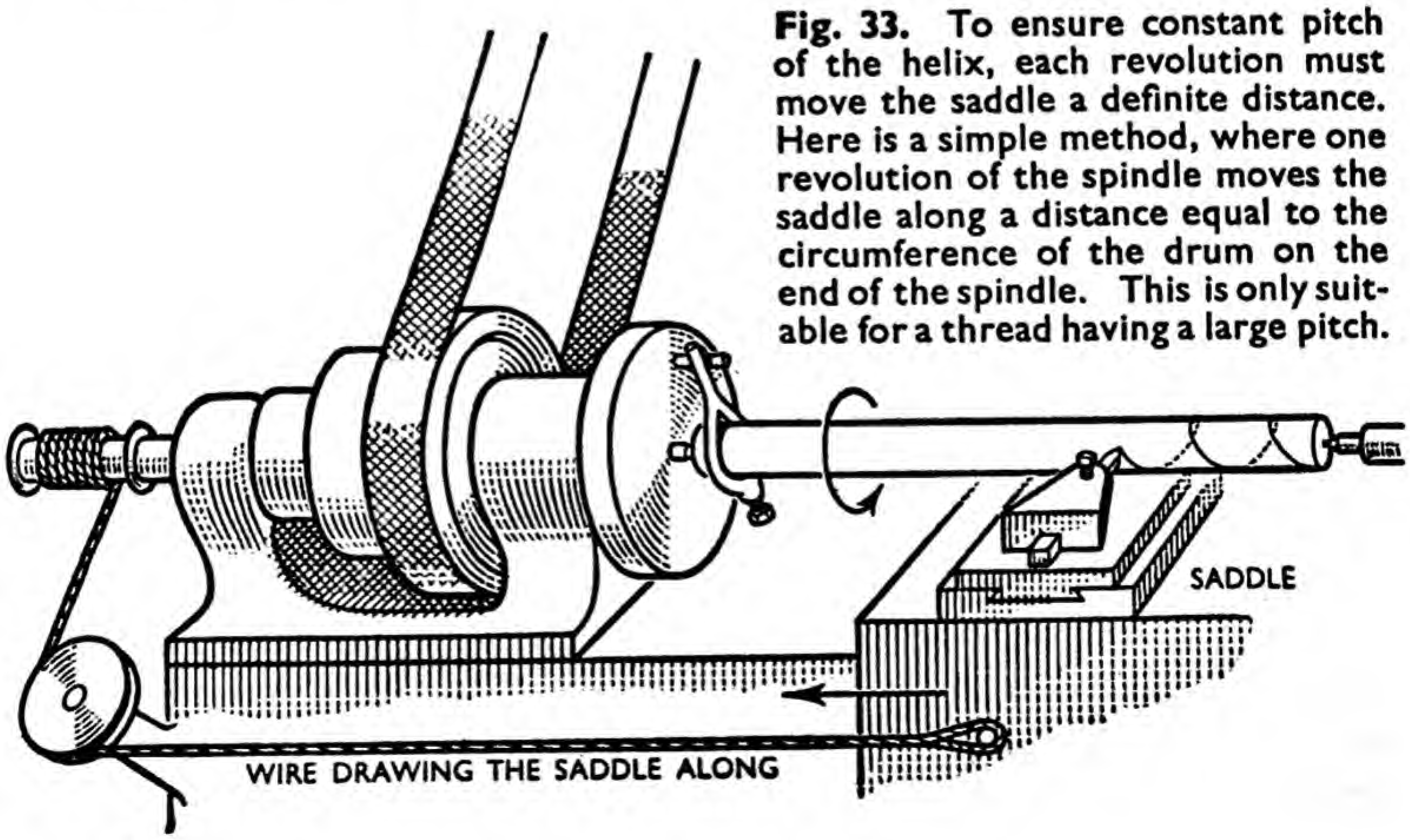
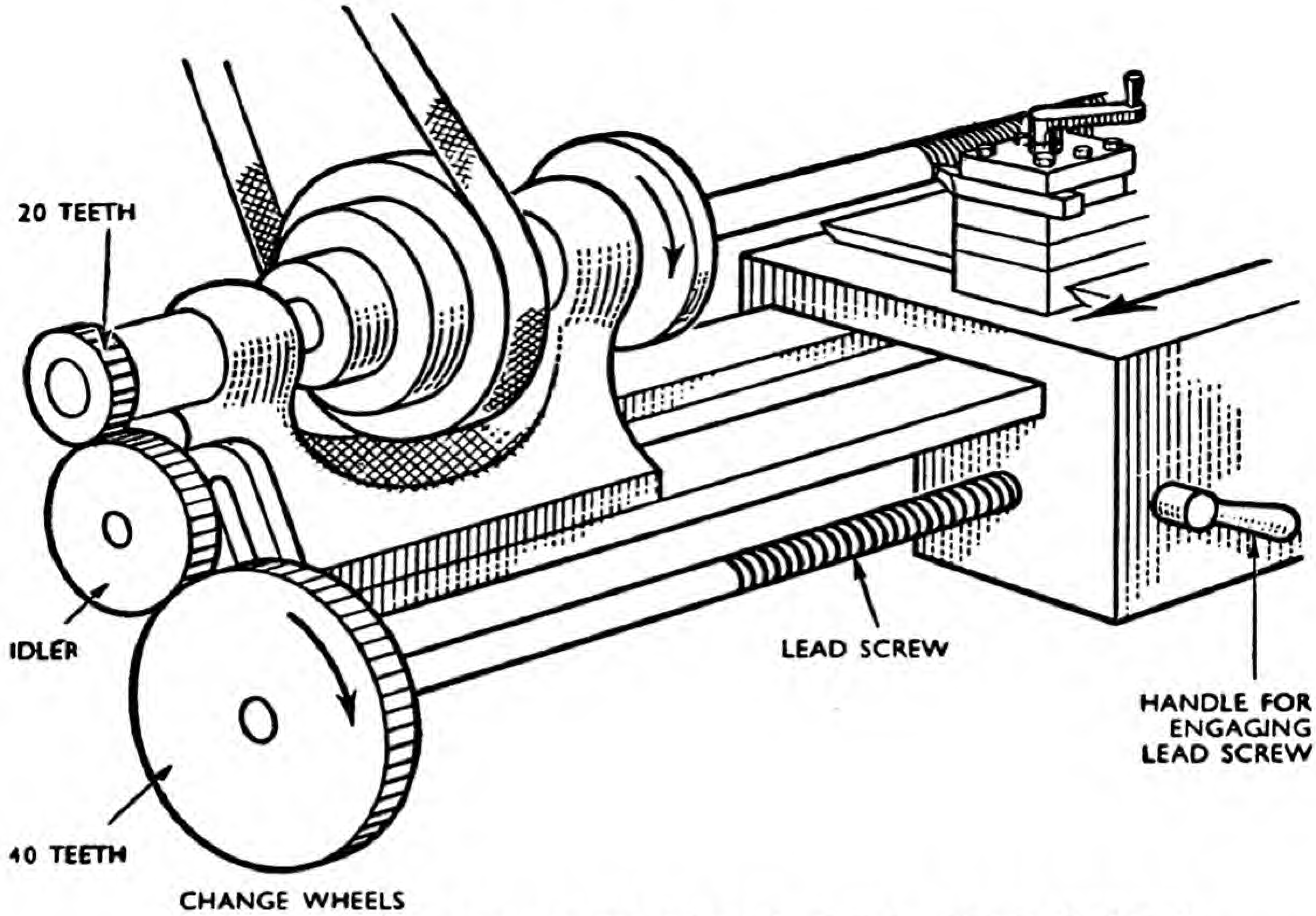


Fig. 33. To ensure constant pitch of the helix, each revolution must move the saddle a definite distance. Here is a simple method, where one revolution of the spindle moves the saddle along a distance equal to the circumference of the drum on the end of the spindle. This is only suitable for a thread having a large pitch.



DRIVING THE SADDLE BY MEANS OF A LEAD SCREW

Fig. 34. The usual method for traversing the saddle along the lathe bed is by means of a lead screw. This lead screw rotates in a nut located in the saddle. The lead screw is geared to the spindle by a number of change wheels, and these wheels can be altered according to the number of threads per inch that are required. The simple train shown enables a screw with 8 threads per inch to be cut with a lead screw of $\frac{1}{4}$-in. pitch.

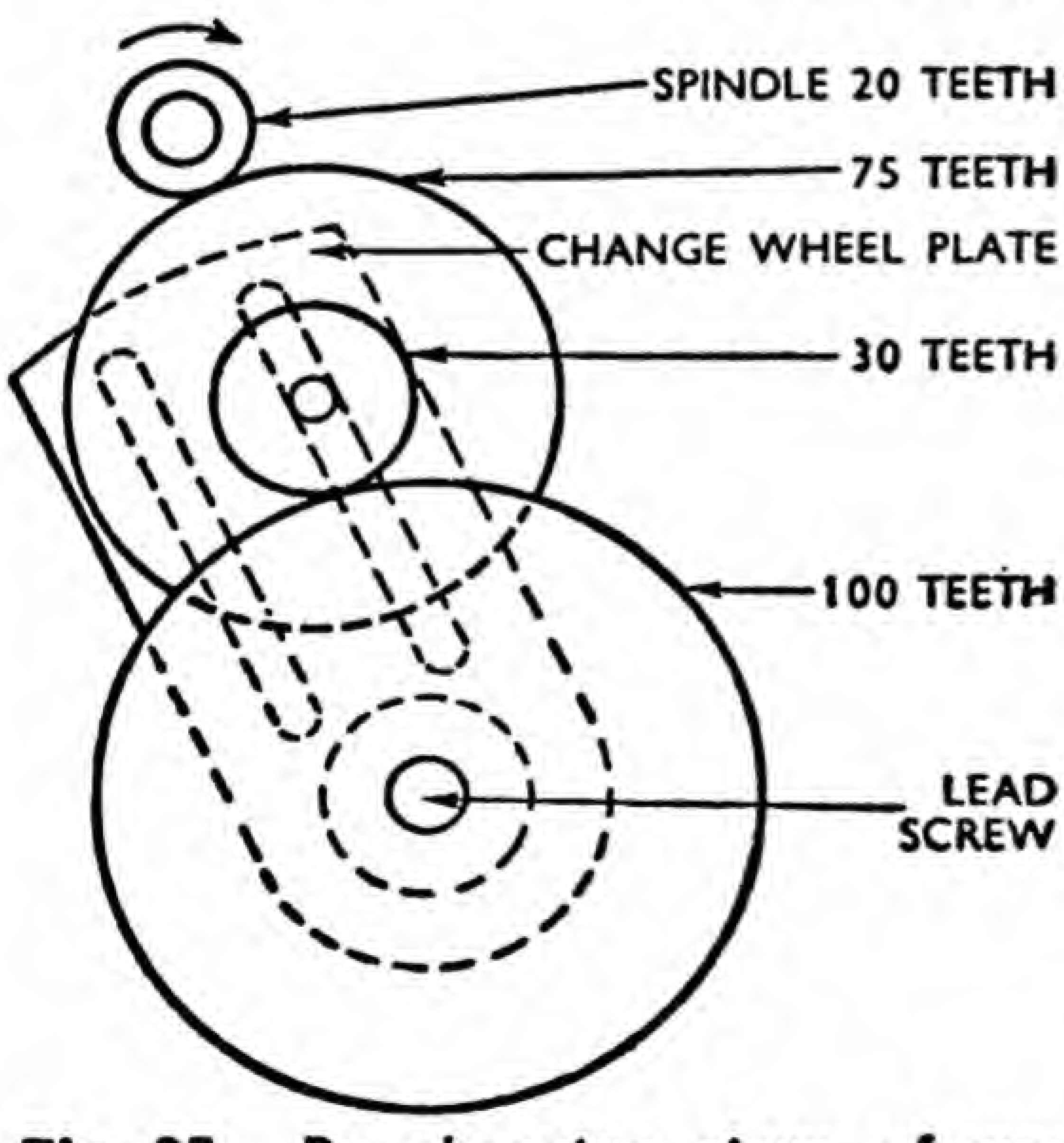


Fig. 35. By changing sizes of gear-wheels, threads of different pitch can be cut with the same lead screw. For cutting 25 threads per inch with a lead screw of $\frac{1}{2}$-in. pitch, wheels are arranged in a compound train to drive lead screw at two twenty-fifths of speed of spindle.

For cutting screw threads, the sliding motion of the saddle is caused by a screw (Fig. 34), the lead screw, which is itself geared to the spindle so as to rotate at a definite speed in relation to the spindle speed. The lead screw usually has either four or two threads per in., viz., $\frac{1}{4}$-in. or $\frac{1}{2}$-in. pitch, so that one complete revolution of the lead screw moves the saddle along the bed of the lathe $\frac{1}{4}$ in. or $\frac{1}{2}$ in. as the case may be. If the lead screw and the spindle are geared together so as to rotate at the same speed, a thread will be cut which has the same pitch as the thread of the lead screw. In fact it will be a copy of the lead screw. If they

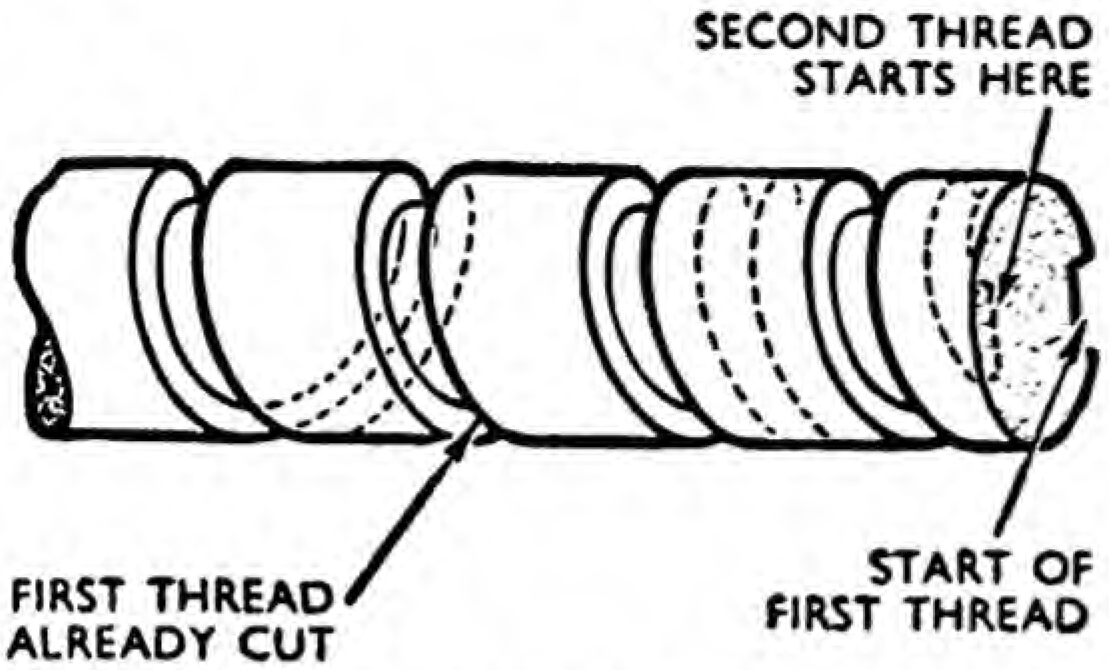


Fig. 36. One thread is cut in the normal way using suitable train of wheels to give large pitch required. Change wheels then disengage while spindle is rotated through 180 deg. before re-engaging for second thread.

are geared together so that the lead screw rotates at one half the speed of the spindle, a thread will be cut which has a pitch one half that of the lead screw.

Choosing Correct Trains

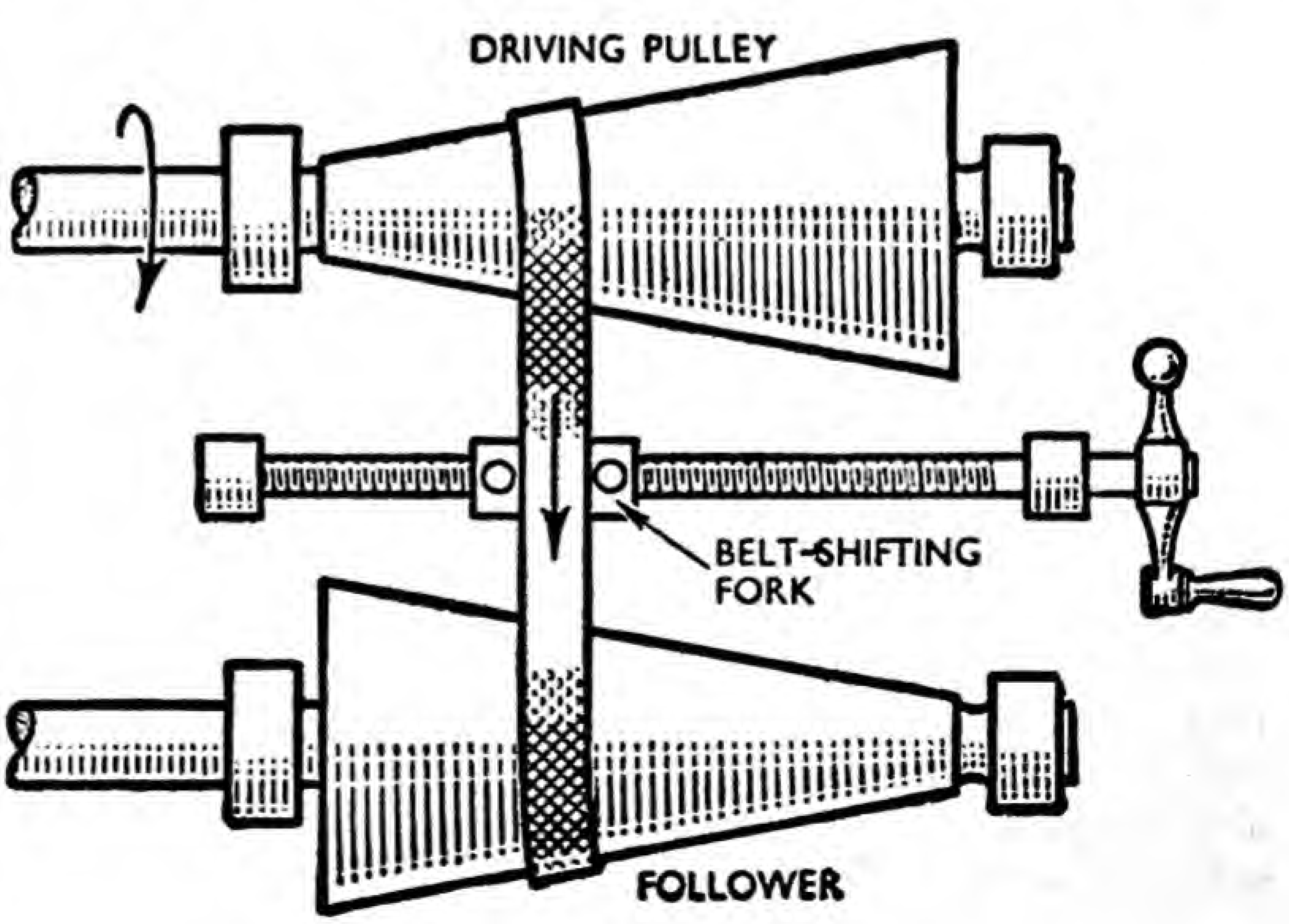
This is obviously a most convenient arrangement, because by changing the velocity ratio of the gear train connecting the spindle and the lead screw, a thread of any desired pitch may be cut (Fig. 35). Every screw-cutting lathe is provided with a set of gears, or change wheels, by means of which a suitable train of gears may be selected. Both simple and compound trains are used, and the diagrams show how they are arranged.

To cut a left-hand thread with a right-hand lead screw, the direction of rotation of the screw must be reversed. This is easily done by including an additional idler in the train. To cut a double thread it is necessary to make two starts (Fig. 36). One thread is cut in the ordinary way, and then the work is rotated half a revolution with the lead screw disengaged. Then the start of the second thread can be made at a point diametrically opposite to the start of the first thread.

Belt Drives

With the gear drives that we have considered, the velocity ratio achieved in practice is exactly the same that is obtained by calculation from the numbers of teeth on the various wheels. The presence of the teeth prevents any possibility of slipping, and, therefore, they are called positive drives. Positive drives must be used where shafts are to maintain exact relative motions, as with the lead screw just mentioned; or the valve timing of an engine where, throughout several thousand revo-

Fig. 37. One way of effecting the speed control of machines is by means of cone pulleys. These taper in opposite directions so that the same belt fits in all positions. When belt is at left, drive is from small diameter to large diameter, giving reduction in speed. By rotating handle of adjusting screw, belt is moved to right by shifting fork and speed of follower increases.



lutions, the camshaft must lift and close the valves at exactly the same points each time. For many purposes, however, the maintenance of the velocity ratio with such exactness is not necessary and here we can use friction drives such as belts and pulleys.

The rule for velocity ratio is similar to that for gears, viz.,

$$\text{Speed of follower} = \text{Speed of driver} \times \frac{\text{Dia. of driver}}{\text{Dia. of follower}}.$$

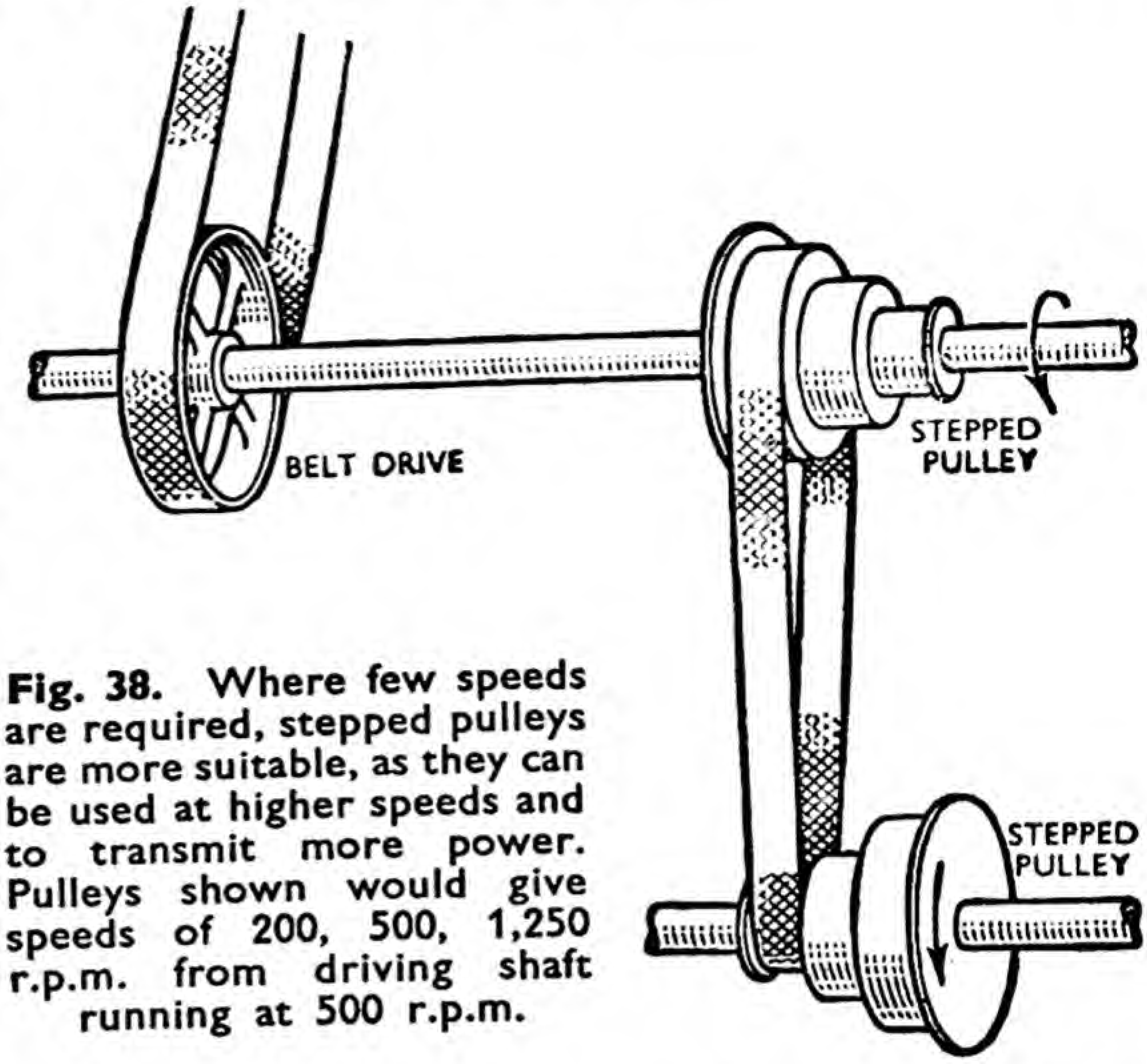


Fig. 38. Where few speeds are required, stepped pulleys are more suitable, as they can be used at higher speeds and to transmit more power. Pulleys shown would give speeds of 200, 500, 1,250 r.p.m. from driving shaft running at 500 r.p.m.

They are particularly convenient for drives to machines when the distance between driving and driven shafts is large or where various speeds are required. For this purpose speed cones or cone pulleys are used, as in Fig. 37. When the drive is from the small diameter of the driving pulley to the large diameter of the follower, a speed reduction is obtained, whereas with the belt at the opposite ends of the pulleys so that a large diameter driver drives a small diameter follower, a speed increase is obtained. Between these two extremes, a continuous variation of speed is possible by moving the belt along by means of a shifting fork and the fine adjusting screw.

For larger powers, and especially for high speeds, the stepped cone pulley is more suitable (Fig. 38). This gives a choice of only three or four speeds, depending on the number of steps, and for many purposes this is quite sufficient, as, for example, for the lathe, because in conjunction with the back gear, six or eight speeds can be obtained. The diameters of the various steps on the pulleys are so chosen that the same belt can be used in all positions without becoming slack or getting unduly tight.

Belt drives also provide one of the simplest means of starting and stopping the drive to a machine. Fast and loose pulleys are used in conjunction with a double-width pulley as shown in Fig. 39. The double-width pulley is keyed to the driving shaft. The fast pulley is keyed to the driven shaft of the machine, whereas the loose pulley can turn freely on a bush without driving the shaft. By means of the belt-shifting fork, the belt can be moved on to the fast pulley or on to the loose pulley, so starting or stopping the machine.

A modification of this arrangement can be used as a reversing gear (Fig. 40). This time the

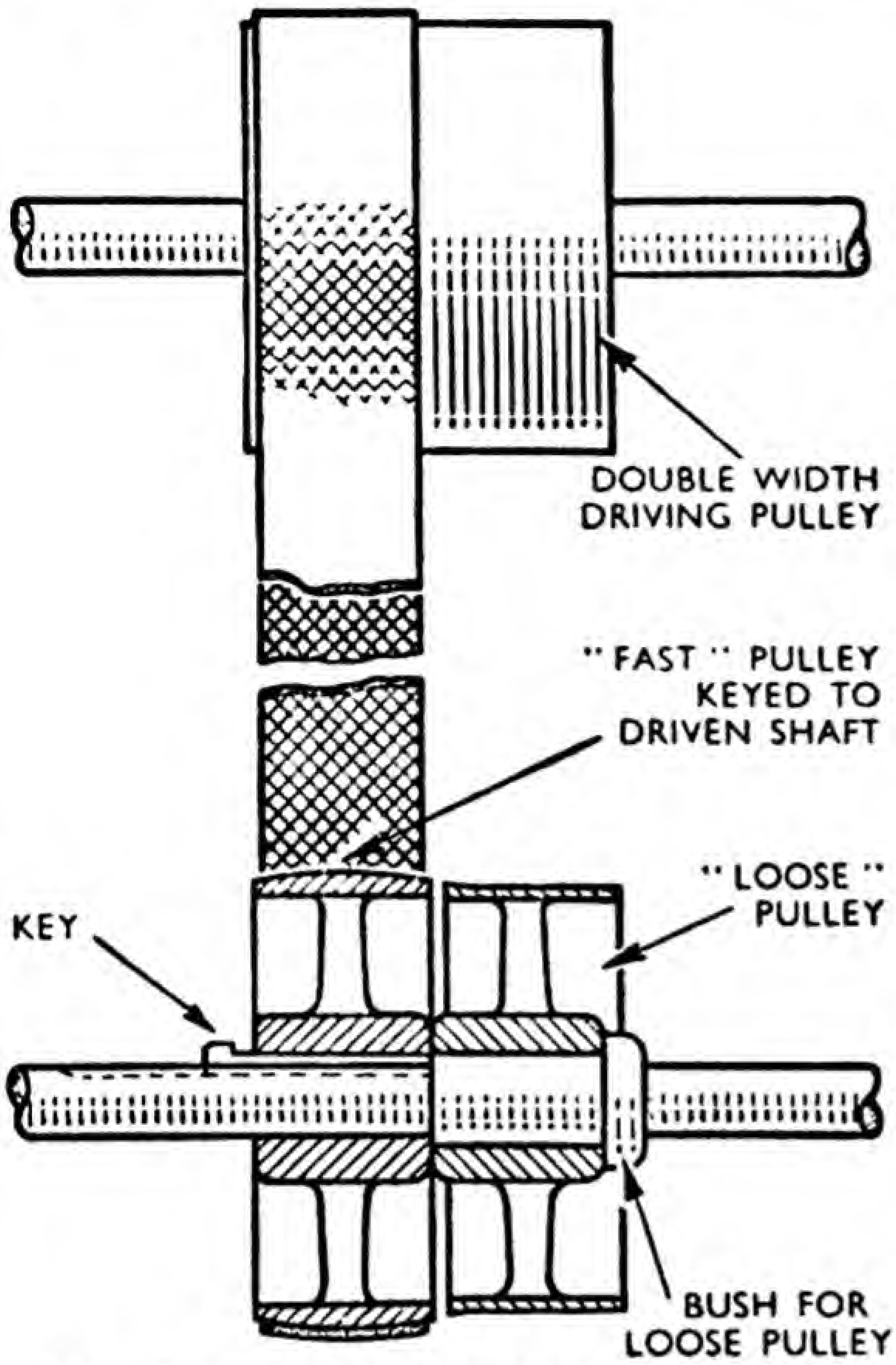
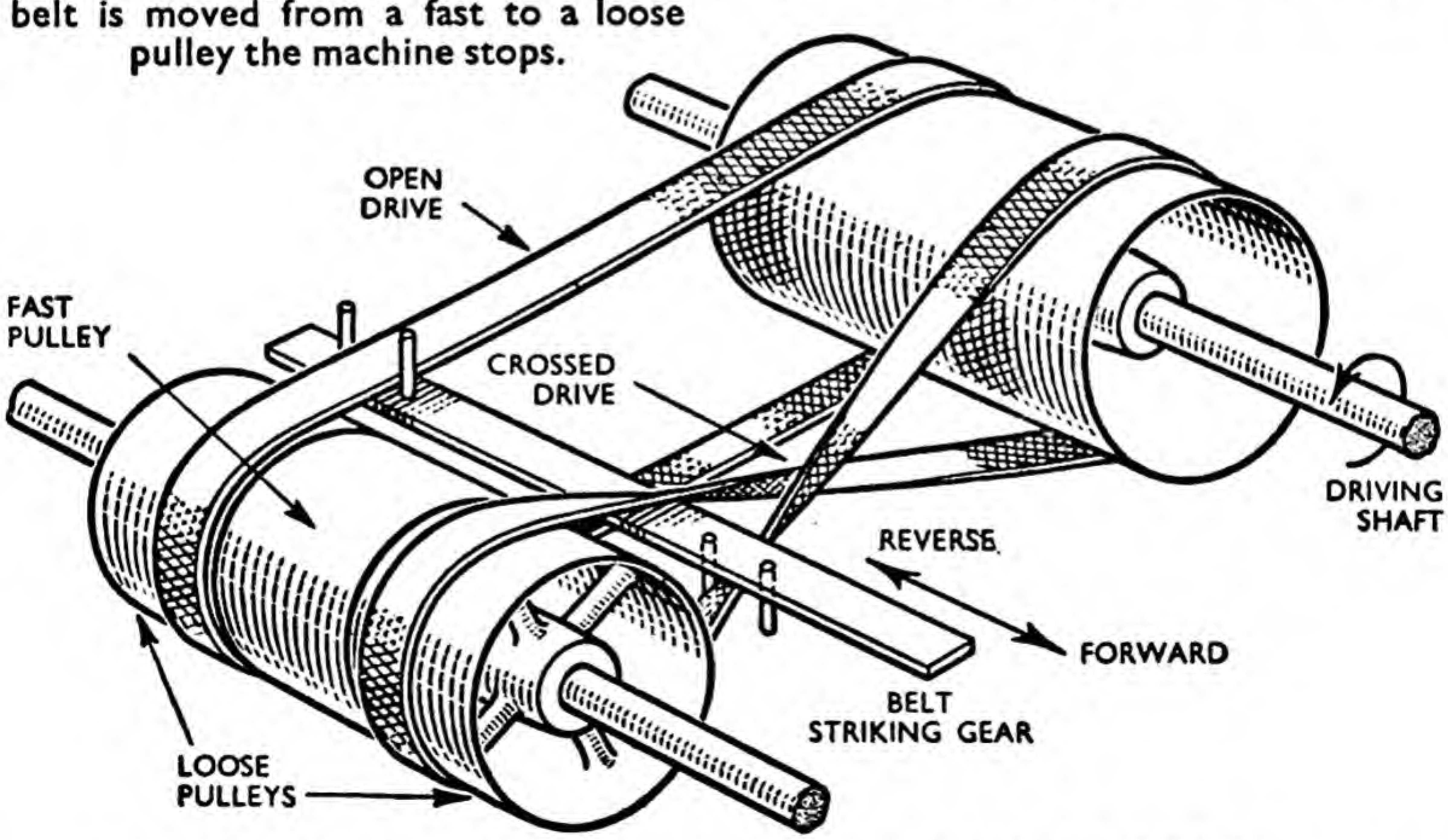


Fig. 39. When several machines are driven from lineshaft, loose pulleys permit any one machine to be stopped without stopping the lineshaft. Machines are provided with fast pulleys keyed to the shaft, and loose pulleys turning freely on bushes. If a belt is moved from a fast to a loose pulley the machine stops.

extra-wide pulley, about five times the belt width, carries two belts, one an open belt and the other a crossed belt. Two double-width loose pulleys are used, and between them a single fast pulley is keyed to the driven shaft. When the open belt is on the fast pulley, the shaft will be driven in the same direction as the driving shaft, and when the crossed belt is on the fast pulley, the direction will be reversed.

Belts can also be used instead of bevel gears to take a drive round a corner (Fig. 41). Here loose pulleys, called guide pulleys, are used to change the direction of the belt. Shafts inclined to each other can be driven by belts instead of skew gears, but great care must be taken to line up the pulleys correctly, otherwise trouble will be experienced due to the belt running off the pulleys. Fig. 42 shows clearly how this should be done.

If an attempt is made to obtain a large velocity ratio with a belt



START, STOP AND REVERSE BY MEANS OF BELTS

Fig. 40. By arranging both a crossed and an open drive to run on loose pulleys with a fixed pulley between them, the driven shaft can be made to rotate in either direction at will by moving the belts either to right or left.

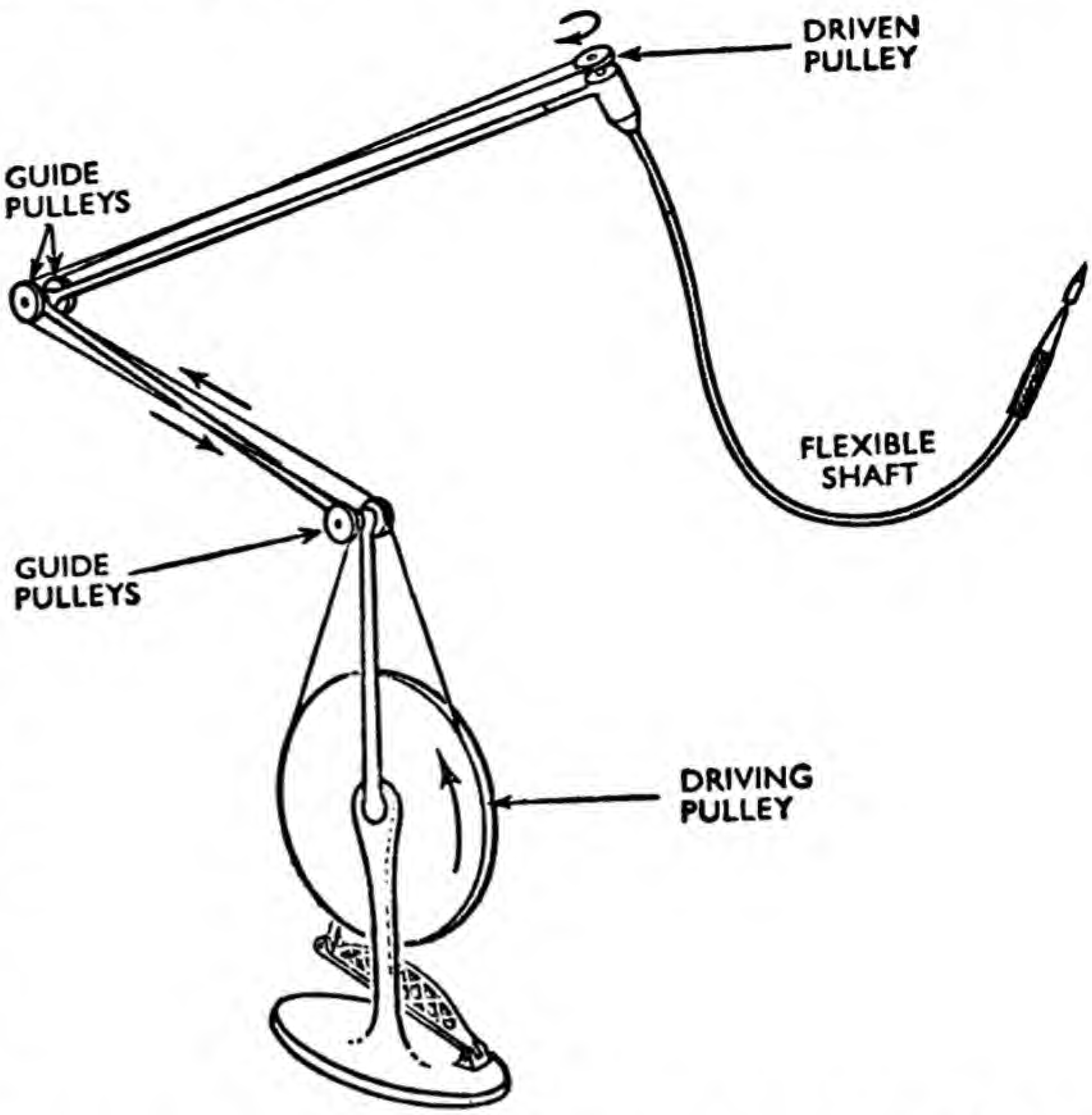


Fig. 41. Belts are normally used to transmit power between shafts which are parallel, but other arrangements are possible with the help of guide pulleys. Belt must travel in approximately a straight line from one pulley to next, and guide pulley enables belt to turn a corner. In dentist's drill shown, hinged arms permit driven pulley to be adjusted into any convenient position. Guide pulleys placed at hinges direct belt and maintain tension in different positions of arms. Guide pulleys are free to turn on their pins and pulleys of each pair rotate in opposite directions.

drive from a small pulley to a very large one, or vice versa, the belt may not grip the small pulley owing to the angle of lap being too small. Power will then be lost due to belt slip. To prevent

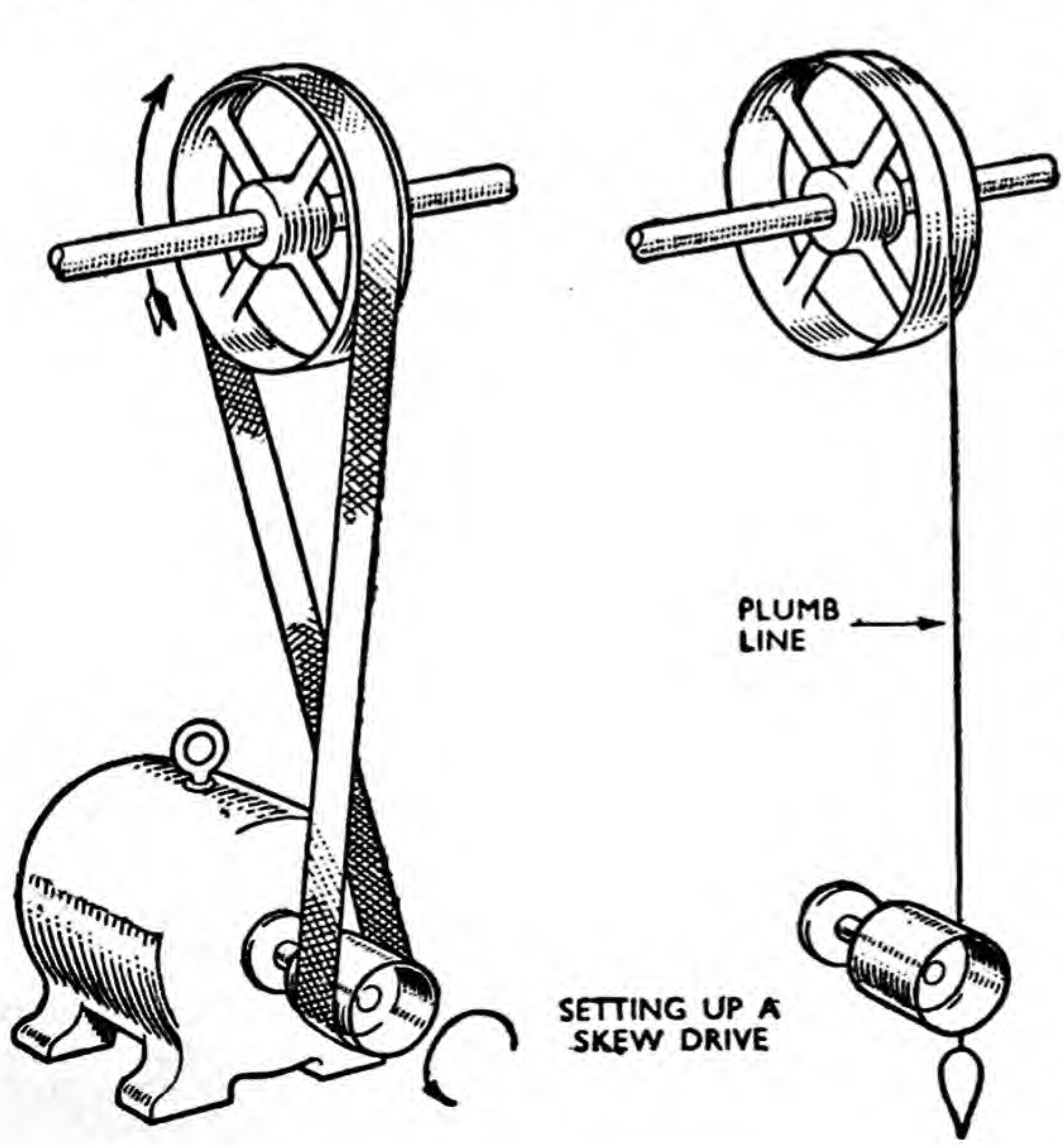


Fig. 42. The skew drive is another example of use of belts to transmit power between non-parallel shafts. It is simpler than the use of guide pulleys, but is not so generally useful, because it is successful only if shafts are skewed in certain directions; it cannot take a drive round a corner and the direction of rotation cannot be reversed, otherwise belt will run off the pulleys. Setting of pulleys is most important and diagram shows how pulleys are aligned with aid of plumb line to ensure correct run of belt.

this, a loose pulley called a jockey pulley can be arranged so as to ride on the back of the belt and thus increase the lap of the belt on the small pulley. By spring-loading or weighting the jockey pulley the correct tension of the belt is maintained automatically (Fig. 43). It is important that we should notice that the jockey pulley should be applied on the slack side of the belt.

Tension Differences

The distinction between the slack and the tight sides of a belt is that the tight side is the driving side, viz., the side of the belt which runs off the follower and on to the driver. In practice, both sides of the belt must be more or less tight, but when the belt is transmitting power, the tension in the tight side must be greater than the tension in the slack side, in fact it is the difference between these tensions which drives the follower round.

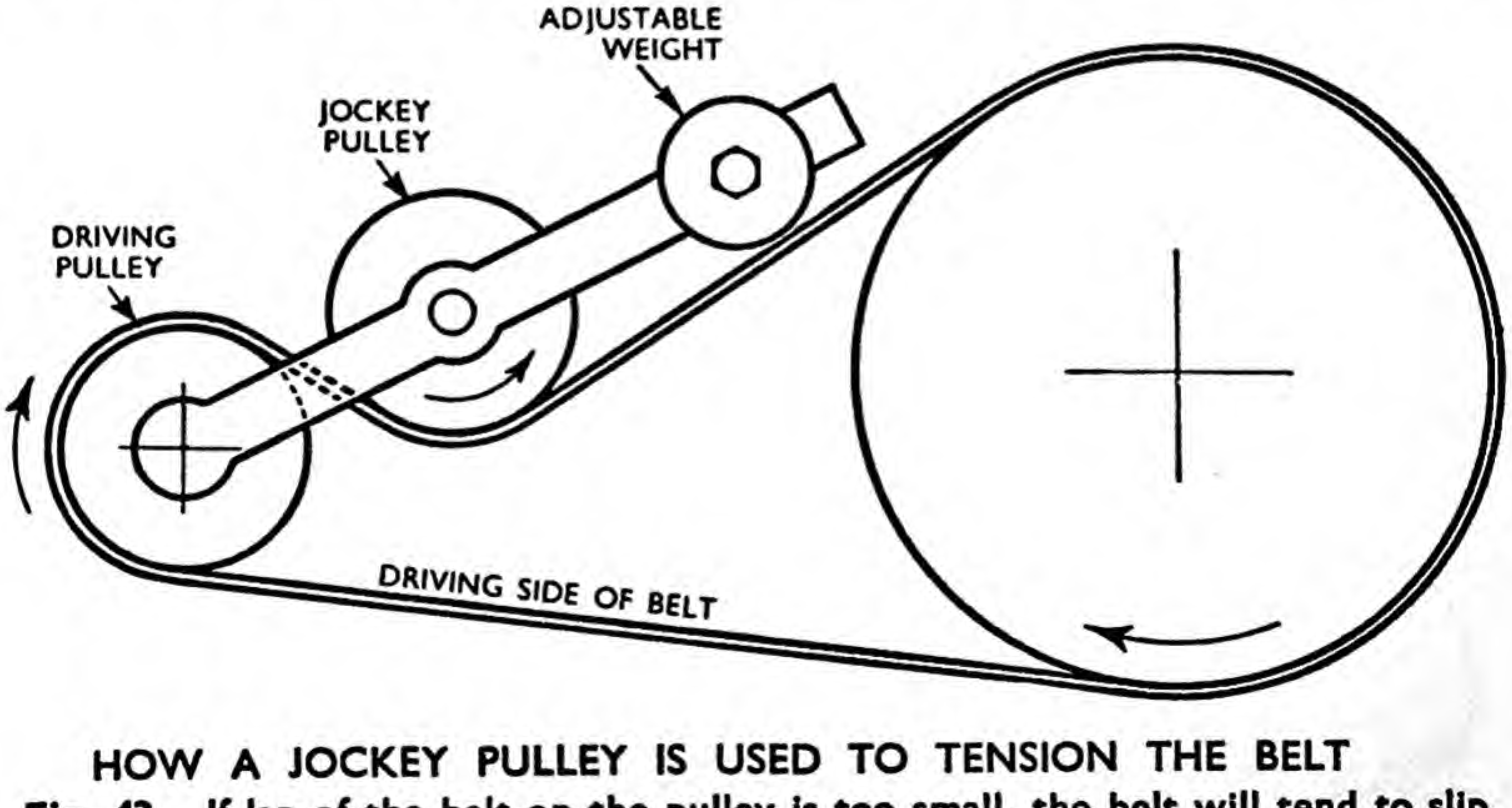
In Fig. 44 the tensions in the two sides of the belt are caused by the weights W and w. It is obvious that if the tension on one side caused by W is greater than the tension on the other side caused by w, the pulley will be rotated by the difference in tension $(W - w)$. In other words, the heavier weight W will fall and w will rise. The effect will be the same as if we applied an effort equal to $(W - w)$ to the rim of the pulley, and the work done will be obtained by multiplying the effort by the distance moved. As the belt is moving continuously round, it is more convenient to consider the distance moved in a given time, viz., the speed of the belt. Then :—

$$\text{Work done per min.} = (W - w) \times \text{Speed of belt in ft. per min.},$$

or

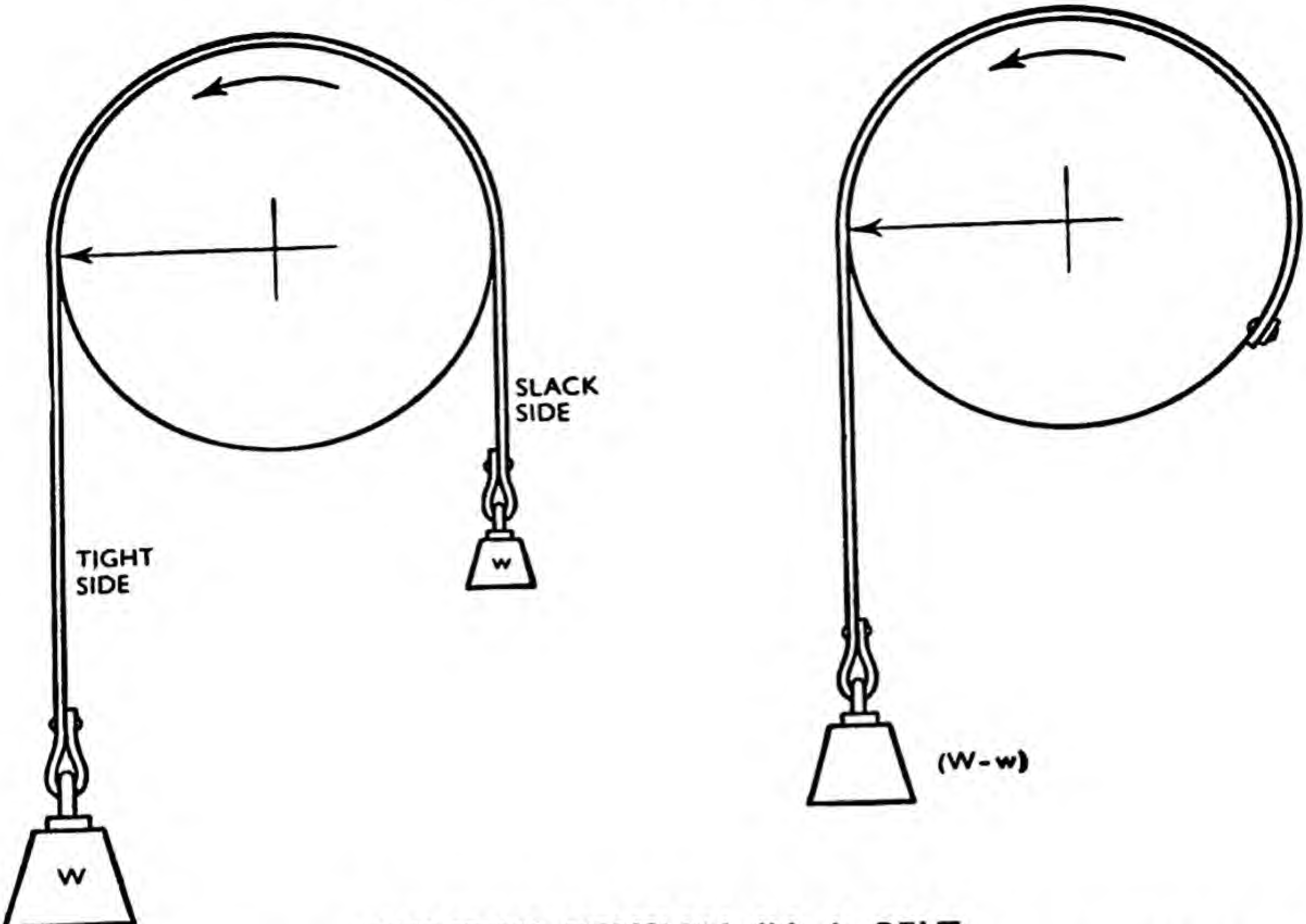
$$\text{Horse-power} = \frac{(W - w) \times \text{Speed of belt in ft. per min.}}{33{,}000}.$$

We can calculate the speed of the belt from the diameter of the pulley and the speed of the shaft in r.p.m. The difference in tension



HOW A JOCKEY PULLEY IS USED TO TENSION THE BELT

Fig. 43. If lap of the belt on the pulley is too small, the belt will tend to slip. This can be prevented by using a jockey pulley which rides on the back of belt. The jockey pulley also prevents slip by maintaining the tension in the slack side.



EFFECTIVE TENSION IN A BELT

Fig. 44. Rotation of the pulley is caused by the difference in tensions of tight and slack sides of the belt. The turning moment which the belt transmits to the pulley is the difference between the moments exerted by these tensions, viz., $WR - wR$. This is equal to $(W—w)R$, and $(W—w)$ is called the effective tension.

between the tight and slack sides of the belt is more difficult to determine, and it will vary automatically with the power being transmitted. If this difference in tension between the two sides becomes excessive, the belt will commence to slip on the pulley. It is useful to know how far we can go without the belt slipping, however, for this determines the maximum power which the belt can transmit. The maximum difference in tension depends upon a number of factors :—

(a) The angle of lap of the belt on the pulley.
(b) The coefficient of friction between the belt and the rim of the pulley.
(c) The maximum permissible tension in the belt.

With a leather belt lapping about half-way round the pulley, the tension on the tight side cannot be much more than three times the tension on the slack side without slipping occurring, viz. :—

$$w = \frac{1}{3} W$$

$$\therefore W - w = \frac{2}{3} W, \text{ approximately.}$$

Belt Speed Important

This means that the maximum effort $(W - w)$ that we can apply at the rim of the pulley is about two-thirds of the maximum tension in the tight side of the belt. A rough rule for the maximum tension is 50 lb. per in. of width of the belt. We can now apply this to a simple problem, as follows.

How much power can a 2-in.

belt transmit to a 15-in. diameter pulley running at 300 r.p.m.?

Speed of the belt

$$= \pi \times \frac{15}{12} \times 300 = 1{,}180 \text{ ft. per min.}$$

Maximum tension in the belt

$$= 2 \times 50 = 100 \text{ lb.}$$

Effort at pulley rim

$$= \frac{2}{3} \times 100 = 66{\cdot}7 \text{ lb.}$$

$$\text{H.p.} = \frac{\text{Effort} \times \text{Speed}}{33{,}000} = \frac{66{\cdot}7 \times 1{,}180}{33{,}000}$$

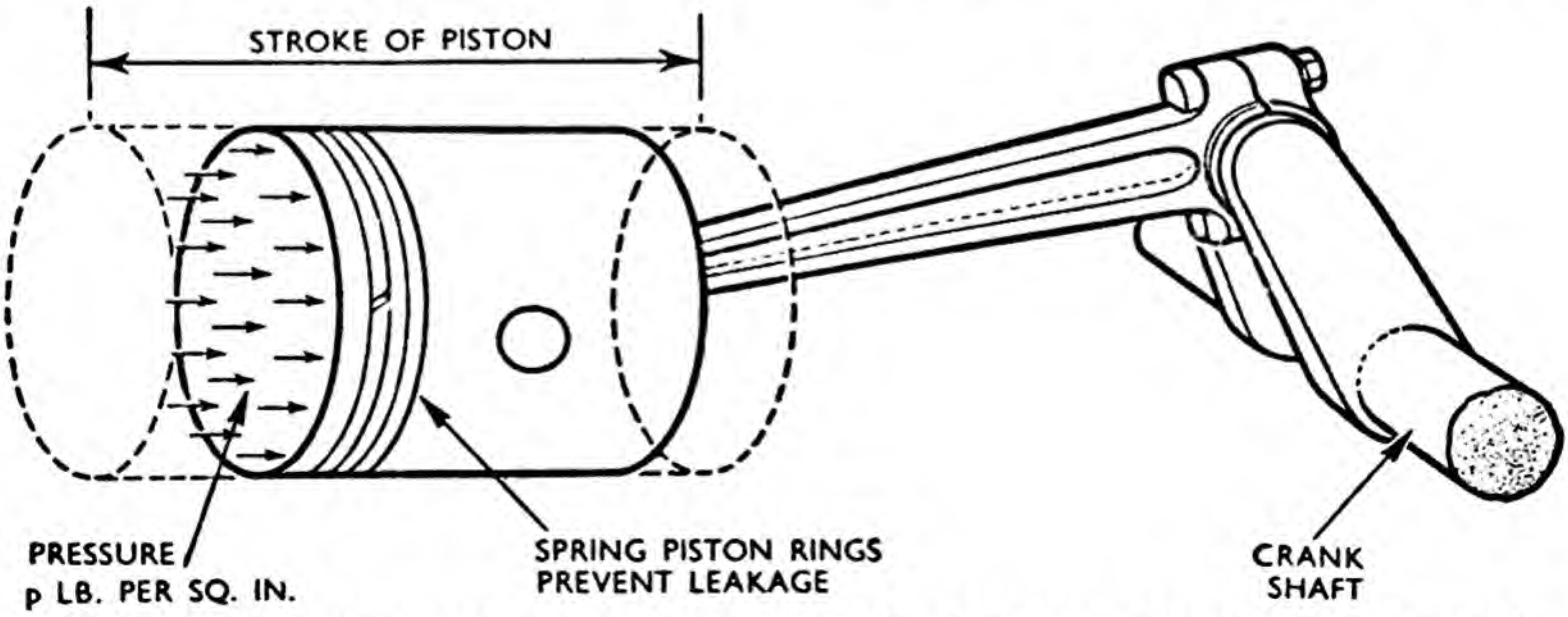
$$= 2{\cdot}4 \text{ approximately.}$$

It will be clear from this example that the same belt and same pulley could transmit twice this power, viz., 4·8 h.p. at 600 r.p.m. Therefore, in practice, small belts running at high speeds are preferable to wider belts running at low speeds. The speed should not be too high, however, as at belt speeds higher than about 3,000 ft. per min. the grip of the belt on the pulley becomes materially reduced owing to the effect of centrifugal force on the belt as it goes round the pulley. This is particularly pronounced with small-diameter pulleys. We have seen, too, that with small pulleys, the angle of lap is frequently reduced unless a jockey pulley is used, and the effort at the pulley rim will be found to be less than half, instead of two-thirds, of the maximum tension if the belt only laps one-quarter of the pulley rim.

Prime Movers

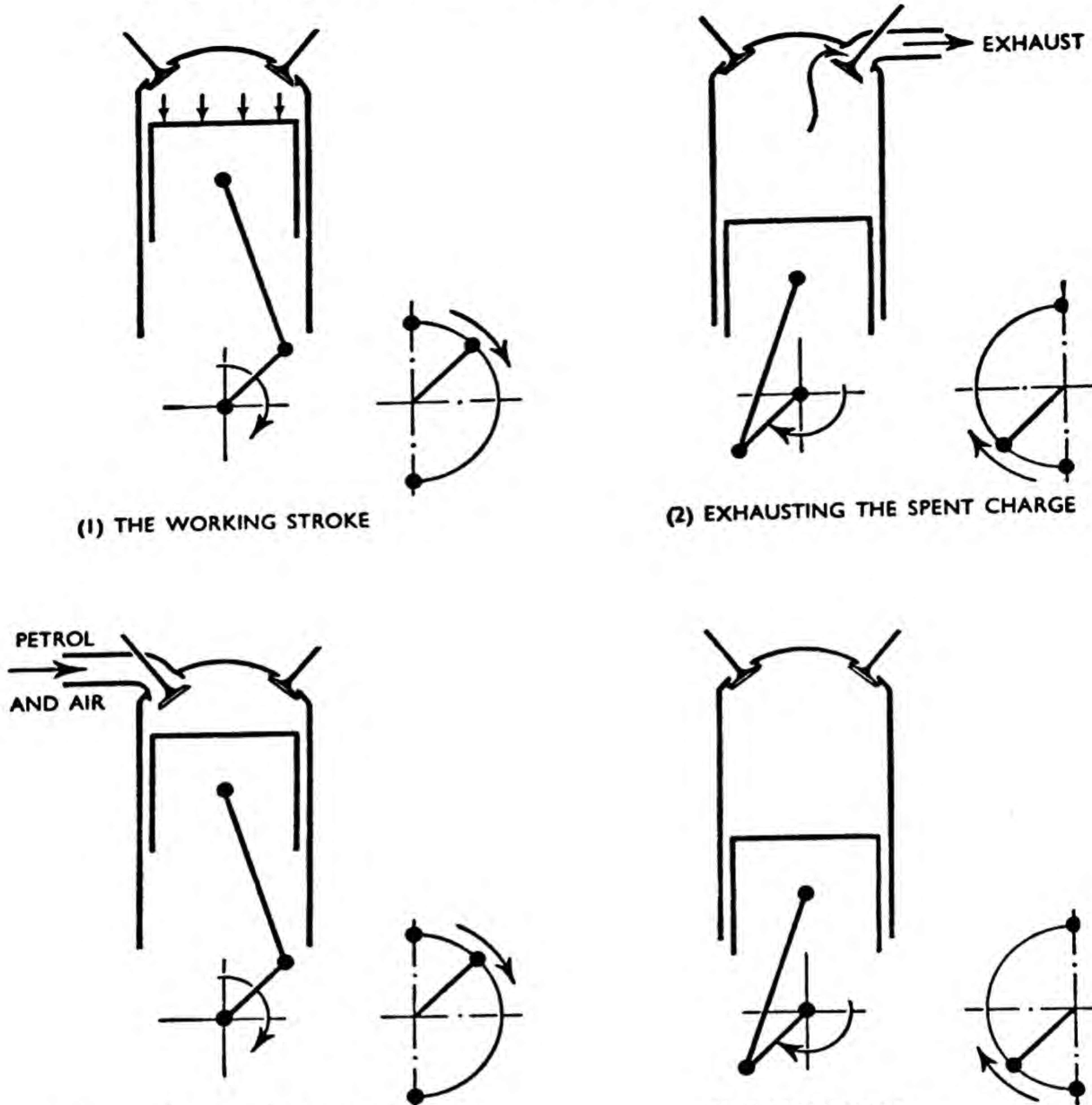
Belts and pulleys and geared shafts serve to transmit the energy that is constantly fed into the driver. This energy comes originally from flowing water or air, or from the combustion of fuel, but before the driver of a machine can make use of it, the energy must be converted into a suitable form by some type of prime mover, a turbine or a reciprocating steam- or internal-combustion engine. The heart of the reciprocating engine is its piston and cylinder (Fig. 45), and this applies equally to the powerful locomotive, the giant Diesel engine or the busy petrol engine which powers alike the smallest motor car or the fastest aircraft.

For work to be done, the effort



HOW PRESSURE IS MADE TO DO WORK IN THE ENGINE CYLINDER

Fig. 45. In the reciprocating engine pressure of steam or gas acting on the crown of the piston urges it forward to do work as the piston slides in the cylinder. Work done per stroke is the product of the mean force on the piston and the distance through which the piston moves.



(1) THE WORKING STROKE

(2) EXHAUSTING THE SPENT CHARGE

(3) INDUCTION DRAWING IN A NEW CHARGE

(4) COMPRESSION

FOUR STROKES OF THE FOUR-STROKE CYCLE

Fig. 46. Most petrol engines operate on a four-stroke cycle in which there is only one working stroke every two revolutions. Remaining three strokes are utilized to exhaust spent charge and to draw in and compress new charge ready for next working stroke. Net output of engine is the useful work done.

of the steam or combustion gases must move through a distance, so that we have :—

Work done
= Effort × Distance moved.

The piston provides a surface on which the pressure can act, and as the piston slides in the cylinder, work is done. During this sliding movement, spring rings called piston rings press outwards against the wall of the cylinder and prevent leakage whilst permitting the piston to slide. The greater the surface presented by the piston to the pressure, the greater will be the effort, and we can write :—

Effort in lb. = Pressure in lb. per sq. in. × Area of piston in sq. in.

The area of the piston will obviously be equal to $\frac{\pi}{4}$ multiplied by the square of the diameter of the cylinder, or, as it is more usually called, the bore of the cylinder.

Determining Horse-power

The distance through which the piston slides in the cylinder is called the stroke, so that the work done per stroke in ft.-lb. is equal to the effort multiplied by the stroke of the engine in ft. If we know how many working strokes there are per min., then we can easily determine the horse-power of the engine by the simple relationship as follows :—

$$\text{h.p.} = \frac{p \times L \times A \times N}{33{,}000}$$

where p = average pressure in lb. per sq. in.

L = stroke in ft.

A = area of piston in sq. in.

$= \frac{\pi}{4} \times$ (bore of cylinder in in.)^2.

N = number of working strokes per min.

You will notice that N is the number of working strokes per minute. This is because in many engines, useful work is done only on every alternate stroke, or on one in every four strokes of the piston. For instance, in the ordinary petrol engine, which is called a four-stroke engine, each working stroke is followed by :—

(b) an exhaust stroke as the piston moves back ;

(c) an induction stroke during which a new charge of petrol vapour and air is drawn in ;

(d) a compression stroke during which the charge is compressed as the piston once more moves back.

At the end of the compression stroke, the charge is exploded by the sparking plug, and it is this pressure, resulting from the explosion of the charge, which acts on the piston during the working stroke. It is clear from Fig. 46 that these four strokes take place during two revolutions of the crankshaft, and, therefore, there is one working stroke every two revolutions. So that if a four-stroke petrol engine is running at 3,000 r.p.m., N, the number of working strokes per minute, is 1,500.

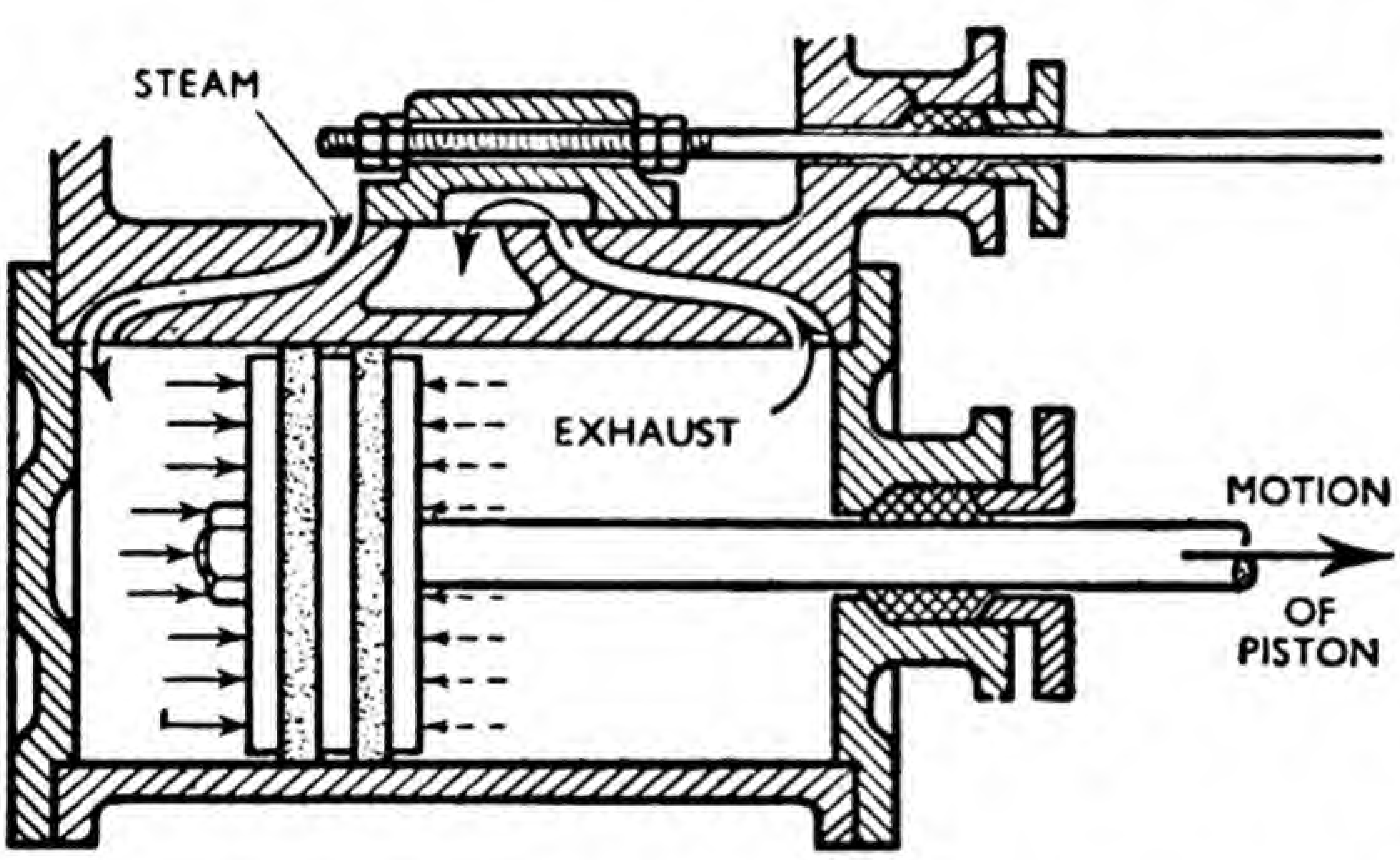


Fig. 47. Steam engines are usually found to be double-acting, i.e., steam acts alternately on each side of piston. Valve admits steam at left, and permits exhaust at right. Effective pressure driving piston forward is difference between pressure of steam on one side and exhaust on other. Valve moves left and cuts off supply; pressure falls as steam expands.

In the two-stroke engine, there is one working stroke every revo-

lution. Instead of using the two strokes (b) and (c) for exhaust of the old charge and induction of the new charge, it is arranged that the new charge enters the cylinder under pressure at the end of the working stroke, and so displaces the old charge through the open exhaust port. In this case, N is the same as the number of revolutions per minute.

The steam engine is different again. Fig. 47 shows the usual arrangement in which the steam acts alternately on each side of the piston. The engine is then said to be double-acting and there are two working strokes per revolution. In this case N equals twice the number of revolutions per minute.

So far we have only considered what goes on in one cylinder. Most engines have more than one cylinder, however, and the total power is then obtained by adding together the power developed in the various cylinders. Frequently the cylinders are all alike, and the power then increases in proportion to the number of cylinders.

Let us take a few examples. The motor car usually has four or six cylinders, sometimes eight or twelve. The bore may be anything from 2½ in. to 3½ in., and these engines almost invariably work on the four-stroke cycle. Aero engines have much bigger cylinders, about 6-in. bore, and there may be 12, 14 or even 24 of them, so that as much as 2,000 h.p. or more can be obtained from a single engine.

The steam locomotive gets its power from a small number of large cylinders. Most locomotives have only two cylinders, but the more powerful ones have three or four. The bore may be as much as 20 in. and the stroke over 2 ft. The engines are always double-acting, and a large modern express locomotive develops about 1,500 h.p.

For ships, the oil engine and the steam turbine are tending to displace the reciprocating steam engines which at one time reigned unchallenged. Many of the oil engines in large motor ships have cylinders over 2-ft. bore and about 4-ft. stroke, so that as much as 1,000 h.p. can be developed in each cylinder. They may have up to eight or even twelve cylinders. Many of the large engines operate on the two-stroke cycle, and some of them are double-acting, so that each cylinder has two working strokes per revolution like a steam engine. Some of the old steam engines had extremely large cylinders, 6-ft. or 7-ft. bore, although these were only suitable for low pressures, and the usual arrangement was to have compound engines, that is engines in which the steam first did work in a small high-pressure cylinder, before being passed into one or more larger low-pressure cylinders.

Mean Effective Pressure

Looking again at Fig. 47, which shows the double-acting steam engine in which the steam acts alternately on each side of the piston, it will be seen that while the inlet steam is doing work on one side of the piston, the spent steam is being exhausted from the other side. In other words, there is pressure on both sides of the piston. This is rather like the case of the belt on the pulley, where there was tension on both sides, and the effort doing work

was the difference between the tensions on the tight and slack sides.

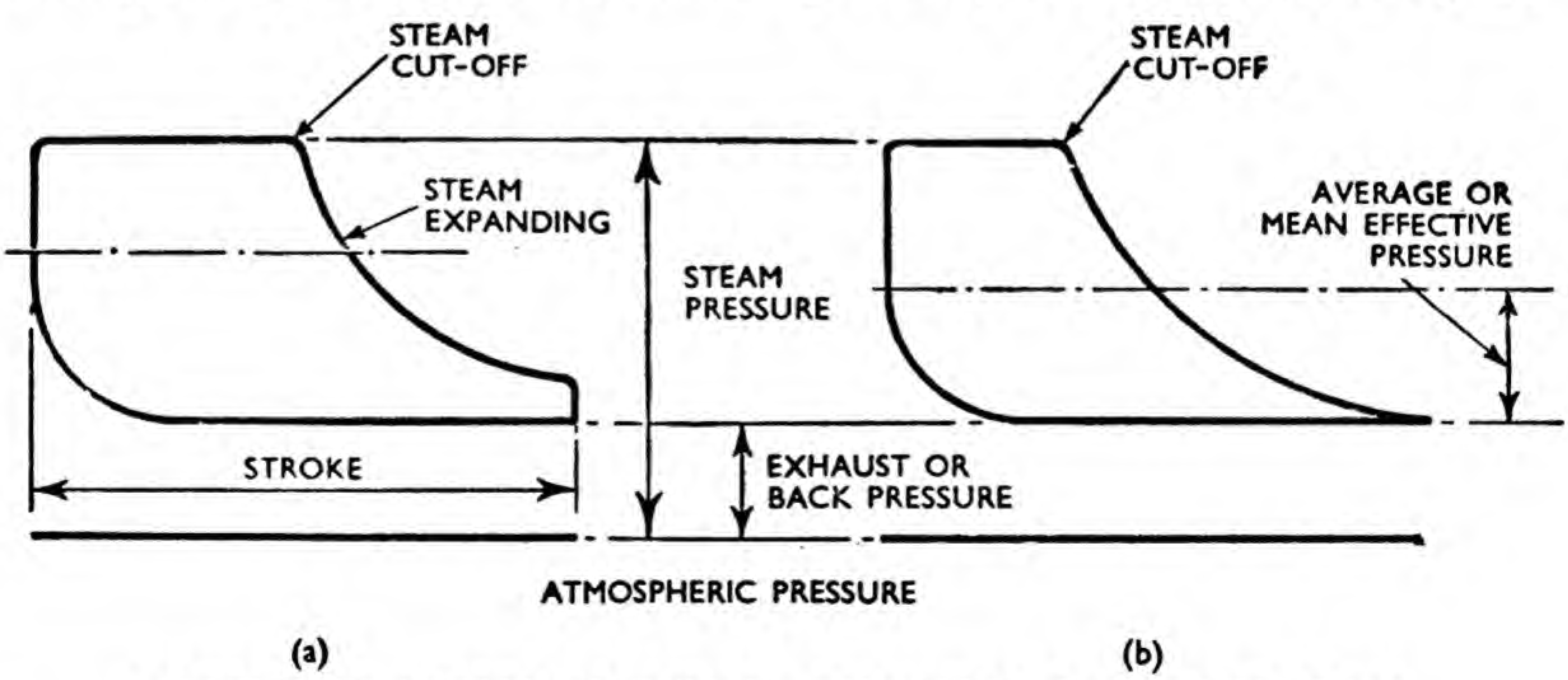
It is just the same with the piston; the effective pressure is the difference between the pressures on the inlet and exhaust sides. In simple engines, especially if the steam is exhausted to a condenser where the steam is condensed to water, so producing a partial vacuum, the pressure on the exhaust side is quite low, but in the high-pressure cylinder of a compound engine where the exhaust steam has to do more work in a low-pressure cylinder, the pressure on the exhaust side of the piston is important when calculating the effective pressure, viz., the pressure difference.

Finding Effective Pressure

For example, the steam may be admitted to the high-pressure cylinder at 150 lb. per sq. in. and then passed to the low-pressure cylinder at 50 lb. per sq. in. to do some more work there. So that all the time the high-pressure steam is driving the piston forward, there is an opposing pressure of 50 lb. per sq. in. on the other side of the piston. The effective pressure, therefore, is only 150 − 50, viz., 100 lb. per sq. in.

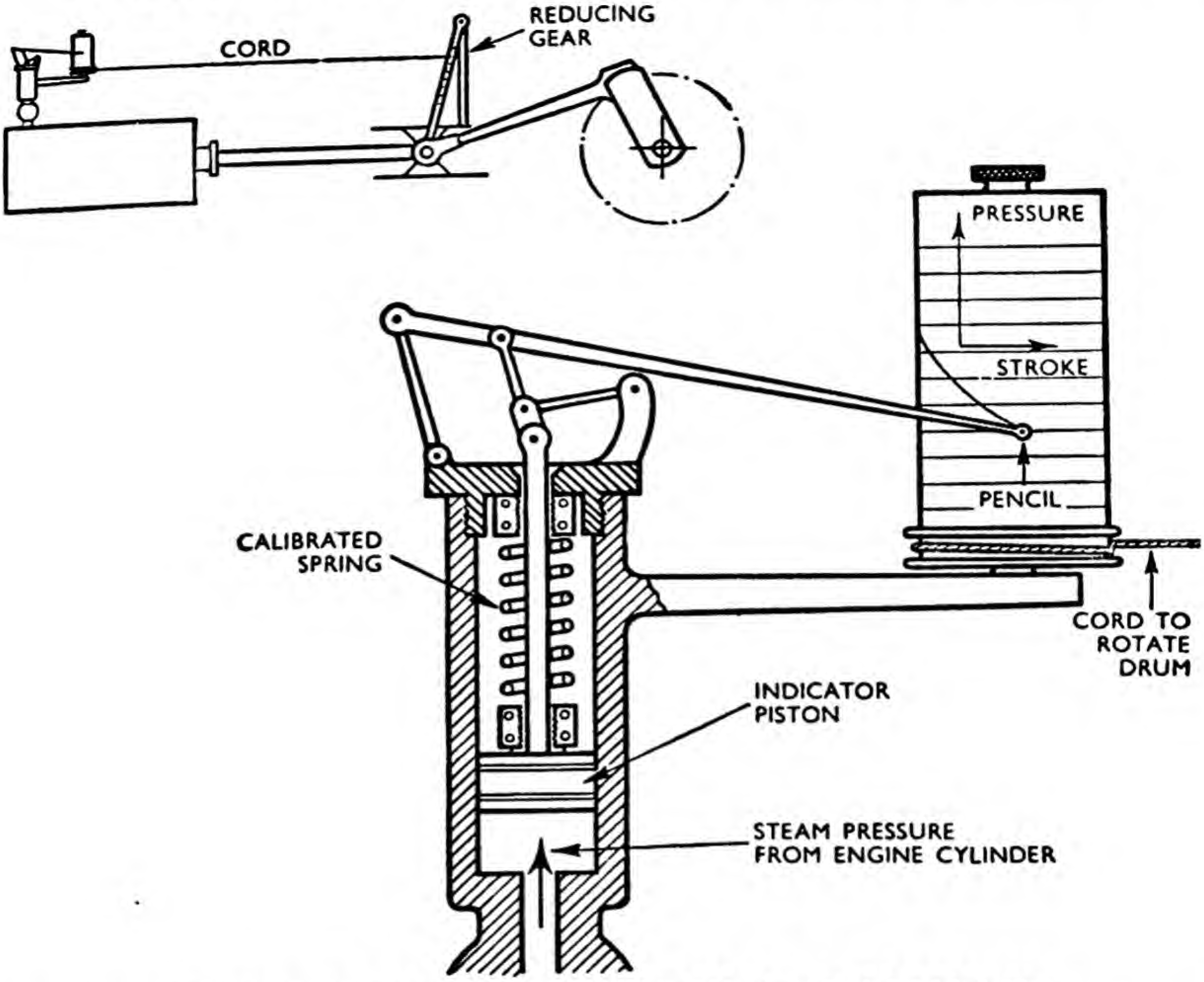
It would be very wasteful to use the full boiler pressure of the steam to drive the piston from one end of the cylinder to the other, because once a certain amount of steam has been admitted to the cylinder, it will continue to drive the piston forward by its own power of expansion without any more steam being admitted. The pressure will fall as the steam expands, of course, but as long as the pressure on the inlet side is greater than the pressure on the exhaust side, the piston will continue to move forward.

For example, consider the compound engine mentioned above. Steam is admitted for half the stroke, and then the valve is closed. The steam will continue to expand until, when the stroke is completed, it has twice its original volume, and the pressure, therefore, will have fallen to one half its original pressure, viz., to 75 lb. per sq. in.



GRAPH OF STEAM PRESSURE—THE INDICATOR DIAGRAM

Fig. 48. Height of the graph shows the steam pressure at any point of the stroke. Cut-off is shown by the point at which pressure starts to fall, and the earlier the cut-off the lower the final pressure. Effective pressure is measured as the pressure of steam in excess of the exhaust or back pressure.



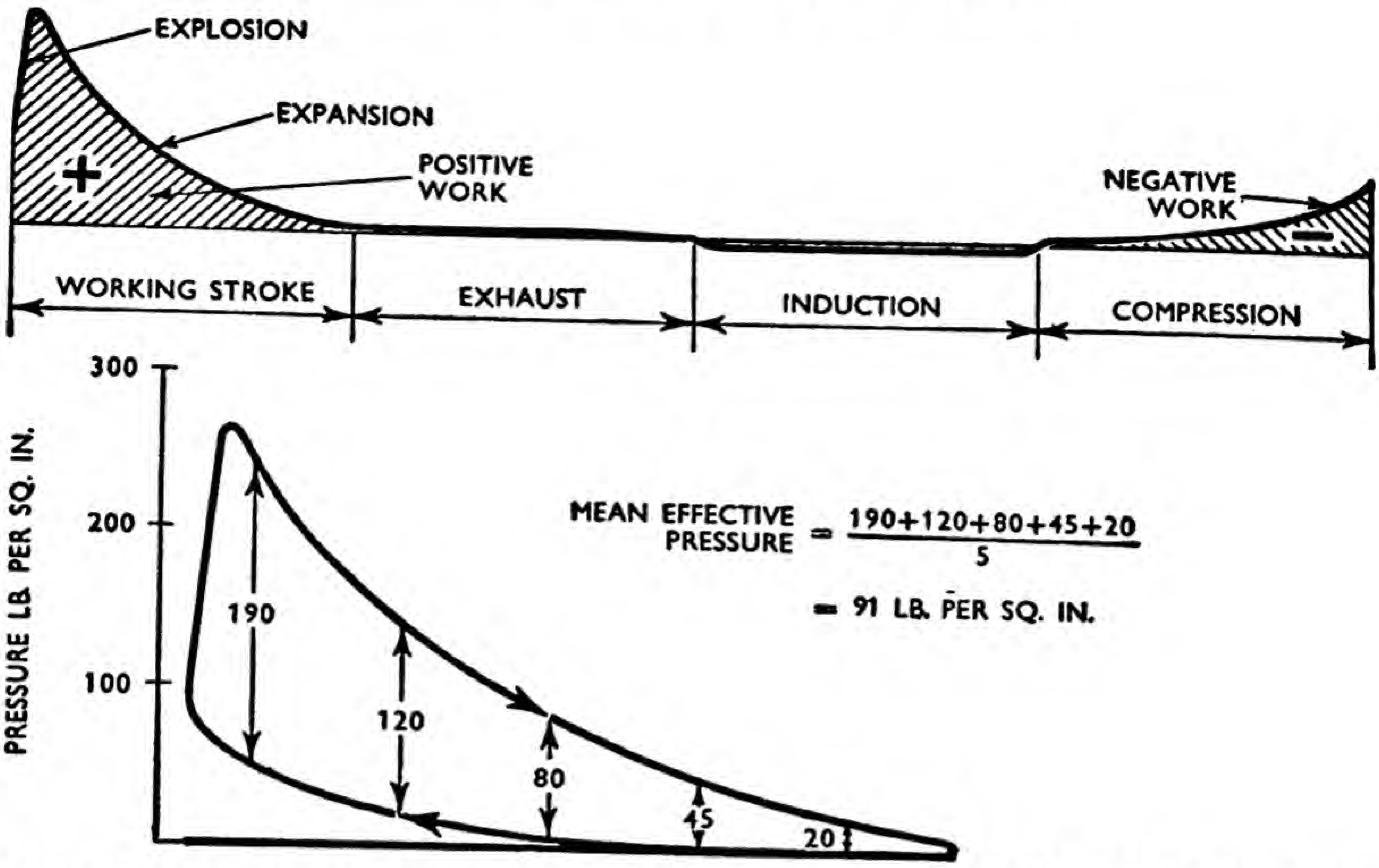
ENGINE INDICATOR AND HOW IT IS ARRANGED

Fig. 49. The engine indicator draws the diagram automatically. It is screwed into one end of engine cylinder so that steam moves indicator piston up and down as pressure rises and falls. Movement of indicator piston operates a light arm carrying a pencil point which draws graph of pressure on card folded round drum. Rotation of drum is proportional to stroke of engine.

But this is still greater than the pressure of 50 lb. per sq. in. on the exhaust side, and the expanding steam will still exert an effective pressure and will continue to do work. The effective pressure will vary from 100 lb. per sq. in. during the first half of the stroke to 25 lb. per sq. in. at the end of the stroke. This can be represented by the diagram of Fig. 48(a).

In a similar way we can show what happens if the steam supply is cut off earlier, say, for example, at one-third of the stroke. The pressure will now have fallen to 75 lb. per sq. in. at two-thirds of the stroke, and to 50 lb. per sq. in. at the completion of the stroke. This is shown at Fig. 48(b). At the end of the stroke, the pressures on both sides of the piston are just equal, and, therefore, there would be no advantage in making the cut-off earlier than one-third of the stroke, because there would then be no effective pressure during the final portion of the stroke.

This kind of diagram which shows how the pressure varies throughout the stroke is called an indicator diagram, and the diagram for an actual engine can be drawn automatically by an instrument called an indicator, which is shown in Fig. 49. This



FOUR STROKES OF THE PETROL ENGINE COMBINED IN ONE DIAGRAM
Fig. 50. In the petrol engine back pressure is due mainly to compression of new charge. Work done during compression must be subtracted from work done during working stroke to get net output. Ordinates, therefore, measured between compression and expansion curves and averaged to get mean effective pressure.

contains a light spring-loaded piston which is acted upon by the same pressure as the engine piston. The indicator piston moves a pencil point up and down as the pressure in the cylinder rises and falls. The spring is carefully calibrated so that 1 in. vertical rise of the pencil corresponds to a definite increase in pressure, say, 50 or 100 lb. per sq. in., and this number, 50 or 100, the spring number, is stamped on the spring. The pressure in the cylinder at any point, therefore, can be found by measuring the height of the indicator diagram and multiplying by the spring number.

When the pressure varies in this way, what pressure must we use in the calculation of the horse-power of the engine? Obviously, we must use the average pressure, or as it is called, the mean effective pressure. This takes account both of the variation of the pressure, and of the presence of the opposing pressure of the exhaust.

Varying Effective Pressure

In the internal-combustion engine, the petrol or the oil engine, the pressure obviously varies much more than in the steam engine. Following the combustion or explosion of the charge at the commencement of the working stroke, a very high pressure, up to 1,000 lb. per sq. in. in some cases, is reached, and this falls rapidly as the piston is driven forward and the gases expand. During the exhaust stroke, the back pressure is so small that we can neglect it, but we do have to take account of the opposing pressure due to the compression of the charge before it is ignited by the sparking plug, and if you have turned a motor-car or motor-cycle engine over compression, you

will know that this pressure is too great to be ignored.

This pressure does not act on one side of the piston while the explosion gases are acting on the other side, however, as in the steam engine. These pressures both act on the same side of the piston, but on different strokes. We get over this difficulty by saying that the expanding explosion gases do positive work during the working stroke, whereas the compression of the charge absorbs work, viz., does negative work. The effective work is the sum of these positive and negative quantities, and Fig. 50 shows how these are combined on one diagram, and how the mean effective pressure is obtained in much the same way as for a steam engine.

Indicated Horse-power

The horse-power calculated in this way from the mean effective pressure (m.e.p.) of the indicator diagram is called the indicated horse-power (i.h.p.). It is the power developed by the steam or explosion gases in the engine cylinder. This power has to be transmitted from the cylinder and piston to the crankshaft, through the connecting rod and crank pin, and the friction in the bearings of these parts will absorb some of the power.

As for other machines, we can make an allowance for the power lost in friction in the bearings and in driving the valve gear, etc., and the actual power obtained from the engine after making these allowances is called the brake horse-power (b.h.p.). Thus the i.h.p. is the power input to the piston, and the b.h.p. is the useful power output from the crankshaft. The ratio of these two quantities is called the mechanical efficiency of the engine, or we can write :—

$$\text{b.h.p.} = \text{Mechanical efficiency} \times \text{i.h.p.}$$

In air compressors and pumps, the process is reversed. Power is put into the crankshaft, and pressures are produced in the cylinder by the action of the piston on the air or water. We can obtain an indicator diagram showing how the pressure in the cylinder varies, in the same way as for an engine, and from the mean effective pressure of the indicator diagram, the indicated horse-power can be calculated. The only difference is that when we are calculating how much horse-power will be required to drive the compressor or pump, we must remember that the power lost in friction will have to be added to the i.h.p. This is equivalent to dividing by the efficiency instead of multiplying. So that :—

$$\text{h.p. required (compressor or pump)} = \frac{\text{i.h.p.}}{\text{Mechanical efficiency}}.$$

Function of the Flywheel

This calculated horse-power is, of course, the average horse-power. But just as the actual pressure on the piston is at times greater or less than the mean pressure, so the actual horse-power developed is greater or less than the average horse-power output. At such times the engine will tend to gain or lose speed accordingly. This is the main function of the flywheel : to store energy, for example during the working stroke of a 4-stroke internal combustion engine, and to give it out again during the three succeeding strokes when no power is actually being developed.

CHAPTER 11

MECHANICS OF FLUIDS

GENERAL PROPERTIES OF FLUIDS. DISTINCTION BETWEEN PRESSURE AND FORCE. PRESSURE DUE TO HEAD. PRESSURE ON SUBMERGED SURFACES. CENTRE OF PRESSURE. DENSITY AND SPECIFIC GRAVITY. BAROMETERS. VARIATION OF ATMOSPHERIC PRESSURE. FLOATING BODIES. METACENTRIC HEIGHT. ARCHIMEDES' PRINCIPLE. VELOCITY OF EXIT UNDER A GIVEN HEAD. FORCE OF A JET. ENERGY OF WATER. BERNOULLI'S THEOREM. FLOW IN PIPES. FRICTION LOSSES IN OPEN CHANNELS.

NO rigid definition is required here as we all have a general idea of what is meant by a fluid, that is, something which can be poured or transferred from one vessel to another.

In general, fluids can be divided into liquids and gases. However, a few special cases arise; for example, pitch is a solid in the ordinary sense at low temperatures, but a thick, slow-moving liquid at higher temperatures. Such a liquid flows so slowly that it would be valueless in a mason's spirit-level, where, as the name implies, a spirit or alcohol is used, because it quickly takes up a new position.

Materials such as pitch have another special property. Even when cold, a stick of pitch held out horizontally will bend down slowly owing to its weight. If, on the other hand, the free end is given a blow, the stick will break instead of bending. The same material thus acts as a slowly moving liquid in the first case, and as a brittle solid in the second.

Gases completely fill the vessels containing them and do not present an upper flat surface as in the case of liquids. From the practical point of view, liquids and gases have much in common, as will be seen below. There is, however, an important difference between them, namely, the density, which may be defined as the mass or quantity of matter contained in a cubic foot.

The density of a liquid is usually far greater than that of a gas; for example, water weighs 62·4 lb. per cu. ft., while air under atmospheric conditions weighs about 0·078 lb. per cu. ft. Gases are highly compressible, which means that the weight per cubic foot, or density, is very sensitive to changes of pressure and temperature. Therefore, it will be seen that the density of a gas cannot be accurately stated unless both the pressure and temperature are known.

Pressure and Force

Before enlarging on these properties we must clearly understand the difference between pressure and force. Let us imagine two pipes, the one having a cross-sectional area of 1 sq. in. and the other of $\frac{1}{4}$ sq. in., leading from the tank under the roof of a house to the kitchen sink, and each with a cork held loosely in its outlet to prevent the water flowing. The forces necessary to hold these corks in place will not be

equal, but will be in the ratio of the areas of the pipes, that is 4 to 1, say, 12 lb. and 3 lb.

When the force is divided by the corresponding area of the circular cross-section of the cork, we obtain the pressure, $\frac{12}{1} = 12$ lb. per sq. in. for the larger pipe, and $3 \div \frac{1}{4} = 3 \times 4 =$ 12 lb. per sq. in. for the smaller pipe. These two values are equal. Therefore, the pressure may be regarded as the force acting on a square inch of surface or area.

Pressures may be expressed in lb. per sq. in. or per sq. ft., and as there are 12×12 or 144 sq. in. in 1 sq. ft., it follows that the pressure in lb. per sq. ft. $= 144 \times$ pressure in lb. per sq. in.

Measurement of Head

Engineers often measure water pressures in terms of head. If we think of a 1-ft. cube of ice as weighing the same as a 1-ft. cube of water, that is, 62·4 lb. (although actually, of course, it weighs less), then, if the block of ice rests on a table, the pressure on the under face is that due to 62·4 lb. acting on 1 sq. ft. of surface, or 62·4 lb. per sq. ft.

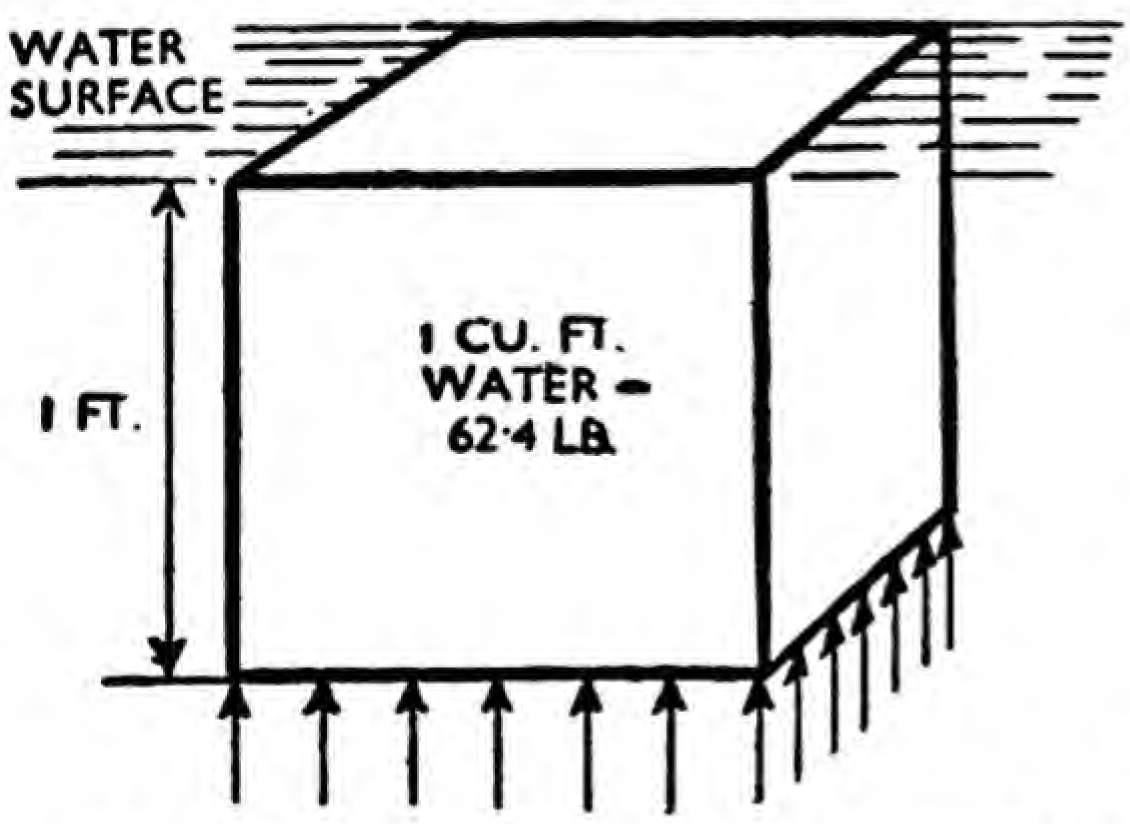


Fig. 1. If we isolate an imaginary 1-ft. cube of water from the water surrounding it, the top face being level with the water surface, the bottom face is subjected to a pressure of 62·4 lb. per sq. ft. This is just sufficient to balance the weight of the imaginary cube, so that a depth or head of 1 ft. is equivalent to a pressure of 62·4 lb. per sq. ft.

If the block is now placed in water, we will see that it floats with its top face just flush with the surface of the surrounding water (Fig. 1), because its weight will then exactly balance that of the water displaced. The pressure on the under face must be the same as that when the cube was resting on the table, viz., 62·4 lb. per sq. ft., and this is produced by the column or head of water measured from the water surface to the bottom of the cube, namely, 1 ft.

The pressure p lb. per sq. ft. under a rectangular block of depth h ft., either greater or less than 1 ft., can be derived as follows :—

1 ft. depth or head is equivalent to 62·4 lb. per sq. ft.

h ft. depth or head is equivalent to $62{\cdot}4 \times h$ lb. per sq. ft.

$= p$ lb. per sq. ft.,

$$\text{so that } \frac{p}{62{\cdot}4} = h.$$

To include fluids other than water, this formula is commonly expressed as $\frac{p}{w} = h$, where w is the density of the particular fluid, or, as already defined, the weight per cubic foot.

We have now seen that the pressure due to a fluid is dependent on the depth. One curious result of this is that the pressure at the level AA in the three vessels (a), (b) and (c) of Fig. 2 is the same, and is independent of the shapes of the vessels. This pressure may be measured either by means of a pressure gauge or a stand glass, as indicated. Thus the pressure on the base of (a) is greater than the weight of fluid in the vessel, and that on the base of (b) is less than

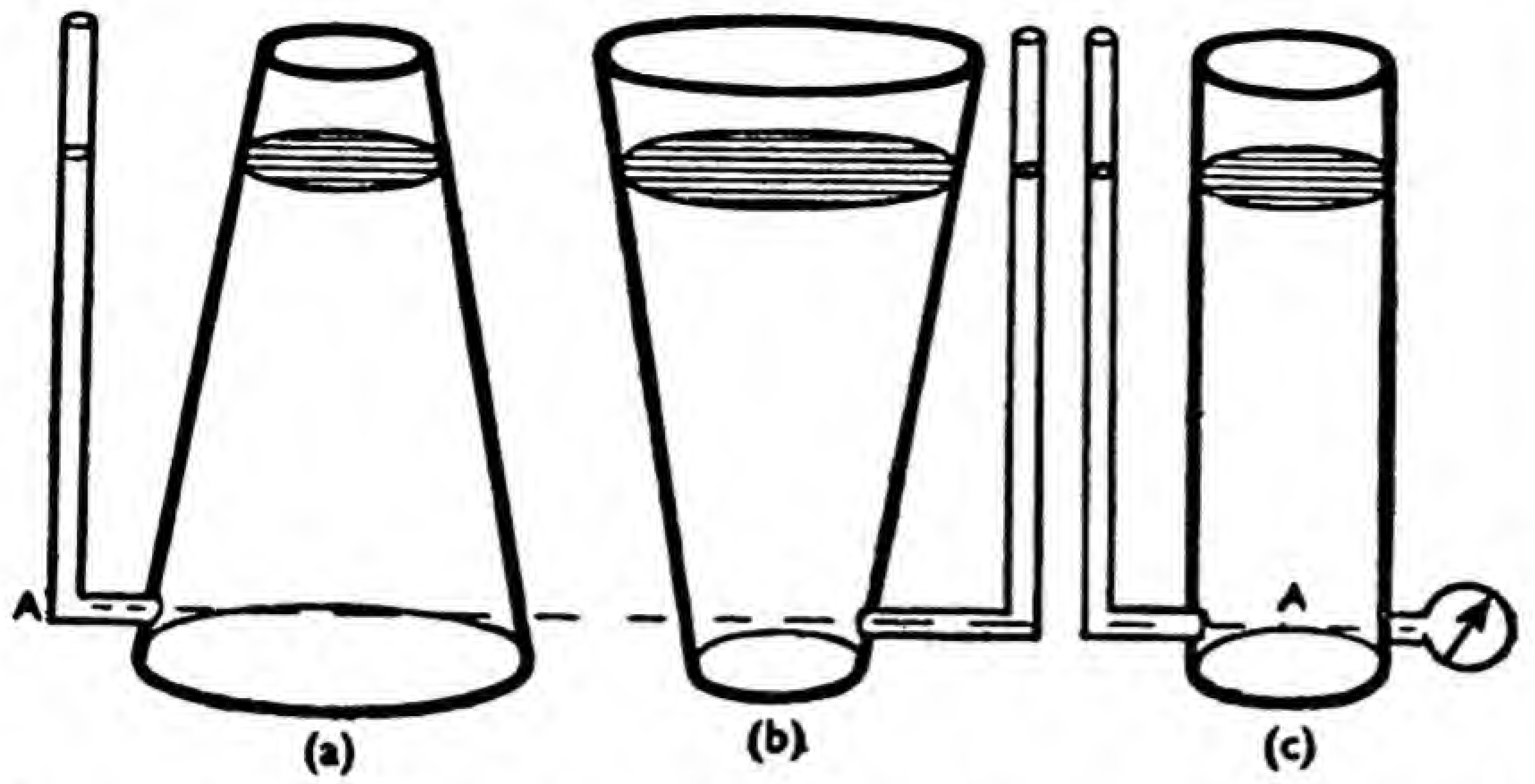


Fig. 2. Pressure at the level *AA* is the same in each vessel as the depth of water is the same in each. It depends only on the depth and is independent of the shape of the vessel.

the weight of fluid in the vessel. This apparently anomalous result is explained by the fact that the fluid also presses on the sides of the vessel in each case, the reaction being downward in case (a) and upward in (b). The vertical component of these reactions in each case accounts for the difference.

Another result is that the force due to the water acting on a dam at the end of a reservoir is quite independent of the surface area of the reservoir. The force is dependent only on the size of the dam and the depth of water behind it (Fig. 3).

Direction of Pressure

In the discussion above, it has been assumed that the pressure on the under face of the block acts perpendicularly to the face.

Referring to Fig. 4, let us consider what would happen if the pressure acted as at *B* instead of perpendicularly to the face as at *A*. Clearly, *B* has a component in the direction *C* which would cause flow or motion of the fluid. No pressure corresponding to *C* can exist in a fluid at rest, and hence the pressure at *B* must be at right angles to the face on which it acts. A similar effect is noticed if we try to walk on ice. While the body is at rest no difficulty arises, but directly we try to produce a horizontal force to cause motion, slipping takes place.

We have seen that the pressure depends solely on the depth or head of fluid. Supposing *A* (Fig. 5(a)) represents a small particle of dust in suspension in the liquid, and that it is of rectangular form as seen magnified in Fig. 5(b). As the particle is at rest, it follows that $p_1 = p_2$. Again, as the vertical dimension is very small,

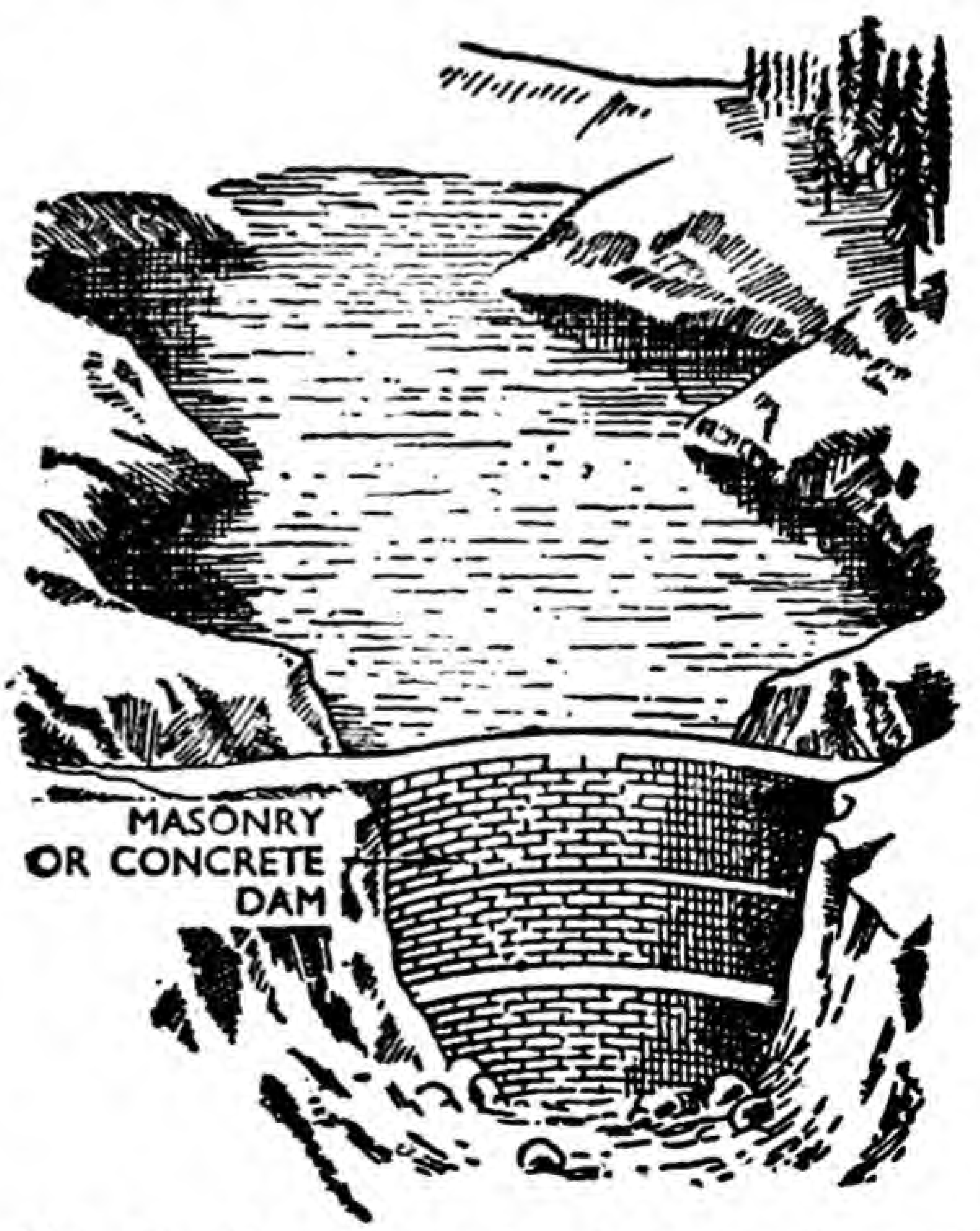


Fig. 3. Force on the dam at the end of a valley due to the water in the reservoir is quite independent of the surface area of the reservoir ; it depends only on the shape or size of the face of the dam and the depth of the water.

$p_3 = p_4 = 62{\cdot}4 \times h = p_1 = p_2$. This argument is true whatever the actual shape of the particle, and it follows that, at a point, the pressure is the same in all directions.

Head may be considered from still another point of view. To transfer 1 lb. of water from the small tank to the large tank of Fig. 6, three methods are available : a pump may be used to force the water in at (1) against the head *h*, or it may be arranged to lift it through the external pipe (2), or the water may be carried up a ladder (3) and poured into the large tank. In all three cases the work done on 1 lb. of water is *h* ft. × 1 lb., or *h* ft.-lb. The head *h* can now be regarded as the work to be done in transferring 1 lb. of water from its own level to that of the surface of the fluid.

Vertical Rectangular Surface

Fig. 7 indicates a rectangular plate mounted on horizontal pivots and just clear of the sides and bottom of a drainage channel. Water is required to fill the channel on one side to a depth of 3 ft., but the rise of level is to be limited to this as a greater rise may cause flooding of the surrounding land. It is interesting to find at what height the pivots must be placed so

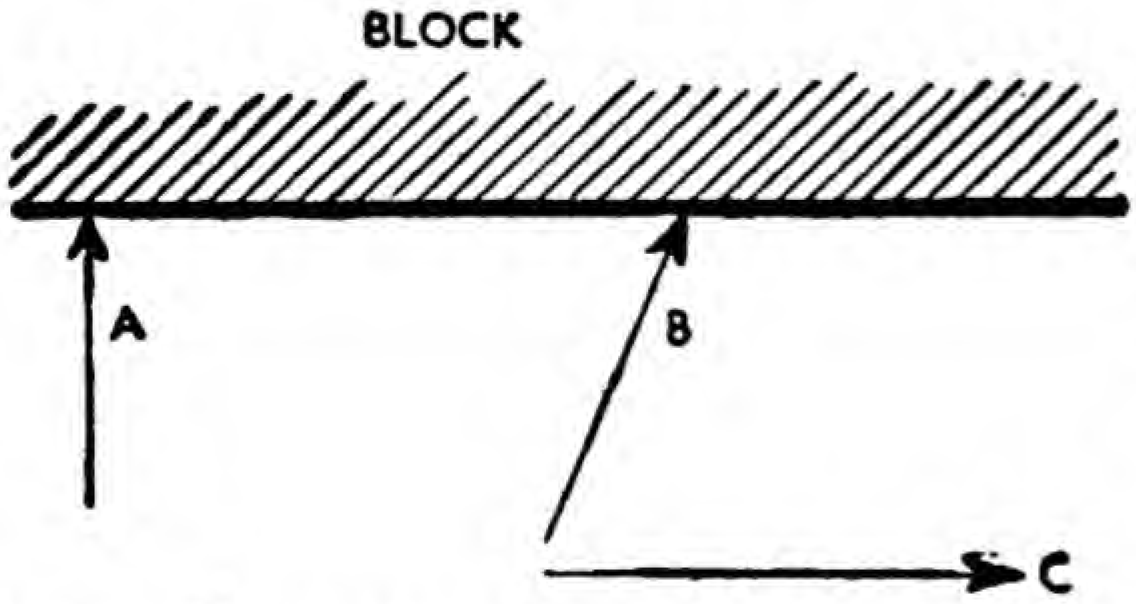


Fig. 4. If water pressure acting against a surface did not press at right angles to the face, as at *A*, but were inclined, as at *B*, it would cause movement of the water as at *C*. This, we know, does not happen.

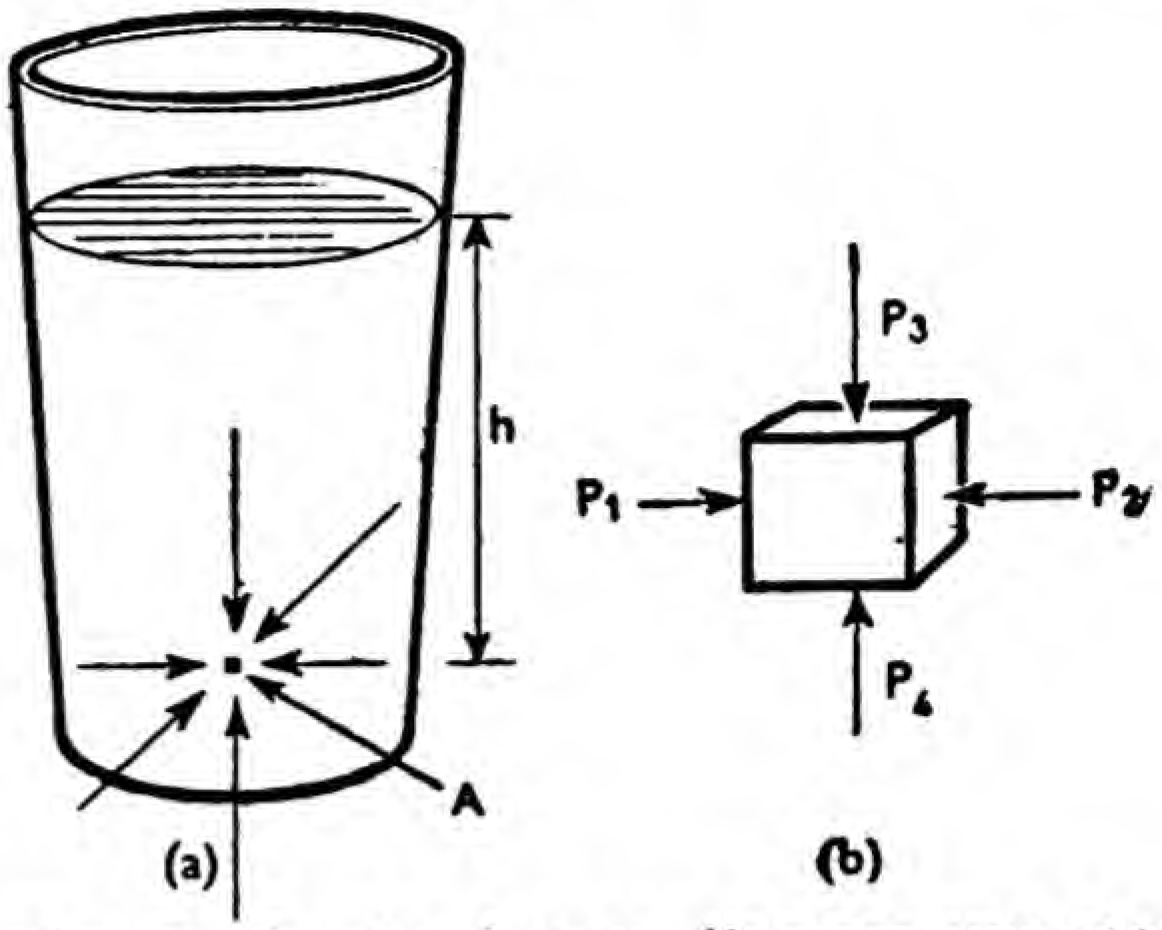


Fig. 5. In the larger diagram we see that the pressures in various directions at a point *A*, actually a particle of dust, depend only on the head *h* and so are all equal. In diagram (*b*), which is an enlarged view of the particle *A*, we see that $P_1 = P_2 = P_3 = P_4$.

that any rise above the specified 3 ft. will cause the plate to topple over and release the water upstream. First it must be noted that when the water is only a few inches deep it will press the bottom edge of the plate against the stop shown in the side view diagram.

If the pressure were the same at all points on the plate, it is clear that the hinges would have to be placed half-way down, viz., level with the centre of the plate. But the side view of the plate shows that the pressure, plotted horizontally, varies with the depth, being zero at the level of the water surface, and steadily increasing with the depth to $3 \times 62{\cdot}4$ lb. per sq. ft. at the bottom edge. The correct position for the hinges is level with the centroid of this triangle, viz., two-thirds of the distance down, or 2 ft. from the water surface.

The mean pressure on the plate is $3 \times 62{\cdot}4 \times 0{\cdot}5$ lb. per sq. ft. acting on an area of 3×2 sq. ft. so that the total force on the plate or hinges is, $1{\cdot}5 \times 62{\cdot}4 \times 6 = 561{\cdot}6$ lb.

It has been seen earlier that in

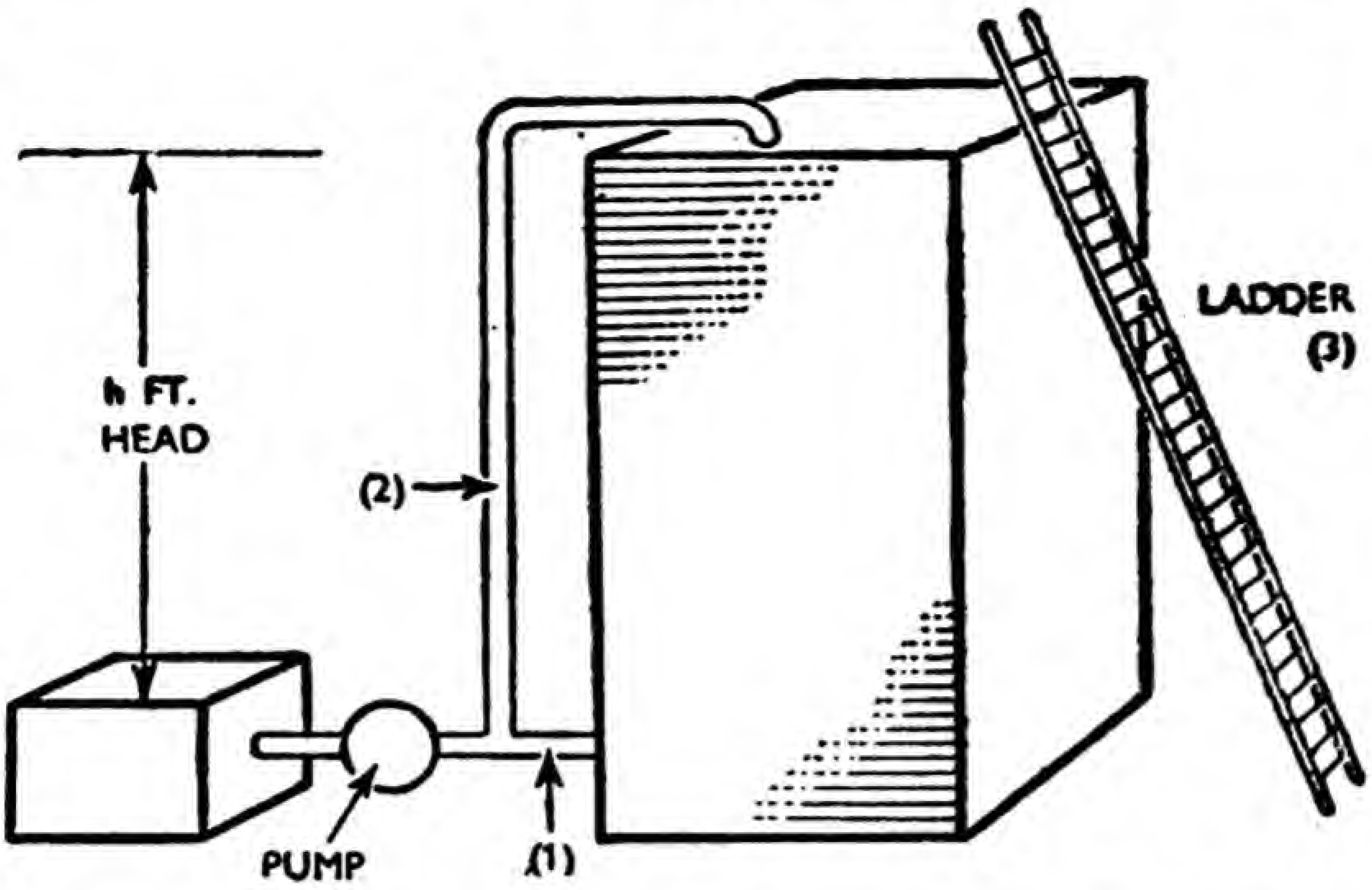


Fig. 6. If we wish to transfer 1 lb. of water from the lower to the upper tank, we may use one of three methods. We may pump in the water at (1), or through the external pipe (2) or we can carry it up the ladder (3) and pour it in. In each case the work done is the same, viz., h ft.-lb. of work per lb. of water.

Static problems the weight of a plate, although distributed over the whole plate, may be considered as acting through the centre of gravity. Here, the resultant force of 561·6 lb. may be considered as acting through a point on the plate, half-way across the width and two-thirds down. This point is named the centre of pressure, and the force will act in a direction perpendicular to the plate.

Another Example

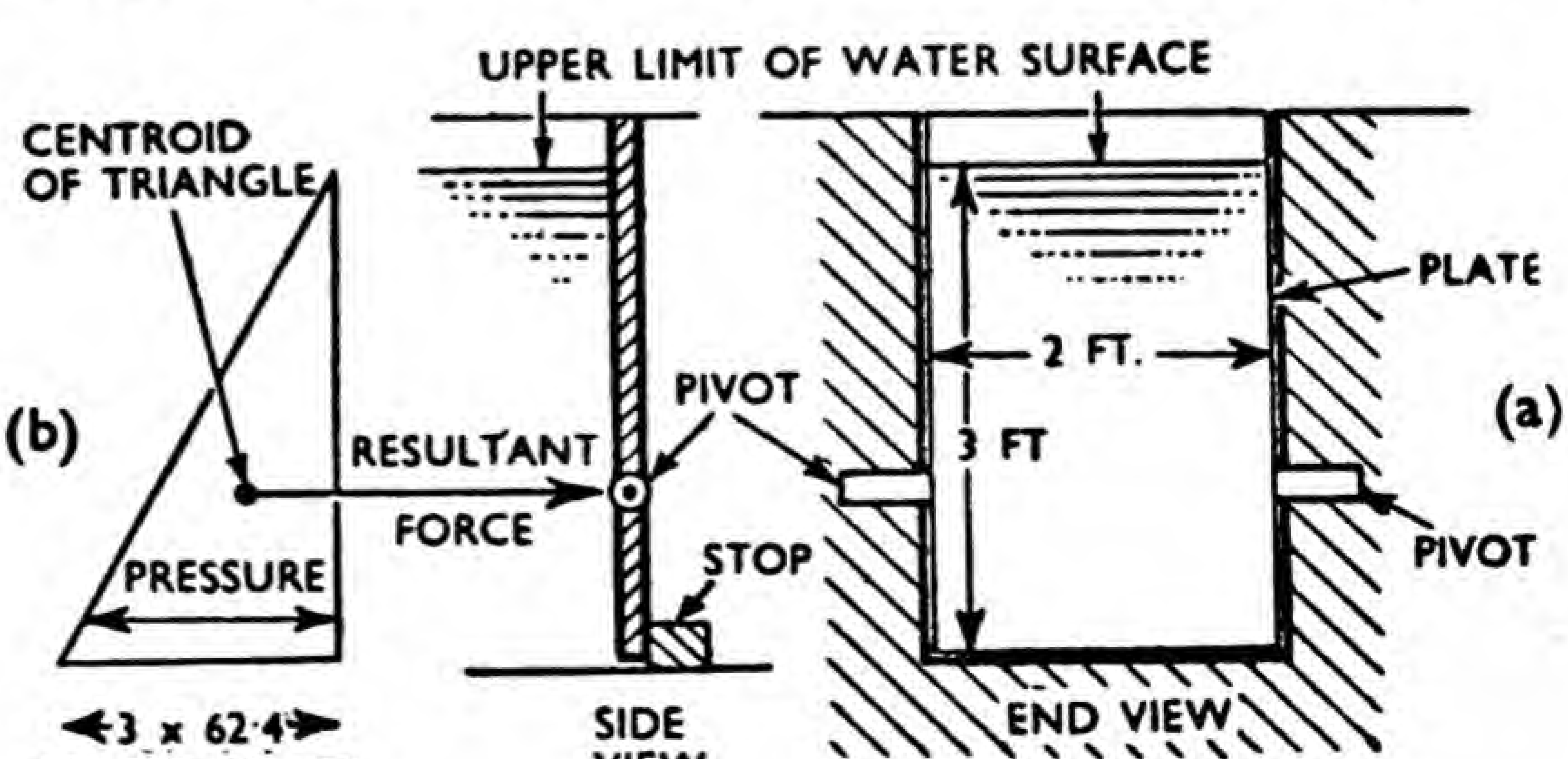
Another example of the forces on a plate may now be discussed. Fig. 8(a) shows diagrammatically a pair of lock-gates, seen from above. In Fig. 8(b) are seen the depths of water on the two sides of one of the gates, together with the corresponding triangles of pressure distribution. These depths are typical of the conditions existing after the passage of a vessel downstream.

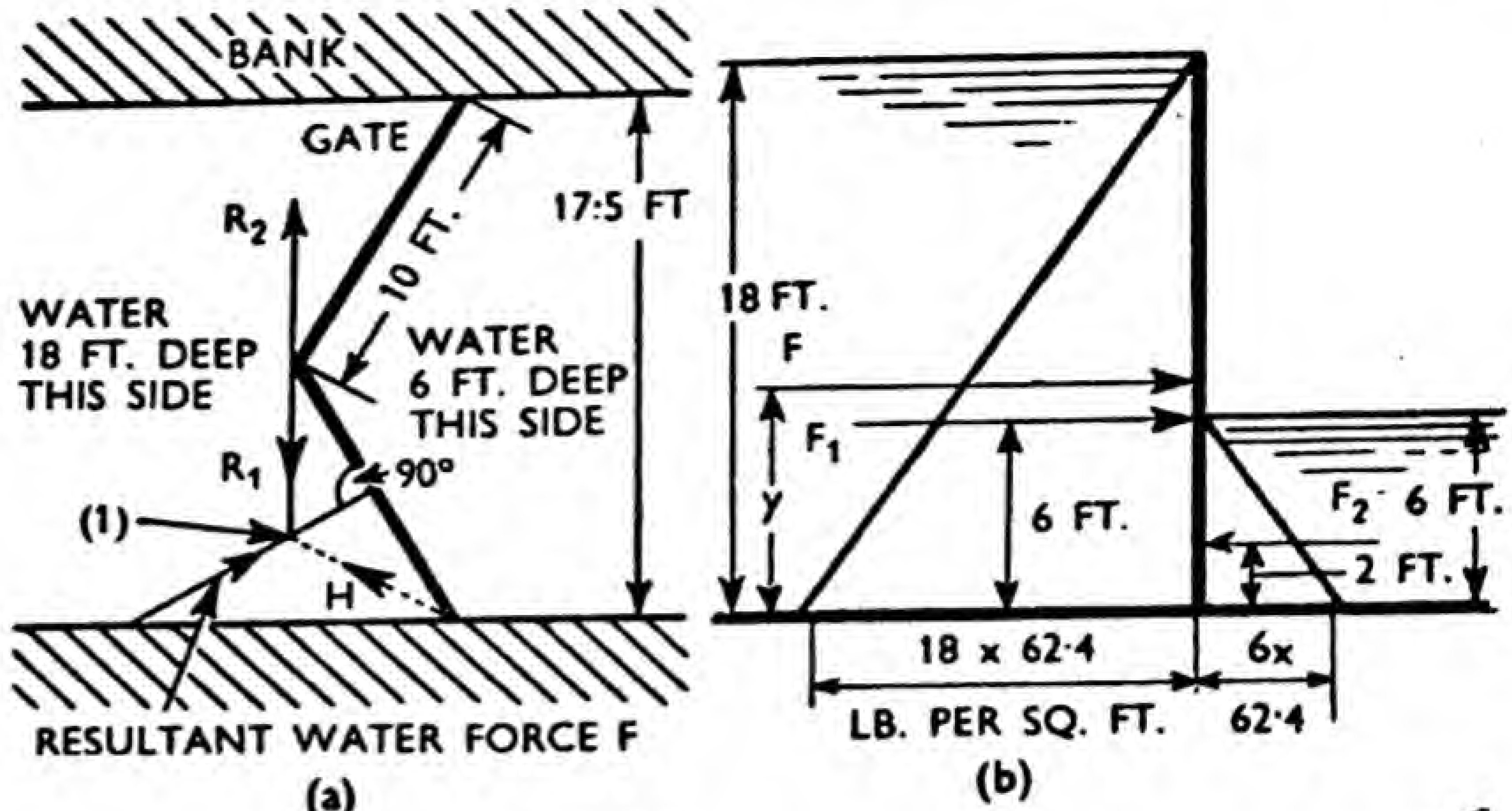
If F_1 is the force on the upstream side of one of the gates, then following the reasoning set out above: $F_1 = (0 + 18) \times \frac{1}{2} \times 62{\cdot}4 \times 18 \times 10 = 101{,}000$ lb. acting $\frac{18}{3}$ or 6 ft. up from the floor of the lock. Similarly on the downstream side: $F_2 = (0 + 6) \times \frac{1}{2} \times 62{\cdot}4 \times 6 \times 10 = 11{,}200$ lb. acting 2 ft. up from the floor. These two forces act perpendicularly to the surfaces and halfway across the 10-ft. width.

The resultant F is the difference between 101,000 and 11,200, and is 89,800 lb. acting in the direction seen in the plan. Taking moments about the base of the gate to obtain y, the height at which the resultant acts, the following is obtained :—

$$F \times y = F_1 \times 6 - F_2 \times 2, \text{ or}$$

Fig. 7. (a) Plate mounted on horizontal pivots to limit depth of water in a rectangular drainage channel. If water rises above limiting level, gate will swing about pivots and release water. (b) Shows that resultant force acts through centroid of pressure distribution triangle.





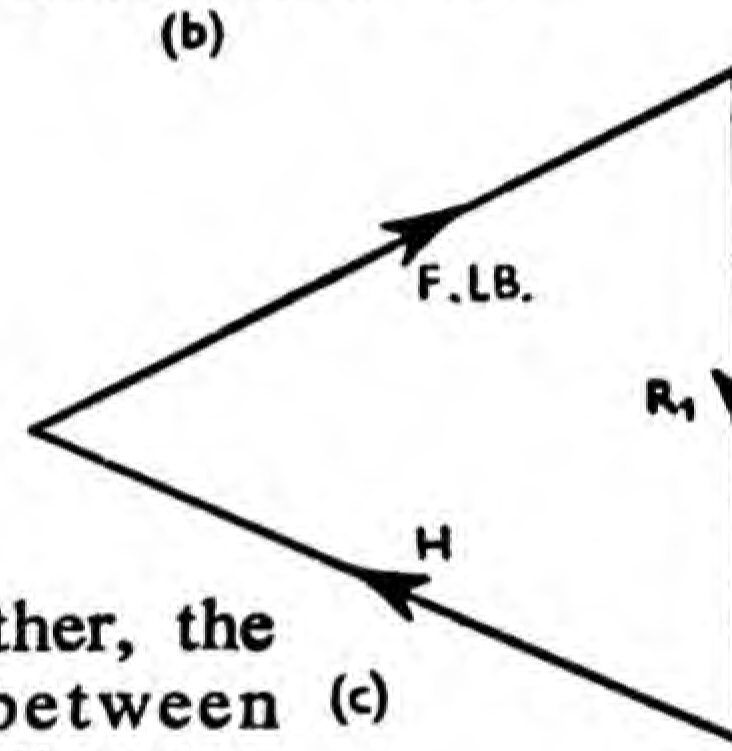


Fig. 8. (*a*) Pair of lock gates is shown diagrammatically in plan. (*b*) The pressure distribution triangles. In the plan we see that the directions of the reaction R_1, between the gates, the resultant water force *F*, and the reaction *H* of the hinges on the one gate, meet at point (1) and are in equilibrium. (*c*) The triangle of forces for the three forces R_1, *H* and *F* acting on the lower gate of (*a*) is shown, in which the three sides are parallel to the directions of the corresponding forces seen in (*a*). As we know the value of *F* by calculation, we can scale the values of *H* and R_1 from the triangle.

$$89{,}800y = 101{,}000 \times 6 - 11{,}200 \times 2,$$

which gives :—

$$y = 6{\cdot}49 \text{ ft.}$$

We must now find the direction of the reaction between the gates, that is, the direction of the force of one gate on the other. If we sit with our elbows on a table and press the forefingers of our two hands together, the reaction between them, or the direction in which one finger presses on the other, will be found to be parallel to the table, as indicated in Fig. 9.

Fig. 9. If we sit with our elbows on a table and press our forefingers together, we find that the reaction of one on the other, R_1 and R_2, are in the directions shown. This tells us the directions of the reactions between two lock-gates Fig. 8(*a*).

The corresponding directions of R_1 and R_2 for the lock-gates is seen in Fig. 8(a). Each gate may now be regarded as subjected to three forces, the resultant water force *F*, the reaction between the gates R_1 in the case of the lower gate of Fig. 8(a), and the reaction of the hinges *H*.

We have already seen that it is a fundamental principle of mechanics that if three forces act on a body and are in equilibrium, their directions must intersect at a point. The directions of *F* and R_1 intersect at (1) of Fig. 8(a), and a line, shown dotted, joining this point to the hinges, gives the direction of the force *H* due to the hinges. The values of R_1 and *H* can now be found in the usual way from the force diagram (Fig. 8(c)), in which the sides of the triangle are drawn parallel to *F*, R_1 and *H*

of Fig. 8(a). The value of F being known, the values of R_1 and H can be measured to scale.

Totally Submerged Surface

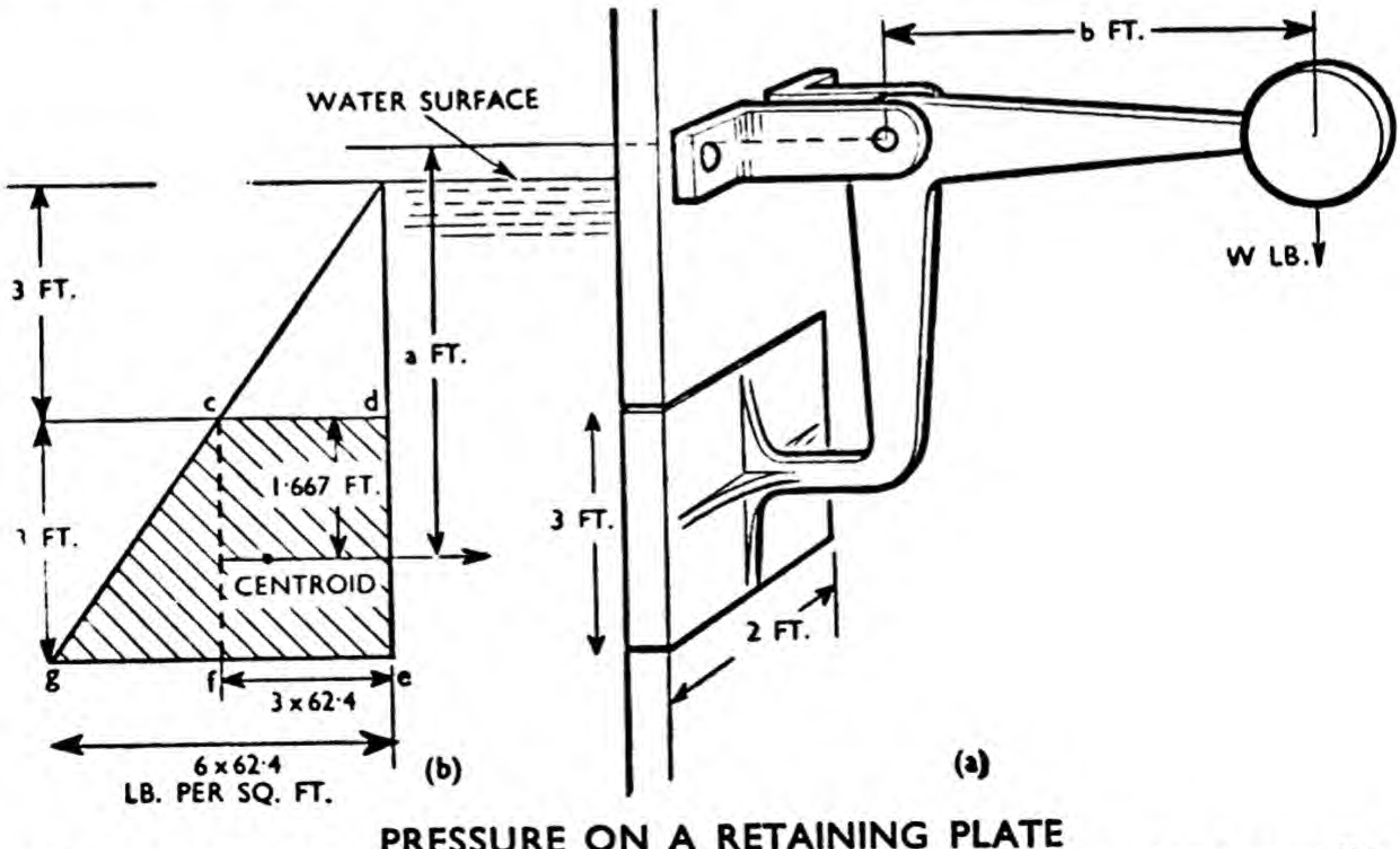
We shall now discuss in brief the case of a surface which is totally under water. Fig. 10(a) indicates a plate held inside an opening in a wall to keep back water, so long as its surface does not rise above some predetermined level, in this case 3 ft. above the upper edge of the plate. The distribution of pressure is seen in Fig. 10(b), and the centre of pressure on the plate is opposite the centroid of the shaded area. Dividing this area into a rectangle *cdef* and a triangle *cfg* by means of the dotted vertical line, we see that the areas are $3 \times 3 \times 62{\cdot}4$, and $3 \times 3 \times 62{\cdot}4 \times \frac{1}{2}$ respectively, while the distances of their centroids from *cd* are $1\frac{1}{2}$ ft. and 2 ft. respectively. Therefore, the distance of the centroid of the whole shaded area from the line *cd* is :

$$\frac{3\times3\times62{\cdot}4\times1{\cdot}5+3\times3\times62{\cdot}4\times0{\cdot}5\times2}{3\times3\times62{\cdot}4+3\times3\times62{\cdot}4\times0.5}$$
$$= 1{\cdot}667 \text{ ft.}$$

For a given position of the pivot of the bell-crank system shown, the distance a from the pivot to the centre of pressure can be determined. The force on the plate acting through this centre of pressure is due to a mean pressure of $(3 \times 62{\cdot}4 + 6 \times 62{\cdot}4) \times \frac{1}{2}$ lb. per sq. ft. acting on 3×2 sq. ft., or a product of $4{\cdot}5 \times 62{\cdot}4 \times 6 =$ 1,684 lb. The weight W required to hold the plate in place, if we neglect the weight of the bell-crank lever itself, is given by $W \times b =$ 1,684a, and so, therefore, for any convenient length b the value of W can be found.

Specific Gravity

It has been seen in previous pages that the weight of material contained in one cubic foot is called the density. It is convenient in some cases to measure density in another



PRESSURE ON A RETAINING PLATE

Fig. 10. (*a*) An illustration of a rectangular plate held in an opening in a wall to retain water. Should the water surface rise above the level shown, the gate will swing open. The pressure distribution diagram is seen at (*b*).

way. We may say, for example, that iron is 7·7 times as dense as water, so that its density is 7·7 × 62·4 or 480 lb. per cu. ft. The number 7·7 is the specific gravity.

The specific gravity of a material, such as iron, may be found in the following manner. A sample of about ½-in. cube is gently lowered into a vessel previously filled to the top with water. The volume of water displaced is caught and weighed. The number of grams weight is also the volume in cubic centimetres, because 1 c.c. of water weighs 1 gram. The sample of iron is now dried and weighed, again in grams. On dividing this weight by the weight or volume of water displaced, the specific gravity of the iron is obtained.

The following example will help to explain this. If the weight of water displaced is 2·05 grams, which is also the volume of the iron in c.c., and the weight of the sample is 15·80 grams, the specific gravity of the metal is :—

$$\frac{15{\cdot}80}{2{\cdot}05} = 7{\cdot}7.$$

Barometers

In many branches of science, a knowledge of the pressure of the atmosphere, or height of the barometer, is of considerable importance.

The barometer in its simplest form, as in Fig. 11, consists of a glass tube about 34 in. long, filled with mercury which has been boiled to expel all dissolved air. The open end of the tube, after filling with mercury, is closed with the thumb. The tube is now inverted and the lower end placed below the surface of the mercury in the reservoir. The thumb is now removed. The pressure of the

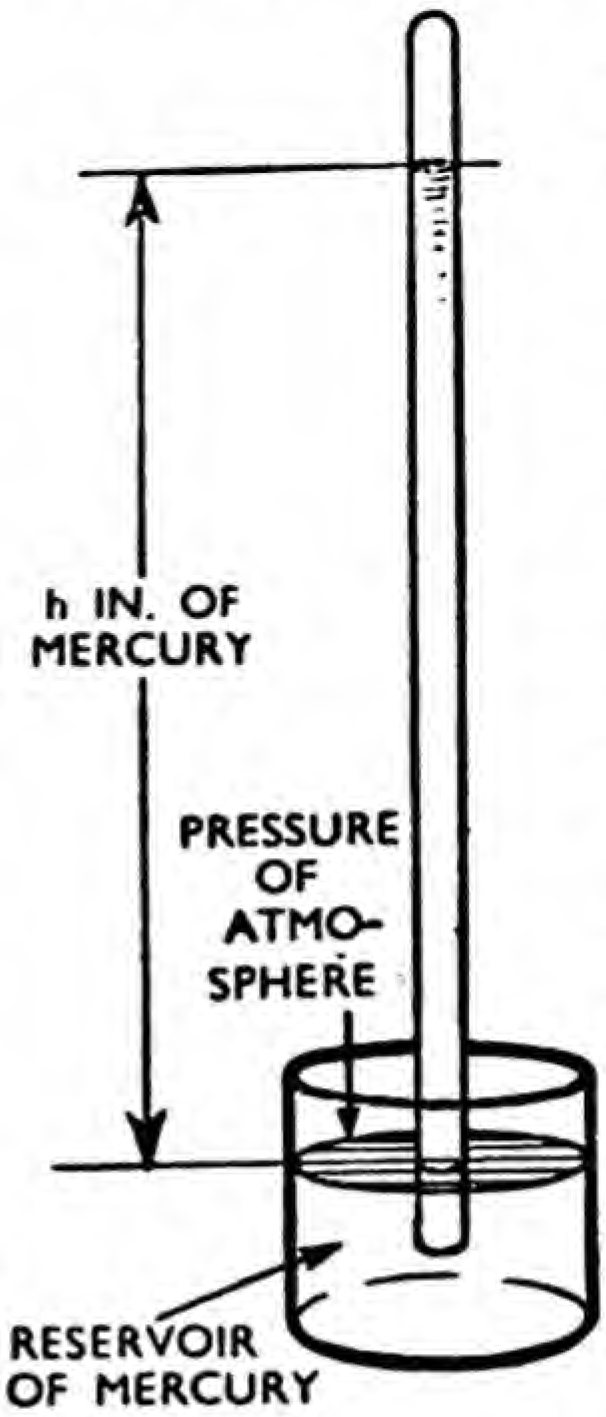


Fig. 11. In the simplest form of barometer, a plain glass tube is sealed at one end and filled with mercury. The thumb is placed over the open end and the tube inverted, the end being placed under the surface of the mercury in the reservoir before thumb is removed. Value of h depends on atmospheric pressure acting on free surface of mercury.

atmosphere acting on the free surface in the reservoir now supports the weight of the column of mercury h in. high. This is the height which is referred to when in simple talk we say that the height of the barometer is, for example, 29·5 in.

The magnitude of the pressure of the atmosphere is not always realized. Mercury is 13·6 times as dense as water and so weighs 13·6 × 62·4 lb. per cu. ft. If the barometer reads 30 in. of mercury, the corresponding head of water is :—

$$30 \times \frac{13{\cdot}6}{12} = 34 \text{ ft.}$$

From the relationship $\frac{p}{w} = h$, which was proved earlier, the pressure of the atmosphere is now found to be :

$$62{\cdot}4 \times \frac{34}{144} = 14{\cdot}72 \text{ lb. per sq. in.}$$

The pressure may be realized better if we consider the following test. A metal sphere 6 in. in diameter is made in two halves, the faces of the joint being ground smooth and given a thin coating of

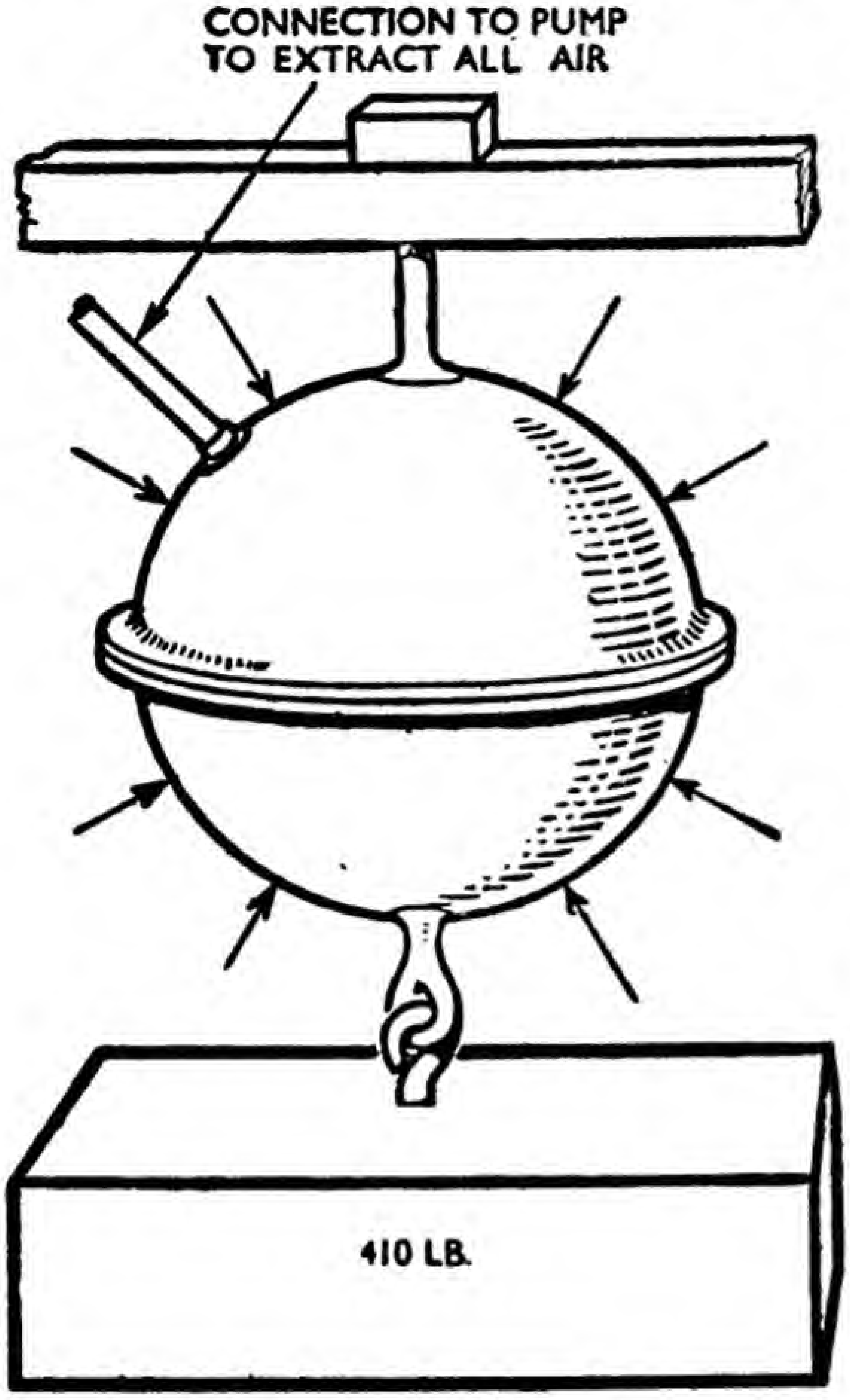


Fig. 12. A 6-inch bell is made in halves with an air-tight joint. The air is extracted and it is found that a 410-lb. load can be suspended from the lower half. The halves of the ball are not connected to one another in any way, the force preventing separation being due to the pressure of the atmosphere acting on the outside.

vaseline to make the joint air-tight. By means of a suitable connexion and tap, the air is withdrawn from the sphere by an air pump, and it will be found that a weight of about 410 lb. may be suspended from the lower half of the ball, as depicted in Fig. 12. This weight is greater than the pull of a cart-horse, and can be supported solely by the excess pressure of the atmosphere acting on the outside of the ball and forcing the two halves together.

When water vapour replaces some of the normal gas of the atmosphere, the barometric pressure falls because the density of water vapour is less than that of air. An increase in the water-vapour content of the atmosphere increases the tendency to rain, and so, to a limited extent, the rise and fall of the barometer can be used to predict the state of the weather.

The type of barometer in general use for this purpose is the well-known weather-glass (Fig. 13). This instrument is also called the syphon barometer, and consists essentially of a U-shaped tube, sealed at the top of the longer arm, and about 34 in. in overall length. The shorter limb of the U-tube has an open end and acts as a reservoir, in which floats a small piece of iron attached by a light cord to the pointer indicated. This pointer moves over a suitably graduated scale showing the difference of level of the two ends of the column of mercury, together with the corresponding state of the weather.

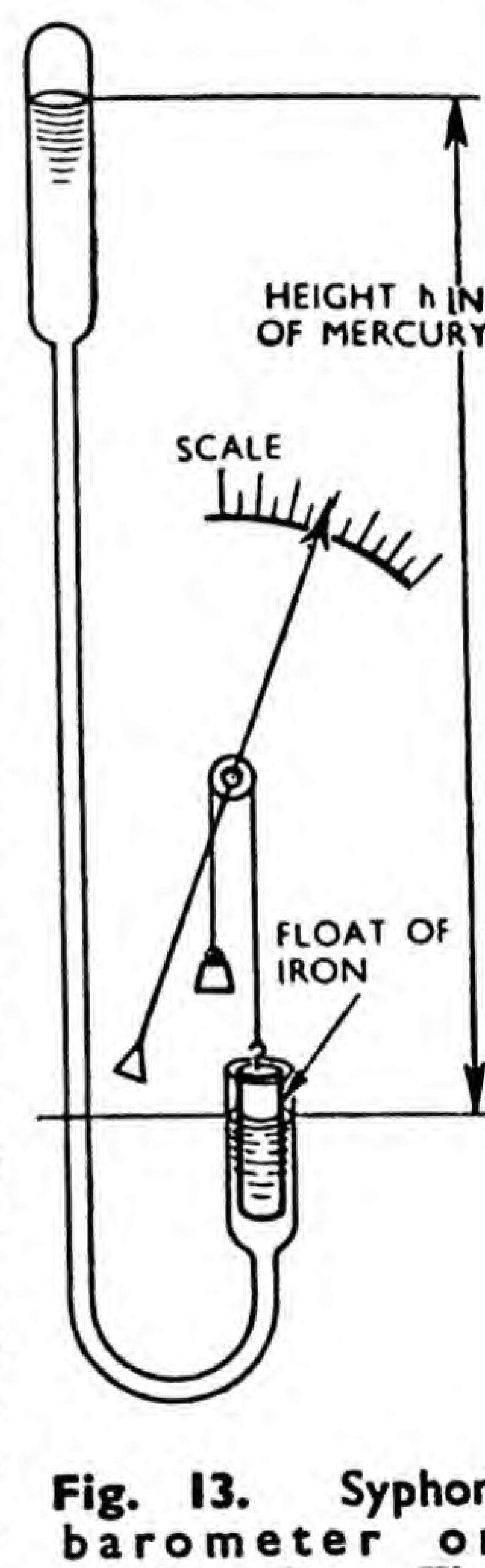


Fig. 13. Syphon barometer or weather glass. The movement of the free surface of the mercury at the open end of the tube causes a small piece of iron to rise or fall. This motion is transmitted to a light pointer moving over a scale suitably graduated.

If a light portable instrument is required, one of the aneroid type is used, in which the essential part is a cylindrical metal

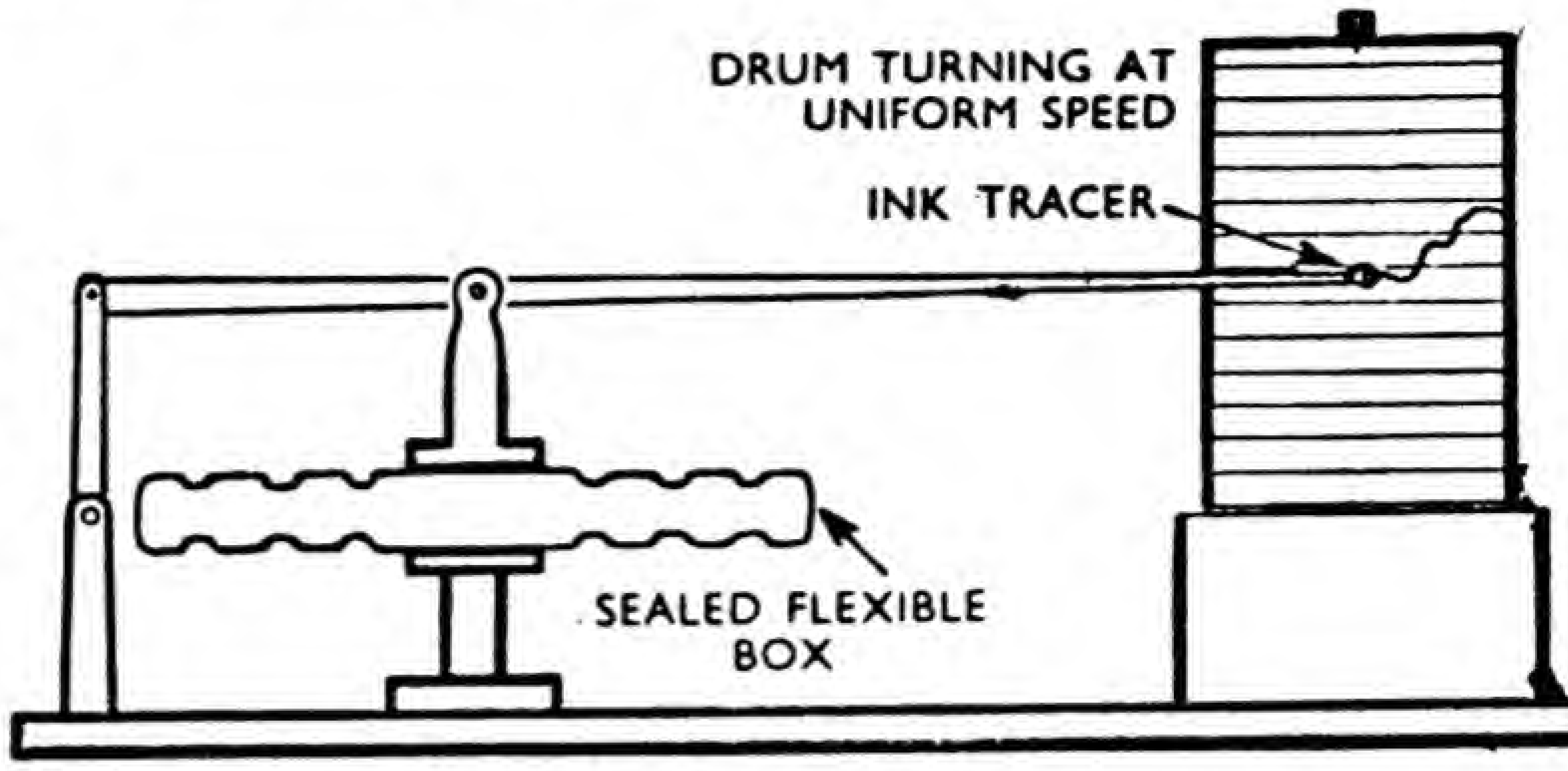


Fig. 14. Aneroid type of barometer. The box seen in section has very flexible covers and is sensitive to changes of atmospheric pressure. The result is traced on the drum.

box, perhaps 3 in. in diameter, having corrugated end covers as shown in Fig. 14. After manufacture, the box is sealed up so that no air can pass in or out. As the covers are very flexible, any change in the external or barometric pressure causes them to bend outward or inward, and the motion is transferred to a suitable light pointer. This reads against a scale graduated to show inches of mercury, or carries an ink marker which touches lightly against a paper-covered drum turning at a uniform rate. This device gives a permanent record of the barometric pressure.

We have seen how the pressure in a tank of water varies with the depth. For the same reason, the atmospheric pressure at the top of a hill on a still day is less than that at the bottom, but the pressure does not vary exactly with the height. Air, unlike water, is very compressible, and this means that the lower air is denser than that higher up the hill, with the result that the law connecting height with pressure is complicated.

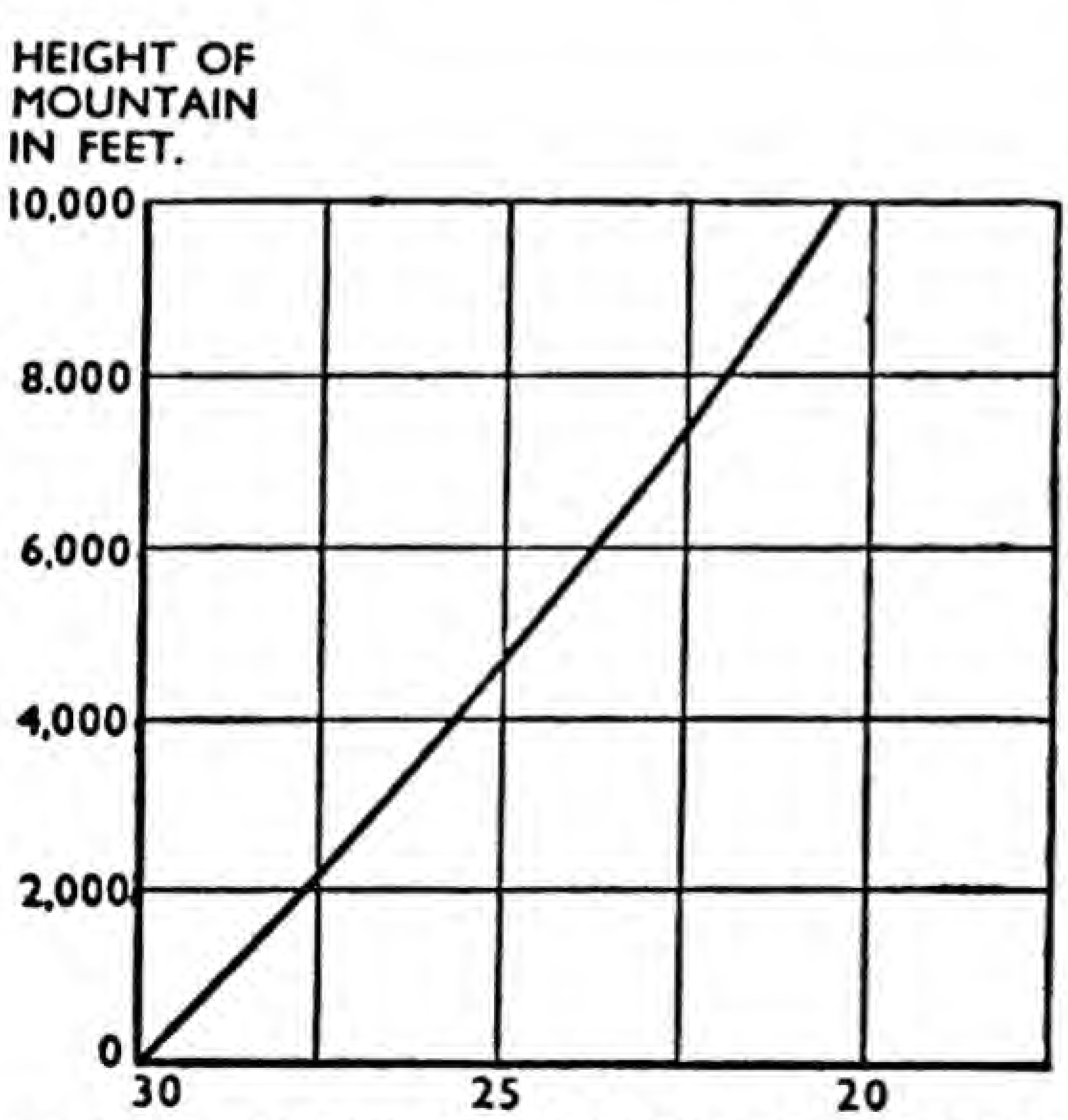


Fig. 15. Showing how the barometric height falls as the instrument is taken up a mountain.

Fig. 15 shows how the barometric pressure varies with height; the graph would be a straight line if air were of a uniform density at all heights. A portable barometer of the type described, if carried up a mountain, will show a fall in pressure, and, from a graph like that of Fig. 15, the height of the mountain can be deduced; thus, for example, a fall of pressure from 30 in. to 25 in. of mercury corresponds to a height of 4,750 ft.

Floating Bodies

The question of the stability of a floating body is fundamental to the safety of all ships, and is clearly of the utmost importance. Let us now consider the conditions of equilibrium of a vessel which is floating freely, as depicted in Fig. 16(a). Here will be seen the section of a vessel whose weight W_1 is just balanced by the upward thrust of the surrounding water, this force being distributed over the

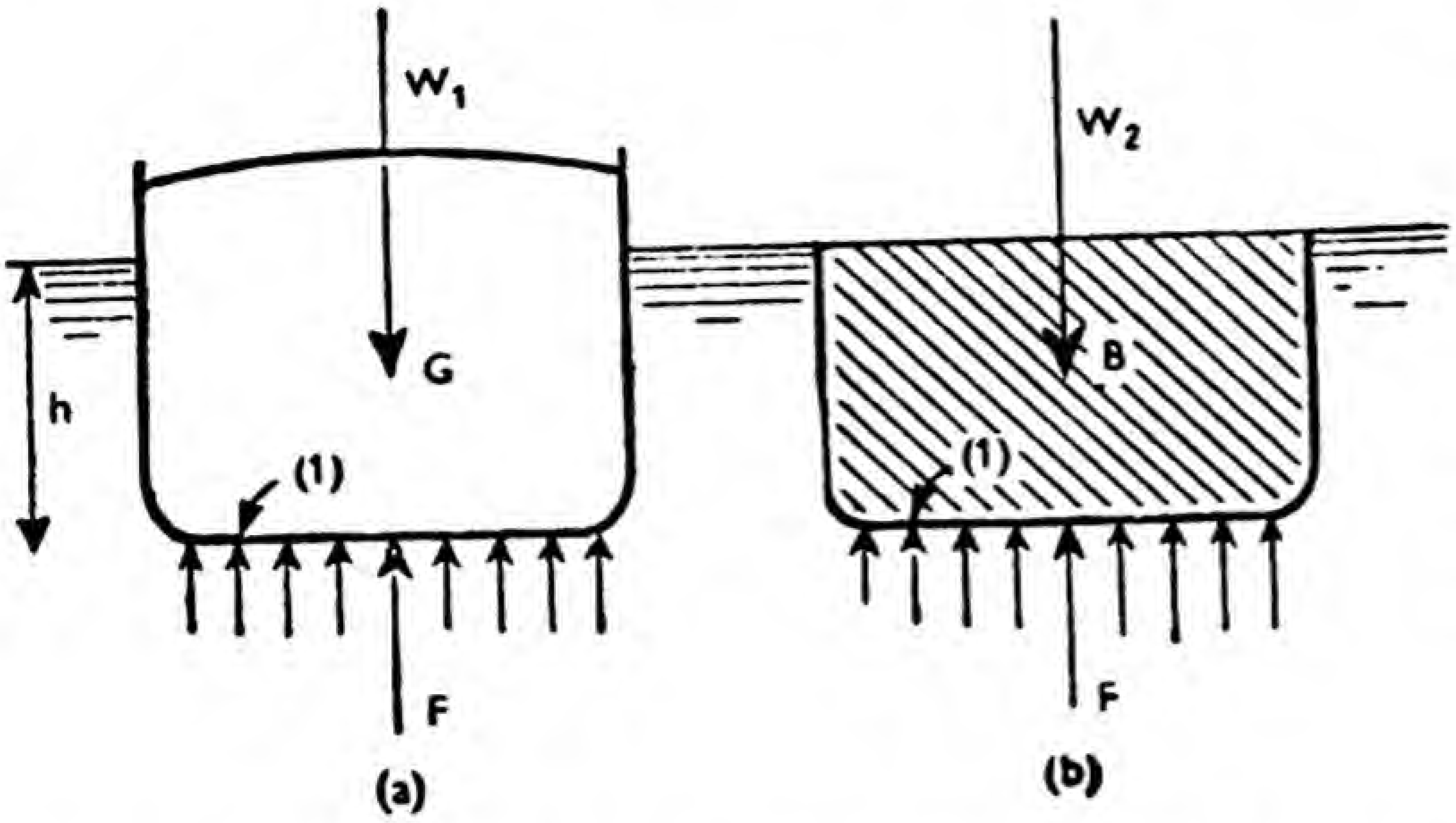


Fig. 16. *(a)* Shows the weight of a vessel W_1, balanced by the upward thrust *F* of the surrounding water, while *(b)* illustrates the weight W_2 of the displaced water balanced by *F*. In both cases, *F* depends on the depth so that $W_1 = F = W_2$.

bottom of the ship as indicated by the small arrow-heads. In Fig. 16(b) we see an imaginary section of the water which has been displaced by the ship. The weight of this water W_2 is also in equilibrium with the upward thrust of the surrounding fluid.

Now the upward pressure at any point 1 on either diagram, as we have previously seen, depends only on the head *h* or depth of the point below the water surface. If *F* is the resultant of these upward forces, we now see that its value is the same for both figures, and that it balances W_1 in the one case and W_2 in the other, with the result that we may write $W_1 = W_2$. Therefore, it may be seen that the weight of the vessel is exactly the same as that of the water displaced.

The original discovery of this equality is accredited to Archimedes, a Syracusan philosopher of ancient times. Fig. 16(a) also shows that the weight W_1 acting downward through the centre of gravity of the ship, is in the same vertical line as *F*, for otherwise these two forces, if displaced laterally from one another, would constitute a couple tending to heel the vessel away from the upright position. Fig. 16(b) shows us that *F* also falls on the same vertical as *B*, the centre of gravity of the displaced water. If the one figure is imagined to be placed over the other so that the two outlines coincide, we see that the point *B*, named the centre of buoyancy, is on the same vertical as the centre of gravity *G*.

We must now discuss what happens if the vessel is slightly displaced from the normal equilibrium position. The new centre of buoyancy B_1, of Fig. 17, that is, the centre of gravity of the water which is displaced under the new conditions, is no longer vertically underneath the centre of gravity *G* of the vessel. There is now a couple $W \times Z$ tending to restore the vessel to its original position. If, on the other hand, the shape of the vessel is such that B_1 falls on the same vertical as *G*, we shall have a condition of neutral equilibrium, as in the case of a ball floating in water, which may be turned in any direction and will remain at rest.

In the case where B_1 falls on the other side of *G* to that seen in Fig. 17, it is obvious that the vessel is unstable, and will move still further from the normal position. For the first of these cases, where there is a restoring couple $W \times Z$, we see that the direction of the upward thrust of the water cuts the

original vertical centre-line at M. This is not a fixed point as it depends on the angle of heel, but it is nearly fixed if the angle of heel is small.

This particular position of the point M is named the metacentre, and the distance GM is called the metacentric height. This height is of great importance to the safety of the ship and to the comfort of the crew, for, as shown in the diagram, it fixes the length Z, and hence the magnitude of the restoring couple. If GM is small, the restoring couple is small, and, although the vessel will roll slowly, it will not be as stable as one for which GM is relatively large.

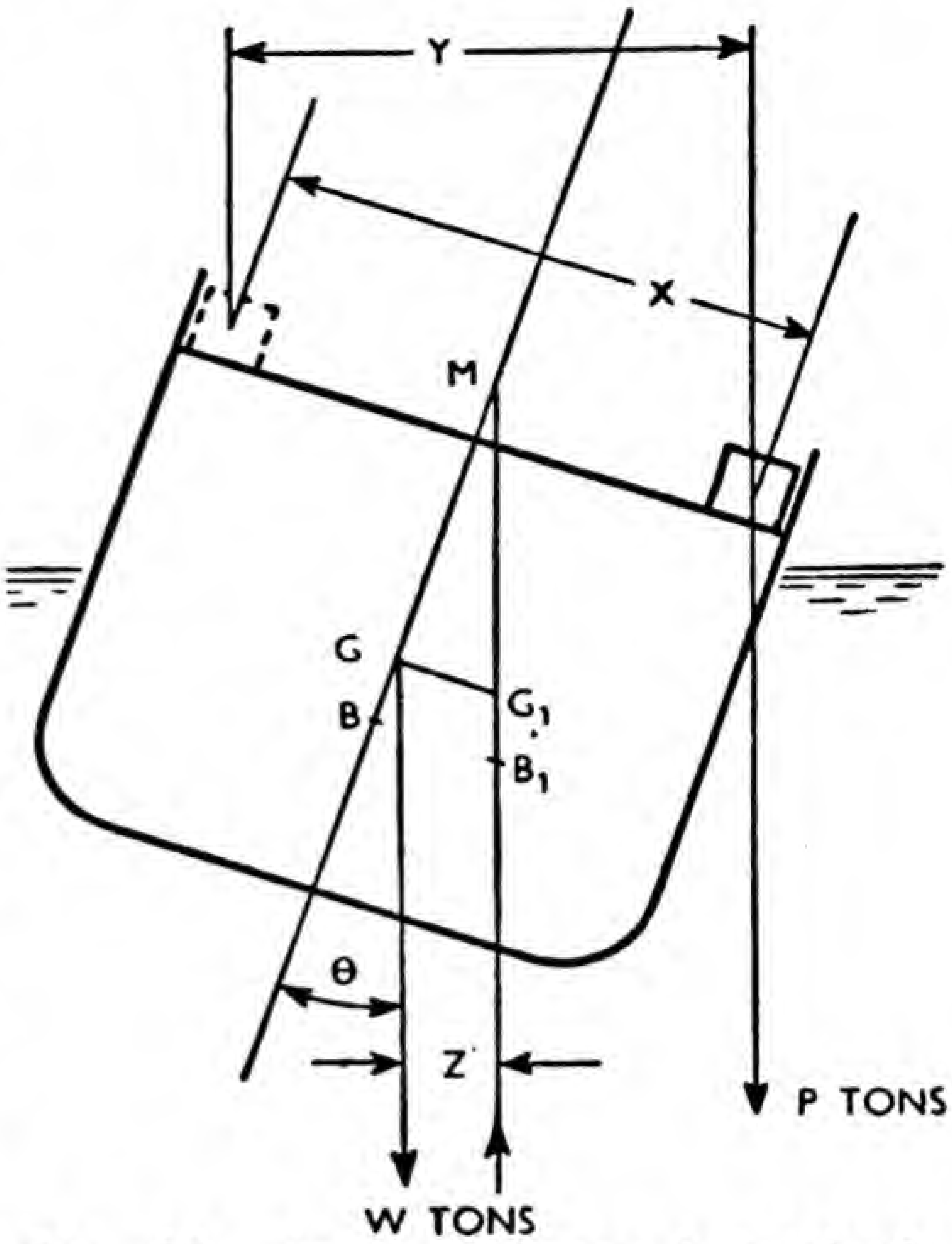


Fig. 18. In the practical method of finding the value of the metacentric height GM of a vessel, a known load P is transferred across the deck by a measured distance X, and the angle of heel θ is noted. The value of GM can be found from θ and GG_1, which is known in terms of P, X and W.

Metacentric Height

After an important ship has been built, it is usual to determine the metacentric height experimentally, and it will be of interest to study the method employed. In the first place, imagine two suitable weights P arranged one on either side of the ship. One of these is now moved through a measured distance X to the other side, as indicated in Fig. 18, and the resulting angle of heel is recorded. The movement of the weight P produces an overturning couple $P \times Y$, and the centre of gravity is transferred from G to G_1, where GG_1 is parallel to X. The value of GG_1 can be found by taking moments, so that $W \times Z = P \times Y$ or $W \times GG_1 = P \times X$, from which we obtain $GG_1 = P \times \frac{X}{W}$.

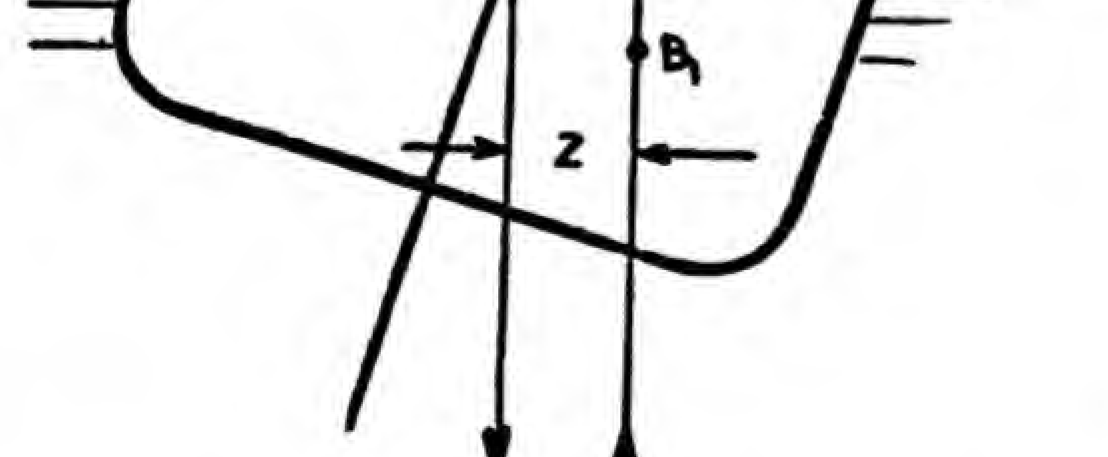


Fig. 17. If the vessel heels over, the new position of the centre of buoyancy B_1 will not be underneath G, the centre of gravity of the vessel. In this case, there is a couple $W \times Z$ tending to right the ship. The angle GMB_1 is named the angle of heel and M the metacentre.

Although the vessel is no longer floating on an even keel, the principle of Archimedes is still valid, so that the upward thrust of the water, W tons, acting through the new centre of buoyancy B_1, must fall on the vertical through G_1, as shown. The triangle GMG_1 can now be drawn to a large scale,

the angle of heel GMG_1 having been measured, and GG_1 calculated by the method explained above, and, by scaling from the triangle, we can obtain the length of GM, thus fixing the position of the metacentre M if that of the centre of gravity G is known. We now see that in loading a ship, care must be taken not to raise the position of G too much, or GM will be reduced to a value which allows rolling to become excessive.

The fact that a couple or torque heels a vessel over, explains why a ship having a single propeller floats in a position slightly displaced from the upright, in the direction opposite to that of the rotation of the propeller. This action will be appreciated if we stand with one hand pressed against a wall, when we feel our body tending to move in the opposite direction to the force applied. An aeroplane having only one propeller suffers from the same effect. In a torpedo, M is at the centre of the geometrical shape of the outside, and as G cannot be kept very low, GM is small. If one propeller only were used, the torpedo would heel over considerably. This is overcome, however, by using two propellers, one behind the other, geared to run in opposite directions. The torque of one just balances that of the other, with the result that the machine sits on an even keel.

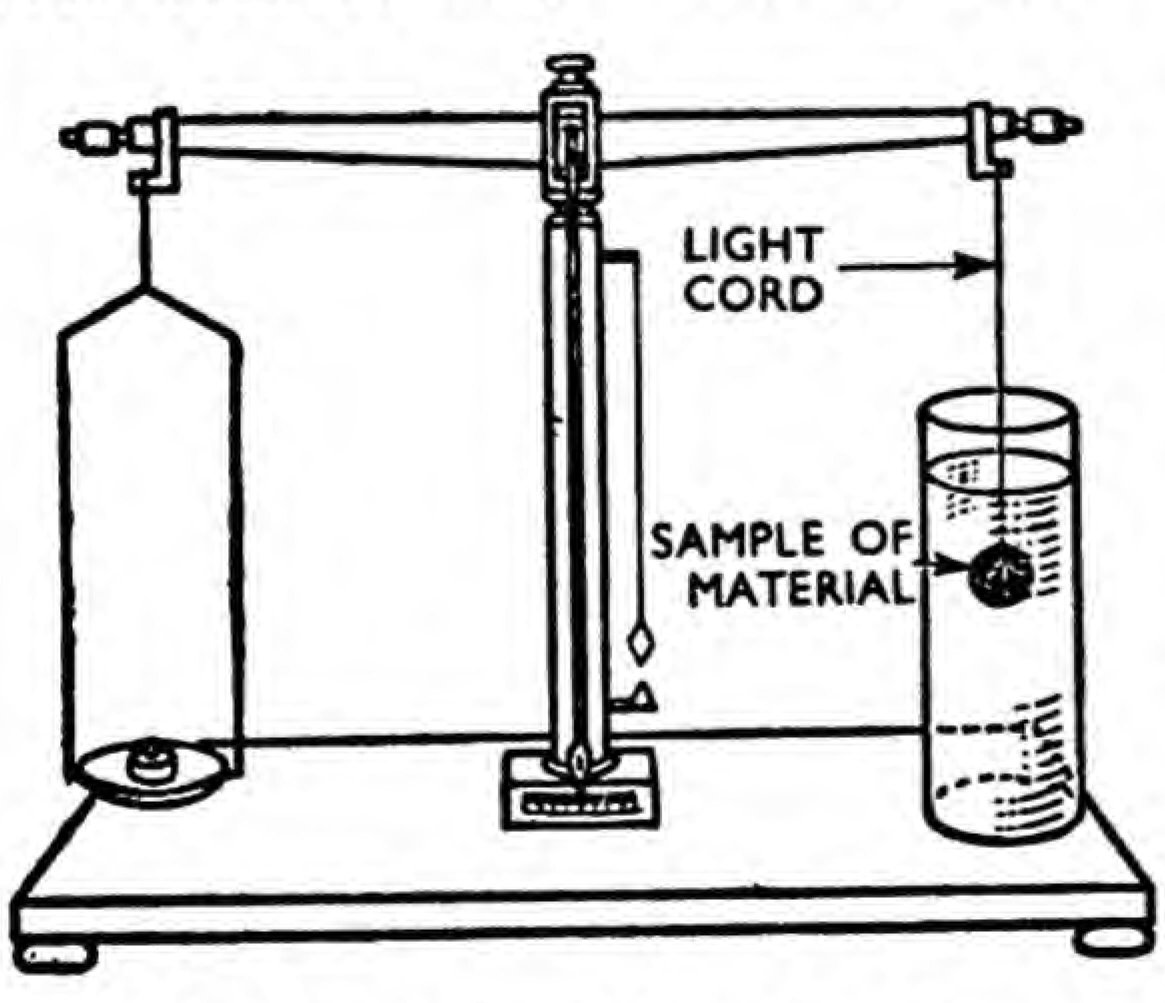


Fig. 19. The principle of Archimedes explains why the weight of a sample of material when immersed in water is less than when weighed in air.

Applying Archimedes' Principle

The method given earlier for finding the specific gravity of a sample of iron is open to experimental difficulties. An extension of the principle of Archimedes provides a method which is both simple and accurate. If a body is immersed in water, the upward thrust of the surrounding fluid is equal to the weight of the fluid displaced. This means that when the sample is weighed suspended in water, its measured weight will be less than normal by an amount equivalent to the volume of the water displaced (Fig. 19).

For measuring small weights and volumes it is convenient to employ the continental system of units, grams and cubic centimetres, a practice followed in all physical laboratories here and abroad.

Let us suppose that the sample weighs W_1 grams in air and has a volume V c.c. Now V c.c. of water weigh V grams, and so the weight of the specimen when immersed is $W_2 = (W_1 - V)$ grams. In the continental system, the density and the specific gravity, given by either $\frac{W_1}{V}$ or $\frac{W_1}{W_1 - W_2}$, are the same numerically since 1 c.c. of water weighs 1 gram. The two fractions tell us actually how many times the material under test is heavier than water, and so give us what is termed the specific gravity. Thus, with a large cast-iron weight, an accurate spring balance and a bucket of water, the density of

cast iron could be found. If the material under test is lighter than water, and, therefore, floats, it is impossible to employ the above method. A cylindrical sample is set to float as in Fig. 20, the length l, diameter d, and depth of immersion x being noted. As was seen above, the weight of the sample in grams is equal to the weight in grams of the water displaced, and is also equal to the volume of water displaced in c.c., which is $\frac{\pi d^2 x}{4}$. Now the volume of the specimen is $\frac{\pi d^2 l}{4}$, so that if we divide $\frac{\pi d^2 x}{4}$ by $\frac{\pi d^2 l}{4}$ we shall obtain the specific gravity which is, therefore, $\frac{x}{l}$. A difficulty arises here, as some cylinders will not float upright. This depends on the relative proportions of diameter and length for a given material, and these must be found by trial so that the specimen will float with its axis vertical.

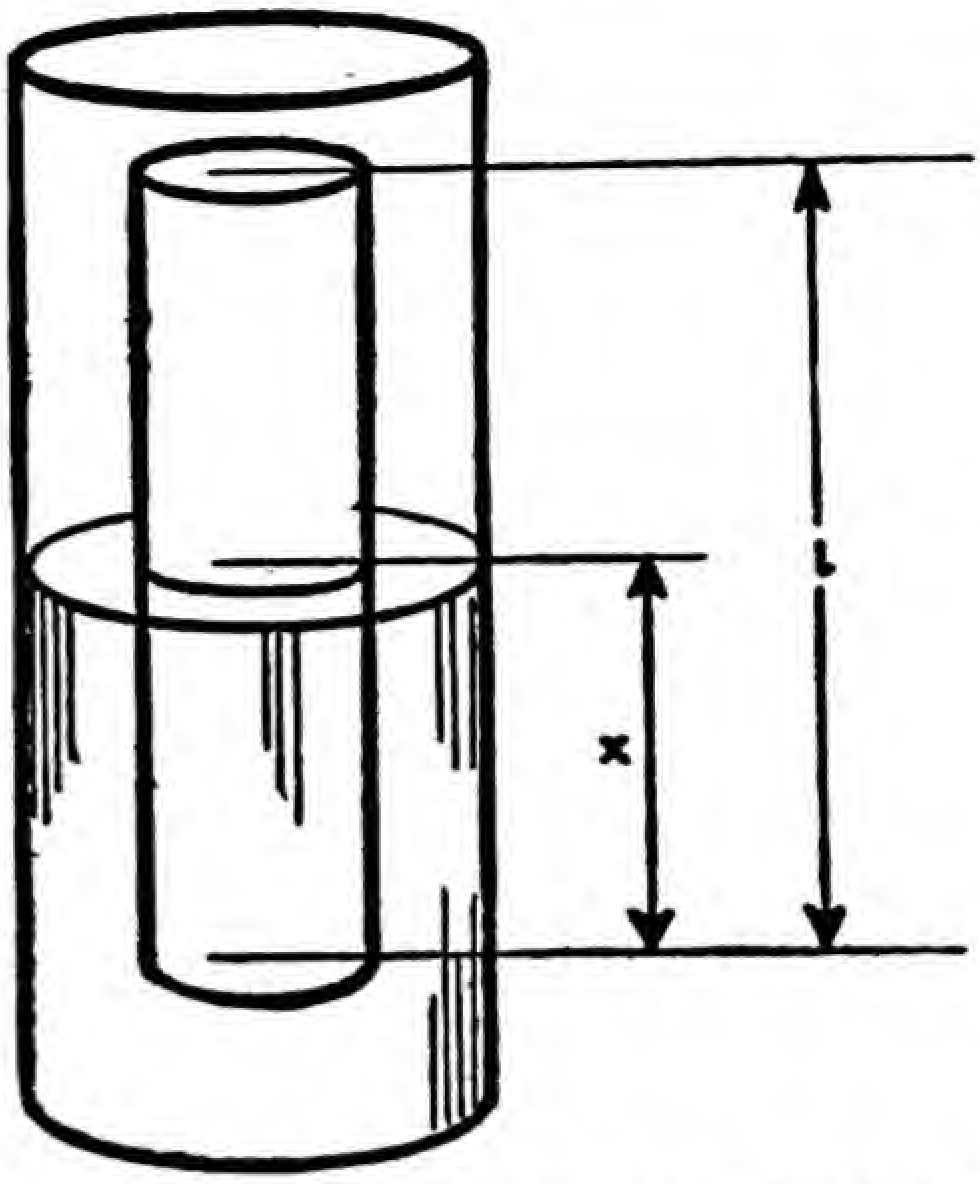


Fig. 20. Showing the method of obtaining the specific gravity of a cylindrical sample of material which is lighter than water. This assumes that the proportions of the cylinder have been chosen by trial so that it floats upright.

Velocity of Jet

In an earlier chapter, we saw that when a stone falls without resistance through a height h ft., it develops a velocity V ft. per sec. due to the steady pull of the earth, such that :

$$h = \frac{V^2}{2g}, \text{ or } V = \sqrt{2gh}.$$

When water flows from an orifice in the bottom of a tank under a head h ft., the same law holds if there is no friction. In the case of water, however, a modification of the formula is required to allow for the internal friction of the fluid. The law now becomes $V = C_V \sqrt{2gh}$, where C_V is about 0·97, and is termed the coefficient of velocity. For viscous fluids like treacle, the velocity of flow for a given head is much smaller, so that the value of C_V is reduced.

The jet of water leaving the orifice is also smaller in diameter than the orifice itself. This is because the streams flowing along the bottom of the tank overshoot the mark, as indicated at (1) in Fig. 21, and do not turn sharply at right angles when they reach the edge of the orifice. As we see at (2) in the second diagram, the section of the jet at which the streams are all parallel is considerably smaller in diameter than the hole. The ratio of the area of the jet at (2) to that of the sharp-edged orifice seen in the figure is about 0·62, and is called the coefficient of contraction.

The discharge of the orifice in cu. ft. per sec. is, $0{\cdot}97 \sqrt{(2gh)} \times 0{\cdot}62$ multiplied by the area of orifice, which is equal to 0·6 multiplied by the ideal discharge per sec., 0·6 being the product of 0·97 and 0·62, and named the coefficient of

discharge C_d. An orifice of this type can be used as a metering device if the head h ft., the diameter of the opening and the value of its C_d are known.

Let us next suppose that a horizontal wire is mounted across the middle of the jet at some convenient point A, and the dimensions x and y are recorded. From these we can determine the value of C_v.

The laws of simple mechanics enable us to calculate the time taken for a particle of water to fall vertically through the distance y, while it travels horizontally through the distance x. Under the action of gravity, if the time of fall is t sec., y ft. $= \frac{1}{2}gt^2 = 16{\cdot}1t^2$, so that $t = \sqrt{\frac{y}{16{\cdot}1}}$ sec. During this interval, we know that the particle of water under discussion has moved through the horizontal distance x with a velocity v ft. per sec., so that, $x = vt$, or $v = \frac{x}{t} = x \times \sqrt{\frac{16{\cdot}1}{y}}$ ft. per sec. We know, however, that the ideal velocity, if there were no friction, would be $\sqrt{2gh}$ ft. per sec., and so $C_v = \sqrt{\frac{v}{2gh}}$.

Force of a Jet

It is well known that a rifle recoils when fired, and for an instant presses heavily against the shoulder. This is due to the rate of change with time of the momentum of the bullet, that is, of the product of the mass and the velocity of the bullet. As an example, let us imagine that a bullet weighing 0·4 oz. has its speed steadily increased in a barrel 2 ft. long, from rest to 1,000 ft. per sec. The average speed is $(0 + 1{,}000) \times \frac{1}{2}$, or 500 ft. per sec., and the time taken by the bullet to travel the length of the barrel, therefore, is $\frac{2}{500}$, or 0·004 sec. The speed of the bullet is increased from zero to 1,000 ft. per sec. in 0·004 sec., corresponding to an acceleration of $\frac{1{,}000 - 0}{0{\cdot}004}$ ft. per sec., every sec., so that by Newton's second law, the force on the shoulder

$$= \text{weight} \times \frac{\text{acceleration}}{g}$$

$$= \frac{0{\cdot}4}{16} \times \frac{1{,}000}{0{\cdot}004} \times \frac{1}{32{\cdot}2} = 194 \text{ lb.}$$

weight. In practice, this would be considerably reduced by the elasticity or give of the shoulder, with the result that, instead of a force of 194 lb. acting for 0·004 sec., the load would be spread out over a relatively long

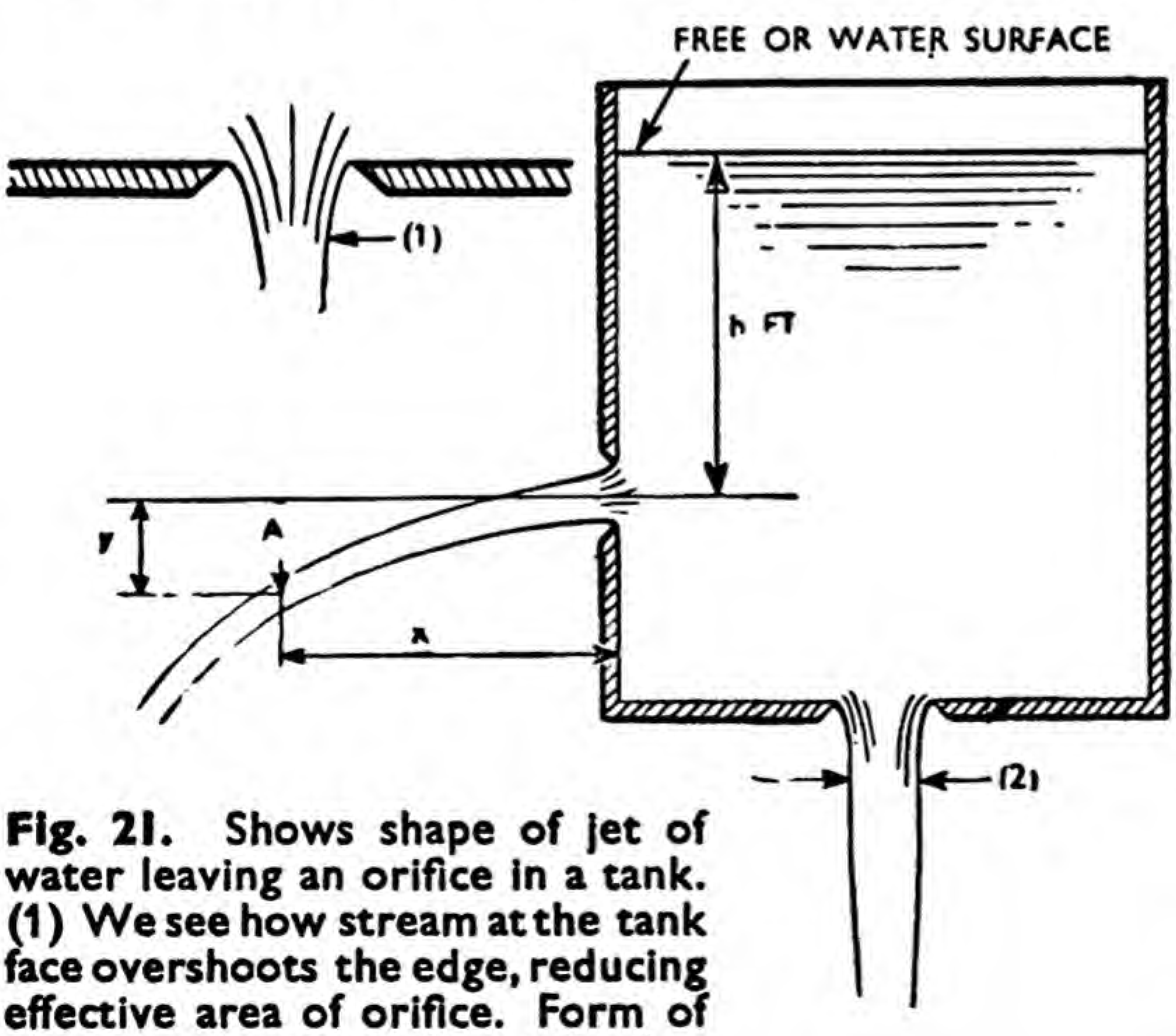
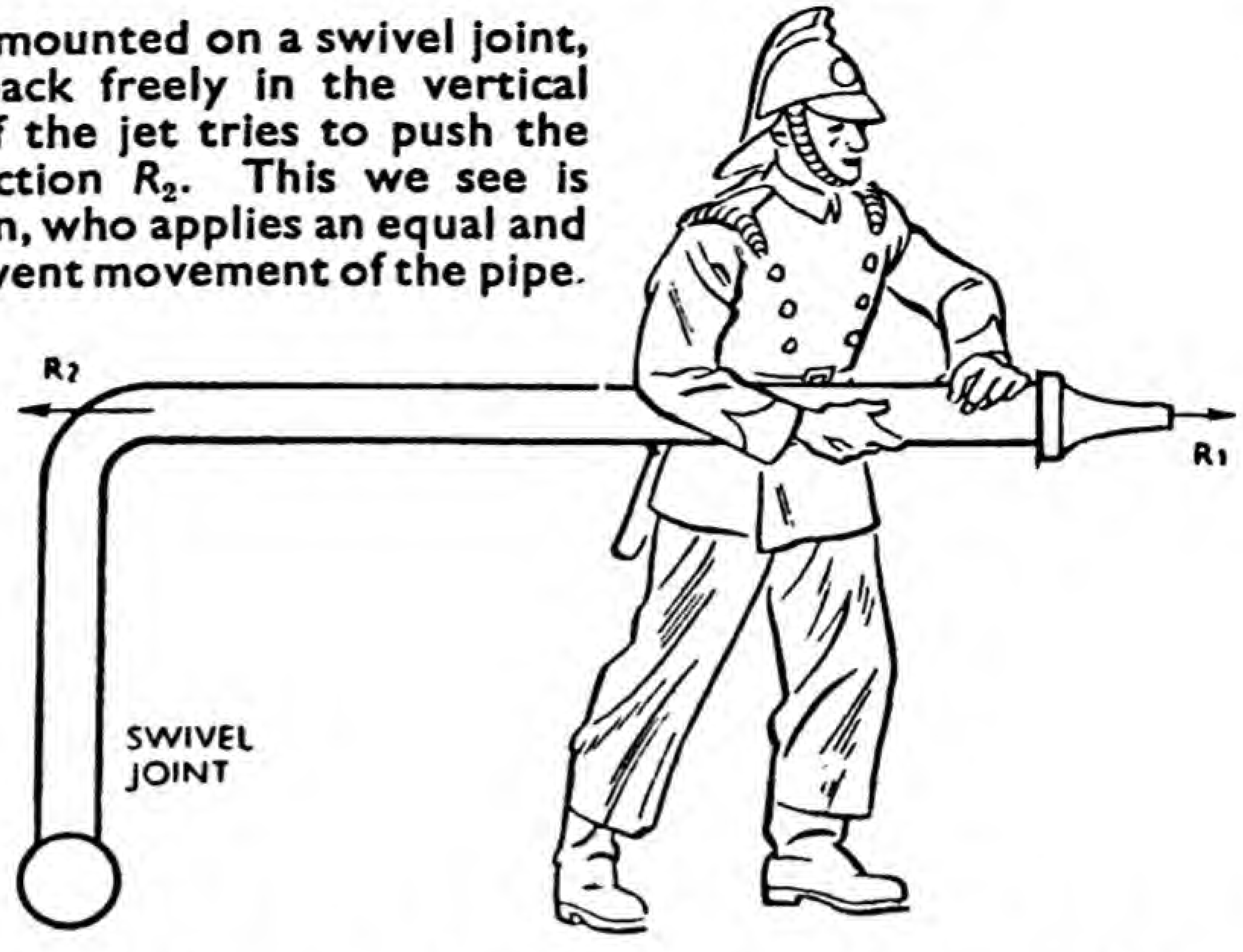


Fig. 21. Shows shape of jet of water leaving an orifice in a tank. (1) We see how stream at the tank face overshoots the edge, reducing effective area of orifice. Form of orifice is termed sharp-edged.

Fig. 22. Pipe shown is mounted on a swivel joint, so that it can swing back freely in the vertical plane. The reaction of the jet tries to push the pipe bend in the direction R_2. This we see is overcome by the fireman, who applies an equal and opposite force R_1 to prevent movement of the pipe.



period with a corresponding reduction in the force.

A more fundamental form of Newton's law is :—

Force in lb. weight = rate of change of momentum. In this case the momentum was originally :—

$$\frac{1}{32{\cdot}2} \times \frac{0{\cdot}4}{16} \text{ lb.} \times 0 \text{ ft. per sec.}$$

and finally:—$\frac{0{\cdot}4}{16}$ lb. $\times \frac{1{,}000}{32{\cdot}2}$ ft. per sec., and the change was effected in 0·004 sec. The rate of change is, accordingly,

$$\frac{0{\cdot}4\,(1{,}000 - 0)}{32{\cdot}2 \times 16} \times \frac{1}{0{\cdot}004},$$

and this number gives the force 194 lb., as before.

If a stream of the above bullets weighing w lb. were fired each second, the change of momentum would be from $\frac{w}{g} \times 0$ to $\frac{w}{g} \times 1{,}000$ in each sec., and the corresponding rate of change would be :—

$$\frac{w\,(1{,}000 - 0)}{32{\cdot}2 \times 1 \text{ sec.}} = \frac{w \times 1{,}000}{32{\cdot}2}.$$

We thus find that a steady pressure of

$$\frac{w \times 1{,}000}{32{\cdot}2} \text{ lb.}$$

would be exerted on the shoulder.

It is not difficult to see the analogy between the force produced by the stream of bullets and the reaction caused by a jet of water emerging from the nozzle of a non-flexible hose, held by a fireman. If the vertical length of pipe of Fig. 22 is attached to a swivel joint so that it can rotate freely in the plane of the paper, the fireman will have to exert a force R_1 to counteract the reaction of the jet on the bend in the pipe, as indicated by the arrow R_2, so that $R_1 = R_2$. Alternatively he may press downwards on the nozzle, exerting a couple about the pivot of the pipe equal to R_2 multiplied by the height of the vertical leg of the pipe. Here the force R_2 will be a steady one, unlike that caused by the stream of bullets, which will give a series of jerks. The water entering the horizontal length of pipe has no velocity in the horizontal direction, but it leaves the nozzle at V ft. per sec. If the rate of flow is w lb. per sec., the equation is :—

$$\text{force } R_2 = \frac{w\,(V - 0)}{g} \text{ lb. weight.}$$

We see that the product $w\,(V - 0)$ has the units $\frac{\text{lb.}}{\text{sec.}} \times \frac{\text{ft.}}{\text{sec.}}$ or lb. $\times \frac{\text{ft.}}{\text{sec.}^2}$, which are the units of mass multiplied by acceleration.

Another example which will help us to understand the action of a jet is seen in Fig. 23. A tank is sus-

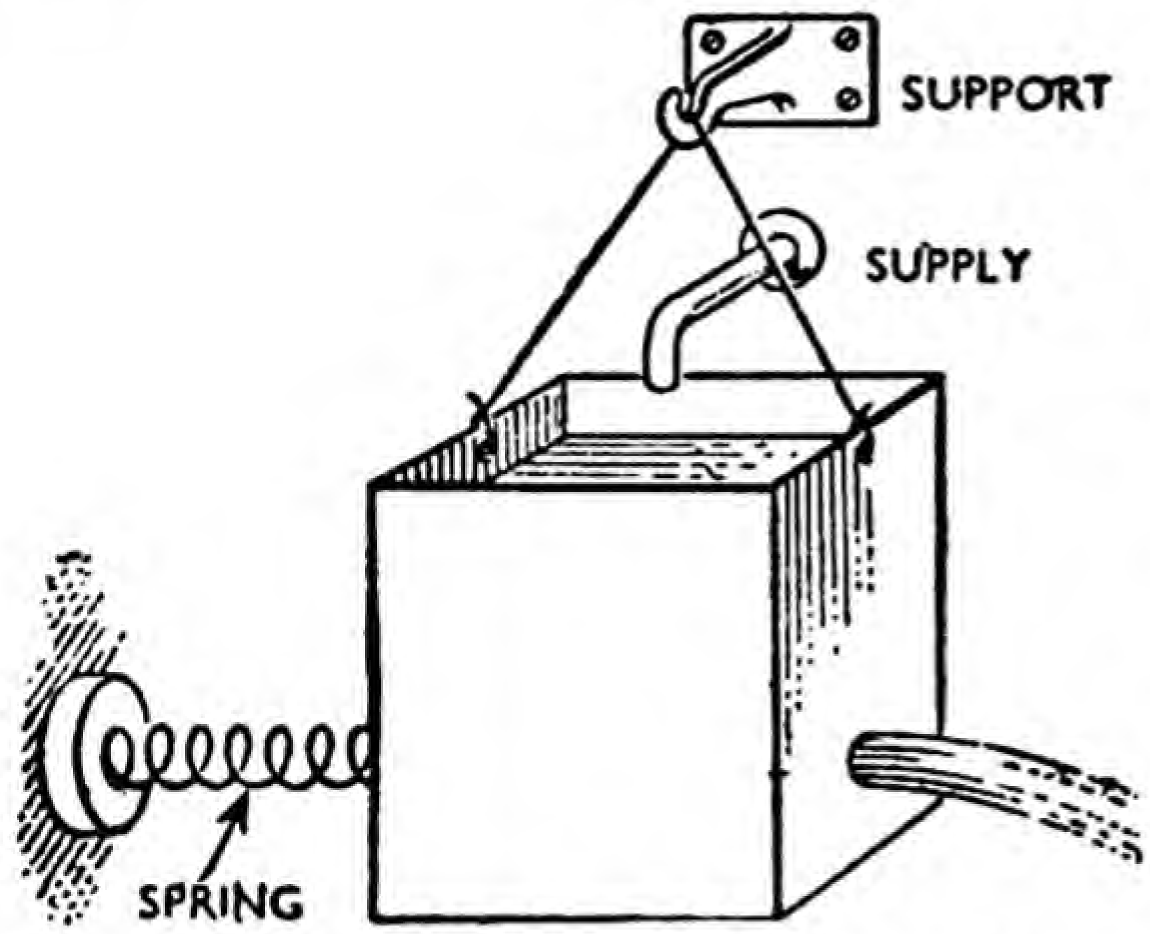


Fig. 23. Tank containing water is suspended freely. If water is allowed to flow through the orifice, the reaction of the jet will make the tank swing slightly to the left. The spring shown is compressed until the tank hangs vertically. Under these conditions, the force in the spring, which we can measure, is just equal to the reactive force of the jet.

pended like a picture, except that it does not rub against a wall so that it is not constrained by friction. We have seen that in the case of the rifle a force was produced in the opposite direction to that in which the bullet moved. In this case the jet issuing from the orifice will produce a similar force on the tank, tending to make it swing to the left.

We will suppose that the spring shown has been compressed just sufficiently to keep the tank upright, so that its weight produces no turning action about the support. Under these conditions, the force in the spring, R lb., just balances the reaction of the jet. Each particle of water has its velocity in the horizontal direction changed from 0 at the entrance to V ft. per sec. at the discharge, and so if w is the quantity flowing in lb. per sec., as before, the reaction of the jet $R = \dfrac{w(V-0)}{g}$ lb.

We will now use a graphical method to study the action of a jet.

If a jet is directed against a fixed plate, as in Fig. 24, the water will flow off radially in all directions, like the spokes of a wheel, and if there is no friction, the speed of the water leaving the plate will be V ft. per sec. at all points, just as in the jet itself. Thus the direction of the water is changed, but not its velocity.

Let us suppose that AB represents V ft. per sec. of the jet to some suitable scale, say, for example, 1 in. = 20 ft. per sec., while BC shows the velocity of the water which moves vertically downward, and BD relates to the water which flows upward.

The change of velocity of the water which leaves the nozzle and flows downward is shown by AC, but the force on the plate will be

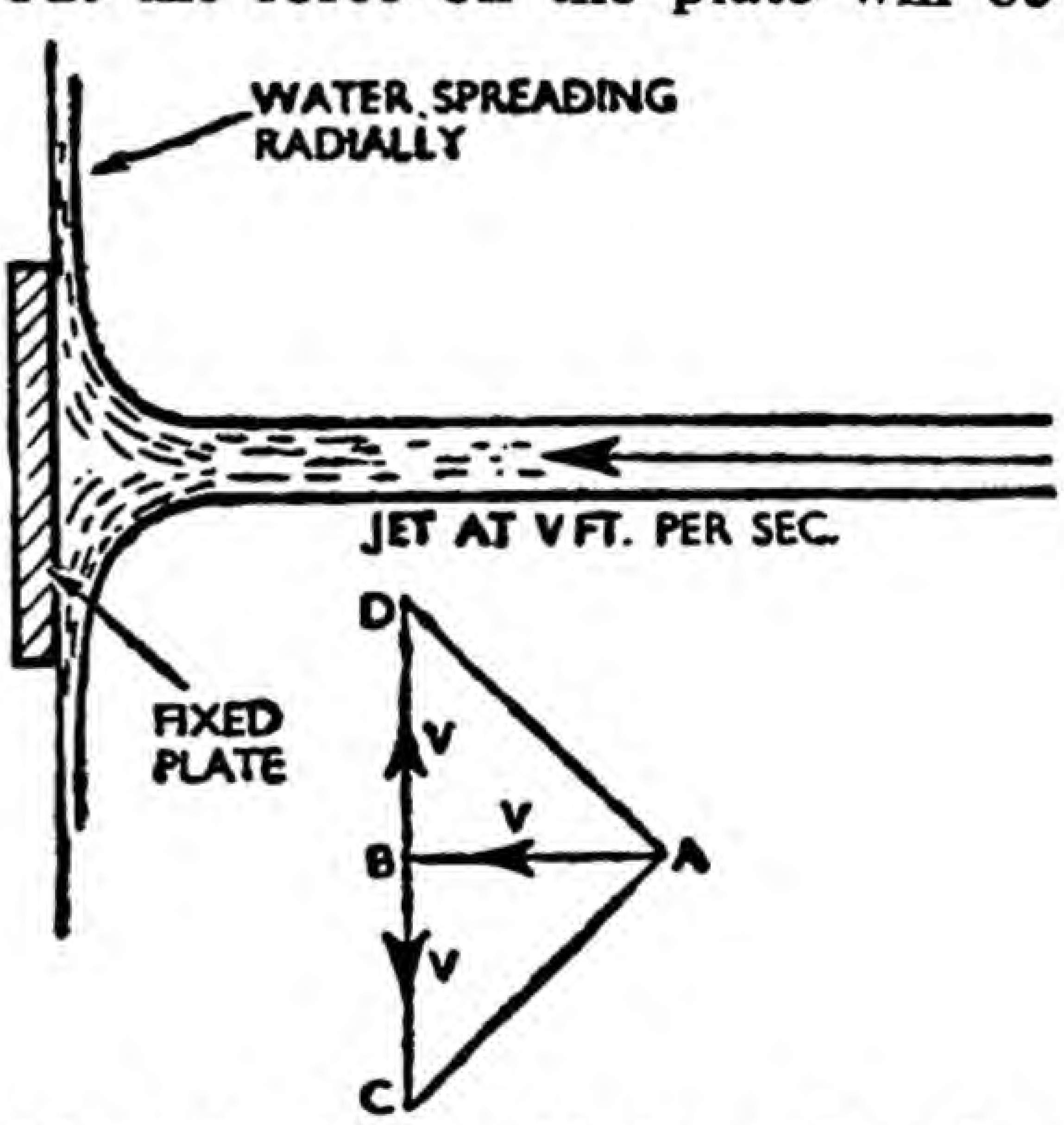
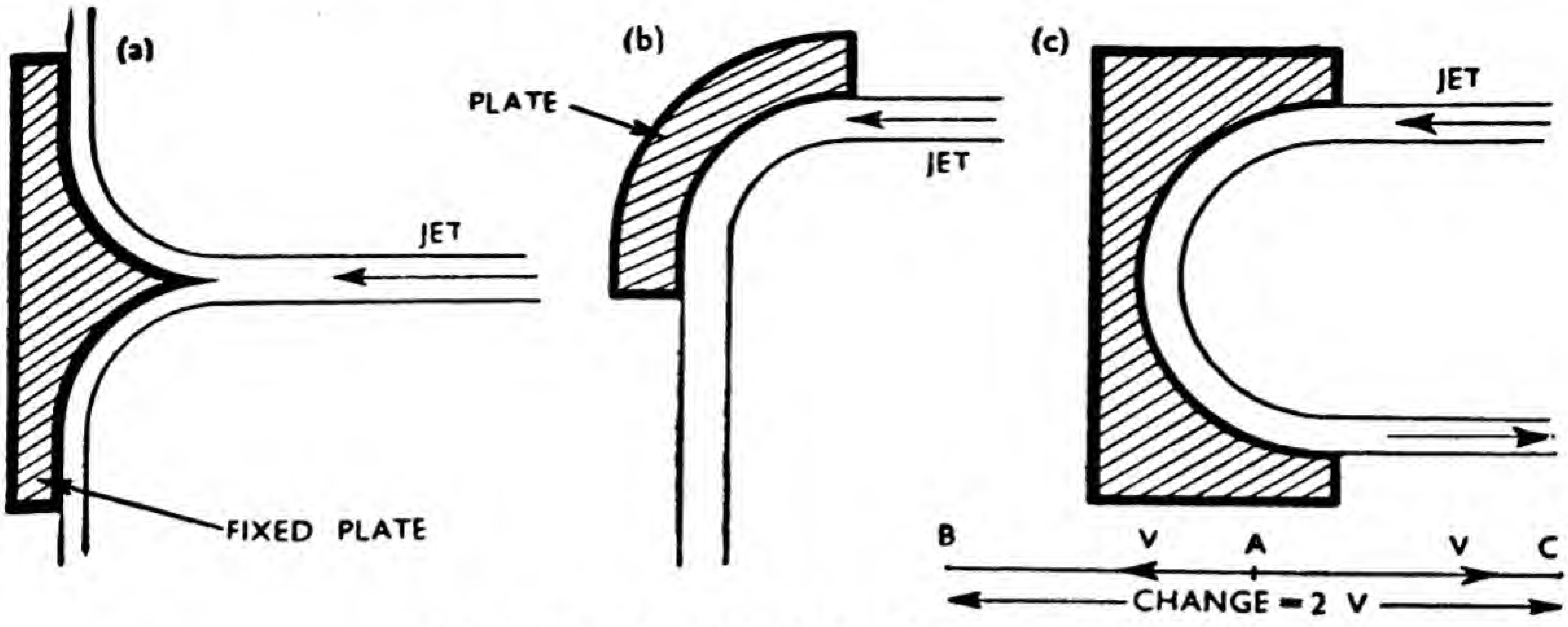


Fig. 24. Jet, on hitting a flat plate, spreads radially in all directions. If there is no friction, the velocity of the water is the same at all points, and we can represent these conditions by means of the figure *ABCD*. *AB* shows us the original velocity of the jet, and *BC* the velocity of that part of the water which flows downwards. The change of velocity of this water is shown by *AC*. The reactive force of the water leaving with the velocity *BC* just neutralizes that leaving with the velocity *BD*. The resultant force on the plate is due solely to the velocity shown by *AB*.



JET SLIDING WITHOUT IMPACT

Fig. 25. If, instead of using a flat plate, we employ one shaped as at (*a*), the push on the plate is increased. The jet slides on this plate without impact. At (*b*) we see a plate which turns the jet through a right angle. The force impressed on the plate in the direction of the jet is exactly the same as in case (*a*). By changing the direction of the jet through 180 deg. as at (*c*) we double the force on the fixed body.

due to the change *AB* only, as the vertical forces due to *BD* and *BC* neutralize one another. The result is that the force on the plate $= w \times \frac{AB}{g} = w \times \frac{V}{g}$ lb., as before, *w* being the quantity of water flowing in lb. per sec., and *V* being regarded as the change of velocity. Therefore, if there is no friction, the force produced on the flat plate is the same as the reaction of the jet itself. This is of interest as it will be noticed later how a simple modification of the shape of the plate enables us considerably to increase the force produced on it.

Pockets of Rotating Water

When we look at the water which has just passed over a weir or waterfall we notice the presence of a number of pockets of rotating water. These are also known as eddies, and they contain a certain amount of energy just like the energy stored in the flywheel of an engine. This energy is converted to heat when the eddies die away, as they do further downstream. If the jet loses some of its energy on account of the formation of eddies, the force acting on the plate will be slightly reduced.

These eddies are formed chiefly at some sudden change of direction of flow, as in the waterfall, and in the case of the flat plate this loss of energy may be reduced by the use of a conical-shaped plate, as in Fig. 25(a), where fewer eddies are formed. Here it will be noted that the jet slides on to the plate rather than hits it.

Another interesting result arises if we use a plate curved in the form of a semicircle as in Fig. 25(c). Here again a graphical solution is convenient. If *AB* represents the velocity *V* of the water leaving the nozzle, or sliding on the plate, and *AC* shows the velocity *V* as it leaves the plate, then the total change of velocity, neglecting friction, is $2V$ ft. per sec., represented by *BC*. The resulting force on the curved plate is now $w \times \frac{2V}{g}$ lb., which is double the previous value.

The addition is easy to under-

stand. If a man runs along the ground and jumps on a light truck, the truck will move in the direction in which he jumped. If he turns round, and this is comparable with the turning of the jet by the cup of Fig. 25(c), and jumps off at V ft. per sec., the truck will have another force impressed on it of the same magnitude as that when he jumped on. The forces are added together as they are in the same direction.

Energy of Water

In comparing the heights of two mountains, we measure each above sea level, as this is a convenient datum. The difference between their heights is independent of the position of sea-level, however. A stone held in the position (1) of Fig. 26 has energy stored in it, but the amount of this energy can be stated only when the height of the stone above the table is known. The top of the table thus acts as the datum level. If we release the stone, it will fall, and when it is in the position (2), it will have acquired some kinetic energy, or energy of motion, at the expense of part of its original potential energy, or energy of position as measured from the same datum.

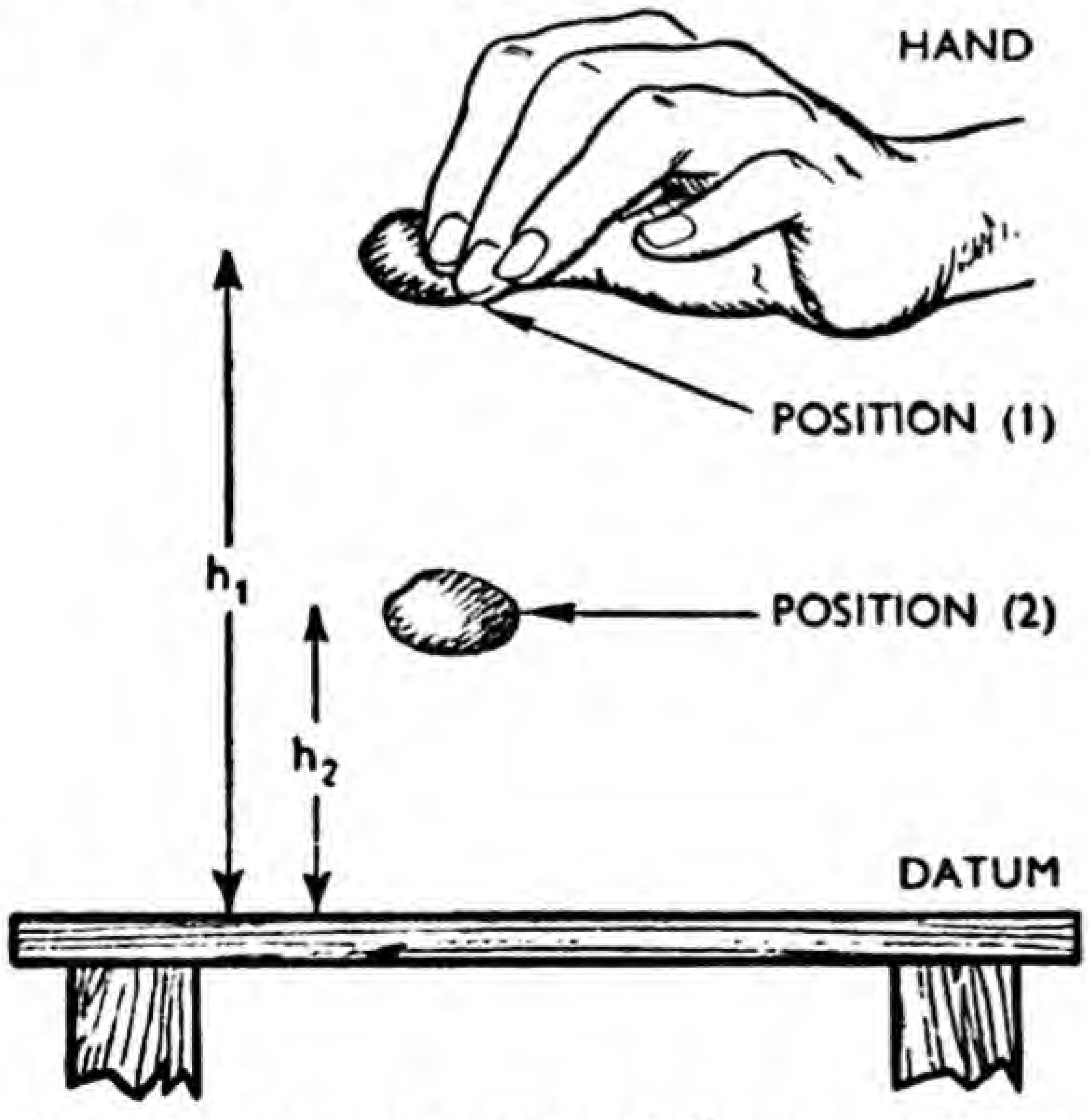


Fig. 26. Stone released from position (1) loses potential energy as it falls, but gains an equivalent amount of kinetic energy. The potential energy must be measured above some chosen datum, and in this illustration it is the table top.

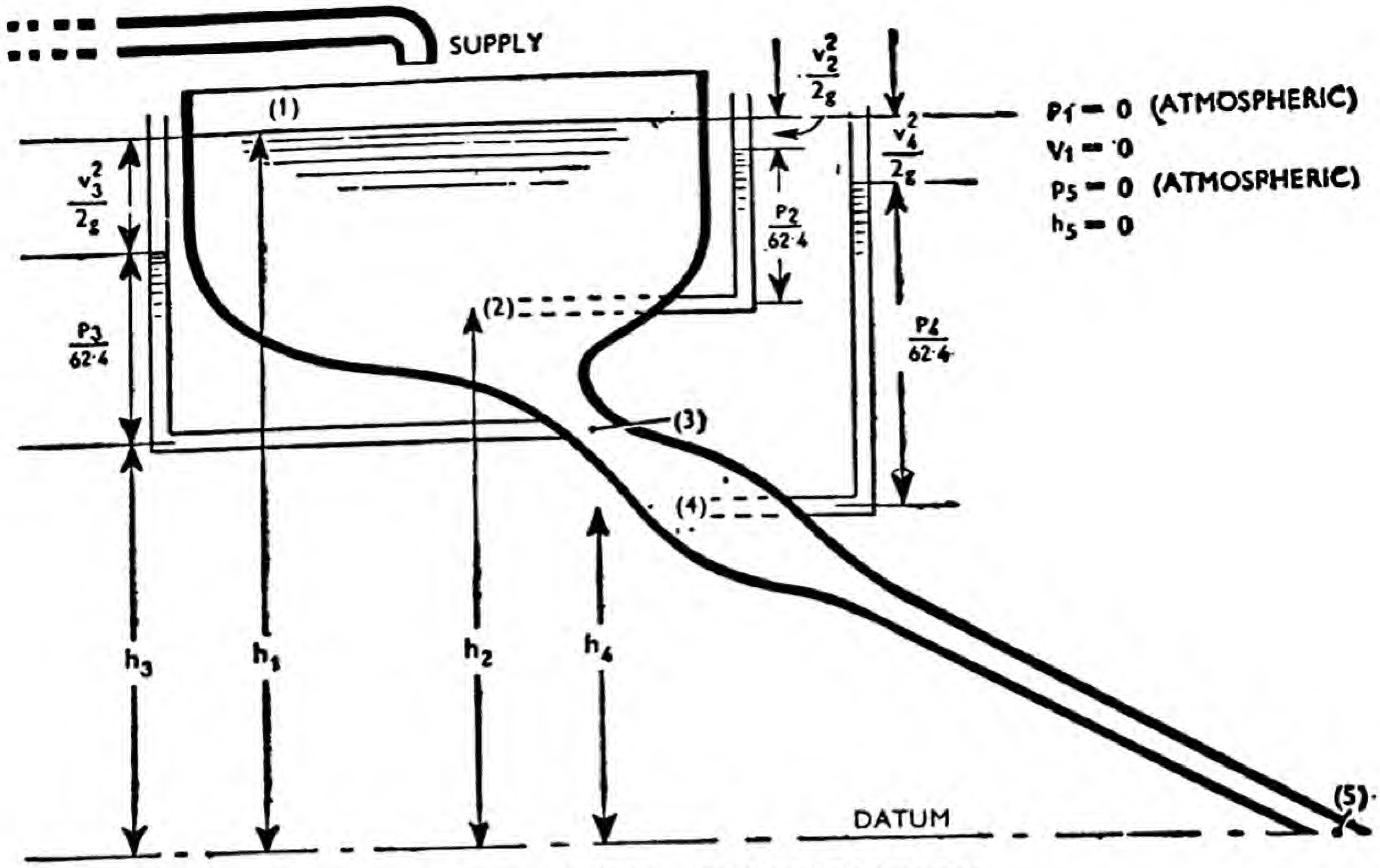
If the stone weighs 1 lb., the work stored at (1) is $(1 \times h_1)$ ft.-lb., while that at (2) is $\left(\frac{1 \times V^2}{2g} + h_2\right)$ ft.-lb., and, if no energy has been lost, we can equate the total energy at the two positions (1) and (2), so that :—$h_1 = \frac{V^2}{2g} + h_2$. It is important for us to note that if the table is lowered by h ft., the values of h_1 and h_2 will be increased by the same amount, and the equation will become :—

$$h_1 + h = \frac{V^2}{2g} + h_2 + h.$$

It will be seen that if the two terms h are cancelled out, we obtain the original equation, from which we infer that the equation is the same whichever datum is chosen.

In considering the flow of water, however, still another form of energy must be taken into account, one which did not exist in the case of the stone. If the tank of Fig. 21 were fitted with a cover and air were pumped in at a pressure of, say, for example, 10 lb. per sq. in., it is obvious that the speed of the jets would be increased. Clearly, it is of no importance whether air or water is pumped in, as long as the additional pressure of 10 lb. per sq. in. is produced at the level of the free surface. Therefore, in calculating the total energy at any point in a fluid we must take into account the energy corresponding to the pressure.

The head or work stored, in ft.-lb. per lb., which is equivalent to the pressure at any point, is easily



PIPE OF VARYING CROSS-SECTION

Fig. 27. In this example a pipe of varying cross-section is joined to a large tank. At each of the points marked 1 to 5 the total energy ($p/62{\cdot}4+V^2/2g+h$) is the same: this is so if there is no friction.

found from the relationship $\dfrac{p}{62{\cdot}4} = h$, which was developed earlier, and this we will name the pressure head to distinguish it from the potential head. At any point in a liquid there are, then, three possible forms of energy; namely, the static head or energy of position measured by the head above the chosen datum, the energy due to the pressure at the point, and, lastly, the kinetic energy. This last term is quite independent of the direction of motion, for, just as in the case of a ball, the energy given to the ball is $\dfrac{WV^2}{2g}$ ft.-lb., whatever the direction in which it is thrown.

In discussing the total energy at various points in the vessel seen in Fig. 27, it is convenient to measure all pressures above atmospheric, so that the pressures at point (1) in the free surface, and at the pipe outlet (5), are both zero. Now, if open-ended glass tubes are connected into the pipe at various points, the water will rise in each to a height equivalent to the pressure at the point.

At point (1) the pressure head as we have seen is zero, and, as the tank is large in diameter in comparison with the final outlet, the speed of flow will be small. Therefore, the quantity $\dfrac{V_1^2}{2g}$ will be unimportant and can be neglected. The whole of the energy at (1) is, thus, in the form of potential energy and is measured by the distance h_1 above the chosen datum.

At point (5), the open end of the pipe, there is again no pressure, but the kinetic head $\dfrac{V_5^2}{2g}$ is relatively large. Here, we see that there is no

potential energy, as the datum passes through the point. At points (2), (3) and (4) all three forms of energy are present.

Bernoulli's Theorem

If no friction exists to destroy any energy, it follows that the total energy at the various points (1), (2), (3), etc., must be equal. In general

$$\frac{p_1}{62 \cdot 4} + h_1 + \frac{V_1^2}{2g} = \frac{p_2}{62 \cdot 4} + h_2 + \frac{V_2^2}{2g} =$$

corresponding terms for points (3), (4), etc. Once again we see that, if the datum is chosen in a position 1 ft. lower, the equation will still hold as each of the heads h_1, h_2, etc., will be increased by the same amount.

The equation above, relating the total heads at the various points, was published in the year 1738, by a Swiss scientist, David Bernoulli. Its importance to hydraulic engineers is great, as it is fundamental to all their problems on the steady flow of a fluid, provided that there is no friction. We shall see later how to modify the equation to allow for friction. Another point of interest may be observed if reference is again made to Fig. 27. In this particular case, we saw that $\frac{p_1}{62 \cdot 4}$ and $\frac{V_1^2}{2g}$ were both zero. The equation, applied to points (1) and (2), becomes

$$h_1 = \frac{p_2}{62 \cdot 4} + h_2 + \frac{V_2^2}{2g},$$

and the diagram now shows us that the vertical distance from the surface of the water in the gauge-glass at (2) to the level of the water in the tank, is equivalent to $\frac{V_2^2}{2g}$. Similarly, the distance above the level in the glass at point (3) indicates $\frac{V_3^2}{2g}$, and so on. At point (5), the water would not rise inside a gauge-glass, as the pipe is here open to the atmosphere. The energy at point (5) is all in the form of kinetic energy, and is equivalent to the height h_1.

In an earlier paragraph we saw, by analogy with the case of a stone falling under gravity without resistance, that a jet of water emerges from an orifice in a tank at a velocity $V = \sqrt{(2gh)}$ ft. per sec., where h is the head of water. This law was discovered by Torricelli, a physicist of the seventeenth century. We see now that it is merely a special application of Bernoulli's law, corresponding to $h_1 = \frac{V_5^2}{2g}$ of Fig. 27.

We will now study the application of this law to the Venturi meter. This instrument is inserted in a pipe-line, replacing a short length of the pipe, and is used for measuring the rate of flow of water in the pipe-line. The device was named after a renowned physicist of the eighteenth century by its inventor, Clemens Herschel, an American engineer.

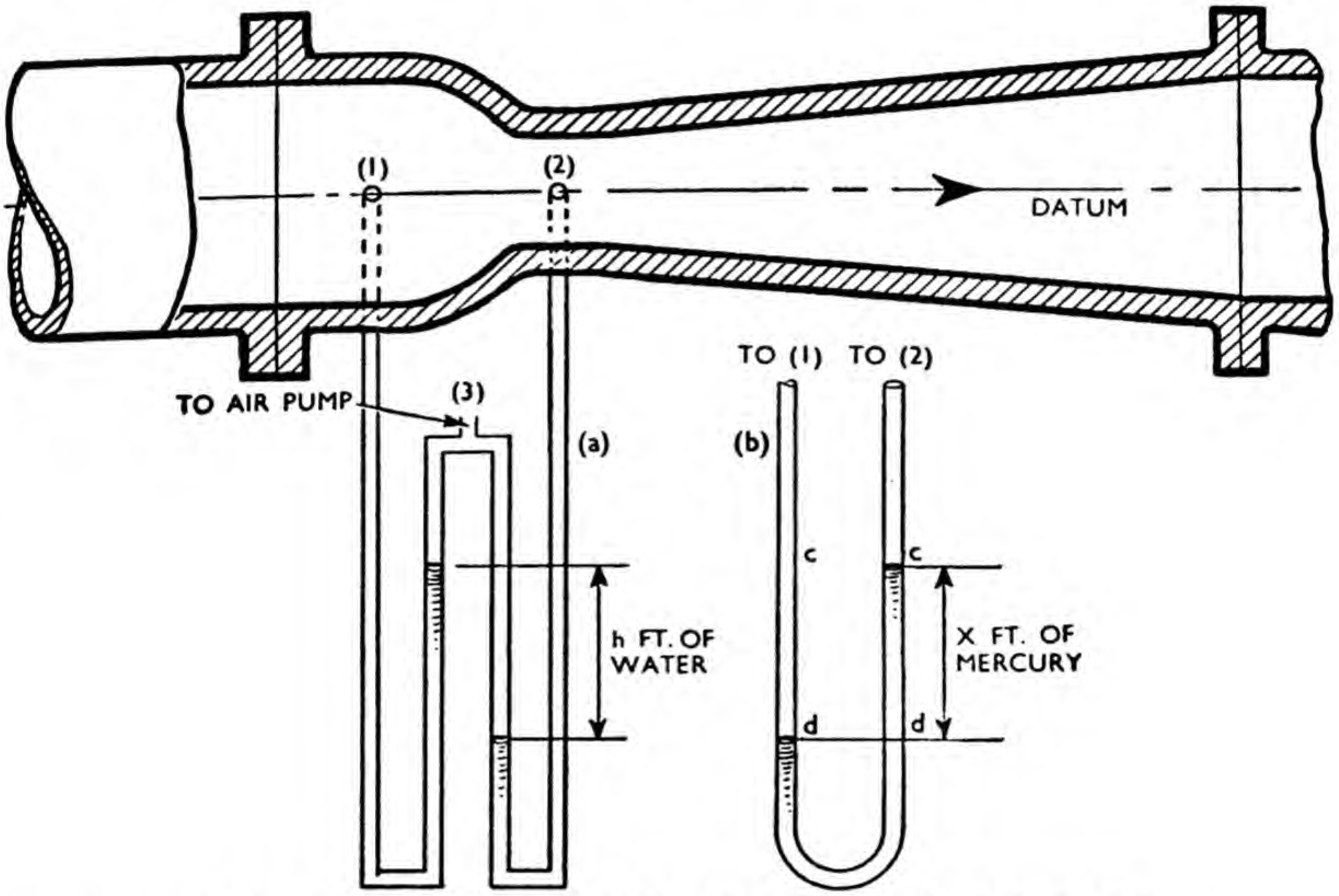
A common form of this meter is seen in Fig. 28, in which the essential parts are the inlet section (1) and the throat or narrow section (2). The long tapered portion which we see on the downstream side is provided to bring the cross-section gradually back to that of the pipe-line with as little loss of head as possible, It will be seen later that a marked loss is incurred here if we suddenly change the section of the pipe.

If we discuss the simplest case, namely, that of a horizontal meter, and we choose the position of the datum at the level of the centre-line, it will be seen that the h terms disappear, and Bernoulli's law as

applied to sections (1) and (2) can be written :—

$$\frac{p_1}{62{\cdot}4}+\frac{V_1{}^2}{2g}=\frac{p_2}{62{\cdot}4}+\frac{V_2{}^2}{2g}.$$

Let us suppose that the areas of the cross-sections at (1) and (2) are known, namely, A_1 and A_2 sq. ft., and that Q is the rate of flow of water in cu. ft. per sec., so that number for any particular meter. We now see why the size of the inlet section was reduced to that at the throat. The water was speeded up from V_1 to V_2 ft. per sec. with a corresponding fall of pressure head, $\left(\frac{p_1 - p_2}{62{\cdot}4}\right)$, which can be measured, and on inserting



PRACTICAL EXAMPLE OF BERNOULLI'S THEOREM

Fig. 28. Practical application of Bernoulli's law is found in the Venturi meter, which we see in section. It is inserted in a pipe-line to measure the rate of flow of water. A change in diameter between the sections (1) and (2) causes an increase of velocity and a corresponding fall of pressure, which depends only on the rate of flow for a given meter. This difference of pressure is measured on a differential gauge.

$V_1 = \dfrac{Q}{A_1}$, and $V_2 = \dfrac{Q}{A_2}$ ft. per sec., then it will be noticed that the equation may be rewritten as:

$$\frac{p_1 - p_2}{62{\cdot}4}=\frac{V_2{}^2 - V_1{}^2}{2g}$$

$$=\frac{Q^2}{2g}\left(\frac{1}{A_2{}^2}-\frac{1}{A_1{}^2}\right),$$

where $\left(\dfrac{\dfrac{1}{A_2{}^2}-\dfrac{1}{A_1{}^2}}{2g}\right)$ is a simple it in the equation above, the value of Q can be found.

The equation shows us that we do not require the actual pressures at (1) and (2), but only their difference, which is conveniently measured by means of a differential gauge. For relatively small differences, a gauge of the type shown in Fig. 28(a) is used, in which air is pumped into the top of the gauge at (3) by means of a bicycle pump, to bring the

water surfaces in the two tubes to suitable positions so that the difference can be measured. If a little greater pressure of air is applied, both levels will be forced down by the same amount, the value of h ft. being unchanged.

If rather larger differences of pressure exist between the inlet and the throat of the meter, a differential gauge of type (b) is used. Here a simple U-tube is partially filled with mercury, the tubes above the mercury being filled with water. Clearly the two columns of water above the level of *cc* balance, while the mercury below *dd* is in equilibrium. The difference of head is now the difference between x ft. of mercury in the right-hand limb, and x ft. of water in the left, and as mercury is 13·6 times as heavy as water, the difference of head expressed as ft. of water is $13{\cdot}6x - x$, or $12{\cdot}6x$ ft.

These Venturi meters are of wide application, the smallest, used in biological tests, having an inlet diameter only a fraction of an inch, while the largest is probably that used for measuring the supply to a power station in Pennsylvania, U.S.A. This meter has an inlet of 41 × 44 ft., tapering in to a circular throat 24 ft. in diameter, its normal capacity being 8,000 cu. ft. per sec. Some idea of the size of this meter can be gained if we think of a river 5 ft. deep and 400 ft. wide, flowing at 4 ft. per sec., which carries about the same amount of water.

At first sight it is curious to find that the larger pressure occurs at the larger section, but we find it reasonable if we argue that some of the pressure energy must be utilized in speeding up the water from V_1 to V_2 ft. per sec. It then follows that p_1 exceeds p_2.

Interesting Example

Another interesting example of the application of Bernoulli's theorem may be found in the case of a stream passing between the

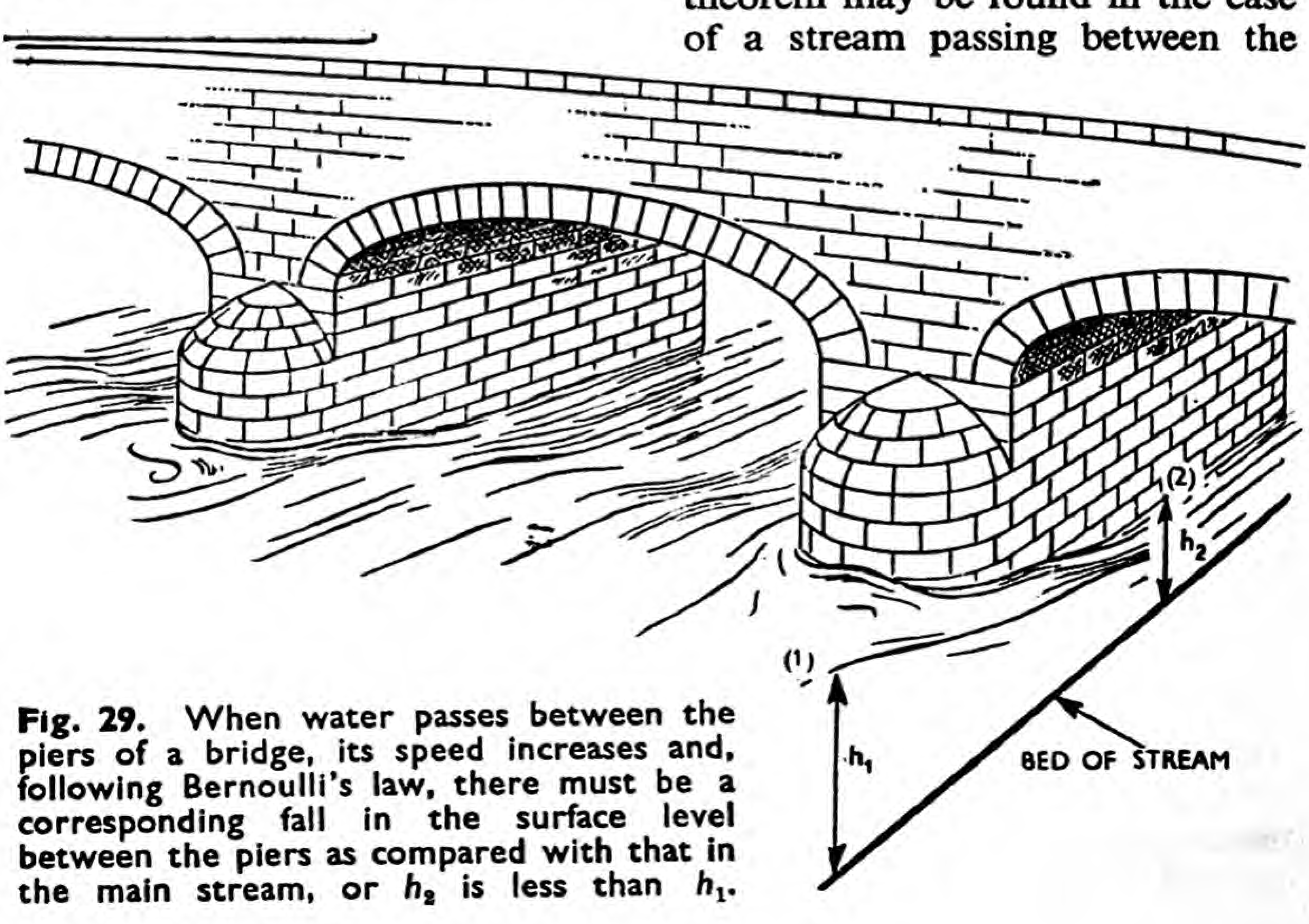


Fig. 29. When water passes between the piers of a bridge, its speed increases and, following Bernoulli's law, there must be a corresponding fall in the surface level between the piers as compared with that in the main stream, or h_2 is less than h_1.

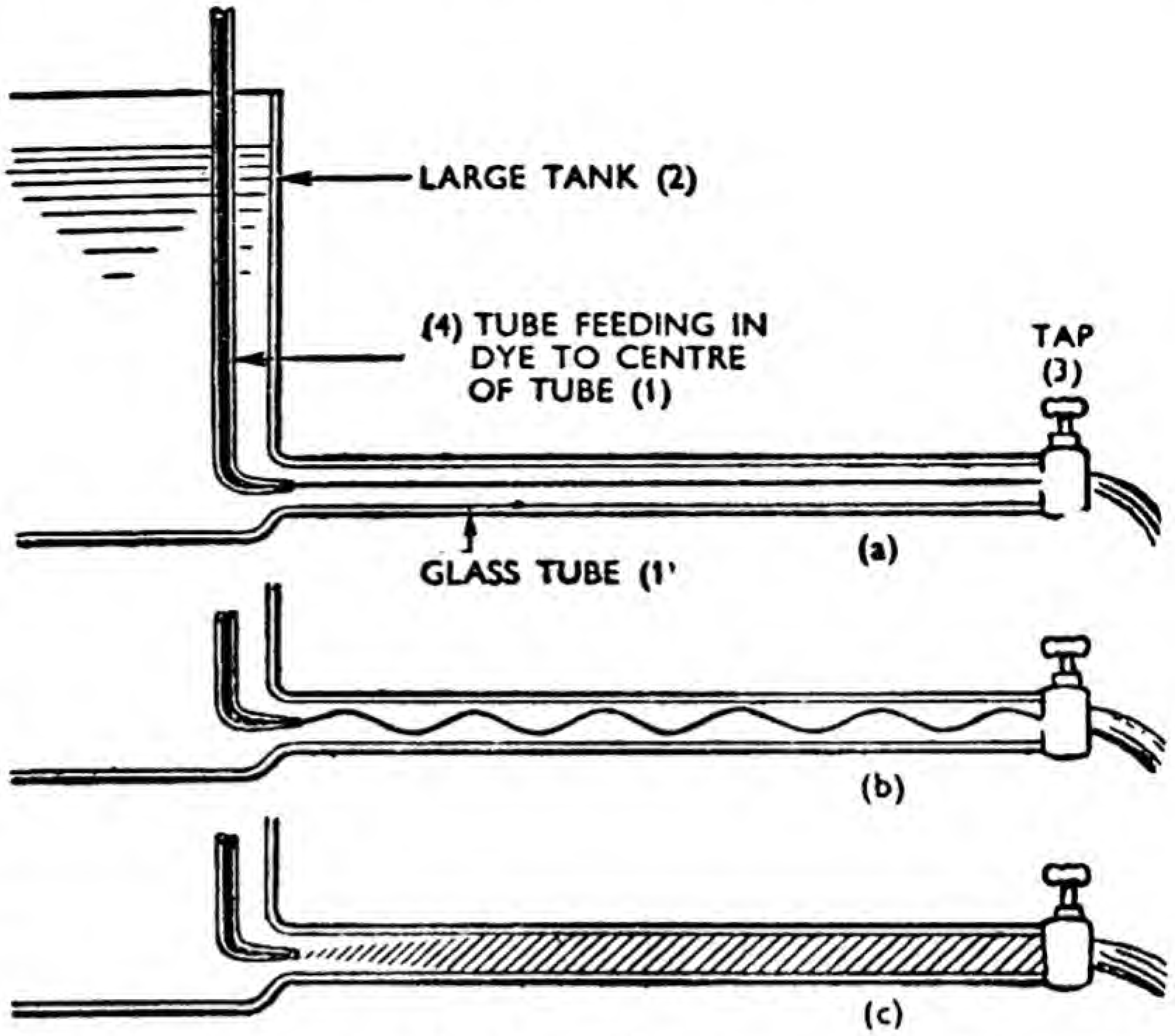


Fig. 30. Water from the tank (2) is passed through the glass tube (1), the speed being controlled by the tap (3). If we inject coloured fluid by means of the tube (4) into pipe (1), we find that at low speeds the jet remains central. This shows us that there are no cross-currents. At a particular speed, named the critical, the jet wavers as at (*b*), and if we increase the speed still further we find that the jet breaks up, colouring all the water in the pipe as indicated at (c).

piers of a bridge, as in Fig. 29. Here it will be seen that the space between the piers at (2) is narrower than that between the centres of the piers at (1), and at first it might be expected that the water would bank up between the piers. However, if the bed of the stream is taken as datum, it will be seen that the potential heads h_1 and h_2 are also the depths of water at the two points under consideration, and, as these points are in the surface, the pressure heads may be omitted, both pressures being atmospheric.

In this example, Bernoulli's equation,

$$\frac{p_1}{62 \cdot 4} + h_1 + \frac{V_1^2}{2g} = \frac{p_2}{62 \cdot 4} + h_2 + \frac{V_2^2}{2g}$$

can be reduced to

$$h_1 + \frac{V_1^2}{2g} = h_2 + \frac{V_2^2}{2g}.$$

We should naturally expect that the velocity at the narrower section would be greater than that at (1), and the above equation tells us that in this case h_2 will be less than h_1, so that a drop in level of the surface will occur between the piers.

The choke tube of a carburetter is another example of the application of this law.

Losses in Pipes

It is known from experience that power is required to force water through a pipe. This is true even when the pipe is horizontal, when no work has to be done in lifting the water against gravity. During the passage of water along a pipe, certain losses occur which are of two types, frictional and shock. We will discuss the frictional losses first as these are usually the more important. It will soon be noticed that the frictional loss depends on the motion of the particles of the water, and this in turn depends to some extent on the speed. This action can most easily be discussed by describing the original experiments on this subject, made by Osborne Reynolds in 1883, at the University of Manchester.

Fig. 30 shows, diagrammatically,

a glass tube (1) with a well-rounded entrance, supplied with water from a tank (2), and provided with a tap (3) to control the rate of flow. Let us imagine that a jet of coloured fluid, for example, red dye, is injected into the entrance of the glass pipe by means of the fine-bore tube (4).

At low speeds of the water, the coloured jet remains central in the glass tube, showing that there are no cross-currents, or in other words, all the particles of water are moving in straight lines parallel to the length of the glass tube. This type of motion, which is called laminar, is indicated at (a). As the control valve in the glass tube is opened up, a point is reached at which the coloured jet suddenly begins to waver like a flag fluttering in a breeze, as indicated at (b). When the flow is increased a little more, the jet of coloured fluid breaks up, as shown at (c), and diffuses throughout the whole of the water in the glass tube.

This shows us that cross-currents exist, and as the motion of the particles of water is complex, we speak of this type of flow as turbulent. Motion of the type which we saw at (b) must be regarded as transitional only between that at (a) and (c). The change-over from laminar to turbulent flow takes place round about a velocity which we term the critical velocity, and is approximately 1 ft. per sec. in a ¼-in. bore smooth pipe, such as drawn brass tube. Tests show us that this velocity diminishes as the size of pipe increases, and it would be ¼ ft. per sec. in a 1-in. diameter pipe. In town mains and pumping systems, where the pipe diameter is seldom less than 1 in. in diameter, velocities are normally about 7 to 10 ft. per sec. These velocities exceed the critical for the pipe sizes used, so that the flow is turbulent.

We will now inquire as to how the friction loss in a pipe depends on the velocity of flow. Fig. 31 shows us the arrangement required for a test on this subject. Water is forced through a long pipe and the difference of head between two points spaced as far apart as possible is read from a differential gauge. The valve seen at the outlet end of the pipe enables us to control the rate of flow.

To assist us in discussing this

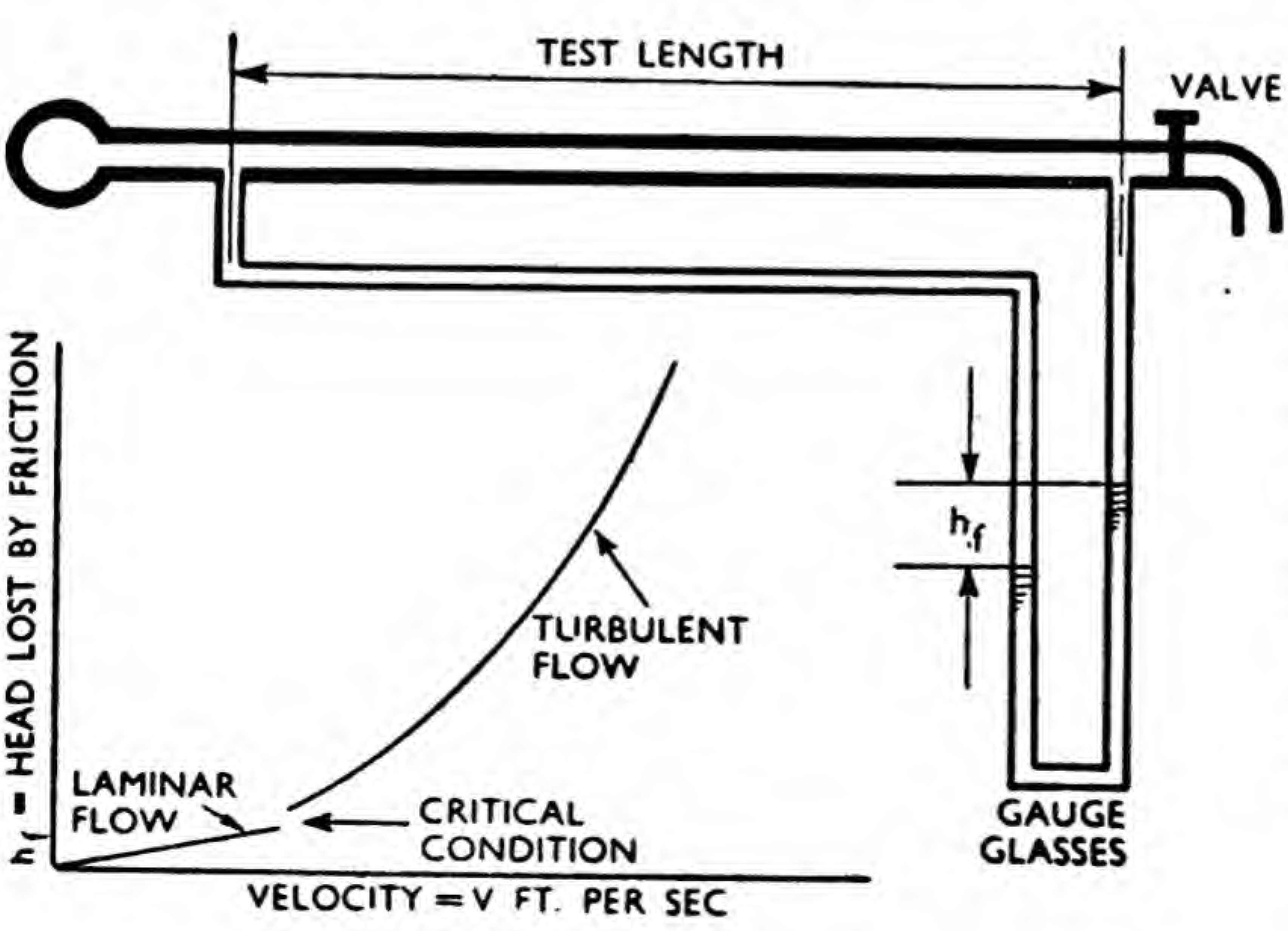


Fig. 31. If we allow water to flow at various speeds through a pipe and measure the head lost in overcoming friction, we find that while the flow is laminar, the friction depends directly on the speed. Under turbulent flow conditions, the friction loss depends nearly on the square of the speed, as indicated by the curved portion of the graph.

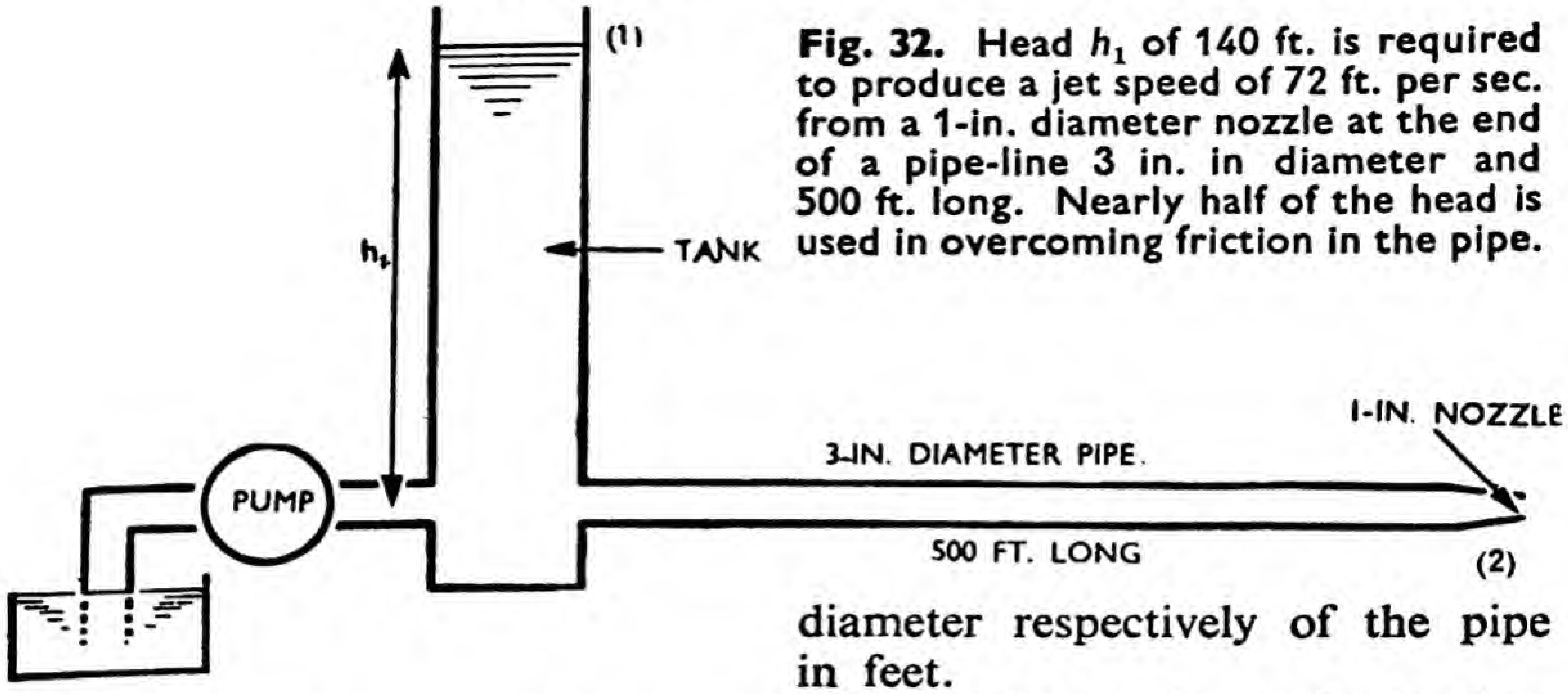


Fig. 32. Head h_1 of 140 ft. is required to produce a jet speed of 72 ft. per sec. from a 1-in. diameter nozzle at the end of a pipe-line 3 in. in diameter and 500 ft. long. Nearly half of the head is used in overcoming friction in the pipe.

question, let us imagine that during the test the rate of flow is steadily increased, and that a graph of the head lost by friction h_f, as read from the gauge, is plotted against the velocity of flow V ft. per sec. The type of graph which will be obtained from the test is also shown in Fig. 31. At low speeds, under laminar flow conditions, we see that a straight-line graph is obtained, but, on opening up the valve, we find that the test points, corresponding to turbulent flow, rise on a curve.

Pipe Roughness

Under these conditions, the loss of head depends on V^n, where n varies between 1·8 and 1·9, depending on the roughness of the inside of the pipe. We know that pipes tend to encrust with age, and that this has the effect of increasing the value of n, and to allow for this it is common practice to make our calculations on the assumption that h_f depends on V^2. Tests also show us that the loss of head may be expressed by the formula :—

$h_f = \frac{4fl}{d} \times \frac{V^2}{2g}$ ft., where f is a coefficient or number, found by test, and l and d are the length and diameter respectively of the pipe in feet.

To assist us in understanding how this formula is applied, let us find what head must be developed by a fire-engine to pump water at 8 ft. per sec. along a 500-ft. length of horizontal pipe, 3 in. or 0·25 ft. in diameter, and eject it through a nozzle 1 in. in diameter (Fig. 32), on the assumption that f, found from a test on a similar pipe, is 0·0075. Here l is 500 ft. and d is 0·25 ft., so that :—

$$hf = \frac{4fl}{d} \times \frac{V^2}{2g} = \frac{4 \times 0{\cdot}0075 \times 500}{0{\cdot}25} \times \frac{8^2}{2 \times 32{\cdot}2} = 59{\cdot}6 \text{ ft.}$$

The head necessary to eject the water from the nozzle must next be found, namely $\frac{V_2^2}{2g}$, where V_2 is the velocity at the nozzle outlet. Now the same quantity of water passes through both the pipe and the nozzle, so that :—

$$\frac{\pi \times 3^2 \times 8}{4 \times 144} = \frac{\pi \times 1^2 \times V_2}{4 \times 144}$$

which gives us :—

$V_2 = 3^2 \times 8 = 72$ ft. per sec., and this requires a head at the nozzle of $\frac{72^2}{2g}$ or 80·5 ft.

For ease of explanation, the pump of Fig. 32 is shown as discharging into a large tank which

in turn feeds the pipe-line. It will be seen that the head in this tank has to overcome the frictional resistance of the pipe and also supply the head necessary to drive the water out from the nozzle, 59·6 ft. and 80·5 ft. respectively, with a total of 140·1 ft. To allow for the head lost in friction, Bernoulli's equation referring to points (1) and (2) must be written as :—

$$\frac{p_1}{62\cdot4} + h_1 + \frac{V_1^2}{2g} = \frac{p_2}{62\cdot4} + h_2 + \frac{V_2^2}{2g}$$

$+ h_f$, and as V_1 in the tank is small, this becomes :—

$$0 + h_1 + 0 = 0 + 0 + 80\cdot5 + 59\cdot6.$$

We see that the nature of the equation is completely changed. Previously the equation was independent of the direction of flow, but if any loss occurs we have to take the direction into account. In our problem, 59·6 ft.-lb. of energy are lost per lb. of water in passing from (1) to (2), and to allow for this we must add 59·6 ft. to the measured conditions at section (2).

The result, however, does not depend on the direction in which the jet plays. If the nozzle were used in a position 20 ft. above the datum, which is again chosen through the centre of the inlet end of the pipe, the equation would have to be modified to read :—

$$h_1 = \frac{p_2}{62\cdot4} + h_2 + \frac{V_2^2}{2g} + h_f = 0 + 20 + \frac{72^2}{2g} + 59\cdot6 = 160\cdot1 \text{ ft.}$$

This would mean that the head at the inlet would have been increased by an amount equal to the vertical lift, 20 ft.

If we now wish to determine the horse-power required in this case to force the water along the pipe, the rate of flow must first be found. The cross-sectional area of the pipe is :—

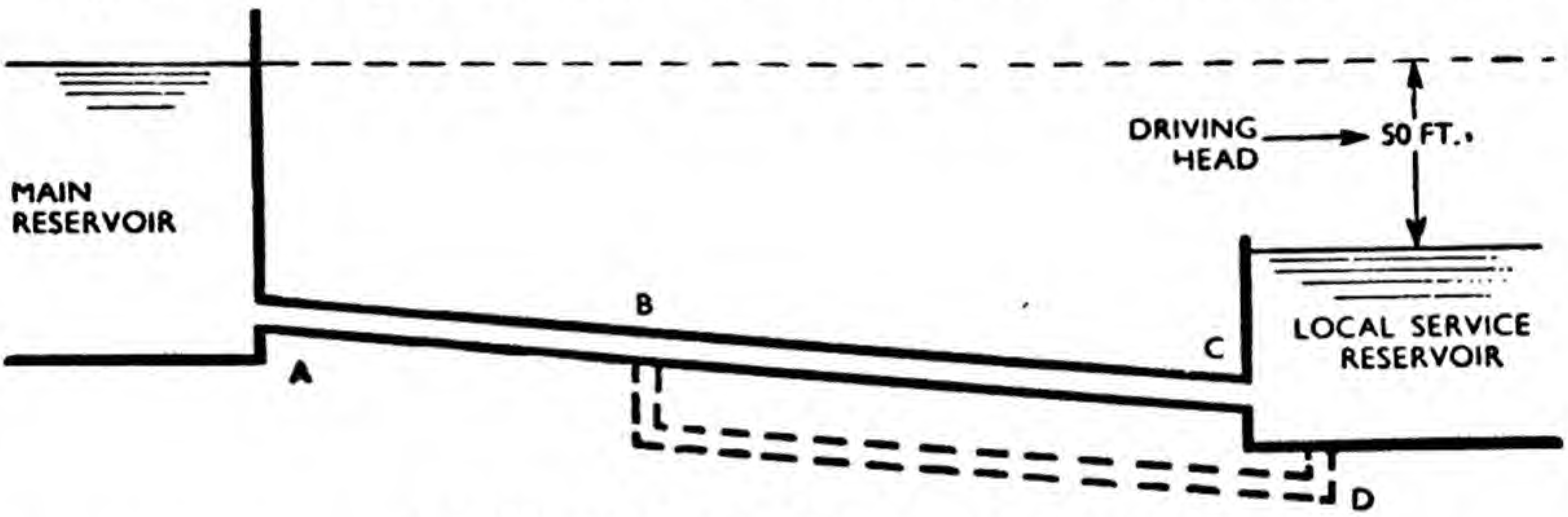
$\frac{\pi}{4} \times \left(\frac{3}{12}\right)^2$ sq. ft., and, with a velocity of 8 ft. per sec., the quantity flowing is :—

$$62\cdot4 \times 8 \times \frac{\pi}{4} \times \left(\frac{3}{12}\right)^2$$

or 24·5 lb. per sec. The horse-power required to pump this quantity against a head of 160·1 ft. is :—

$$\frac{24\cdot5 \times 160\cdot1}{550} = 7\cdot14.$$

As the question of the rate of flow through a pipe system is of



REDUCTION OF PIPE RESISTANCE

Fig. 33. In this diagram, different scales are used vertically and horizontally. If we neglect all losses except pipe friction, we see that the difference of levels in the reservoirs overcomes friction of the pipe system. In our first problem, one pipe only, *AC*, is used, and on putting a second pipe, *BD*, in parallel with part of *AC*, we see that 50-ft. head overcomes resistance of *AB* and *BC* or *AB* and *BD*.

such importance in practice, we will discuss a further example. Let us examine the problem of the flow through a main 2 ft. in diameter and 5,000 ft. long, connecting a main-storage reservoir in the country to the local-service reservoir of a town, as in Fig. 33. Here it will be found that the quantity $\frac{V^2}{2g}$ of the water leaving the pipe is quite small in comparison with the loss of head caused by friction, and may be neglected, unlike the term $\frac{V_2^2}{2g}$, the kinetic head at the nozzle of the last problem.

Rate of Flow

We may regard the whole of the difference of level of the water surfaces in the two reservoirs as used up in overcoming frictional resistance, as long as the pipe is free from any pronounced bends. If it is assumed that the value of f is again 0·0075 and that h is 50 ft., we have :—

$$50 = h_f = \frac{4 \times 0{\cdot}0075 \times 5{,}000}{2} \times \frac{V^2}{2g}$$

which gives us :—$V = 6{\cdot}54$ ft. per sec.

The corresponding value of $\frac{V^2}{2g}$ is $\frac{6{\cdot}54^2}{64{\cdot}4}$, or 0·66 ft., which has been neglected in comparison with the driving head of 50 ft. The rate of flow due to a speed of 6·54 ft. per sec. through a pipe 2 ft. in diameter is :—$\frac{\pi}{4} \times 2^2 \times 6{\cdot}54$, or 20·5 cu. ft. per sec. An interesting problem arises if we put a second pipe of 2-ft. diameter and 5,000 ft. in length into service connecting the two reservoirs, so that it operates in parallel with the first pipe, when we see that the 50-ft. difference of level also acts as its driving head against friction. This second pipe will increase the total flow by a further 20·5 cu. ft. per sec.

It may happen in practice that one pipe will not carry all the water required, but that it is impracticable to put a second pipe alongside the first over the whole length AC of Fig. 33. As an example, we will suppose that a main 1·5 ft. in diameter is connected to the pipe AC at B, and taken to the lower reservoir, the distance AB being 2,000 ft. while BC is 3,000 ft. in length. The driving head of 50 ft. is lost in friction along the pipes AB and BC, or along AB and the new pipe, referred to as BD.

If we use the symbols V_{AB}, V_{BC} and V_{BD} to denote the velocities in the pipes AB, BC and BD respectively, and apply our equation $h_f = \frac{4 \times f \times l \times V^2}{d \times 2 \times g}$ to each pipe, we obtain

$$50 = \frac{4 \times 0{\cdot}0075 \times 2{,}000 \times V_{AB}^2}{2 \times 2 \times g} + \frac{4 \times 0{\cdot}0075 \times 3{,}000 \times V_{BC}^2}{2 \times 2 \times g},$$

which we can reduce to

$$V_{AB}^2 + 1{\cdot}5 V_{BC}^2 = 107{\cdot}3,$$

named equation (1), and

$$50 = \frac{4 \times 0{\cdot}0075 \times 2{,}000 \times V_{AB}^2}{2 \times 2 \times g} + \frac{4 \times 0{\cdot}0075 \times 3{,}000 \times V_{BD}^2}{1{\cdot}5 \times 2 \times g},$$

which becomes

$$V_{AB}^2 + 2V_{BD}^2 = 107{\cdot}3,$$

named equation (2). We have also a third equation, which arises from the fact that the quantity flowing per sec. in AB is the sum of the quantities passing through BC and BD, so that :—

$$\pi \times \frac{2^2}{4} \times V_{AB} = \pi \times \frac{2^2}{4} \times V_{BC} + \pi \times \frac{1{\cdot}5^2}{4} \times V_{BD}, \text{ or}$$

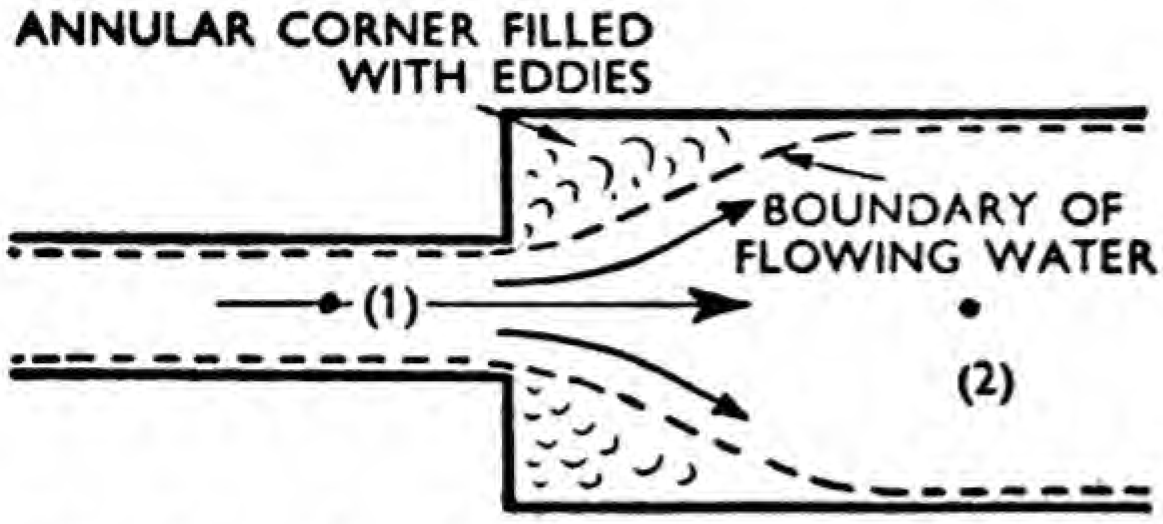


Fig. 34. Sudden enlargement of the pipe produces a so-called shock loss. The water tends to flow within the dotted boundary, leaving an annular pocket filled with eddying water.

$4 \times V_{AB} = 4 \times V_{BC} + 2{\cdot}25 \times V_{BD}$, which we will name equation (3).

These three equations are most easily solved by trial and error. Let us try, for example, the solution $V_{AB} = 7$ ft. per sec. On substituting this in equation (1), we find that $V_{BC} = 6{\cdot}23$ ft. per sec. while equation (2) gives $V_{BD} = 5{\cdot}39$ ft. per sec. If we now substitute these values of V_{BC} and V_{BD} in equation (3), we find that V_{AB} is 9·26 ft. per sec. instead of the assumed value of 7 ft. per sec. A few trials will give us the correct value of 8·0 ft. per sec. for V_{AB}, from which we see that the quantity flowing per sec. through AB is $\frac{\pi}{4} \times 2^2 \times 8$, or 25·12 cu. ft. as compared with 20·5 cu. ft. when a single 2-ft. pipe was used over the whole length.

Shock Losses

Wherever there is a sudden change in the area of cross-section of a pipe, as in Fig. 34, we get what are termed shock losses. The water coming from the smaller pipe at V_1 ft. per sec. cannot turn suddenly at right angles so as to fill the corner completely, but follows a boundary such as that shown dotted. The stream gradually opens out until it fills the larger pipe. The annular space or corner between the body of the stream and the larger pipe is filled with dead water, which has numerous eddies in it of the type which were discussed in connexion with the Venturi meter.

These, it was seen, mean a loss of energy, a fact which we can explain by a simple analogy. If a man in running through a doorway hits his shoulder against the door-post, the impact will cause him to rotate slightly, and some energy will be used up in this turning motion. A simple method of determining the loss involved at a sudden enlargement, is to imagine a man walking in the direction of flow at V_2 ft. per sec. alongside the pipe which we must suppose to be transparent. He will see a particle of matter suspended in the water in the smaller pipe, moving, apparently, at $(V_1 - V_2)$ ft. per sec., while the particle will appear to be at rest a moment later when it reaches point (2), so that he sees the speed of the water changed from $(V_1 - V_2)$ to 0 ft. per sec.

Now it is known that the energy which has to be removed from a stone moving at V ft. per sec., in order to bring it to rest, is its kinetic energy, or $\frac{WV^2}{2g}$ ft.-lb., where W is its weight. In our

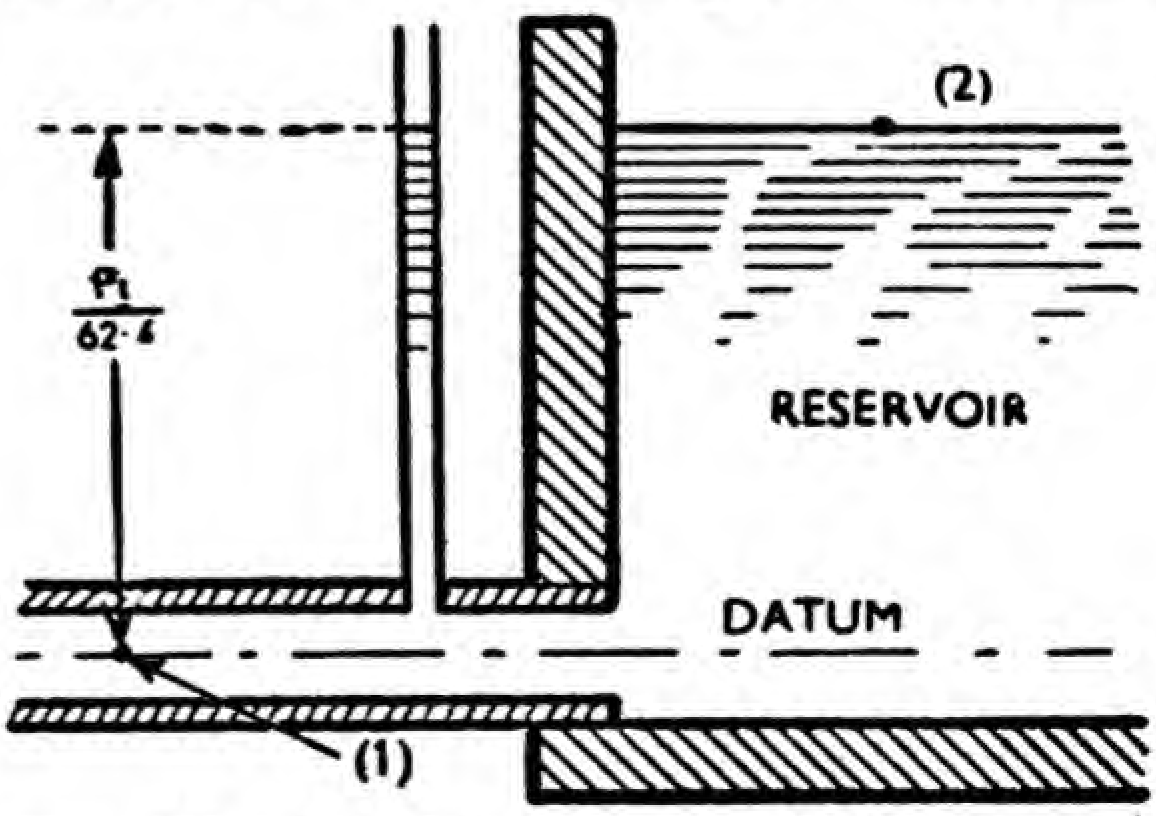


Fig. 35. Level in a gauge glass close to end of the pipe entering a reservoir is the same as in the reservoir itself.

problem, the work which has to be removed from each lb. of water is

$$1 \text{ lb.} \times \frac{(V_1 - V_2)^2}{2g} \text{ ft.-lb., so that}$$

the loss of head is $\frac{(V_1 - V_2)^2}{2g}$ ft.

We must be careful not to confuse this loss of head with the difference of pressure head between the two sections (1) and (2). If a smooth pipe having a contour such as that followed by the dotted curves were used, no corner would be left in which eddies could be formed, and as the losses would now be small, we could write Bernoulli's law as :—

$$\frac{p_1}{62{\cdot}4} + \frac{V_1^2}{2g} = \frac{p_2}{62{\cdot}4} + \frac{V_2^2}{2g},$$

choosing the horizontal centre-line as datum. The theoretical increase of pressure is reduced in our case, however, by the shock loss, so that :—

$$\frac{p_1}{62{\cdot}4} + \frac{V_1^2}{2g} = \frac{p_2}{62{\cdot}4} + \frac{V_2^2}{2g} + \frac{(V_1 - V_2^2)}{2g} \text{ or } \frac{p_2 - p_1}{62{\cdot}4}$$

$$= \frac{(V_1^2 - V_2^2)}{2g} - \frac{(V_1 - V_2)^2}{2g}.$$

Finding Level Attained

We can apply this result to find the level to which the water will rise inside a gauge glass connected into a pipe close to the point where it empties into a large reservoir as in Fig. 35. Equating the energies per lb. of water at points (1) and (2), and allowing for the shock loss :—

$$\frac{p_1}{62{\cdot}4} + \frac{V_1^2}{2g} + h_1$$

$$= \frac{p_2}{62{\cdot}4} + \frac{V_2^2}{2g} + h_2 + \frac{(V_1 - V_2)^2}{2g}.$$

Now (1) is on the chosen datum and h_1, therefore, is zero, while p_2 is atmospheric and is counted as zero. Again, V_2 in the reservoir is very small, so that the equation reduces to :—

$$\frac{p_1}{62{\cdot}4} + \frac{V_1^2}{2g} = h_2 + \frac{V_1^2}{2g},$$

or :—

$$\frac{p_1}{62{\cdot}4} = h_2.$$

This means that the water in the gauge glass will be level with that in the reservoir.

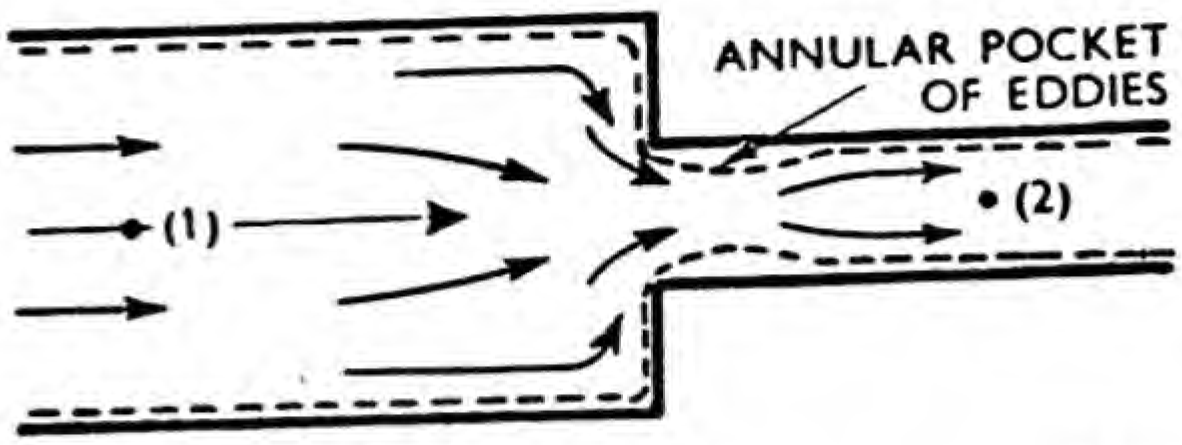


Fig. 36. Conditions of flow at a sudden contraction in a pipe are quite different from those at an enlargement. Eddies are formed in the smaller pipe, and the loss of head is about $\frac{1}{2} \times V_2^2/2g$ ft.

Fig. 36 shows us that an entirely different set of conditions arises at a sudden contraction of the section of a pipe as compared with an enlargement. The pocket of eddies is formed in the smaller pipe, and tests show us that the loss of head is about $\frac{1}{2} \times \frac{V_2^2}{2g}$ ft., which is the shock loss at the inlet to the smaller pipe.

We now have the data necessary to enable us to examine more fully the flow through a uniform pipe joining two tanks, as seen in Fig. 37. The difference of level in the tanks overcomes the losses at the inlet to the pipe, along its length and at the exit, that is :—

$$\frac{1}{2} \times \frac{V^2}{2g} + \frac{4fl}{d} \times \frac{V^2}{2g} + \frac{V^2}{2g}.$$

The pressure just inside the inlet end of the pipe will be less than that

corresponding to the depth of water in the tank by $1\frac{1}{2} \times \frac{V^2}{2g}$ ft.

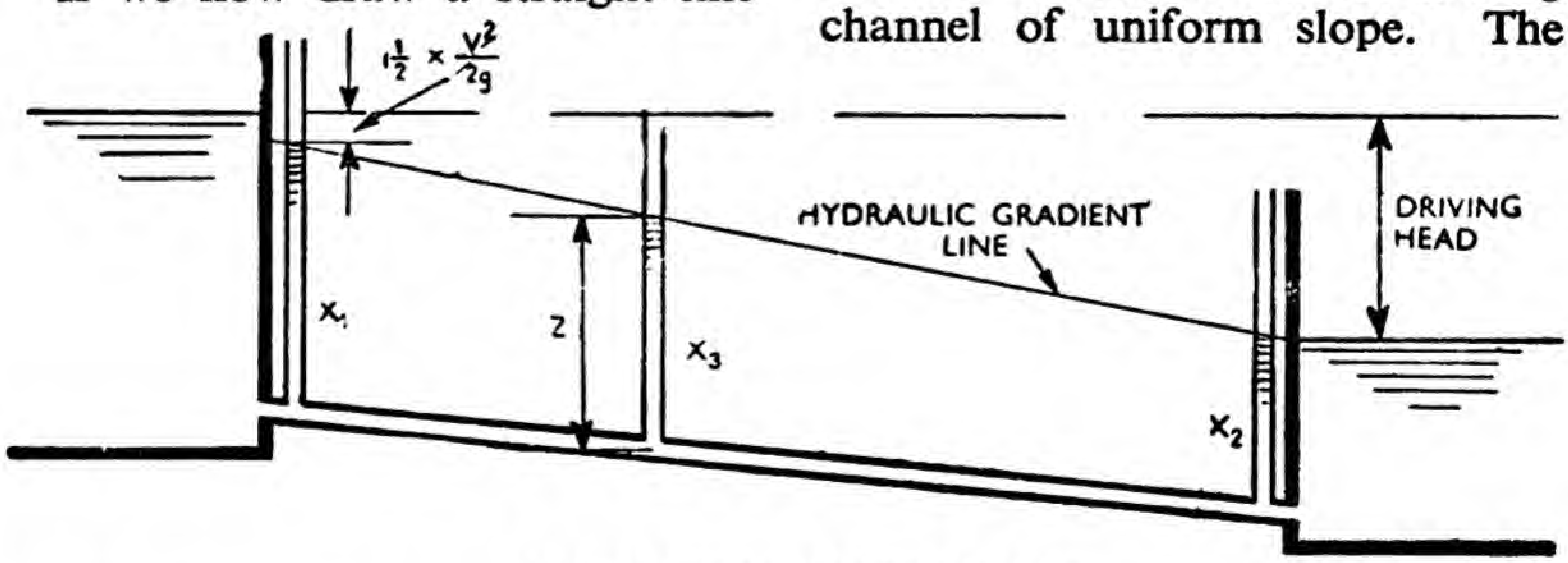
It is important for us to realize the true meaning of this factor, $1\frac{1}{2}$. To set the water in the pipe into motion needs a head of $\frac{V^2}{2g}$ ft. This is simply a conversion of energy from the potential to the kinetic form, and does not involve any loss. There is, however, a loss of $\frac{1}{2} \times \frac{V^2}{2g}$ ft. head, the shock loss at the entrance to the pipe, so that the water level in a gauge-glass connected in the pipe close to the inlet is $1\frac{1}{2} \times \frac{V^2}{2g}$ ft. below that in the tank. At the exit end, as was seen above, the level in the gauge glass will be the same as that in the lower tank. The difference of levels of the water in the gauge glasses X_1 and X_2 at the ends of the pipe-line of Fig. 37, is the loss in friction along the length of the pipe, and, if the pipe is of uniform bore, this loss will be uniformly distributed along its length.

If we now draw a straight line joining the surfaces in X_1 and X_2, named the hydraulic gradient line, we see that it marks the level to which the water will rise in any other gauge glass, such as X_3, where the pressure head in the main will exceed atmospheric by the head Z ft.

In Fig. 38, we assume that the ground rises between the reservoirs, and to keep down the cost of excavation of the pipe trench, the pipe is laid at some points above the hydraulic gradient line. The height Y shows us that the pressure at this point is Y ft. of water head below atmospheric. If this height exceeds about 24 ft., air in solution in the water will be released and will rise to the highest point in the pipe-line, reducing the area of section of the pipe available for the passage of water, and, therefore, the rate of flow.

Flow in Channels

The laws of friction for open channels and for pipes which do not flow full of water are, for the most part, beyond the scope of this book. One important case which we can discuss is that of a long channel of uniform slope. The



ALL LOSSES CONSIDERED

Fig. 37. Here, as distinct from the case which we examined in Fig. 33, all losses are taken into account. These include the inlet loss $\frac{1}{2} \times V^2/2g$ ft., the friction loss h_f, and the exit loss $V^2/2g$. The level in the glass X_1 is $1\frac{1}{2} \times V^2/2g$ below that in the upper reservoir, while that in glass X_2 is at the level of the surface in the lower reservoir. At any intermediate point, the water will rise in a gauge glass X_3 to the position of the hydraulic gradient line.

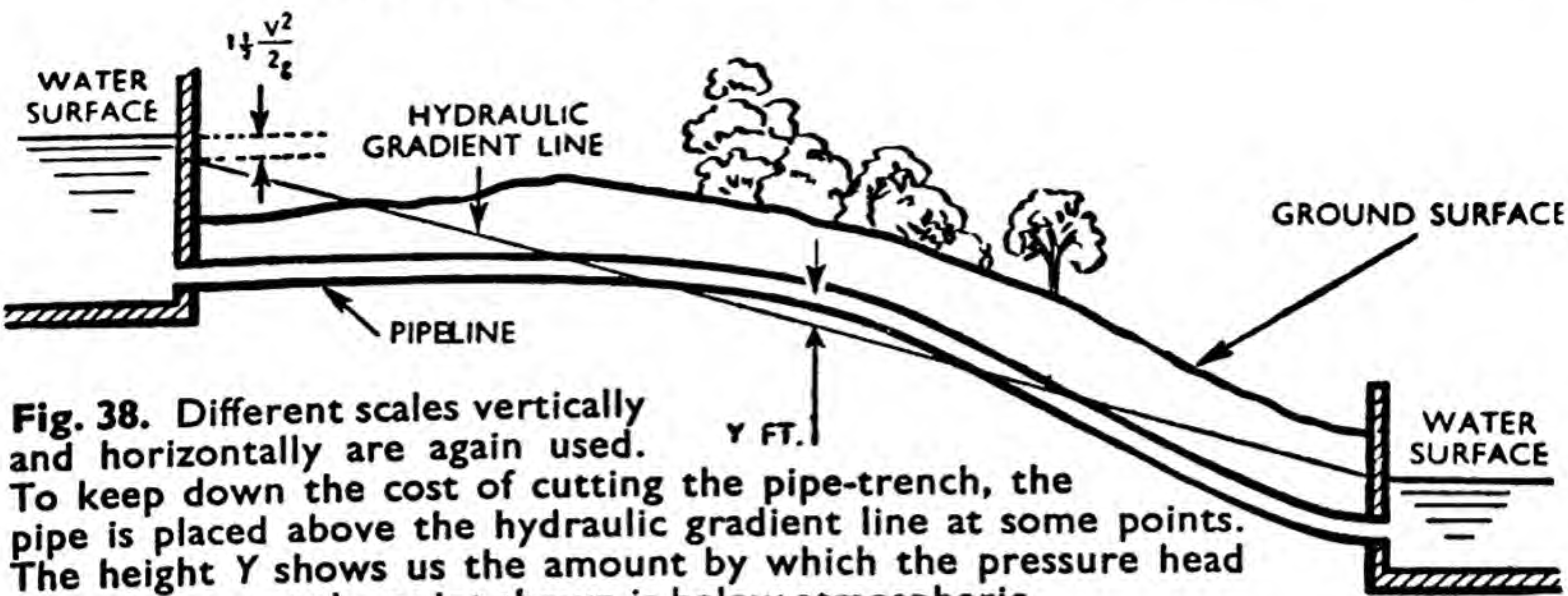


Fig. 38. Different scales vertically and horizontally are again used. To keep down the cost of cutting the pipe-trench, the pipe is placed above the hydraulic gradient line at some points. The height Y shows us the amount by which the pressure head at the point shown is below atmospheric.

conditions of flow can be judged from a simple analogy. A cyclist arrives at the top of a long hill of uniform slope, and, on starting to free-wheel, he will gradually move faster and faster until he reaches a steady speed. If the slope of the hill is 1 in 10, then, over a length of 10 ft., the rider will drop vertically through 1 ft. The work done on him by gravity will be $W \times 1$ ft.-lb., where W is his weight in lb., and this will just counterbalance the work done against wind resistance over the 10-ft. length. This resistance depends on the square of the speed, and may be expressed as KV^2 lb., where K is a number determined by test.

Now we know that work is equal to force multiplied by distance, so that we may write, $W \times 1 = KV^2 \times 10$ ft.-lb., and this gives a definite value of V if W and K are known. It will change only if the slope of the hill is altered.

By analogy, the water in a channel of fixed slope and roughness of wall, will move at a uniform velocity, and, if the width is constant, the depth of water must be uniform. Experiment shows that the speed of the water is given by :—

$$V = C\sqrt{\frac{A \times i}{P}} \text{ ft. per sec.,}$$

where C is a coefficient depending on the nature of the surface of the channel, i is the slope of the channel, A the cross-sectional area of the stream in sq. ft. and P the length in ft. of that part of the circumference of the section of the channel which is in contact with the water.

A simple example will help to make this clear. A long rectangular channel 10 ft. in width carries water at a depth of 5 ft. Let us find how much water flows per sec. if C is 100. Fig. 39 shows us that $A = (10 \times 5)$ sq. ft., $P = 5 + 10 + 5 = 20$ ft., so that if the slope i is 1 in 500,

$$V = 100\sqrt{\frac{10 \times 5 \times 1}{20 \times 500}} = 7{\cdot}07 \text{ ft.}$$

per sec., which gives us

$Q = 7{\cdot}07 \times 50 = 345$ cu. ft. per sec.

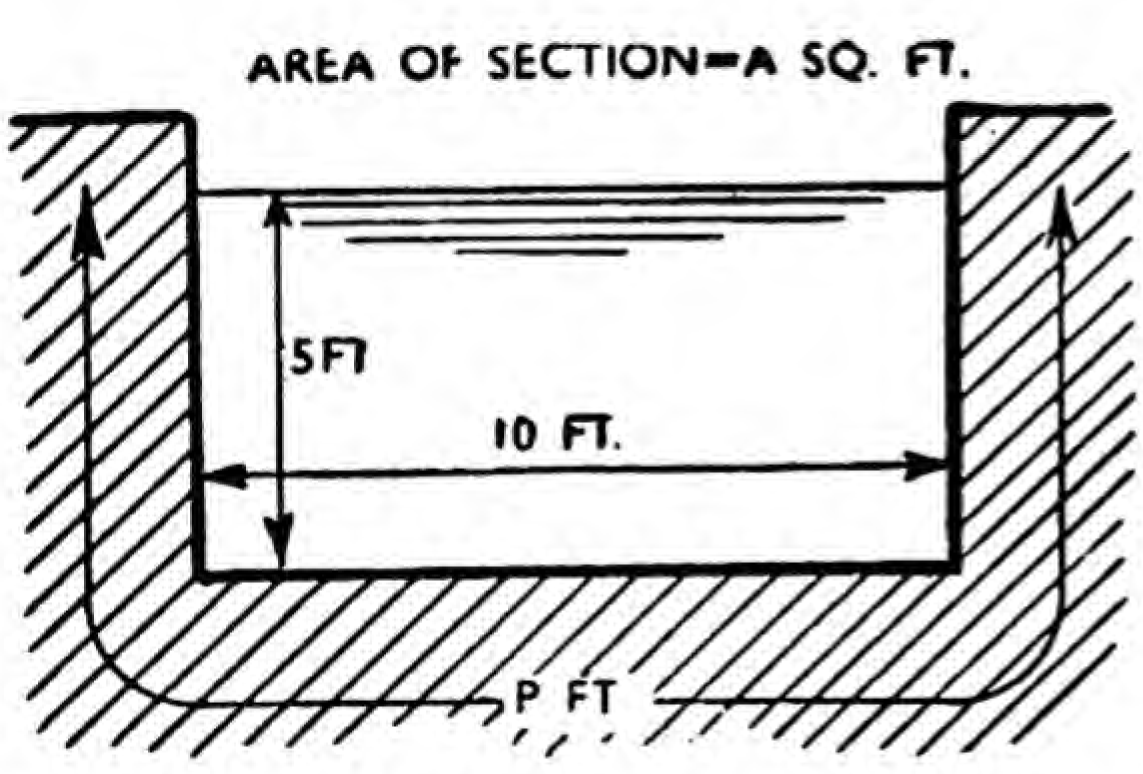


Fig. 39. Length P of the portion of the circumference of a section of a water course which is in contact with the fluid, is named the wetted perimeter.

CHAPTER 12

HYDRAULIC MACHINERY

HYDRAULIC TRANSMISSION. HYDRAULIC PRESS. ACCUMULATORS. LIFTS AND CRANES. HYDRAULIC JACKS. SUCTION AND FORCE PUMPS. THREE-THROW RECIPROCATING PUMPS. WATER WHEELS. IMPULSE TURBINES. PELTON WHEEL. REACTION TURBINE. FRANCIS MACHINES. GOVERNOR CHANGING ANGLE OF IMPELLER GUIDE VANES. CENTRIFUGAL PUMPS.

SINCE earliest times, man has usually founded his settlements and townships close to a stream of drinkable water. The presence of a stream naturally gave rise to the idea of generating power and this, in turn, led to the development of the simple water wheel of the country mill and, later, to the modern water turbine. These machines have grown considerably in size of recent years. This has been made possible only because the turbine drives an electrical generator and it is relatively easy to distribute electricity over long distances by means of the overhead power lines with which we are all familiar.

In many towns, power is distributed through underground mains in the form of high-pressure water. London, for example, has a service of about 800 lb. per sq. in. This is available for operating machinery such as lifts and cranes, and it will be of interest to discuss this matter in more detail.

In the last chapter, it was seen that if the datum were chosen at the centre of the pipe, the total energy per lb. of water was measured by $\frac{p}{62{\cdot}4} + \frac{V^2}{2g}$. In pressure mains, the speed of water is commonly about 8 ft. per sec., and so it is found that the relative values of the two terms above are $800 \times \frac{144}{62{\cdot}4}$ and $\frac{8^2}{2 \times 32{\cdot}2}$, or 1,848 ft. and 1 ft. respectively. From these we see that the quantity 1 ft., representing the velocity head, is so small in comparison with the pressure head that it may be neglected.

If we imagine, for a moment, that there is no loss by friction in the main, we find that the rate of flow at 8 ft. per sec. through a pipe of diameter d ft. is :—$\frac{\pi}{4} d^2 \times 8 \times 62{\cdot}4$ lb. per sec., and the horse-power transmitted is given by :—$\frac{\pi}{4} d^2 \times 8 \times 62{\cdot}4 \times \frac{1{,}848}{550}$.

Where Friction Exists

Now let us examine the case where friction exists, the length of the pipe being l ft., and 1,848 ft. the head at the inlet end. In the last chapter it was seen that the loss of head h_f, caused by pipe friction, was given by $\frac{4fl}{d} \times \frac{V^2}{2g}$ ft., so that the head at the point where the power is used, as in the cylinder of a lift, is here :—$\left(1{,}848 - \frac{4fl}{d} \times \frac{V^2}{2g}\right)$ ft., and the h.p. transmitted is :—$\frac{\pi}{4} d^2 \times V \times \frac{62{\cdot}4}{550}\left(1{,}848 - \frac{4fl}{d} \times \frac{V^2}{2g}\right)$.

To fix ideas, it will be of interest to

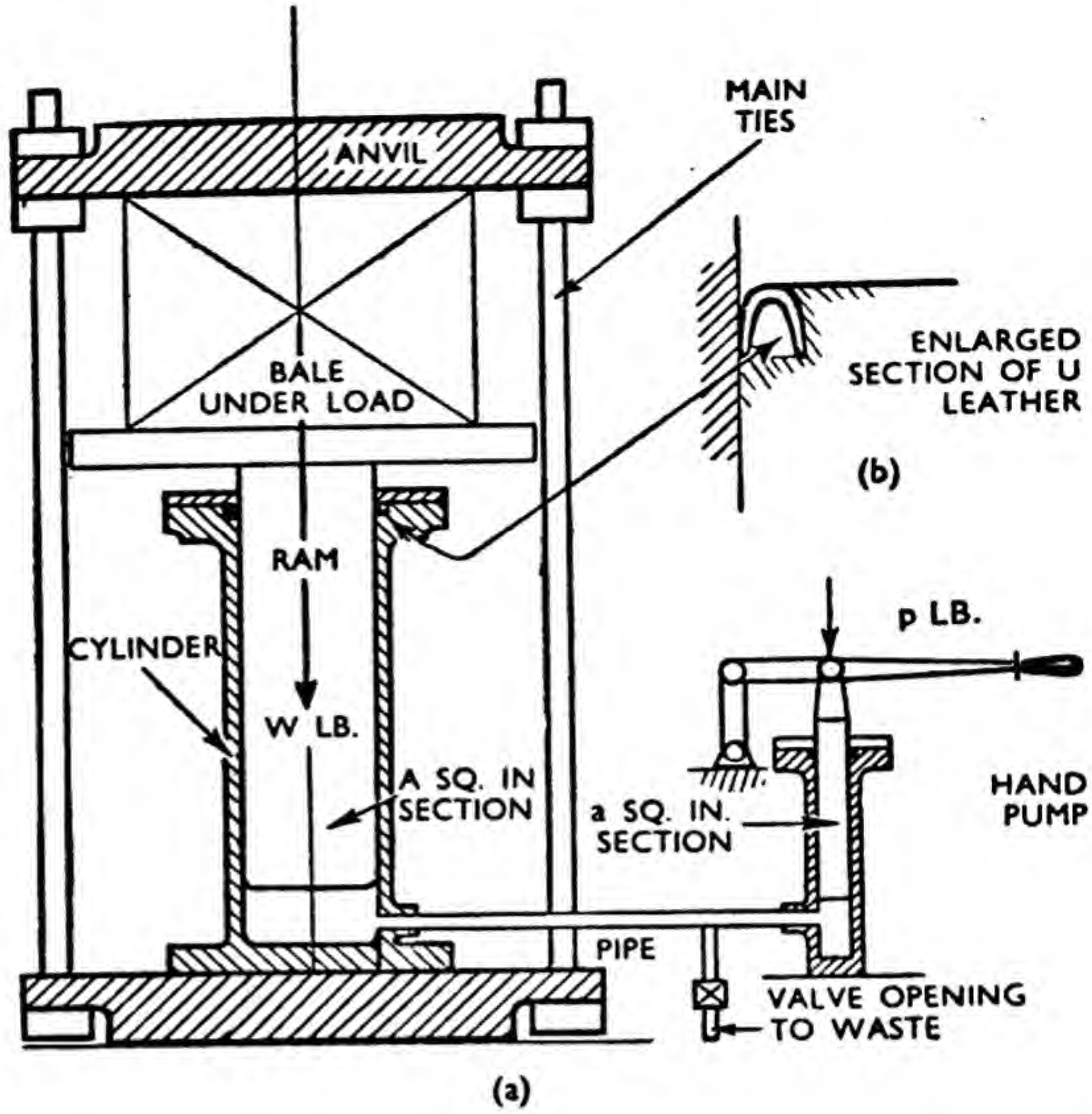


Fig. 1. (*a*) Here we see a section through a small hydraulic press, operated from a hand-pump. The pressures in the two cylinders are equal, so that $\frac{p}{a} = \frac{W}{A}$. By making the ratio A/a large, the value of W, the maximum load which the press can produce, can be made as large as desired. (*b*) U-leather packing ring is seen. This prevents leakage taking place between the ram and cylinder.

find the power transmitted through 1,000 ft. of 3-in. diameter pipe at 8 ft. per sec., if f, the coefficient of friction, is 0·0075.

Here, h_f, the head lost in friction, is :—$4 \times \frac{0{\cdot}0075}{3} \times 1{,}000 \times 12 \times \frac{8^2}{64{\cdot}4} = 119$ ft. The head at the machine end of the pipe is, accordingly, 1,848 − 119, or 1,729 ft., and as the rate of flow is :—$\frac{\pi}{4} \times \left(\frac{3}{12}\right)^2 \times 8 \times 62{\cdot}4$ or 24·5 lb. per sec., the h.p. transmitted is :—$24{\cdot}5 \times \frac{1{,}729}{550} = 77$, while the wastage by friction is :—$24{\cdot}5 \times \frac{119}{550} = 5{\cdot}3$ h.p.

If we wish to lay a new main, the question arises as to whether a large, and hence relatively expensive pipe should be used, so as to reduce the velocity and the friction loss, or whether a smaller and cheaper main should be employed with a correspondingly greater frictional loss. We can solve this type of problem only by experience and a comparison of the relative cost of waste power and the initial cost of the pipe-line.

Hydraulic Press

A hydraulic press is shown diagrammatically in Fig. 1(a), where we see that the essential parts are a ram, which is forced upward by high-pressure water supplied to the cylinder by means of a pipe. The material to be compressed is placed between the top of the ram and the anvil, which is held in the main frame.

Such machines are particularly suited to the production of large compressive forces where only a slow rate of loading is required, as in the process of cotton baling or pressing the domed ends of boiler

shells from flat plates. Water may be supplied from a hand-pump, as indicated in the figure, or in the case of a large plant, from a hydraulic main.

Method of Operation

The method of operation is quite simple. After the desired load has been applied, the supply of high-pressure water is cut off and the pressure is released by opening the valve to waste, when the weight of the ram will cause it to withdraw into the cylinder again.

If the frictional loss is neglected in the short length of pipe connecting the hand-pump to the press cylinder, and the symbols shown in the figure are used, we see, on equating the pressures in the pump and press cylinders, that $\frac{p}{a} = \frac{W}{A}$ lb. per sq. in. By increasing the ratio of A to a sufficiently, the value of W can be made as large as desired. If a pressure of 800 lb. per sq. in. is applied to a ram of 12-in. diameter, we find that the resulting load is :—

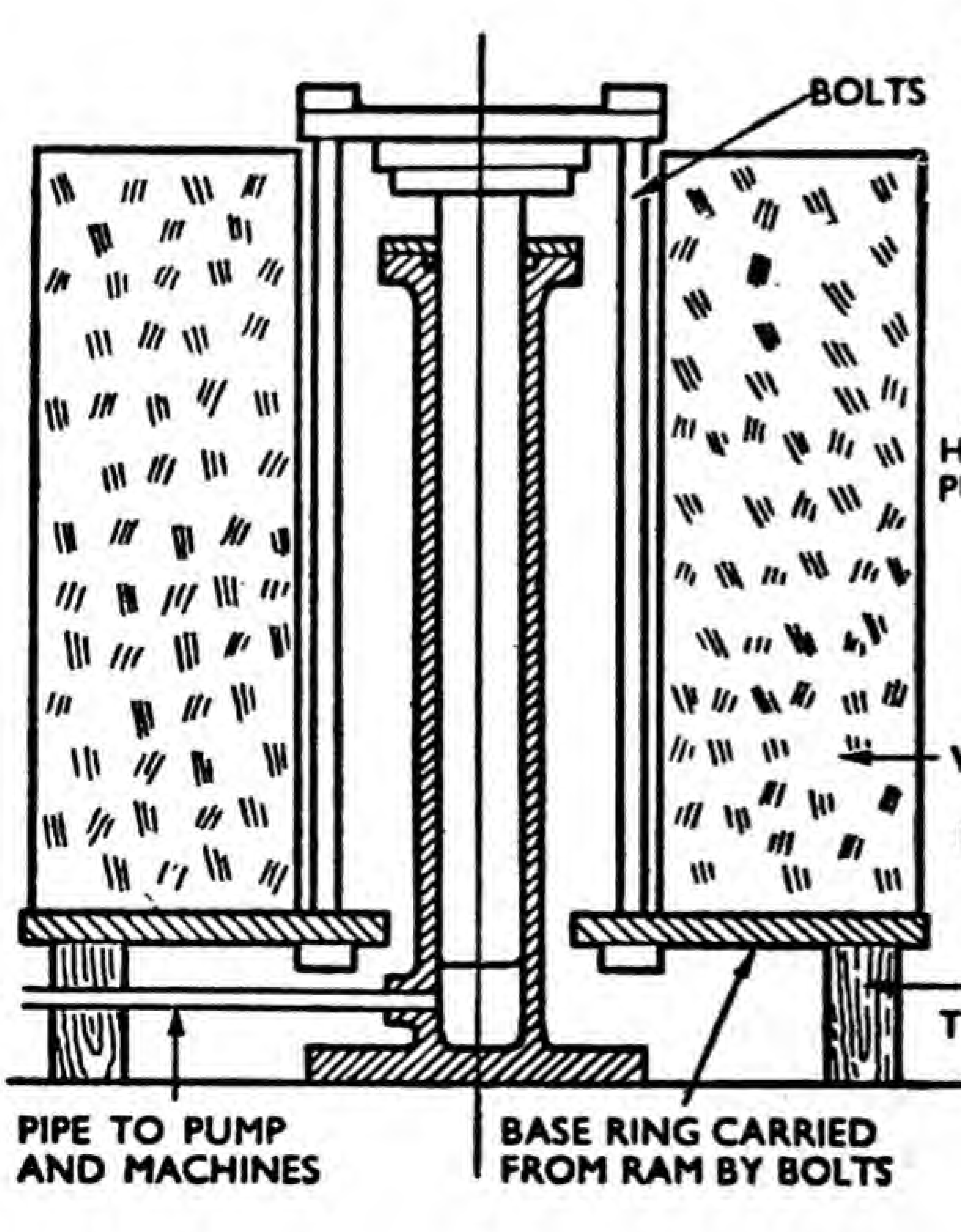
$$\frac{\pi}{4} \times 12^2 \times 800, \text{ or } 90{,}000 \text{ lb.}$$

The method of sealing the joint between the moving ram and the stationary cylinder is of interest. A ring of leather is employed (Fig. 1(b)), the section of which is in the form of a U. Water under pressure leaks into the inner part of the U and forces the U to open slightly, the one side coming into close contact with the ram and the other with the cylinder, thus forming a joint free from leakage.

Machinery such as hydraulic cranes, presses and lifts do not function continuously, and this means that they require high-pressure water only intermittently. If a pump is connected directly to a press, for example, the pump has to be sufficiently large to supply a volume equal to that of the press cylinder, in the relatively short time that the press is in operation.

Hydraulic Accumulator

A cheaper plant results if an accumulator is inserted between the pump and the press, for, as we shall presently see, a smaller pump can be installed. This will run



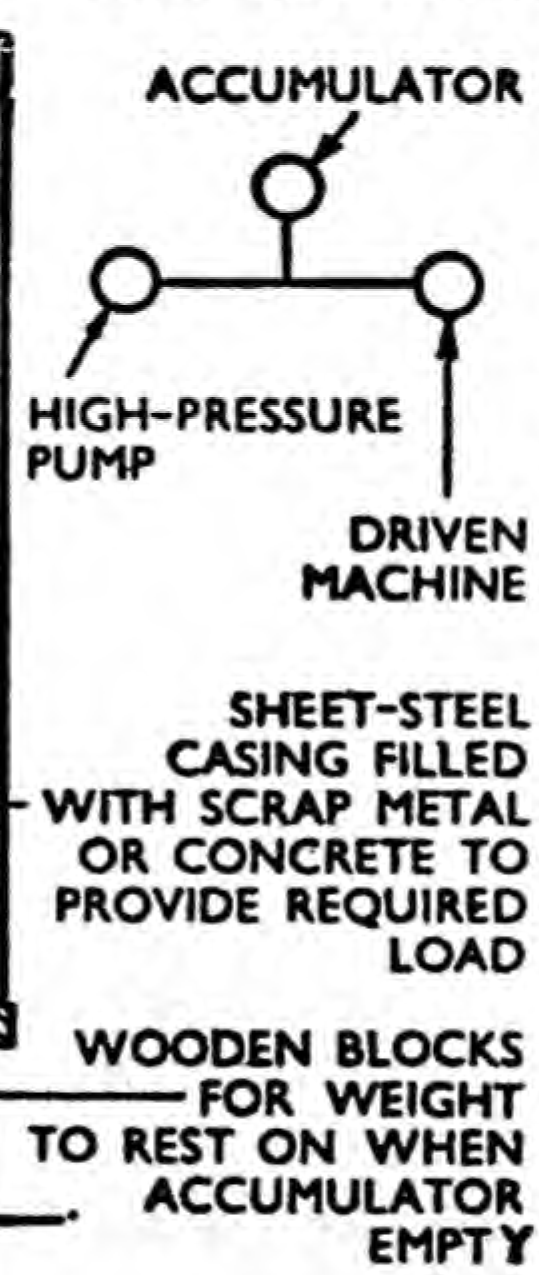


Fig. 2. Illustrating the essential parts of an accumulator, which is used to store energy in the form of high-pressure water. The ram has suspended from it a large weight, and is lifted when the supply pump is delivering water at a greater rate than that demanded by the machine, such as a crane, which it drives.

more or less continuously, and when the press is not in use, the pump will feed the accumulator, lifting the ram and storing energy for later use. The hydraulic accumulator is, thus, a device for storing energy in the form of high-pressure water, and, as Fig. 2 reveals, it has something in common with the press.

A central ram is lifted against the resistance of a large weight and, according to whether the ram is rising or falling, more energy is being stored or high-pressure water is being drawn off for use in some machine. A numerical example will help us to understand how it works.

Let us imagine that the ram diameter is 10 in., the stroke 10 ft., and the working pressure 800 lb. per sq. in. We will find what dead load must be carried by the ram, and the amount of energy stored when the ram is at the top of its stroke. The area of the cross-section of the ram is :—$\frac{\pi}{4} \times 10^2$ or 78·5 sq. in., and so the dead load required to produce a pressure of 800 lb. per sq. in. is :—$78{\cdot}5 \times \frac{800}{2{,}240}$ or 28 tons. The energy stored when the accumulator is fully loaded is that corresponding to a load of 28 tons lifted through 10 ft., or $28 \times 2{,}240 \times 10 = 626{,}000$ ft.-lb.

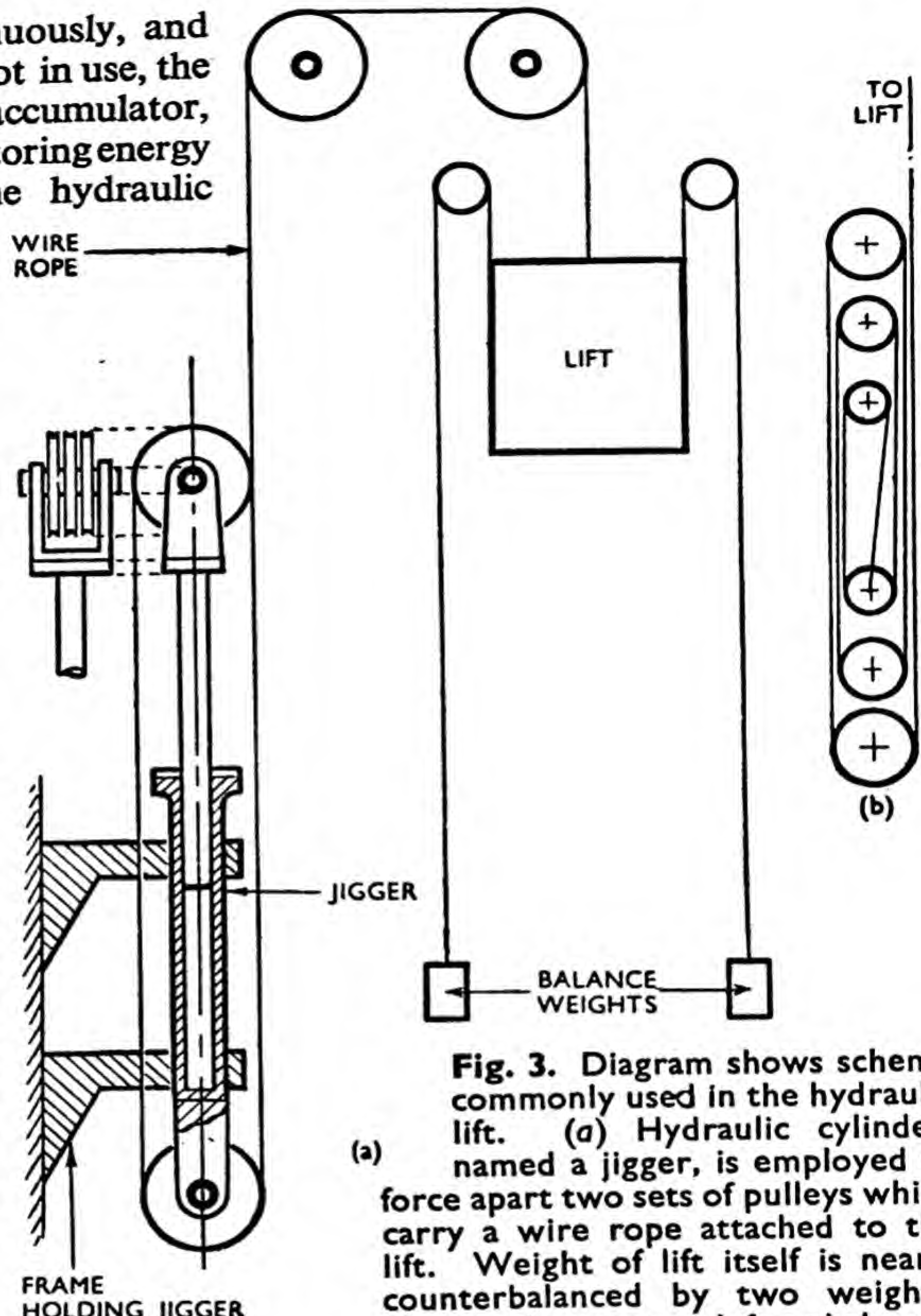


Fig. 3. Diagram shows scheme commonly used in the hydraulic lift. (*a*) Hydraulic cylinder, named a jigger, is employed to force apart two sets of pulleys which carry a wire rope attached to the lift. Weight of lift itself is nearly counterbalanced by two weights, shown, the amount left unbalanced being sufficient to force jigger to close up when water pressure is released. (*b*) Indicates how the rope is arranged over pulleys, so that there are six effective ropes over the jigger.

Influence on Pump Size

Now let us examine how the size of pump is affected by the introduction of an accumulator. We will suppose that a crane cylinder of 10 cu. ft. capacity is to be charged in 2 min., followed by a

dwell or idle period of 6 min. The required capacity of the pump, if no accumulator is used, is :— $\frac{10}{2} = 5$ cu. ft. per min., whereas the duty is reduced to :— $\left(\frac{10}{2+6}\right) =$ 1·25 cu. ft. per min. in the other case.

Over the period of 2 min. during which the crane is working, the pump will deliver 1·25 cu. ft. per min. × 2 min. = 2·5 cu. ft., while the capacity of the crane cylinder is 10 cu. ft. The required capacity of the accumulator is, accordingly, 10 − 2·5 = 7·5 cu. ft., this being the amount by which the supply from the pump is below that required by the crane during the working period.

Lifts and Cranes

Lifts and cranes, as Figs. 3(a) 3(b) and 4 show, are merely special applications of the principle of the press. One of the commonest arrangements of lift is shown in Fig. 3(a), where the outer ends of both the ram and cylinder are provided with a group of two or three sheaves. A wire rope, secured at one end to the cylinder, is carried over the pulley system and then attached to the lift. If there are three sheaves in each block, there will be six ropes carrying the load, as shown in Fig. 3(b), where the pulleys are separated for convenience of explanation.

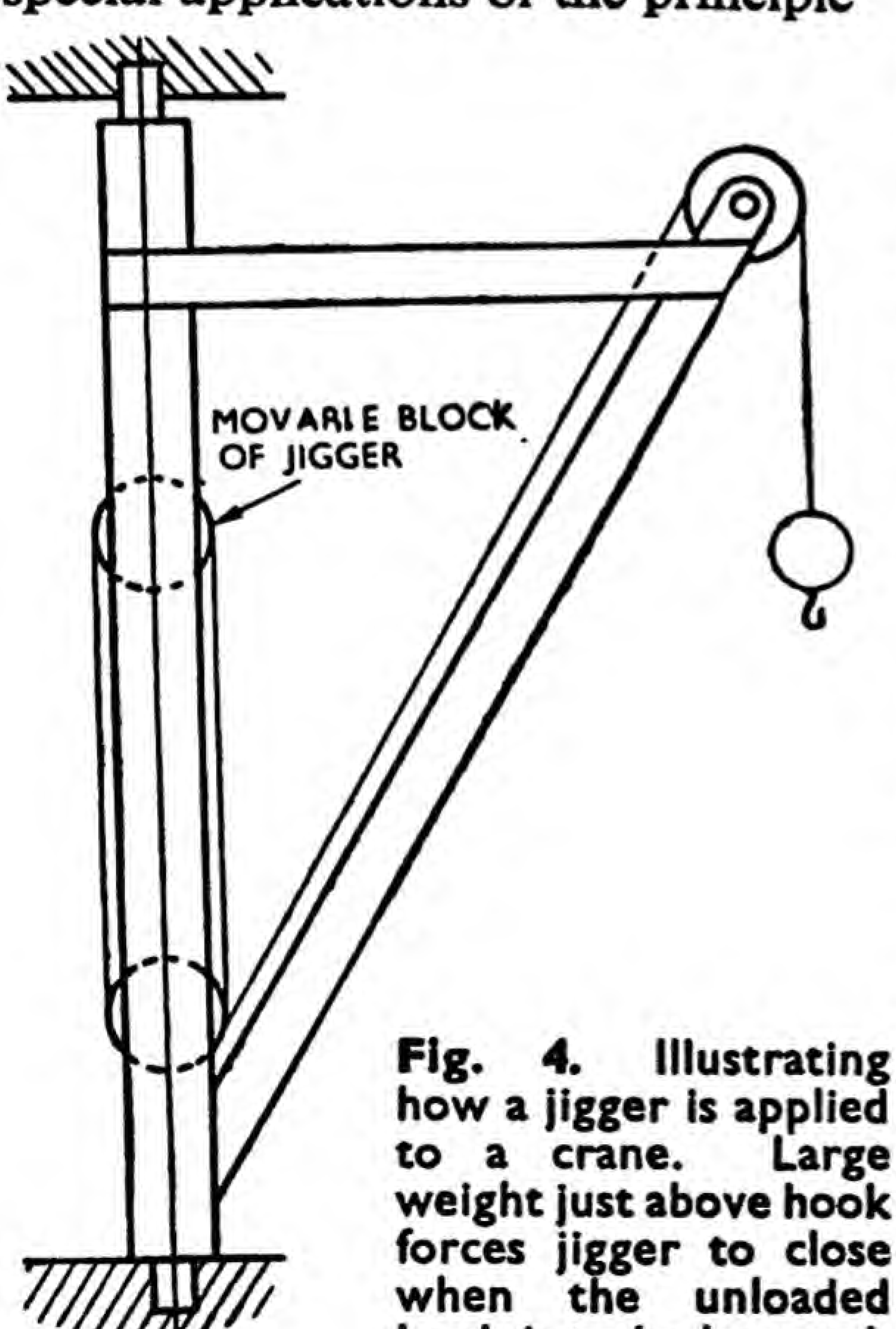


Fig. 4. Illustrating how a jigger is applied to a crane. Large weight just above hook forces jigger to close when the unloaded hook is to be lowered.

In this case, the load on the ram is six times the weight of the lift, but the distance moved by the ram is only one-sixth of that travelled by the lift. The advantage of this design is the cheapness of construction of the ram and cylinder with its pulley system, named the jigger, compared with that of the long cylinder which would be required if no pulleys were used. The long cylinder would also entail the provision of a deep well to house it. The jigger, if desired, may be mounted horizontally. There is a marked disadvantage, however, as there is a considerable loss of efficiency due to the friction of the pulley system.

Pulleys Reduce Efficiency

If we assume, for sake of explanation, that the rope attached to the lift of Fig. 3(b) carries a load of 1 ton, and that the efficiency of each sheave is 0·94, then the rope on the extreme left will carry a pull of $\frac{1}{0{\cdot}94} = 1{\cdot}063$ tons. On passing over the uppermost sheave, the rope will carry a still larger pull, $\frac{1{\cdot}063}{0{\cdot}94}$ or $\frac{1}{0{\cdot}94^2}$ tons. For six effective ropes, we find that the total

load on the ram is $\frac{1}{0{\cdot}94} + \frac{1}{0{\cdot}94^2} + \frac{1}{0{\cdot}94^3} + \frac{1}{0{\cdot}94^4} + \frac{1}{0{\cdot}94^5} + \frac{1}{0{\cdot}94^6} = 8{\cdot}127$ tons, instead of 6, the value if there were no friction. The efficiency is, accordingly, $\frac{6}{8{\cdot}127}$, but this has to be reduced still further, by about 5 per cent, to allow for the friction of the packing of the ram, so that the final value is $0{\cdot}95 \times \frac{6}{8{\cdot}127}$ or $0{\cdot}7$.

Fig. 4 illustrates the application of the same type of cylinder to a crane. Here it is necessary to provide a large weight, as seen just above the crane hook, in order to force the ram back into the cylinder when the high-pressure water is released.

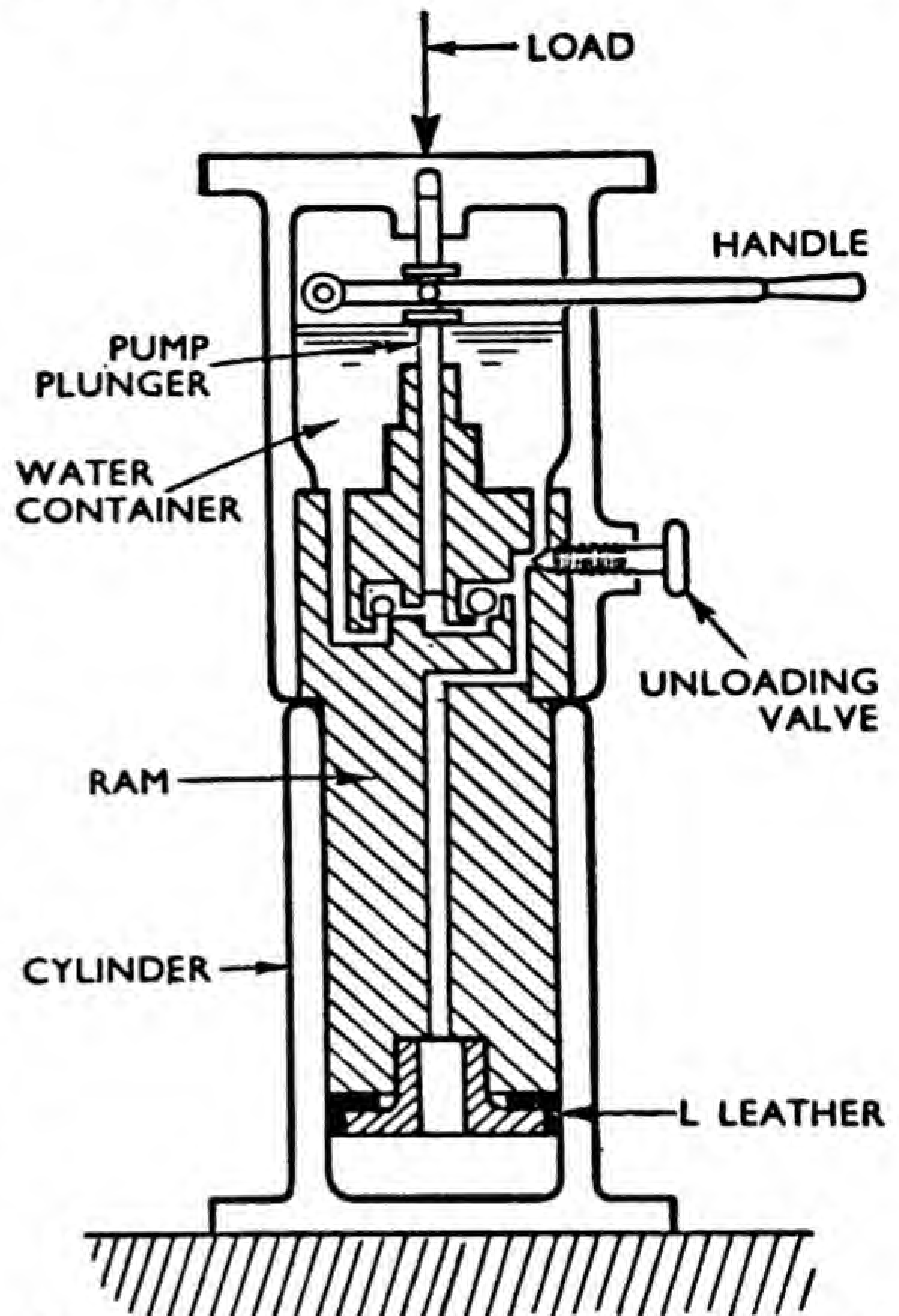


Fig. 5. Hydraulic jack is a portable form of ram and cylinder, used on temporary work for lifting heavy weights. The necessary water is carried in the head of the tool.

Hydraulic Jacks

The hydraulic jack is a self-contained and portable form of ram used for lifting heavy weights. In the form depicted in Fig. 5, the base acts as the cylinder, and the upper or movable portion contains the hand-pump for operating the tool and a supply of water. Here another form of packing is seen, a collar of leather, whose section, it is noticed, resembles the letter L.

On raising the pump plunger, water is drawn into the body of the pump from the container, through a non-return suction valve. This consists of a phosphor-bronze ball held down by a light spring, not shown in the figure. On the following down-stroke of the plunger, the water is forced out through the right-hand or discharge valve, and thence down through the central hole to the underside of the ram, thus lifting the ram and the applied load.

To release the load, a screw is provided, which opens a port connecting the underside of the ram to the container. The same water is used over and over again. A jack of the type shown is employed on temporary work, such as the erection of machinery, and will handle a load of 12 to 15 tons if the ram diameter is $5\frac{1}{2}$ in.

In suction and force pumps a piston is driven up and down in a cylinder with a motion similar to that of the piston of a steam engine. Fig. 6 shows a suction pump which is employed to draw water from a supply tank or well, and deliver it at atmospheric pressure at the level of the pump itself. The simple village pump is of this type.

In the suction pipe is a non-return valve which opens upward, the discharge valve being fitted in the piston itself, now named the bucket, which is pierced with suitable holes for the passage of the water. The action is that during an upstroke of the bucket, the discharge valves close and the movement of the piston tends to produce a steady fall of pressure in the cylinder. This,

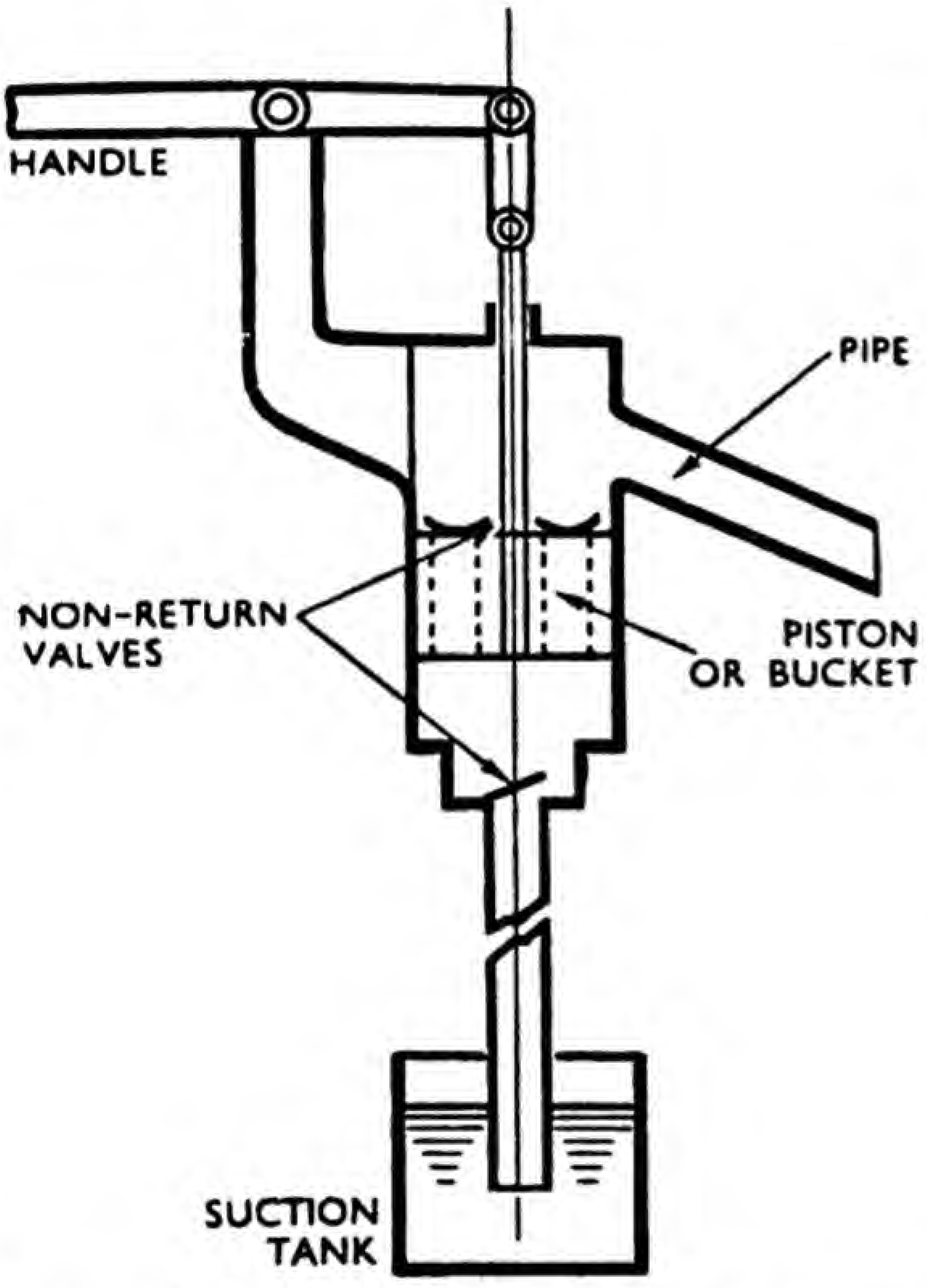


Fig. 6. In the suction pump, the upward movement of the piston or bucket tends to reduce the pressure in the cylinder. The difference between the pressure of the atmosphere, acting on the surface of the water in the suction tank, and that in the cylinder, forces water to flow up through the inlet valve past the non-return valve. On the following downstroke of the piston, the water trapped by the non-return valve passes up through the dotted ports in the bucket and is prevented from returning by the non-return valves. This water now flows away freely through the pipe. These pumps cannot lift water through more than about 24 ft. as air is then released from solution in the suction pipe.

however, causes the water to be drawn up through the suction pipe, filling the cylinder. During the down-stroke of the piston, the suction valve closes, and the water now trapped in the cylinder passes through the valves in the bucket and we notice it flows away freely through the pipe during the next upstroke.

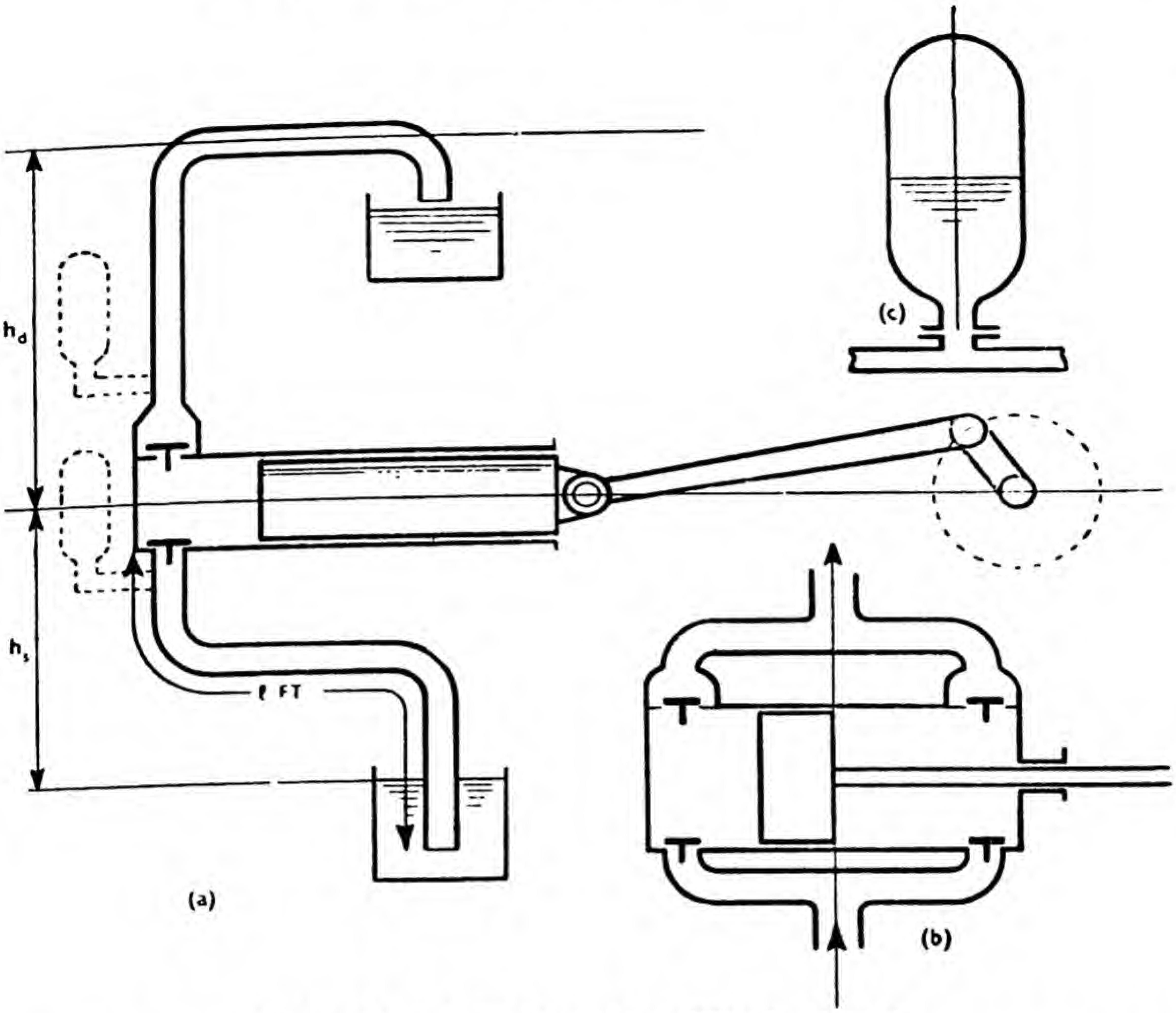
The name suction pump is really a misnomer, for it is the difference between the pressure of the atmosphere acting on the free surface of the water in the supply tank, and that in the pump cylinder which causes the flow of water through the inlet pipe. This difference is in no way produced by suction.

If the pump has not only to lift the water from the supply tank, but also to discharge it under pressure, a force pump must be used, as indicated in Fig. 7. The essential difference from the suction pump is that the discharge is here under a pressure greater than atmospheric. We see that there are no valves in the bucket, independent suction and discharge valves being provided in the body of the pump.

The action of these pumps is the same as that in the suction type. They may be made single-acting, as in Fig. 7(a), so that the one end of the cylinder only is operative, or they may be double-acting, as in Fig. 7(b), in which case the one end of the cylinder is filling while the other is discharging, and vice versa.

Head for Suction Lift

Before we discuss the pressure in the cylinder at various instants during the movement of the piston, it will be instructive to consider the pressure required to suck a drink through a straw, as indicated in Fig. 8. The pressure in the mouth



SINGLE- AND DOUBLE-ACTING PUMPS

Fig. 7. Type of pump shown is employed instead of a suction pump when the discharge pressure is above atmospheric. These pumps are suited to discharge pressures up to 1,200 lb. per sq. in. or more, and may be single-acting as in (*a*), or double-acting as in (*b*), so that one end is filling while the other is discharging. The intermittent nature of the flow can be smoothed out to a large extent by the use of air-vessels, as shown dotted in (*a*). (*c*) Pressure in the air-vessel in the discharge pipe is increased slightly at the start of the discharge stroke, and then tends to force the water up at a steady rate.

was originally atmospheric, that is, 34 ft. of water head. If we draw the fluid up just to taste it, the pressure in the mouth must be reduced to $34 - h_s$ ft., where h_s is the vertical suction lift. If we draw the liquid up steadily, the pressure in the mouth must be reduced still further by h_f, the loss of head by friction in the straw; and, finally, if we apply a sudden suction, the flow will be accelerated. In this case, the pressure in the mouth must fall to $34 - h_s - h - h_a$ ft., where h_a is termed the acceleration head.

Pressure Alters with Volume

Referring to Fig. 9(a) it will be seen how the above analogy helps us to understand the way in which the pressure in a pump alters with changes in the rate of movement of the piston. If water is drawn up very slowly, so that the velocity of flow, and hence the frictional loss in the suction pipe, is negligible, the pressure will be shown by the

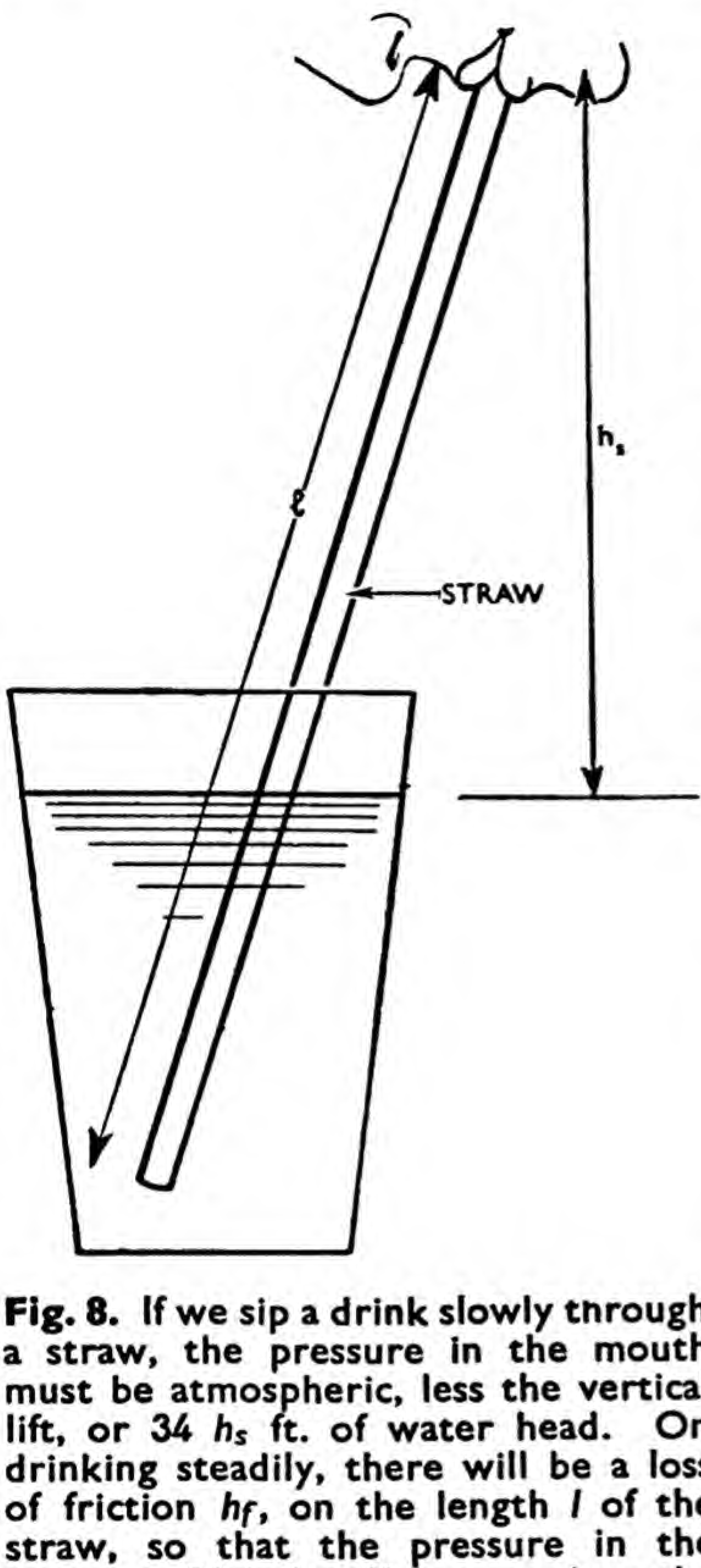


Fig. 8. If we sip a drink slowly through a straw, the pressure in the mouth must be atmospheric, less the vertical lift, or 34 h_s ft. of water head. On drinking steadily, there will be a loss of friction h_f, on the length l of the straw, so that the pressure in the mouth is 34 h_s-h_f. If we speed up the flow, the pressure must fall to 34-h_s-h_f-h_a, where h_a is the acceleration head.

line *AB*, which, as was seen above, will be h_s ft. below atmospheric.

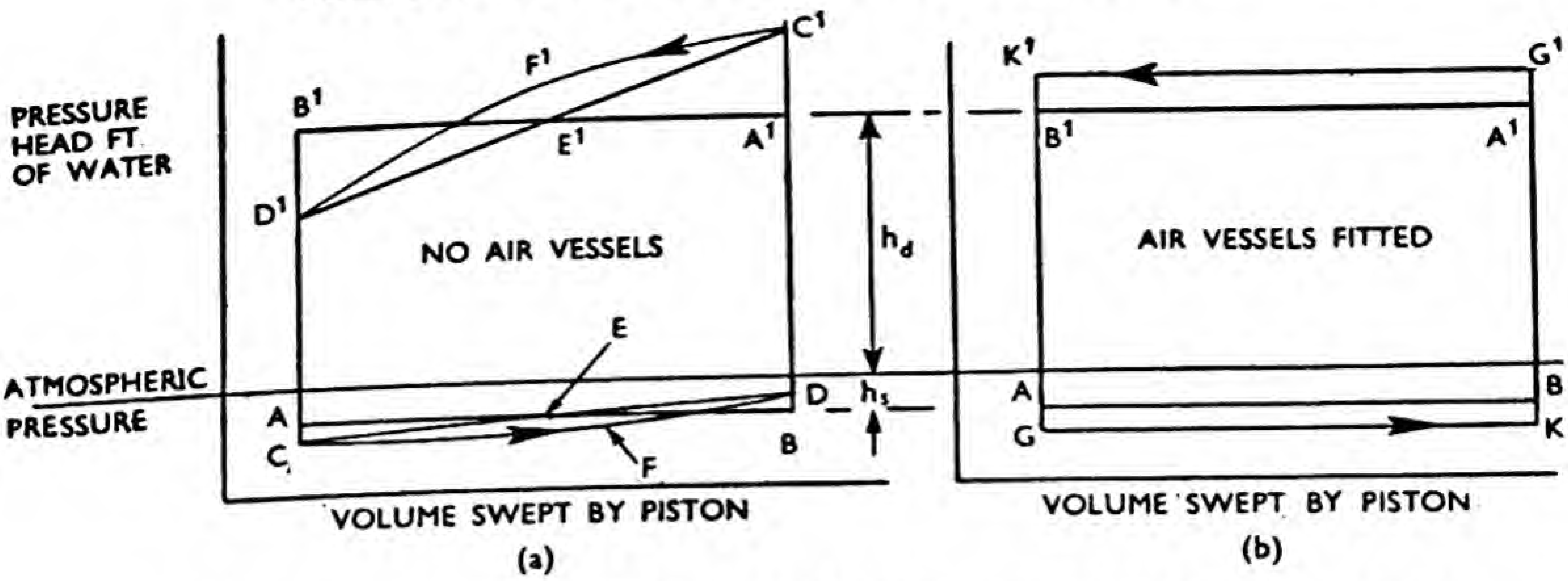
Now, the piston has a velocity which changes from instant to instant, for its motion is similar to that of a pendulum. We may equate the quantity of water flowing per sec. in the suction pipe, to the volume swept out by the piston per sec. at the same instant.

If a and v are the area of the cross-section in sq. ft. and the velocity at any instant in ft. per sec. in the suction pipe, while A and V are the corresponding values for the piston, then $a \times v = A \times V$, which gives us the instantaneous velocity in the pipe as $V \times \frac{A}{a}$ ft. per sec. This quantity will be zero at each end of the stroke, when the piston comes momentarily to rest.

At the centre of the stroke, the velocity of the piston is given by $2\pi rn$ ft. per sec., where n is the speed of the driving crank in revs. per sec. and r is its radius in ft. We see now that the speed of the water in the pipe-work is also zero when the piston is at either end of its stroke, and has the value $\frac{A}{a} \times 2\pi rn$ ft. per sec. for the mid-position of the piston. The corresponding values of the head lost in friction h_f, are :—O and $\frac{4fl}{d} \times \left(\frac{A}{a} \times 2\pi rn\right)^2 \times \frac{1}{2g}$ ft.

Acceleration of Water

Next to be considered is the question of the acceleration of the water in the pipe, and the head necessary to produce it. As in the case of a pendulum, the acceleration of the piston is proportional to its distance from the central position, where it is zero, the maximum values occurring at the ends of the stroke. This maximum acceleration is given by :—$(2\pi n)^2 \times r$ ft. per sec. per sec., and if the same argument is used as was employed to obtain the speed of the water in the pipe from that of the piston, we find that the acceleration is $\frac{A}{a} \times 4\pi^2 n^2 r$ ft. per sec. per sec. This quantity changes from plus to minus as the piston moves over its



PRESSURE VOLUME RELATION IN A RECIPROCATING PUMP

Fig. 9. *AB* is h_s ft. head below atmospheric in both cases and represents the steady pressure corresponding to the suction lift. (*a*) *AC* is the acceleration at the start of the outstroke, and the vertical distance between *CFD* and *CED* gives us the friction in the suction pipe. In (*b*), relating to a pump with suitable air-vessels, the constant frictional loss on the suction side is shown by the height *AG*. For the discharge side, the frictional loss is shown by A^1G^1, while in (*a*), at the start of the discharge stroke, A^1C^1 gives the acceleration, and the height between $C^1F^1D^1$ and $C^1E^1D^1$ the friction, when no air-vessels are provided.

stroke, a point which will be readily understood if we think of a weight of, say, 5 lb. held from a spring balance, as in Fig. 10.

On moving the hand up and down, the balance will be found to read more than 5 lb. at the beginning of the up-stroke and less than 5 lb. at the top of the stroke. The balance will only register 5 lb. when the hand is at the centre of its motion. This effect is due to the acceleration and deceleration of the suspended weight. The actual value of h_a, the acceleration head, can now be calculated. The weight of water in the suction pipe is $62{\cdot}4 \times a \times l$ lb., where l is the length in ft. If Z is the acceleration, the force required to speed up the flow is, $\dfrac{62{\cdot}4alZ}{g}$ lb.

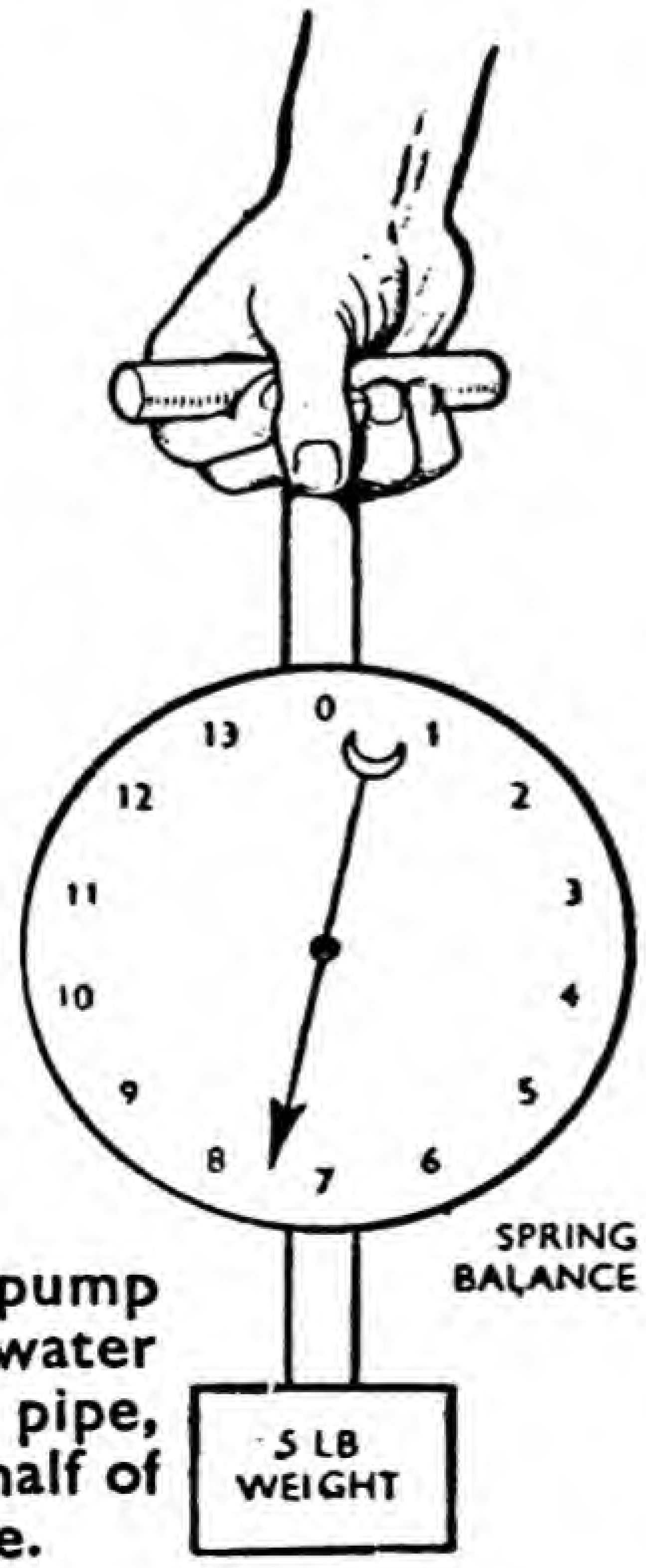


Fig. 10. If a weight of, say, 5 lb., suspended from a spring balance, is moved up and down with a motion similar to that of a piston in a cylinder, the balance will read more than 5 lb. when the weight is below its mid-position and vice versa. This helps us to understand why an extra head is developed in the pump to accelerate the water in the discharge pipe, during the first half of the in-stroke.

Friction and Acceleration Heads

Now let us imagine a small piston to be fitted in the pipe and given the appropriate acceleration. The pressure on it will be :—

$62{\cdot}4\,\dfrac{alZ}{ga}$ or $\dfrac{62{\cdot}4\ lZ}{g}$ lb. per sq. ft.,

and the corresponding head h_a, found from $\dfrac{p}{62{\cdot}4} = h$, will be $\dfrac{lZ}{g}$ ft. On substituting for Z, we find that :—$h_a = l \times \dfrac{4\pi^2 n^2 r}{g}$ ft. We see

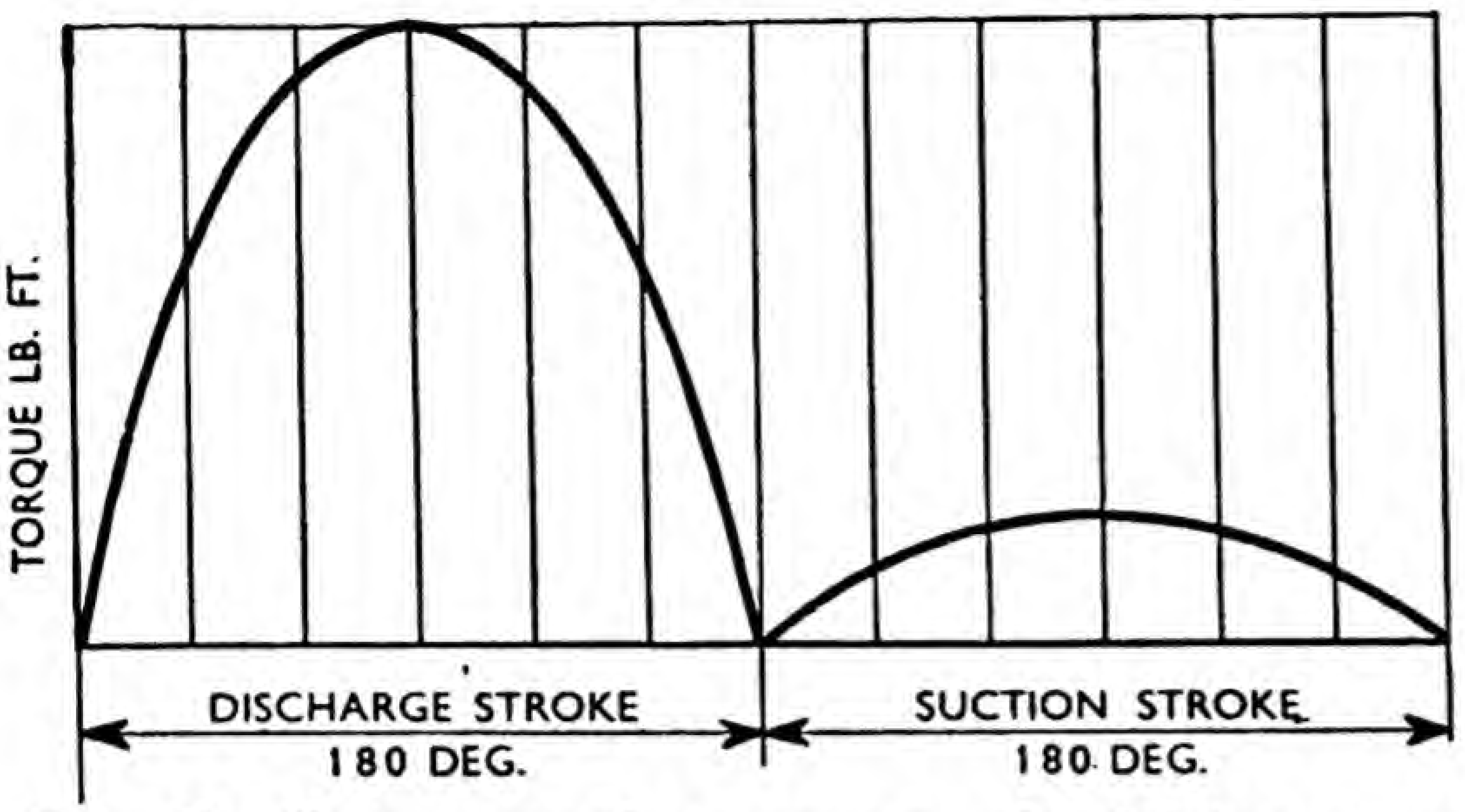


Fig. 11. Shows how the torque on the driving motor of a single-acting single-cylinder pump varies with the angle through which the crank turns.

the acceleration head h_a, in the suction pipe, represented on the pressure head volume plotting of Fig. 9(a) by the straight line *CED*, while the vertical distance between *CED* and the curve *CFD* shows us the corresponding changes in the friction head h_f.

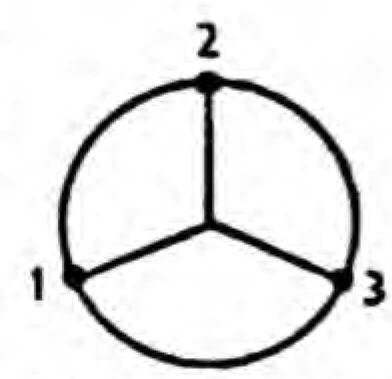
Similar heads are produced on the discharge side, A^1B^1 showing the discharge head in a slow-moving pump, h_d ft. above atmospheric. $C^1E^1D^1$ is a straight line and shows us the acceleration head for the weight of water in the outlet pipe, while the vertical height of the curve $C^1F^1D^1$ above $C^1E^1D^1$ is the friction loss h_f, for the discharge side.

Reducing Pressure Fluctuations

It is to be noted from the diagram that, under certain conditions of acceleration, which depend on the square of the rotational speed of the crank, a very high head may be obtained at the point C^1, which may cause damage to the pump. Again, a very low pressure may result near the point *C*, in which case separation, as it is termed, may occur in the column of water in the suction pipe. This means that air will be drawn out of solution from the water and a gap will be formed. The water will still be in motion and will hit the piston later in its stroke when it is slowing down, and an objectionable hammer action will occur.

Both the acceleration heads at *C* and C^1 may be diminished to a large extent by the introduction of an air-vessel into each pipe-line, close to the pump, as seen dotted in Fig. 7. These are merely air-tight vessels which act as springs, as shown in the enlarged view in Fig. 7(c).

During the rapid acceleration of the piston at the beginning of the discharge stroke, water is forced



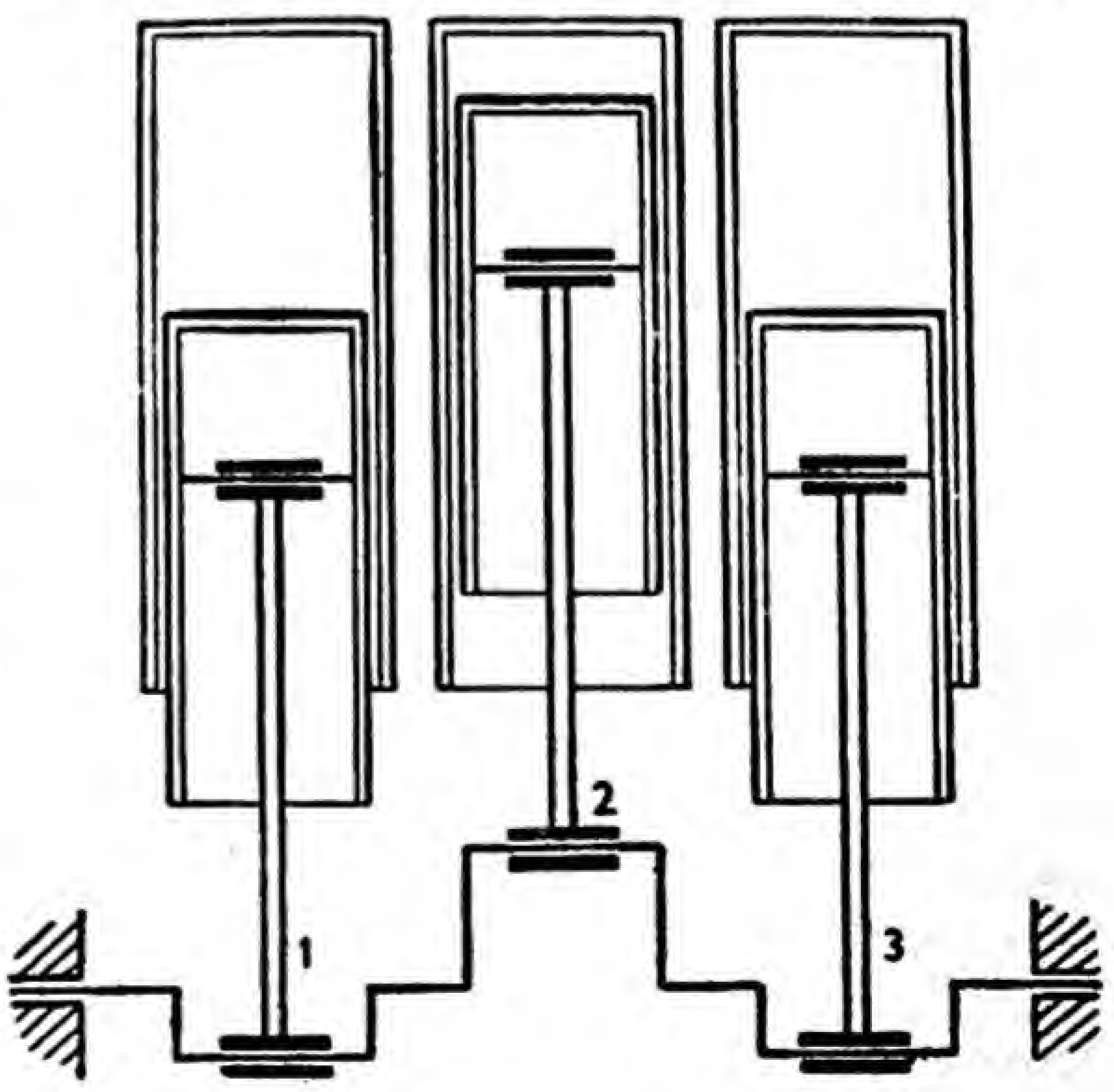


Fig. 12. A three-throw single-acting pump is commonly used for high-pressure work. If the three cranks are arranged at 120°, as shown, the torque on the driving motor is nearly uniform.

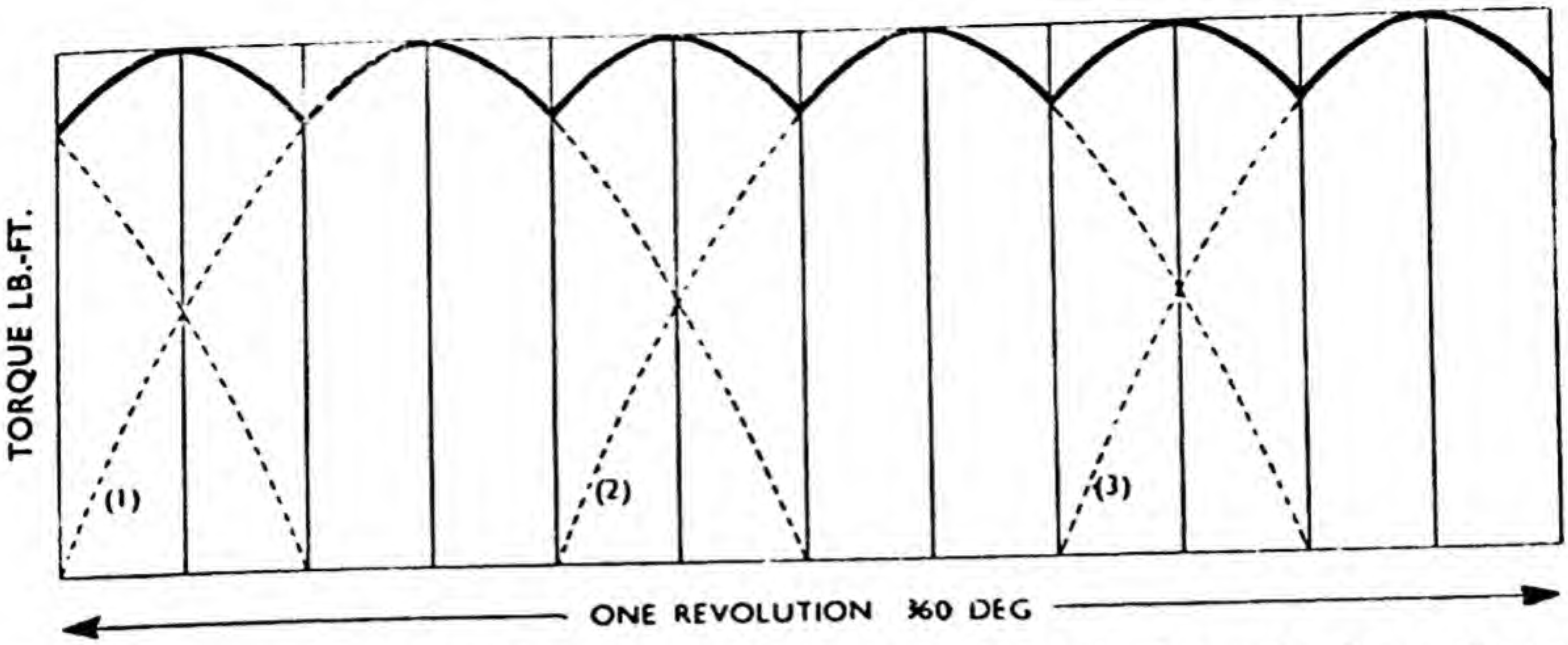


Fig. 13. Illustrating the torque of the three separate cylinders of a three-throw pump, marked (1), (2) and (3). The full line curve gives the resultant torque to all three cylinders ; this is nearly steady, and is obtained by adding the ordinates of the individual dotted curves. In Fig. 11 we saw that the driving torque of a single-cylinder pump was far from steady.

into the corresponding air-vessel, compressing the air which expands at a later point in the working. This tends to make the flow in the pipe more steady, so that there is very little acceleration if the air vessel is sufficiently large. The piston of a single-acting pump will sweep out a volume $A \times 2rn$ cu. ft. per sec., and the mean speed in the pipe is, accordingly, $\frac{A}{a} \times 2rn$ ft. per sec. The corresponding friction loss h_f, is given by :—

$$\frac{4fl}{d}\left(\frac{A}{a} \times 2rn\right)^2 \times \frac{1}{2g} \text{ ft.}$$

In Fig. 9(b), we see the pressure-volume diagram for a pump provided with suitable air-vessels, AB and A^1B^1 showing us, as before, the static suction lift and discharge head respectively. The lines GK and G^1K^1 indicate the constant friction heads in the two pipes.

Diagram of Work

We have seen earlier that we can quickly convert a head-volume diagram to a pressure-volume figure by means of the relationship $\frac{p}{62 \cdot 4} = h$, the dimensions of pressure $\times$ volume being, $\frac{\text{lb.}}{\text{ft.}^2} \times \text{ft.}^3$ or ft.-lb., the same as those of work. It follows that the area of the diagram is proportional to the work required to drive the pump, and if we draw to scale Figs. 9(a) and 9(b), it will be found that the area of the latter is slightly smaller. This tells us that the introduction of the air-vessels also reduces the cost of running the pump.

Load not Uniform

It will be evident that if one of the pumps we have just discussed is driven by an electric motor, through a crank and connecting-rod mechanism, the load on the motor will be far from uniform. The larger loop of Fig. 11 indicates the torque on the motor during the discharge stroke, while the smaller loop shows us the corresponding values for the following suction stroke of a single-acting pump.

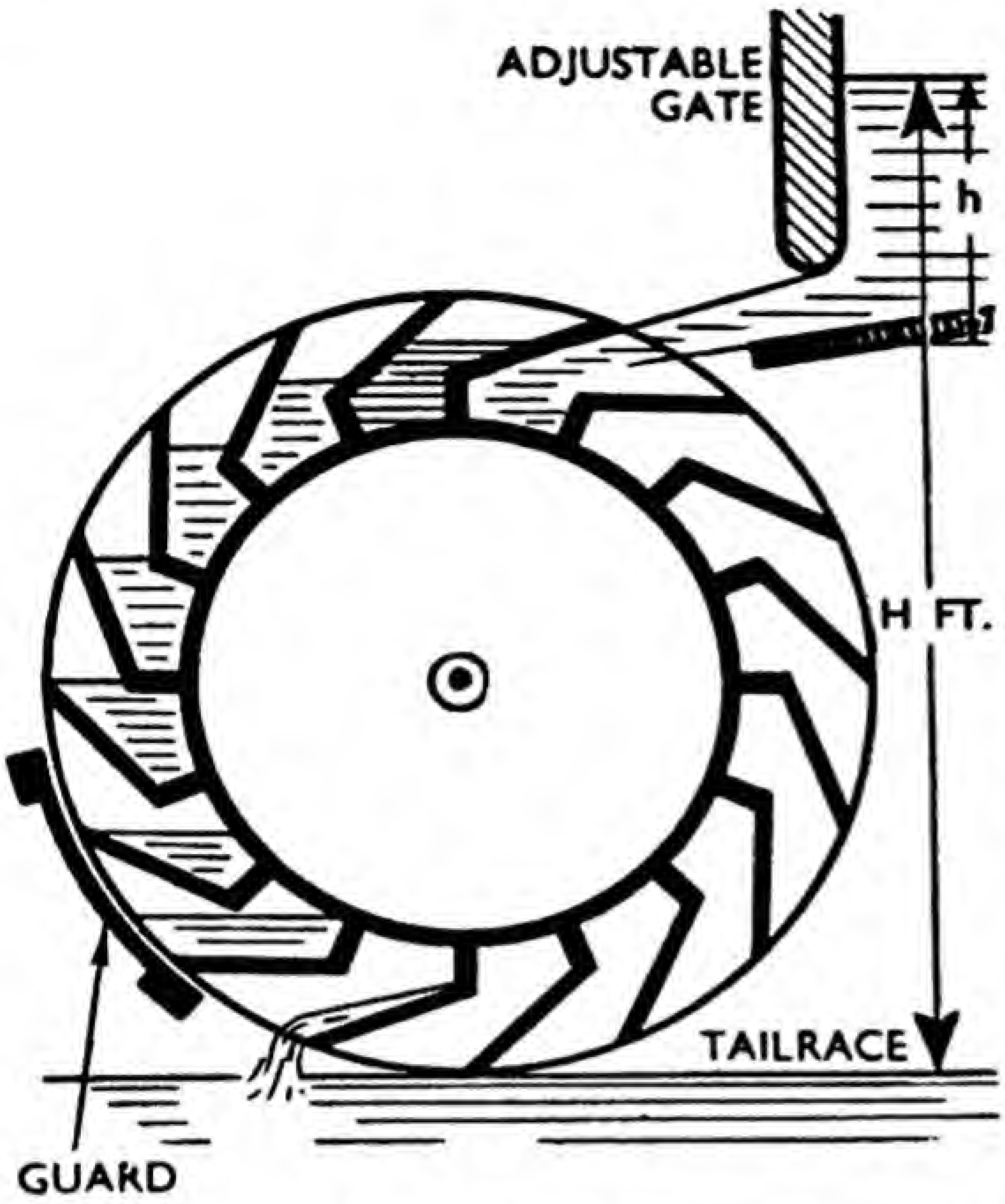


Fig. 14. The overshot water wheel is often used for driving small mills, where a head *H* of 20 to 30 ft. is available. The bulk of the work is done by the weight of the water trapped in the buckets. This is prevented from escaping too readily by the guard shown.

We can make the torque more nearly uniform by using a three-throw pump, the cranks of which are arranged mutually at 120 deg. (Fig. 12). For ease of manufacture, these pumps are usually made single-acting. For the purpose of explanation, we will assume that the torque during suction is relatively small, as occurs in practice when the head on the discharge side is large in comparison with the suction head.

Resultant Torque

The three dotted curves (1), (2) and (3) of Fig. 13 show the corresponding torque-angle plottings for such a pump, spaced 120 deg. apart. The resultant torque on the motor is given by the full line, which is obtained by adding the heights of the other curves on any given vertical line. Clearly the rise and fall of this resultant curve

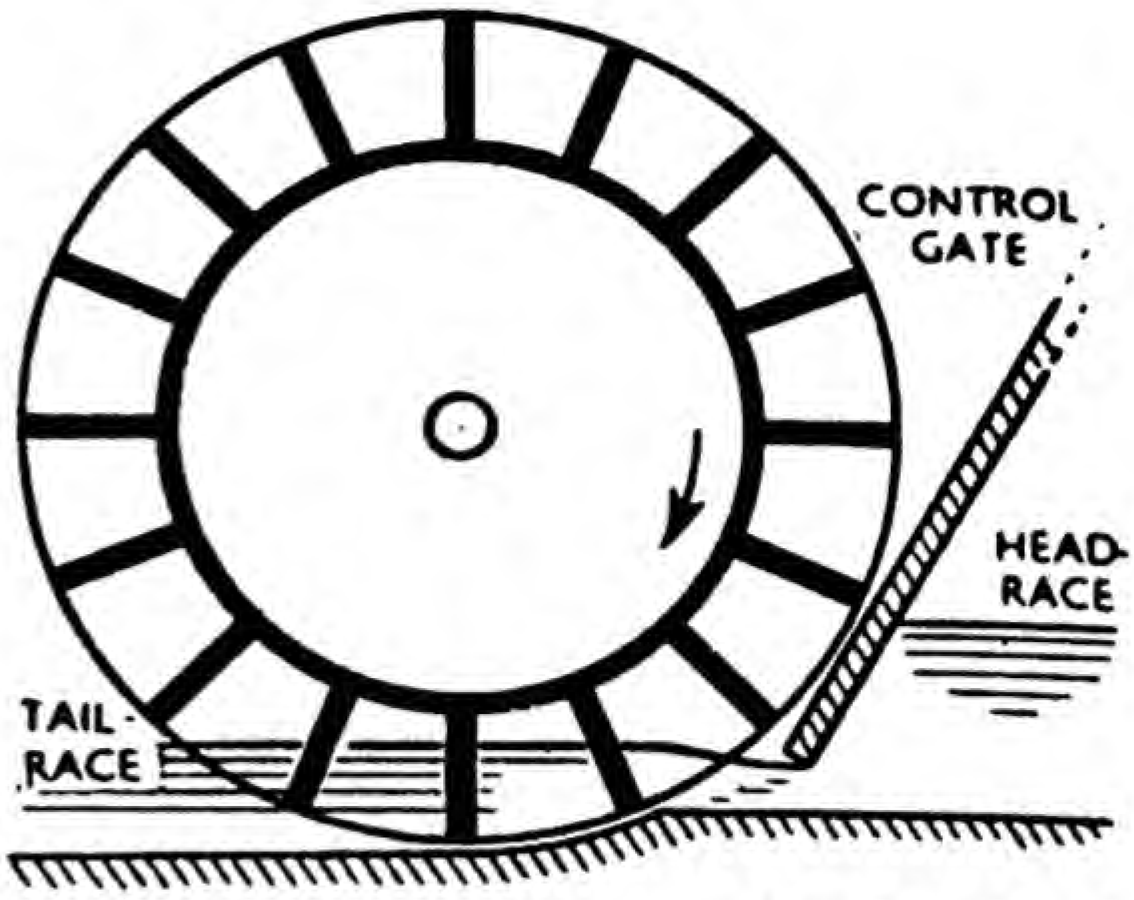


Fig. 15. For rather smaller heads, the undershot wheel is used instead of the overshot. The radial blades do not give an efficient design, but the cost of construction is low.

above and below the mean torque line is here relatively small.

Water wheels are used for relatively small powers, of, say, 10 to 30 h.p., and as they are unimportant in comparison with turbines, their working will only be described in brief. They have the advantage of being cheap in construction and upkeep. Fig. 14 shows an overshot wheel, for which tests tell us that the best speed of the buckets is about 90 per cent of that of the jet of water, or $0{\cdot}9\sqrt{2gh}$.

Under good conditions, the efficiency, which we may define as the ratio of the actual work done

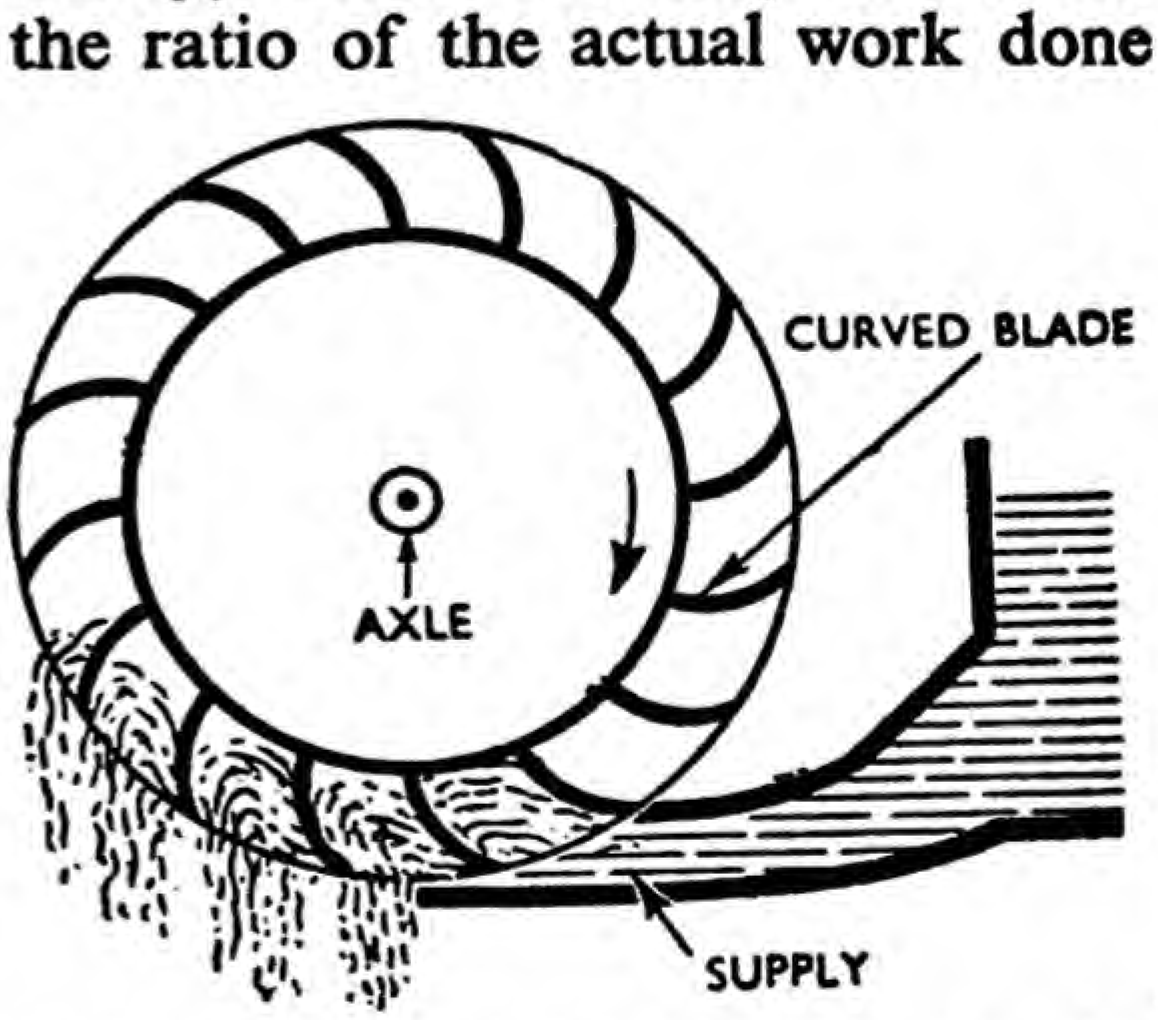
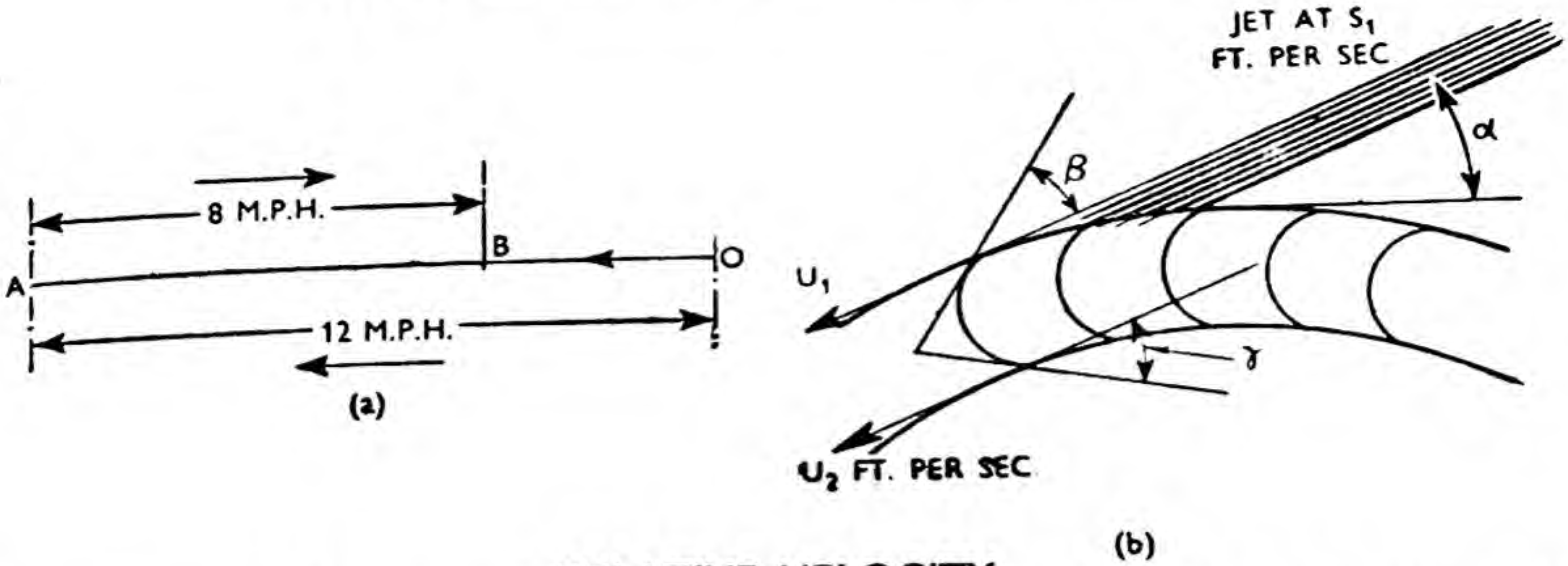


Fig. 16. Efficiency of an undershot wheel is raised considerably by the use of curved blades.



RELATIVE VELOCITY

Fig. 17. (*a*) If *OA* represents the speed of a train (1) and *AB* that of a train (2) moving on a parallel set of rails, *AB* being set off in the reverse direction to the actual motion of (2), *OB* shows us the speed of train (1), as observed by a man in train (2). (*b*) Shows the notation used for the velocity triangles at inlet and outlet of the blades of a water wheel. A stream or jet moving at S_1 ft. per sec. at an angle α to a group of blades is to slide on to the blades without shock. To do this, the relative velocity at the point of entry must be inclined to the blade motion at the point at an angle β, which is the inclination of the inlet edge of the blade to its direction of motion. U_1 and U_2 are the inlet and discharge blade edge speeds respectively, and the water will leave a blade at an angle γ to the tangent. This gives us the direction of the relative speed at discharge. For clearness in the diagram U_1 and U_2 are shown to one side of the point where the jet enters and so are not horizontal.

per lb. of water flowing to the work available per lb., or the head *H* ft., may reach 50 per cent.

Blade Interference

In discussing the action of a jet of water passing over a plate, we saw in the previous chapter that, as it left the blade, it impressed a force on the plate. If the water leaving a blade hits against another blade mounted on the same wheel, however, this reactive force is destroyed, for it presses the one blade backward with the same force that it presses the other forward, the one effect neutralizing the other.

In the diagram we see that the stream of water shoots into the uppermost bucket, but the stream then hits the back of the next oncoming bucket, and this leads to a considerable loss of effort. The bulk of the work is obtained simply from the weight of the water suspended in the various buckets. To prevent the water escaping too easily towards the end of its fall, a fixed guard is provided, made of wooden planking and mounted just clear of the wheel.

For rather smaller heads, the undershot wheel is used, one form of which we see in Fig. 15. Here, flat radial blades, mounted on the wheel, are driven forward by the stream of water. The construction is cheap but the efficiency, however, is low, seldom exceeding 30 per cent, even when the blades are running at the optimum speed, which is about $0{\cdot}4\sqrt{2gh}$ ft. per sec. A rather higher efficiency results, up to 50 per cent, if curved blades are employed, as in Fig. 16.

Before discussing the more efficient water turbine, we must investigate, in greater detail, the action of the water on the blading.

As an introductory step, let us discuss the relative speed of two

trains, moving in the same direction on parallel sets of rails. If we are in train (1) moving at, say, 12 m.p.h., and the other train (2) is moving at 8 m.p.h., we shall appear, to a man in train (2), to be moving in the same direction as his train at 4 m.p.h.

Let us now represent this graphically. If OA is marked off to represent 12 m.p.h. in its actual direction, as in Fig. 17(a), and then AB is set off to show 8 m.p.h. in the reverse direction to the actual motion, the length OB will scale 4 m.p.h. and will be in the same direction as AB, and so gives us the relative velocity. Here, then, is a rapid method of finding the relative velocity of two moving bodies. Chapter 4 deals more completely with this aspect.

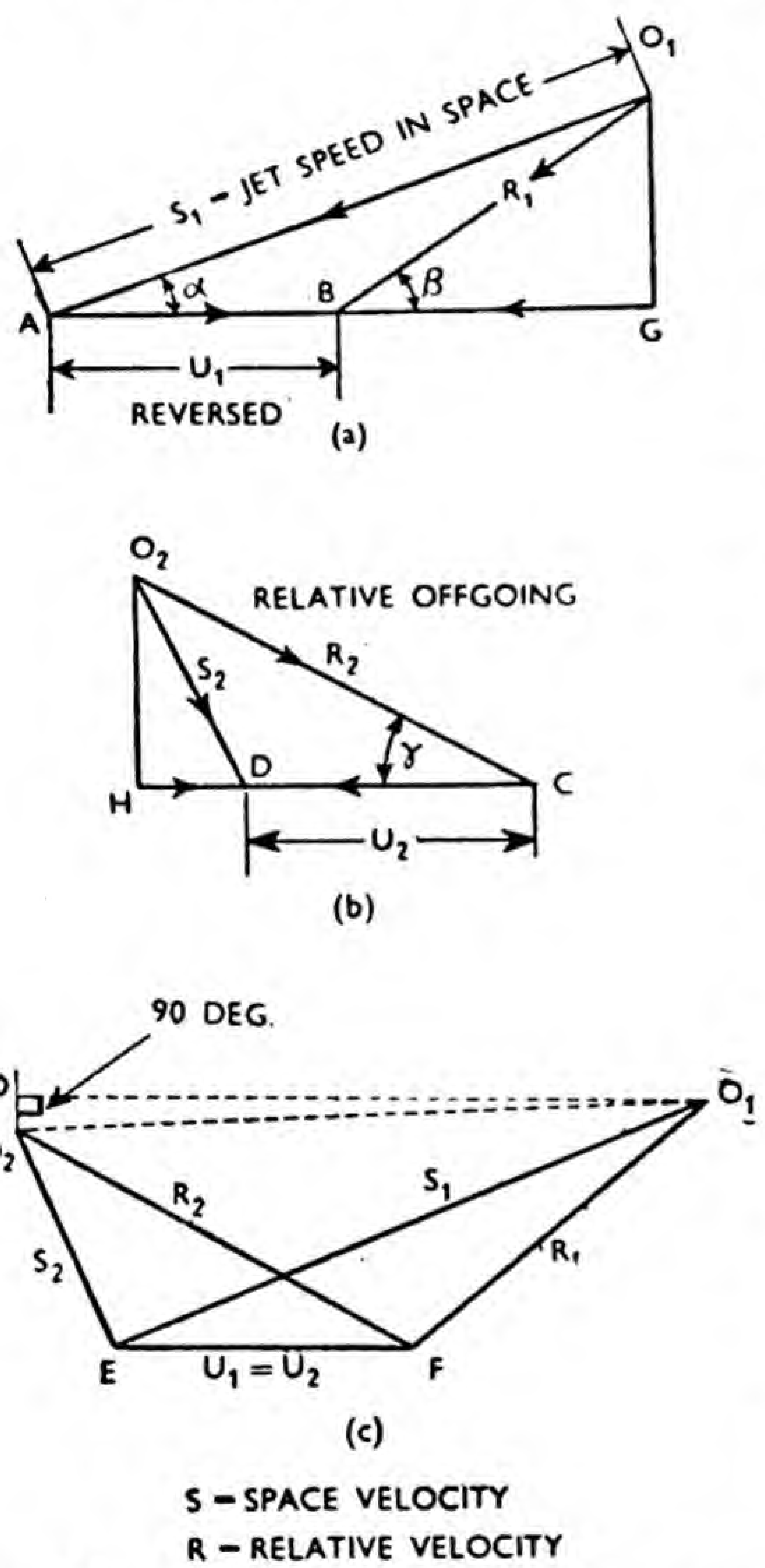
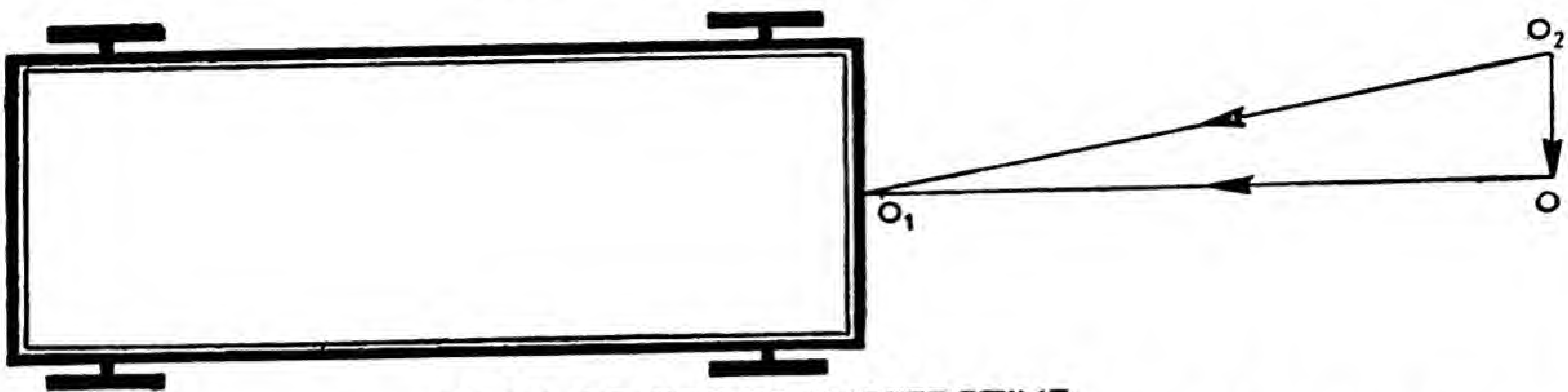


Fig. 18. (*a*) Is the velocity diagram at inlet, and (*b*) is the velocity diagram for outlet of the blades, the inclinations of R_1 and R_2 being the angles of the blades at inlet and discharge respectively. (*c*) Shows how we can overlap (*a*) and (*b*) on one another in the simplified case where the blade speeds U_1 and U_2 are equal. The actual change of velocity is shown to scale by the length O_1O_2, but the useful part of this is the projection O_1O drawn parallel to U_1 or U_2.

Blade Speed Reversed is Added

In order to find the speed of a stream of water relatively to a moving blade, we must, therefore, add on the blade speed reversed. Referring to the notation of Fig. 18, it will be seen that S_1 is the speed in space of the ongoing stream of water, inclined at an angle α to the direction of motion of the inlet edge of the blade, which has a velocity of U_1 ft. per sec. Now mark off O_1A to a suitable scale, as in Fig. 18(a), to show S_1 at an angle α to AB, which is drawn to represent U_1. The length O_1B gives us R_1, the relative inlet speed, and the angle β from the figure tells us the angle at which the inlet edges of the blades should be set to allow the stream of water to slide on to the blades without shock.

If, now, the water leaves the blades with a relative speed R_2, at an angle γ to the direction of motion of the outlet edges of the blades, as in Fig. 17(b), the corresponding outlet velocity triangle can be drawn. We set off O_2C to represent R_2, as in Fig. 18(b), and add on U_2, shown by CD, in its correct direction, and it will be noticed that S_2 gives us the speed of the water in space, as it leaves the blading. The



PART OF FORCE INEFFECTIVE

Fig. 19. Point raised in Fig. 18 (c) is made clearer if we refer to the above illustration, where it is seen that part of the force O_2O_1 acting on a truck is ineffective, the resolved portion O_2O tending to push the truck off the rails, and so doing no useful work.

reason that U_1 was added with its direction reversed while U_2 is set off in its true direction is that in Fig. 18(a) we were stepping from a space to a relative velocity, whereas in Fig. 18(b) we are finding a velocity in space from a relative velocity.

For ease of explanation, now assume that U_1 and U_2 are equal, so that AB may be superposed on CD, as in Fig. 18(c). If E is thought of as the origin of this velocity figure, we see that the stream has had its velocity in space changed from EO_1 to EO_2, the change itself being O_1O_2 ft. per sec.

If, on the other hand, it is imagined that we are moving with the blades, F must be treated as the origin of the diagram, when it will be seen that the speed of the water has been changed from FO_1 to FO_2. Once again the change of velocity is O_1O_2. For a rate of flow of water of w lb. per sec., the rate of change of momentum is, $\frac{w}{g} \times O_1O_2$, and the corresponding force on the water is, $w \times \frac{O_1O_2}{g}$ lb., while the force impressed on the blade is in the opposite direction, or, $w \times \frac{O_2O_1}{g}$ lb.

Next consider the force corresponding to O_2O_1 as acting on the simple truck, seen in Fig. 19. Clearly, work is done only by the force corresponding to the projection OO_1, the force corresponding to O_2O tending to push the truck off the rails, and doing no useful work.

In the case of our turbine, work is done by the change OO_1 of Fig. 18(c). This change is named the velocity of whirl, and the work done by each lb. per sec. of water is, $U_1 \times \frac{OO_1}{g}$ ft.-lb. per sec. If the more complicated case is taken, where U_1 and U_2 are not equal, the above formula must be modified to read :—$\left(\frac{U_1 \times AG - U_2 \times DH}{g}\right)$, where AG and DH are obtained from Figs. 18(a) and 18(b). It will be noticed that we have not, so far, discussed how the length R_2 is obtained from R_1.

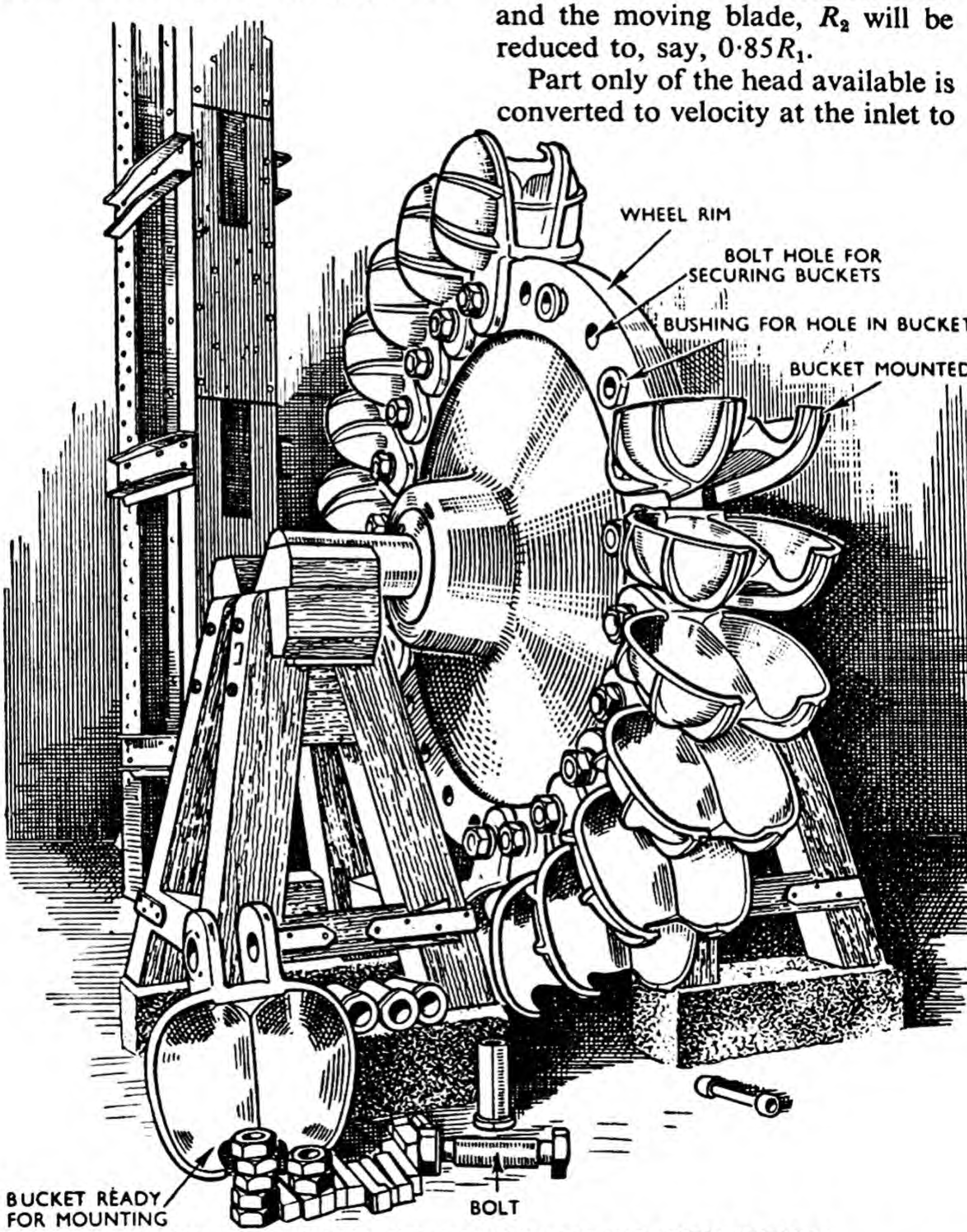
Impulse and Reaction Machines

There are two main types of turbine, named impulse and reaction machines, the latter being more suitable where the head available is relatively low.

In the impulse type the whole of the head available, H ft., is converted into velocity in a nozzle, following the relationship, $S_1{}^2 =$

$2gH$, as we saw in Chapter 11, the nozzle being very similar in shape to that used by a fireman. A jet of water slides as a band at atmospheric pressure over the various blades in turn, and as the whole of the head available has already been converted into kinetic energy, there is no energy available to speed up the water once it is in contact with the moving blade. Therefore, if there is no friction, $R_2 = R_1$. If, on the other hand, there is friction between the water and the moving blade, R_2 will be reduced to, say, $0{\cdot}85R_1$.

Part only of the head available is converted to velocity at the inlet to



PELTON WHEEL IN COURSE OF CONSTRUCTION

Fig. 20. The wheel is carried on a horizontal shaft resting on trestles and the buckets are attached by means of bolts passing through the rim. The central fin for causing the jet to split into two is seen clearly on the bucket in the lower left-hand corner.

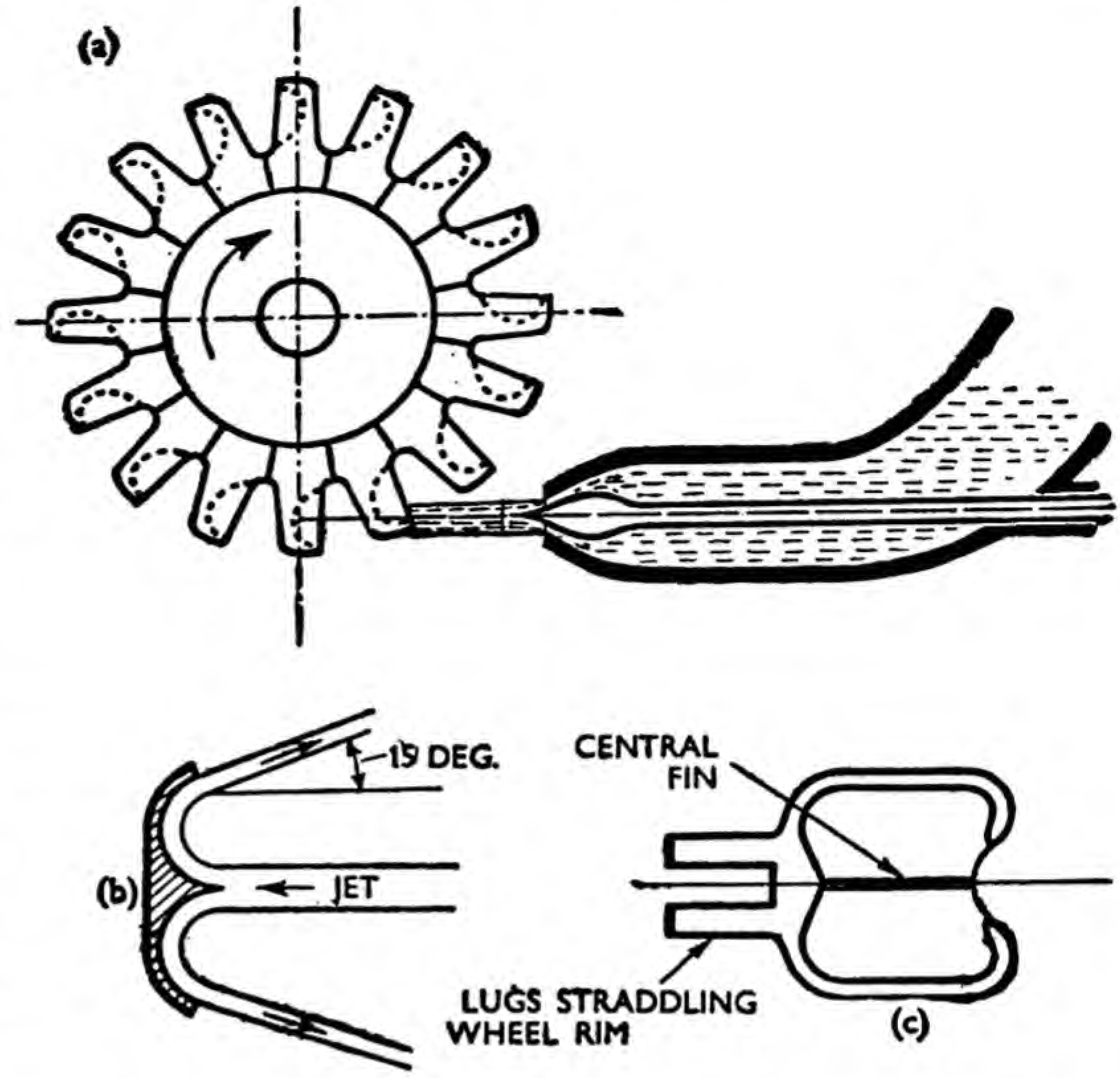


Fig. 21. Best known impulse turbine is Pelton wheel. (*a*) Nozzle with central needle controlled by governor, directs jet tangentially on to blading. (*b*) Jet is divided and turned through 165°. Cannot be turned 180°, as it would hit back of oncoming blade. (*c*) Frontal view of blade shows central dividing fin, and way in which bottom edge of bucket is cut away so that jet shall not hit blade until the face is nearly perpendicular to jet. This is to keep up the efficiency. A diagrammatic illustration only.

the reaction machine, the remainder being released inside the moving blade passage, thus increasing the relative velocity, so that R_2 is greater than R_1. This can happen only if all the passages between the blades are full of water. We know that a difference of head is possible in a Venturi meter, but it could not occur in an open channel, as the air over the water would balance up any difference of pressure between the ends. This means that the rotating portion of the reaction turbine, the runner, must be drowned.

The Pelton Wheel

The most important type of impulse machine is the Pelton wheel, in which one, or more, jets play tangentially on to a series of buckets mounted radially on a wheel, as in Figs. 20, 21(a) and 21(b). These turbines are suitable for relatively high heads of 400 ft. or more. It will be noted that the buckets are shaped in pairs, with a central dividing fin. This, as we saw in the last chapter, gives a larger effort than results from the jet hitting a flat plate. It might appear that the maximum force would be obtained if the jet were turned through 180 deg., but it is clear that this is not so, as the water leaving the bucket, which is in action at the moment, would hit the back of the next oncoming blade. This would cause a large loss of power, and so the jet, in practice, is turned through about 165 deg. The maximum force will be produced if the jet hits the blade squarely and, to this end, a portion of each bucket, as shown in Fig. 21(c), is cut away, otherwise earlier contact between jet and bucket will result, when the bucket is inclined at a considerable angle to its best position.

We must next sketch the type of

velocity diagram, as in Fig. 22, which will be obtained for, say, the half of the water which acts on the near side of the bucket of Fig. 21(a), making $R_2 = k \times R_1$, to allow for friction, but noting that the true value of k is not yet known. Let us imagine that $k = 0{\cdot}85$, and that the speed of the jet S_1 is fixed, while the load on the wheel is adjusted so that it runs at a series of different speeds with the jet control fully open throughout.

Maximum Output

A series of velocity figures can be drawn, as in Fig. 22, each with a different value of U, ranging from $U = 0$ to $U = S_1$. In each case we must measure the length OO_1, and calculate how much power is generated for a flow of 1 lb. per sec. of water, namely, $U \times \frac{OO_1}{g}$ ft.-lb. per sec., as was seen in the foregoing text.

It will be found that the maximum output occurs when $U = 0{\cdot}5 \times S_1$. This result is independent of the value of k chosen, for it will be found that the optimum value of U is $0{\cdot}5 \times S_1$, if the triangles are drawn again, for example, with $k = 0{\cdot}9$. Using the value $k = 0{\cdot}85$, we shall find that the maximum length of OO_1 is then $0{\cdot}91 \times S_1$.

The power is then :—$0{\cdot}91 \times S_1 \times \frac{U}{g}$, or, $0{\cdot}91 \times \frac{0{\cdot}5}{32{\cdot}2} \times S_1^2$ ft.-lb. per sec. for a flow of 1 lb. per sec. of water. If instead of a flow of 1 lb. per sec. a flow of 1 lb. per min. is considered, the same quantity of work will be developed in a minute.

It follows from this that we can omit time from both quantities and say that $0{\cdot}91 \times \frac{0{\cdot}5}{32{\cdot}2} \times S_1^2$ is the work in ft.-lb. per lb. of water supplied. The work available in the jet is the kinetic energy, $\frac{S_1^2}{64{\cdot}4}$ ft.-lb. per lb. of water, and the ratio gives us the maximum efficiency, namely, $0{\cdot}91 \times \frac{0{\cdot}5}{32{\cdot}2} \times S_1^2 \times \frac{64{\cdot}4}{S_1^2} = 0{\cdot}91$. In practice, two modifications are necessary. Tests show us that the best ratio $\frac{U}{S_1}$ is $0{\cdot}47$ instead of $0{\cdot}5$, and, in the above discussion, all mechanical losses due to windage of the wheel and bearing friction have been neglected.

Finding Value of k

We must now arrange a test to give us the value of k. Fig. 23 shows a suitable layout, with a scale model of the proposed blade mounted over a jet moving at R_1 ft. per sec. In a practical case, both the jet speed S_1, and the best blade speed $U = 0{\cdot}47\,S_1$ are known, while we see from Fig. 18 that, as the angle α is zero, $R_1 = S_1 - 0{\cdot}47 S_1 = 0{\cdot}53 S_1$.

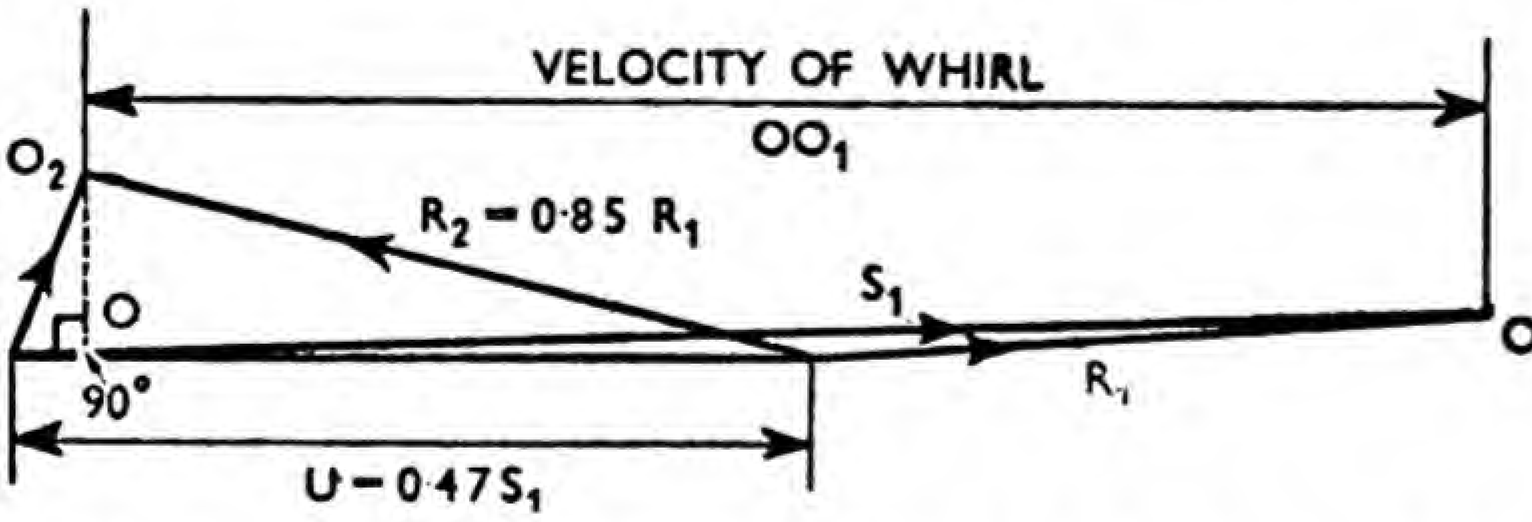


Fig. 22. In order to prevent confusion, the jet velocity S_1 is shown in this illustration at a small angle to the blade speed U, although the angle is actually zero. The angle between R_2 and U is 15°. The efficiency according to the diagram is $U \times OO/g \div S_1^2/2g = 0.91$.

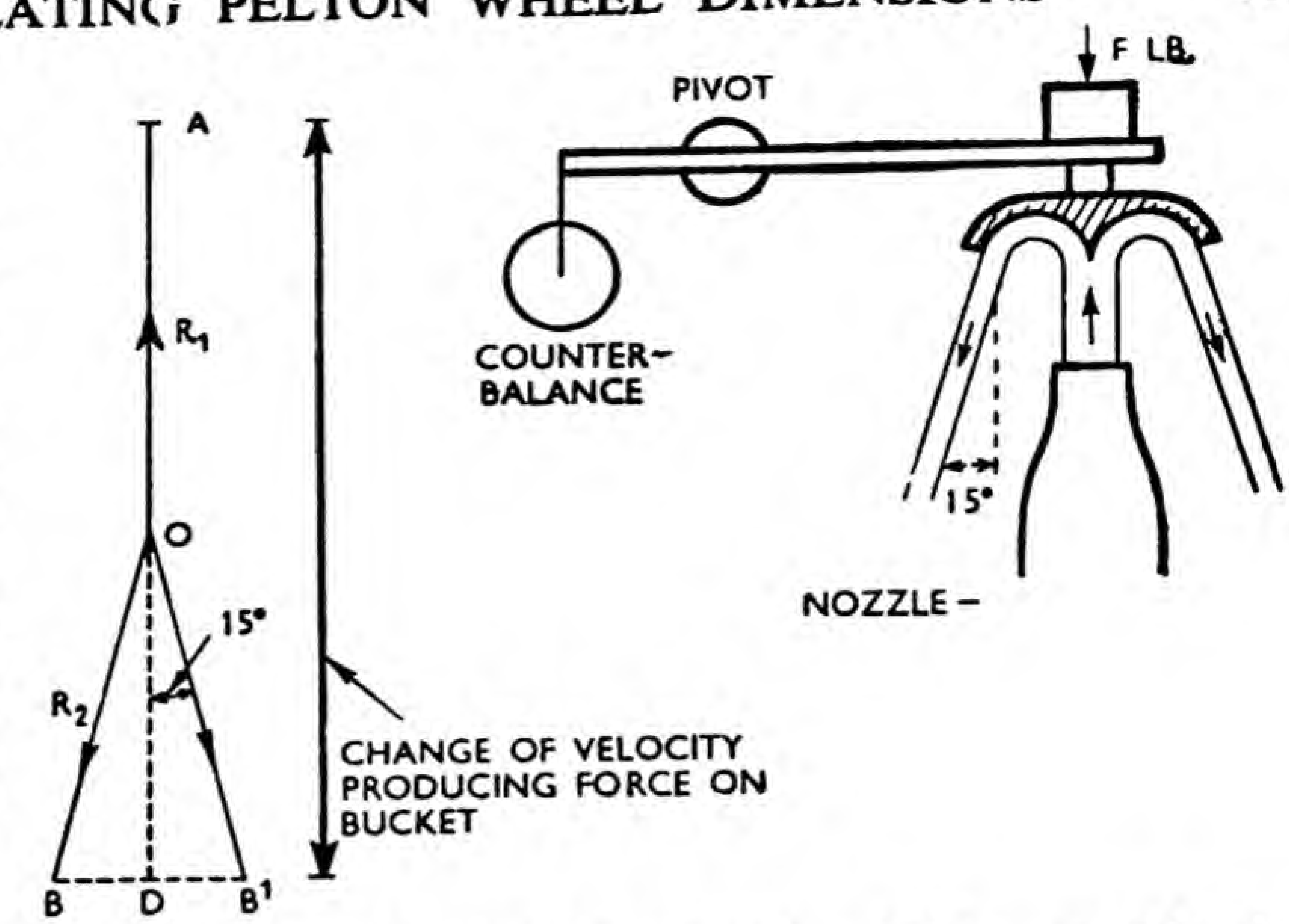


Fig. 23. Diagrammatic illustration of a weigh-arm carrying a model blade mounted over a jet moving at the relative velocity at inlet to the actual turbine. By weighing the force required to balance the thrust on the blade we can calculate the loss by friction as the jet travels over the surface of the bucket.

This is the value of the jet speed which must be used in our test, for it will be seen that as the blade is here stationary, we must now work in terms of the relative velocities instead of the velocities in space.

The blade is carried from a weigh-arm and the force F lb., required to balance the rate of change of momentum of the jet, can be measured. As the rate of flow W lb. per sec., from the nozzle, can also be measured, the formula

$$F = W \times \frac{\text{change of velocity}}{g} \text{ lb.,}$$

enables us to calculate the change of velocity, which is represented by the length AD of Fig. 23. Setting off OA to represent R_1, which is known, the length OB drawn parallel to the direction in which the water leaves the bucket can be scaled. This gives us R_2, and so the ratio $\frac{R_2}{R_1}$ or k can be found.

It will be of interest to calculate the main dimensions of a Pelton wheel to the following particulars: h.p., 1,000; total head available, 800 ft.; loss allowed in pipe-line, 50 ft.; value of f, 0·0075, the length of the pipe being 1,200 ft.; r.p.m., 500; coefficient of velocity of the nozzle, 0·97; mechanical efficiency, 0·94; efficiency from velocity triangles, 0·91.

The head available at the nozzle is 800 − 50, or 750 ft., and if Q is the quantity of water required in cu. ft. per sec., $Q \times 62{\cdot}4 \times \frac{750}{550} \times 0{\cdot}91 \times 0{\cdot}94 = 1{,}000$, giving us Q = 13·76 cu. ft. per sec. If d is the diameter of the pipe-line in ft., the velocity is $\frac{13{\cdot}76 \times 4}{\pi \times d^2} = \frac{17{\cdot}5}{d^2}$ ft. per sec., and on substituting this in $h_f = \frac{4fl}{d} \times \frac{V^2}{2g}$, we obtain

$$50 = \frac{4 \times 0{\cdot}0075 \times 1{,}200}{d} \times \left(\frac{17{\cdot}5}{d^2}\right)^2 \times \frac{1}{64{\cdot}4} = \frac{171{\cdot}6}{d^5}.$$

From this we find that $d^5 = \frac{171{\cdot}6}{50} = 3{\cdot}43$, and $d = 1{\cdot}28$-ft. diameter.

Quantity of Water Flowing

It has been assumed that the coefficient of velocity of the nozzle is 0·97, so that the speed of the jet is $0{\cdot}97\sqrt{2g \times 750}$, or 213·5 ft. per sec. Now the product of the area of the nozzle and the speed of the

jet gives us the quantity of water flowing per sec., Q, so that area of nozzle $= \frac{13{\cdot}76}{213{\cdot}5} = 0{\cdot}0645$ sq. ft., or 9·28 sq. in. The corresponding nozzle diameter is 3·44 in. The best bucket speed is given by $U = 0{\cdot}47S_1 = 0{\cdot}47 \times 213{\cdot}5 = 100$ ft. per sec., and so, if the diameter of the wheel is D ft. running at 500 r.p.m., we have $\pi \times D \times 500 = 60 \times 100$ ft. per min., or $D = 60 \times \frac{100}{\pi} \times \frac{1}{500} = 3{\cdot}83$ ft.

Reaction Turbine

The simplest type of reaction machine is the Francis turbine (Fig. 24), which is suitable for a head of 150 to 250 ft. We have seen earlier that in this type of machine we do not convert all the head available at inlet into the form of velocity and, in order that there may be a difference of pressure across the moving blade, the blade passages must be full of water. In other words, the runner or rotating portion must be drowned, and guide vanes, which correspond to the nozzle of the Pelton wheel, must be arranged round the complete circumference, as indicated in Fig. 25(a). The supply is led into a spiral casing, so designed that the tangential velocity is the same at all points, and is directed by the guide vanes on to the runner. The first question which will confront us is, why is such a costly casing used instead of a simple circular one?

The answer is clear if we look at Fig. 25(a), where it will be seen that the water has one general direction of flow at all points. In Fig. 25(b), however, it is found that, on the left-hand side, the water has to double back on itself, and this means that there is a large shock loss, with a corresponding loss of efficiency. We see also that the general direction of flow is inward, so that the spiral casing has to be made sufficiently strong to withstand the pressure of the supply. This is done on account of the governing of the machine.

Nearly all water turbines are used to drive electric gene-

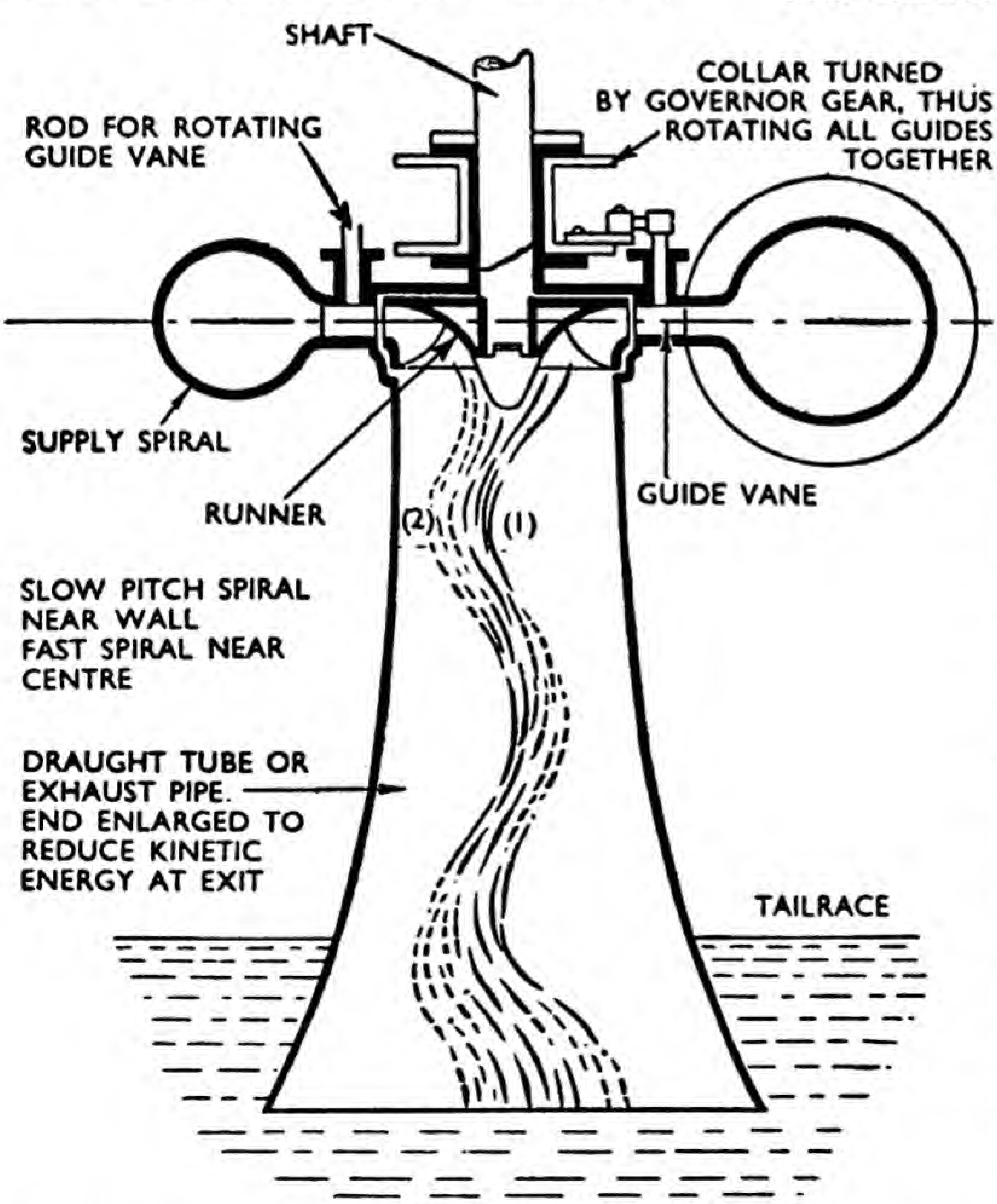
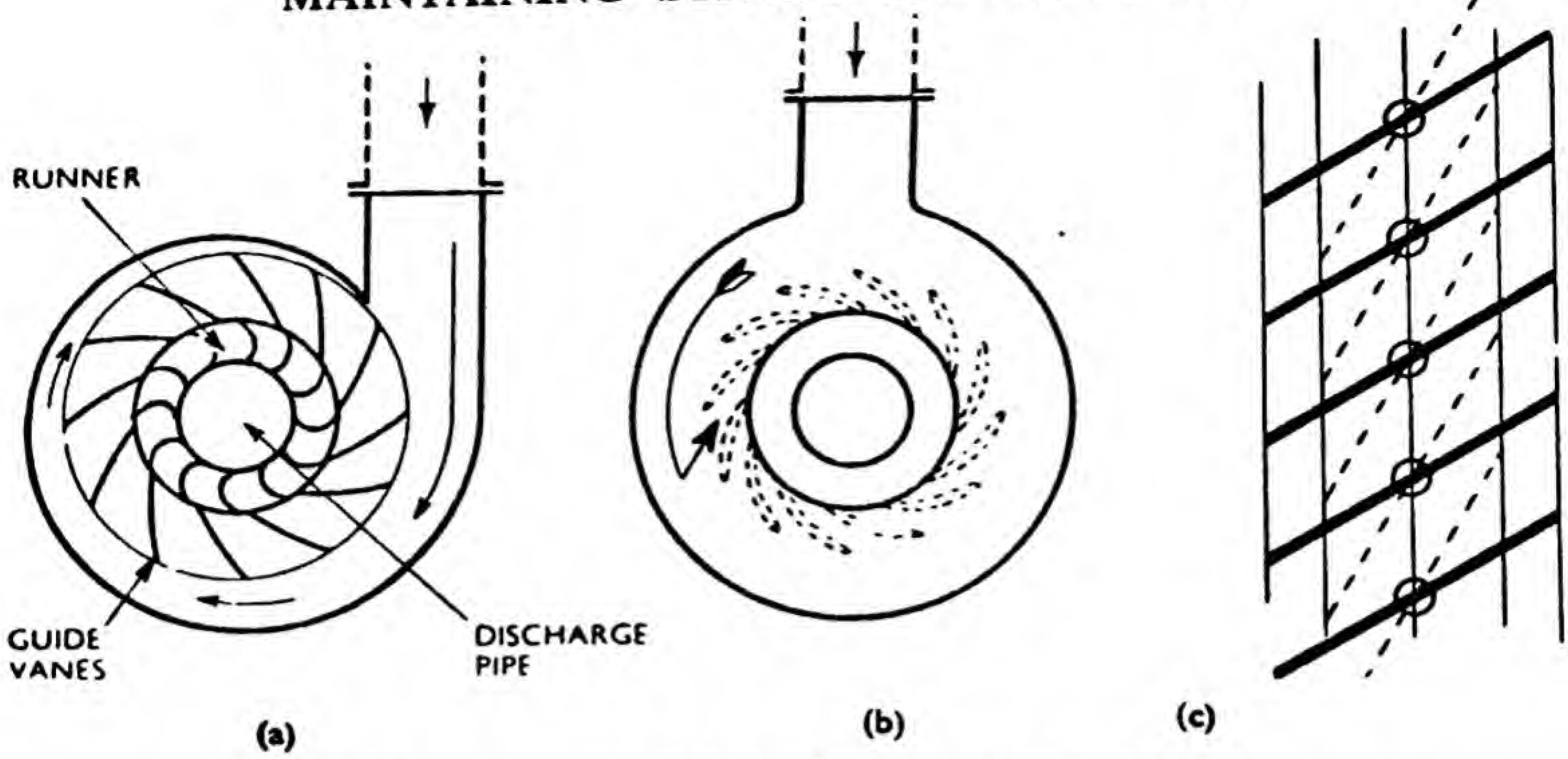


Fig. 24. Typical simple Francis reaction turbine. Water is led in through a spiral casing and passes radially through the runner, exhausting vertically.



GUIDE VANES OF A REACTION TURBINE

Fig. 25. (*a*) Shows type of spiral casing used to lead water up to guide vanes of a reaction turbine. It is designed to give a uniform tangential velocity at all points. (*b*) We see that a concentric casing around the ring of guide vanes, although far cheaper to manufacture, leads to large shock loss. Arrow on left-hand side shows how water at this point has to double back on itself on entering the guide passage, and we have seen that shock losses depend on the squares of changes of velocity. (*c*) Gives a diagrammatic arrangement of a louvre type of ventilator. The opening is far less when the plates are in the dotted than in the full-line position. Rotation of the guide vanes of the turbine has a similar effect in changing the area available for passage of water.

rators, and a steady speed is essential. Let us suppose that some of the load is thrown off, by switching off motors connected to the generator. The speed of the turbine will rise slightly before the governor has time to act, and this will increase the centrifugal action. We all know that the pull in a string, which is whirled round with a weight attached to its free end, increases with the speed. There is a similar action on the water inside the runner, and an increase of speed increases this effect, tending to resist the inflow of the water.

If, on the other hand, we build a machine in which the general direction of flow is outward through the runner, an increase of speed will tend to produce a still larger flow, and this in turn will increase the speed still further. Therefore, it is easier to maintain a steady speed in the inward-flow machine.

The governor itself controls the speed by changing the angle of the guide vanes. Each vane carries a tail-rod, which projects through a gland in the casing, as in Fig. 24, and all the vanes are turned together, so that the resultant area of flow is adjusted to pass the quantity of water necessary to keep the speed steady. The action is very similar to that in a louvre type of ventilator, the opening of which is adjusted by altering the slopes of the plates, as indicated in Fig. 25(c).

Vertical Shaft

Most turbines are arranged with the shaft vertical. This allows the use of a simple form of draught or discharge tube, without any bend. When we dealt, in the previous chapter, with the problem of the flow in a pipe joining two reservoirs, we found that there was a loss at the exit end equivalent to

the kinetic head, $\frac{V^2}{2g}$ ft.-lb. per lb. of water.

To reduce this loss to a minimum, the outlet pipe of our turbine is made bell-mouthed to provide as large a cross-section as possible, and so reduce the final velocity of discharge.

The exhaust pipe is arranged to discharge under the surface of the water in the tail-race. This not only ensures that the turbine casing is kept full of water or drowned, but allows the column of exhaust water to help to draw the water through the turbine by producing a pressure at the top of the pipe, which is below atmospheric.

This could not be done in the case of the Pelton wheel, where the machine ran in a casing open to the atmosphere. Here, we have one of the reasons why the reaction machine is better suited to low heads than the impulse type, for there is no loss of head due to the machine being mounted above the tail-race level.

Another reason is, that for a given head and power, the rotational speed of the reaction machine, for maximum efficiency, is rather greater than that of the impulse type. This means that a cheaper electric generator can be employed to absorb the power. As in the case of the suction pipe of the reciprocating pump, separation or cavitation will occur in the exhaust pipe if it is too long, exceeding, say, 20 ft. Certain conditions of flow in this pipe will aggravate matters, and these we will discuss with reference to a simple analogy.

Increasing Inertia

Let us imagine an acrobat, spinning round on a vertical axis, on a ball-bearing mounted in the heel of one of his shoes. If he has a certain angular velocity when his hands are at his sides, and he then stretches out his arms, he will increase his inertia, and his speed of rotation will fall. Actually, his angular momentum, $I \times \omega$, where I is his inertia and ω his angular velocity, will be the same for both positions of his arms.

In the case of the discharge pipe, if the water leaving the runner has any velocity of whirl, that is, any tangential velocity, the whole of the water in the discharge pipe will flow down along a series of spirals. The angular velocity, about the vertical centre line will be greatest near the centre. Now let us consider the points (1), near the centre, and (2), near the wall, of the exhaust pipe of Fig. 24. These points are on a horizontal, so that Bernoulli's law tells us that p_1 must be less than p_2, since we have just shown that V_1 exceeds V_2.

A very low pressure may result near the

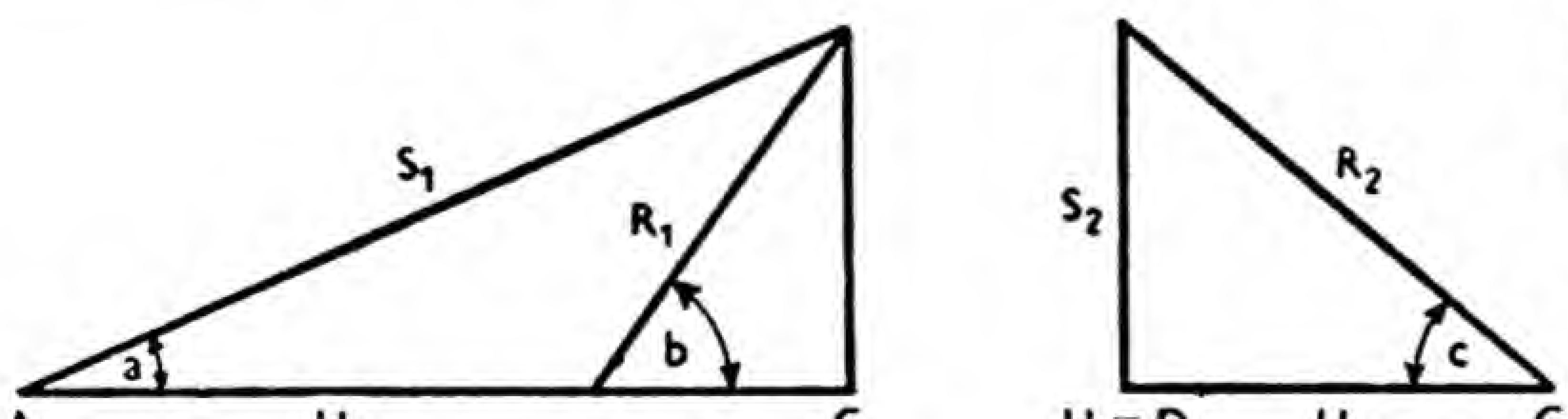


Fig. 26. Typical velocity figures for reaction turbines. To prevent rotation of the water in the exhaust pipe, the water leaving the runner of a Francis turbine should have no velocity in the tangential direction. This means that S_2 is at right angles to U_2 and the velocity of whirl HD at exit (compare Fig. 18(b)) is zero. $(R_2^2-R_1^2)/2g$ is the head required to speed up the water in its passage over the moving blades.

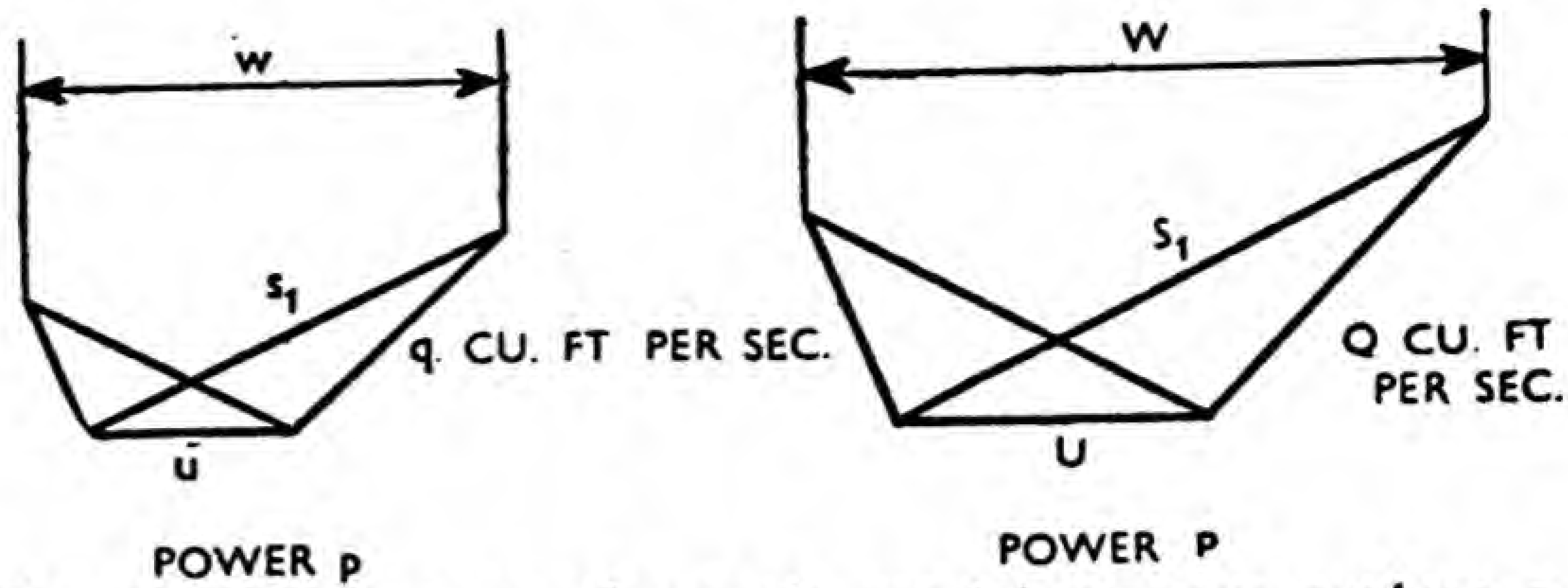


Fig. 27. By means of a scale model we can analyse a proposed design of turbine. It is not necessary that the driving heads on the two machines should be to the same geometrical scale as their linear dimensions, but we must make the ratio s_1/S_1 or $u/U = \sqrt{h/H}$. The analysis gives us a clear guide as to the most suitable type of machine for any given set of conditions of head, power and speed.

centre line at the top of the pipe. If cavitation then occurs, the water in the discharge pipe will break away and fall down, the syphon action at the outlet will be destroyed and the head on the turbine suddenly reduced. The governor will automatically open out to keep up the speed, admitting more water, and this will fall on top of the broken column with a heavy blow. Dangerous stresses may result, and so it is desirable to arrange the exit blade angles so that the water leaves the runner radially. The water in the draught tube will then flow axially without rotation, as shown by Fig. 26.

Scale Model Tests

In the reaction machine, the flow is more complicated than in the Pelton wheel and it is not easy to separate the various losses into mechanical, and hydraulic in the guide passages, runner and exhaust. Particulars for building a large turbine are usually obtained from tests on a scale model of, say, 20 h.p., in which modifications of the runner design can be tried out at a relatively low cost.

In order to analyse this method, we will discuss two geometrically similar Pelton wheels, as this case is rather easier to follow than that of the reaction machine, although the same argument applies for both impulse and reaction turbines.

If both machines are operating under their best conditions of speed it follows that the water must slide on to both sets of blades with the minimum of shock, and the various angles of the velocity figures will be exactly those of the machines. It is not necessary that the two driving heads should also be geometrically similar, but we must use the same design of nozzle in both cases, so that if $S_1 = k\sqrt{2g \times \text{head}}$, k is the same for both designs.

If we refer to Fig. 27, we now see that the ratio of the lengths of any corresponding pair of velocity lines depends on the square root of the ratio of the heads; thus, for example, $\dfrac{u}{U} = \sqrt{\dfrac{h}{H}}$. The ratio of the diameters of the two nozzles will be the same as that of the two wheels, or $\dfrac{d}{D}$, and so the ratio of the two quantities of water flowing per sec. will be :—

$\dfrac{q}{Q} = \dfrac{\pi d^2}{4} \times \sqrt{h} \div \dfrac{\pi D^2}{4} \times \sqrt{H} = \dfrac{d^2}{D^2} \times \sqrt{\dfrac{h}{H}}$. The ratio of the h.p. from the relationship,

power = lb. per sec. of water × blade speed × $\dfrac{\text{velocity of whirl}}{g \times 550}$,

is $\dfrac{p}{P} = \dfrac{d^2}{D^2} \times \sqrt{\dfrac{h}{H}} \times \dfrac{u}{U} \times \dfrac{w}{W}$, but we see from the triangles that $\dfrac{u}{U} = \dfrac{w}{W} = \sqrt{\dfrac{h}{H}}$, and so $\dfrac{p}{P} = \left(\dfrac{d}{D}\right)^2 \times \left(\dfrac{h}{H}\right)^{3/2}$. Again $\dfrac{u}{U}$ or $\sqrt{\dfrac{h}{H}}$ is also $\dfrac{\pi dn}{\pi DN}$, where n and N are the respective r.p.m., from which we find that $\dfrac{d}{D} = \sqrt{\dfrac{h}{H}} \times \dfrac{N}{n}$. On making this substitution for $\dfrac{d}{D}$ we have, $\dfrac{p}{P} = \dfrac{h}{H} \times \left(\dfrac{N}{n}\right)^2 \times \left(\dfrac{h}{H}\right)^{3/2}$

$$= \left(\frac{N}{n}\right)^2 \times \left(\frac{h}{H}\right)^{5/2} \quad \dots \text{(1)}$$

Now let us imagine that another model is built of such a size that it develops 1 h.p. when it operates under a head of 1 ft. At what speed should it run for greatest efficiency?

Use of Specific Speed

If we take the test results of a number of similar Pelton wheels, and insert the values of P, N and H for the best working of each machine in equation (1) above, it will be found that in each case the optimum speed of our special model is given by $\dfrac{1}{P} = \left(\dfrac{N}{n}\right)^2 \times \left(\dfrac{1}{H}\right)^{5/2}$, the value of both p and h being 1. This reduces to:—

$$n = \frac{N\sqrt{P}}{H^{5/4}} \text{ r.p.m.}$$

As this number is the same for all the wheels tested, it is named the specific speed, being specific to any one type of runner. We now have a convenient method of deciding which type of machine is best suited to a given set of conditions of power, speed of rotation and head. Let us turn back for a moment to the design of the Pelton wheel which was made earlier, when the specific speed is found to be

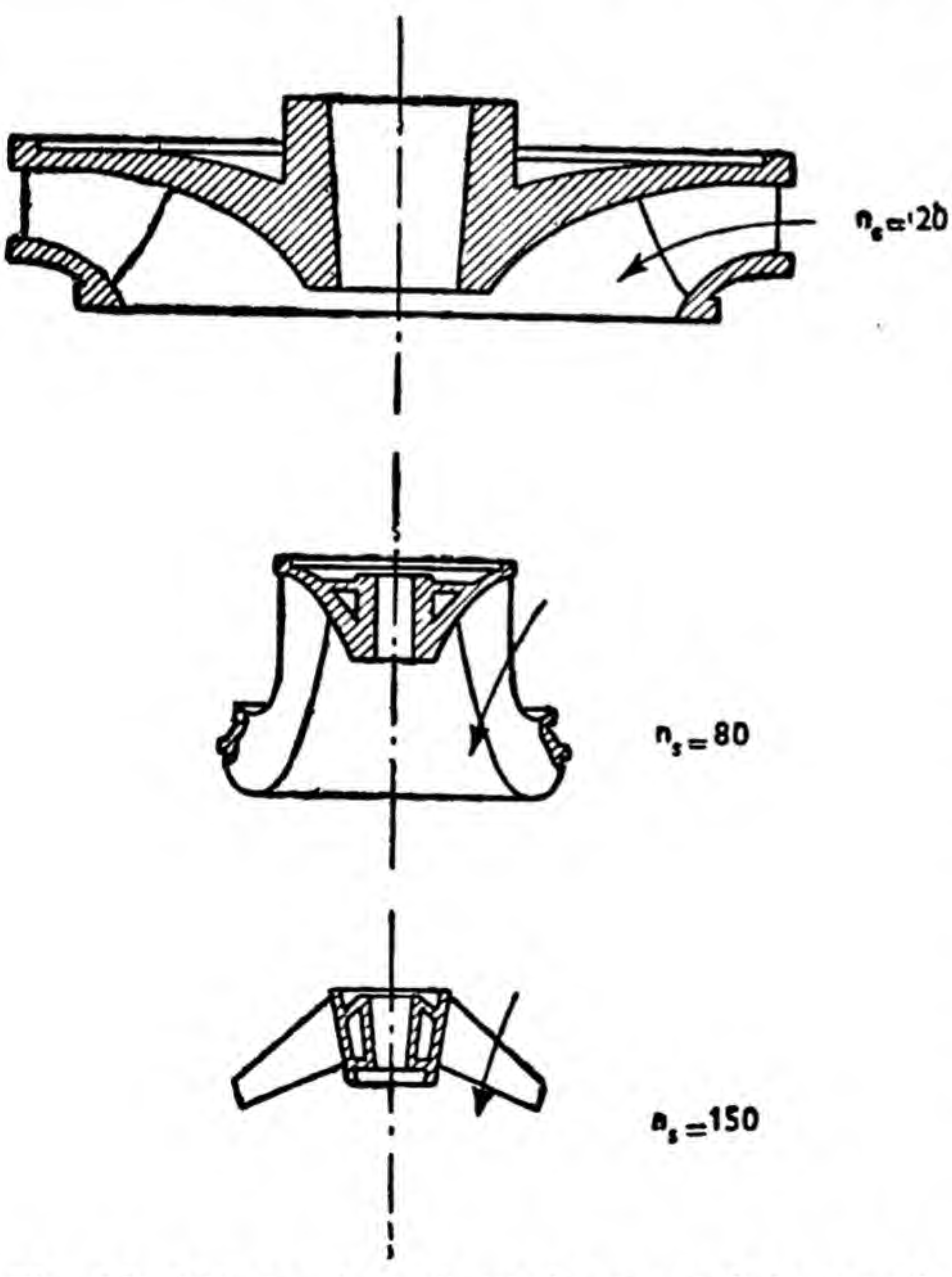


Fig. 28. We see here how the general design of the specific runner of the reaction type, that is, a machine which will develop one horse-power under a head of one foot, changes with the speed of rotation, viz., as the values of n_s increase, the direction of flow changes from radial to axial.

$$n = \frac{500\sqrt{1{,}000}}{750^{5/4}} =$$

4·02.

The specific speed of a good design of Pelton wheel falls between about 3·5 and 4, so that our value of 4·02 is close to the upper limit. If, on the other hand, the value were found to be, say, 15, a simple Francis reaction turbine would have to be used for efficient operation. Fig. 28 shows us that there is a considerable latitude of design in the case of the reaction machine, as the specific speeds range from 10 to about 150.

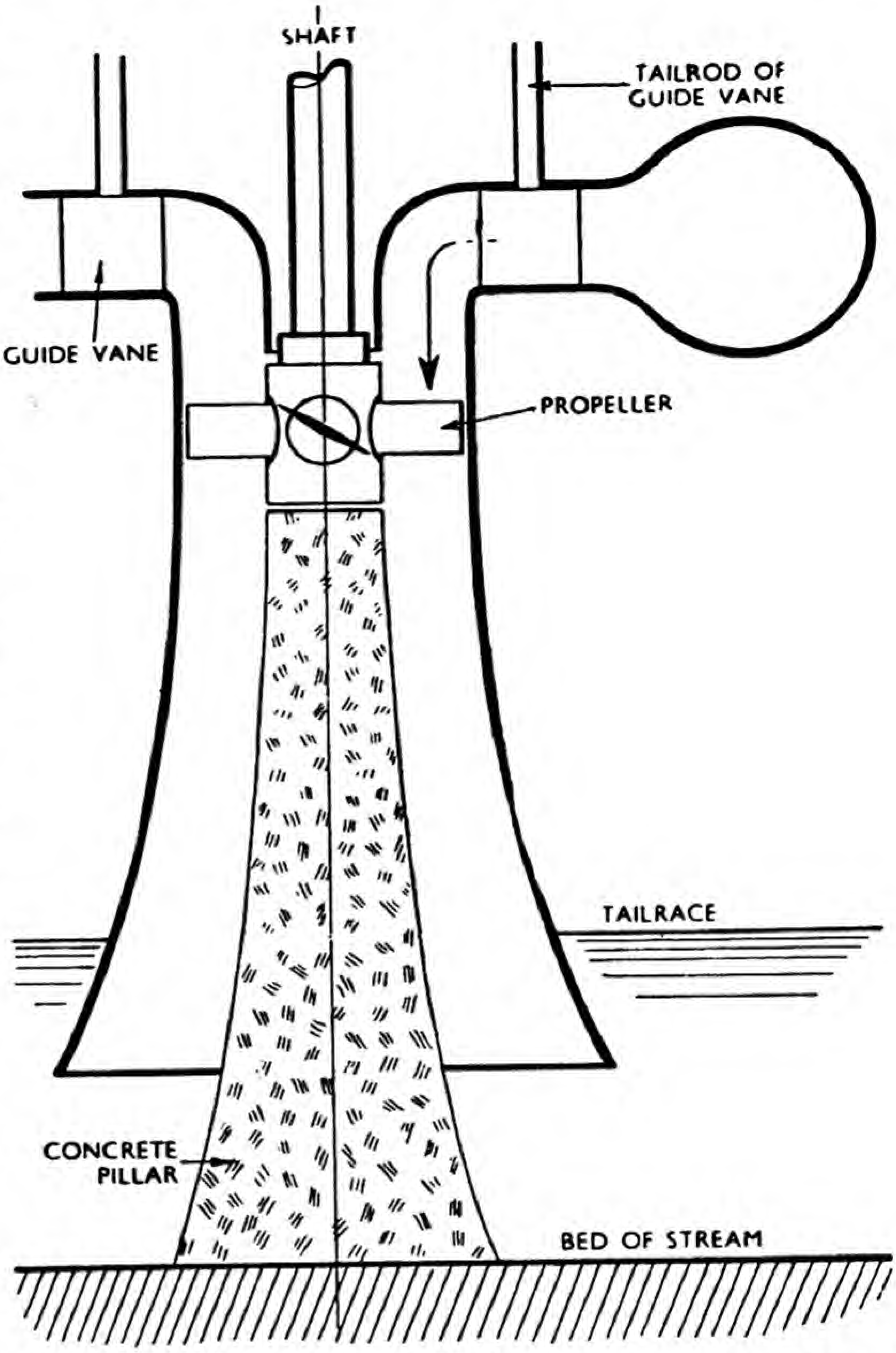


Fig. 29. For the highest specific speeds, a propeller type of runner is used. The guide vanes and intake details are very similar to those of a Francis machine. To avoid cavitation a central pillar of concrete is often built in the draught tube. This reduces the speed of rotation of the water near the centre of the pipe and prevents the formation here of too low a pressure.

Propeller Turbine

The greater the specific speed, the smaller the head on which the particular machine will work most efficiently, the value of 150 corresponding to a head of, say, 60 ft. If a still smaller head has to be utilized, of, say, 35 ft., a propeller type of turbine is used. The spiral casing and guide-vane system which is illustrated in Fig. 29 is very similar to that used in the Francis machine, but the water leaving the guides flows in the form of a spiral passing more or less axially through the runner. This, as Fig. 30 shows, is very similar in appearance to the propeller of a ship.

In practice, the load which a turbine carries is usually well below its full capacity. During a normal day, the bulk of the output will be absorbed in driving electrical motors in works, but when, in the

late afternoon, a heavy lighting load is switched on in addition, our machine has to develop its full output. Under partial load, the conditions of flow do not lend themselves to giving us the maximum efficiency.

Increasing Efficiency

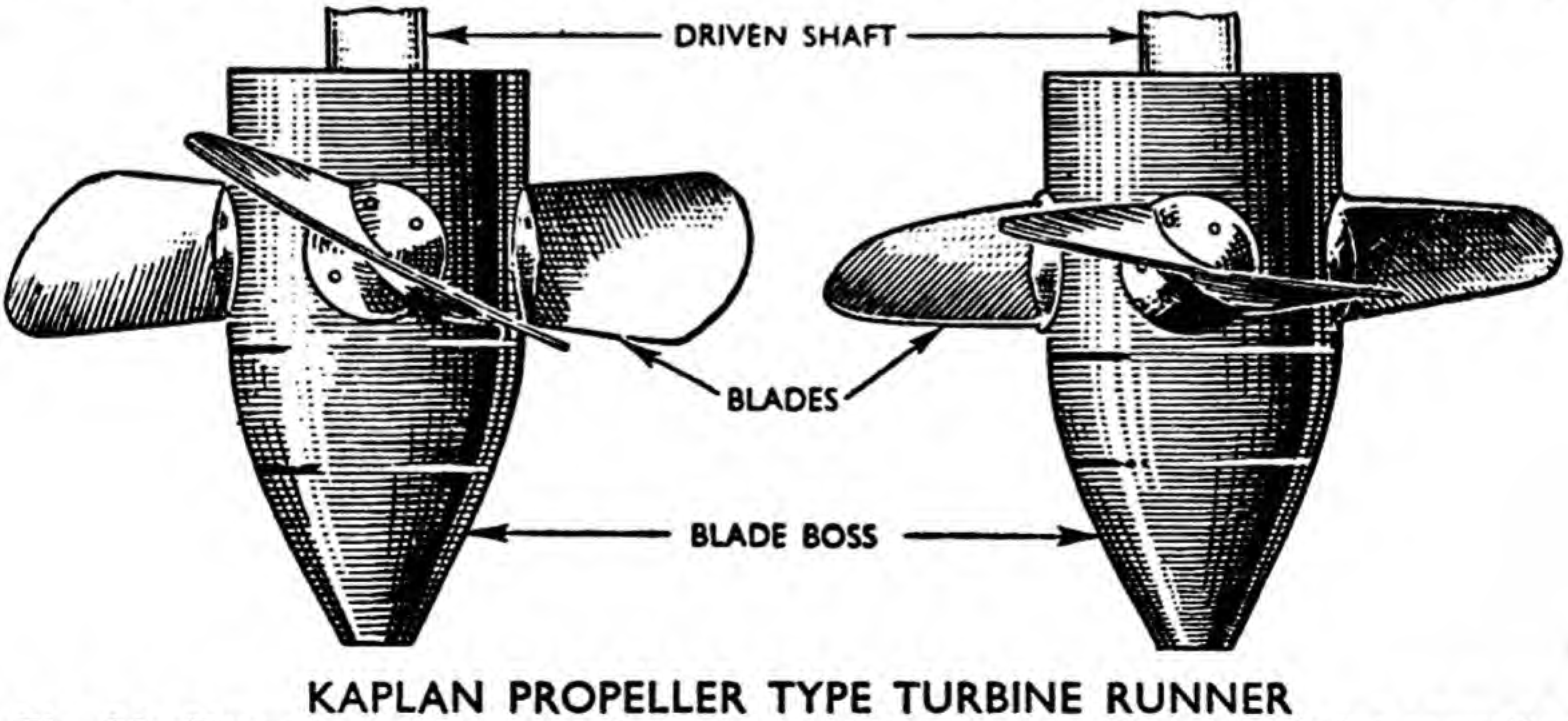
If we refer to Fig. 31, we see that when the guide vanes are turned by the governor gear, changing the ingoing velocity line S_1, from the full to the dotted position, the new or dotted position of the relative velocity does not overlap its original setting.

This means that the water entering the runner will not slide on to the blading, but will enter at an angle, so producing a shock loss, for the blade was originally made to suit the full line position of the line R_1. If we now swing the moving blade round so that it is parallel at the inlet to the dotted position of R_1, we shall eliminate this shock effect, and so increase the efficiency of the machine.

Control, by the governor, of both guide and runner vanes is effected in the Kaplan turbine. Fig. 30 indicates two settings of the blades of a runner of this type. With this design, specific speeds of 200 may be reached. To reduce the chances of cavitation in the exhaust pipes of these machines, the turbines are mounted, when possible, only a few feet above the tail-race. If the discharge pipe is long, however, the production of a high vacuum can be minimized by arranging a central column of concrete in the pipe, as shown in Fig. 29. We saw earlier that the pressure tended to reach a minimum near the centre line of the pipe, and this tendency is now reduced, for the velocity of the water at the minimum radius will now be diminished by frictional drag against this central pillar.

Centrifugal Pumps

The three-throw pumps, which we discussed earlier, were specially suited to the supply of high-pressure water for use in hydraulic machinery such as presses, etc. For pumping large volumes of drinking water against a head of, say, 120 ft., it is often convenient to install a three-throw reciprocating pump



KAPLAN PROPELLER TYPE TURBINE RUNNER

Fig. 30. Two views of a propeller type turbine runner. Runner blades as well as guide vanes in the casing are controlled by the governor. Object is to improve efficiency on partial loads. Left-hand diagram shows blade setting for full output and, on the right hand side, setting of the blades for a small output.

driven from a triple-expansion steam engine.

Where a supply of electricity is available, a centrifugal pump is generally employed for heads ranging from 30 to 300 ft. Unlike the turbine, the pump requires no governor, its speed being controlled by the driving unit. For the smaller rates of discharge, a single-entry pump is used, as in Figs. 32(a), (b), (c) and (d).

The water in the supply pipe enters the centre of the impeller, as the rotating member is named, in an axial direction and then spreads out radially into the blading, like the spokes of a wheel.

If the velocity diagram for point (1) of Fig. 32(b) is drawn, Fig. 32(e) is obtained, in which S_1, drawn radially, is the inlet or pipe velocity, and to this we add the inlet blade speed U_1 reversed, in order to obtain the relative velocity at the inlet R_1. This gives us the corresponding angle b of the blade at the inlet. At the point (2) at the discharge, we have to add the blade speed U_2 to the relative velocity R_2 to give us the velocity of the water in the space at the exit S_2. The height f_2 of the outlet-velocity triangle is seen to be at right angles to the blade speed, and so represents the radial component of the velocity at the exit from the runner.

By making the circumferential area at the inlet equal to that at the outlet, viz., $2\pi r_2 b_2$, we have $f_2 = S_1$. This fixes the value of R_2, if the outlet blade angle c is known, so allowing us to complete the triangle.

It is generally found that S_2 is relatively long compared with S_1, showing us that the water leaving the impeller at the larger radius has a considerable amount of kinetic energy stored in it. This should be expected if the problem is compared with that of a stone whirled round on the end of a string, when we know that the longer the string, the greater the speed of the stone, for a given speed of rotation.

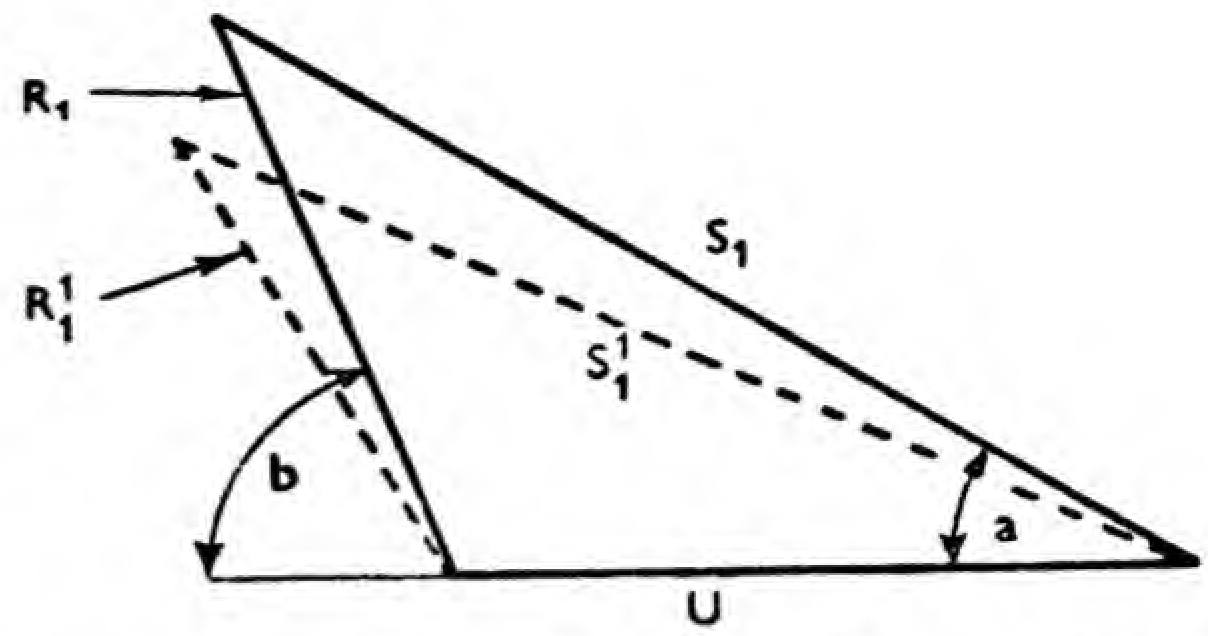


Fig. 31. On partial load, the governor normally changes only the guide vane angle a, displacing S_1 to $S^1{}_1$. The new position of R_1, that is $R^1{}_1$, no longer agrees with the inlet angle of the blades b, which is made to suit R_1. A shock loss then occurs at inlet, which is eliminated if the governor gear also swings the runner blades around to suit. This is done in the Kaplan turbine to keep up the efficiencyon partial loads.

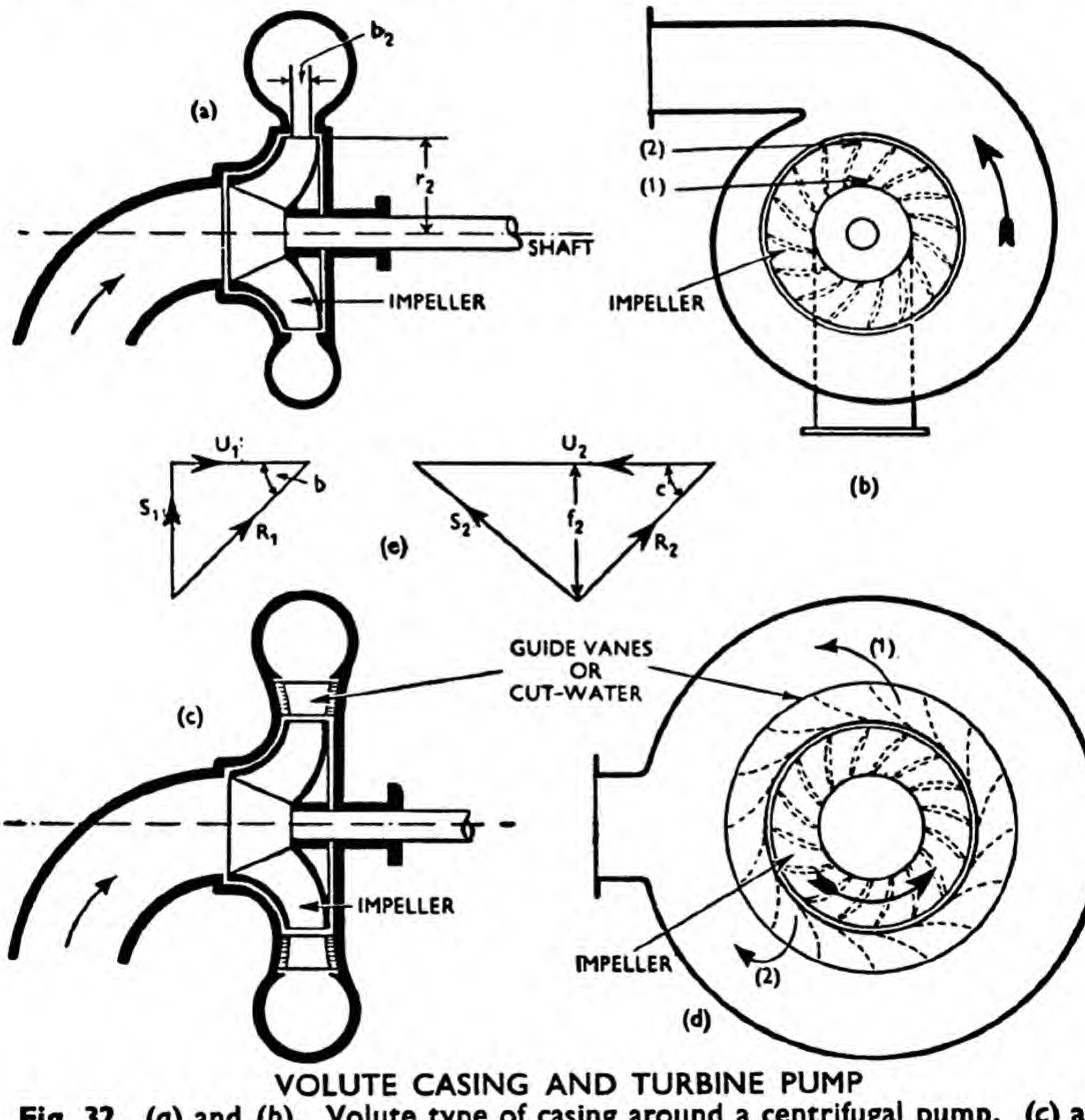
Energy Conversion

Our next problem is to convert as much as possible of this kinetic energy of the water leaving the impeller into pressure head. In the last chapter we saw how a trumpet-shaped pipe was fitted beyond the throat of a Venturi meter to slow down the water, and so build up the pressure head.

Two methods are adopted in the design of a pump casing to effect this conversion of energy. One is by means of the volute casing, seen in Figs. 32(a) and (b), in which the tangential speed of the water is constant as in the spiral of the Francis turbine. In the other, as in Figs. 32(c) and (d), the impeller discharges into a set of guide vanes, the passages between which act in the

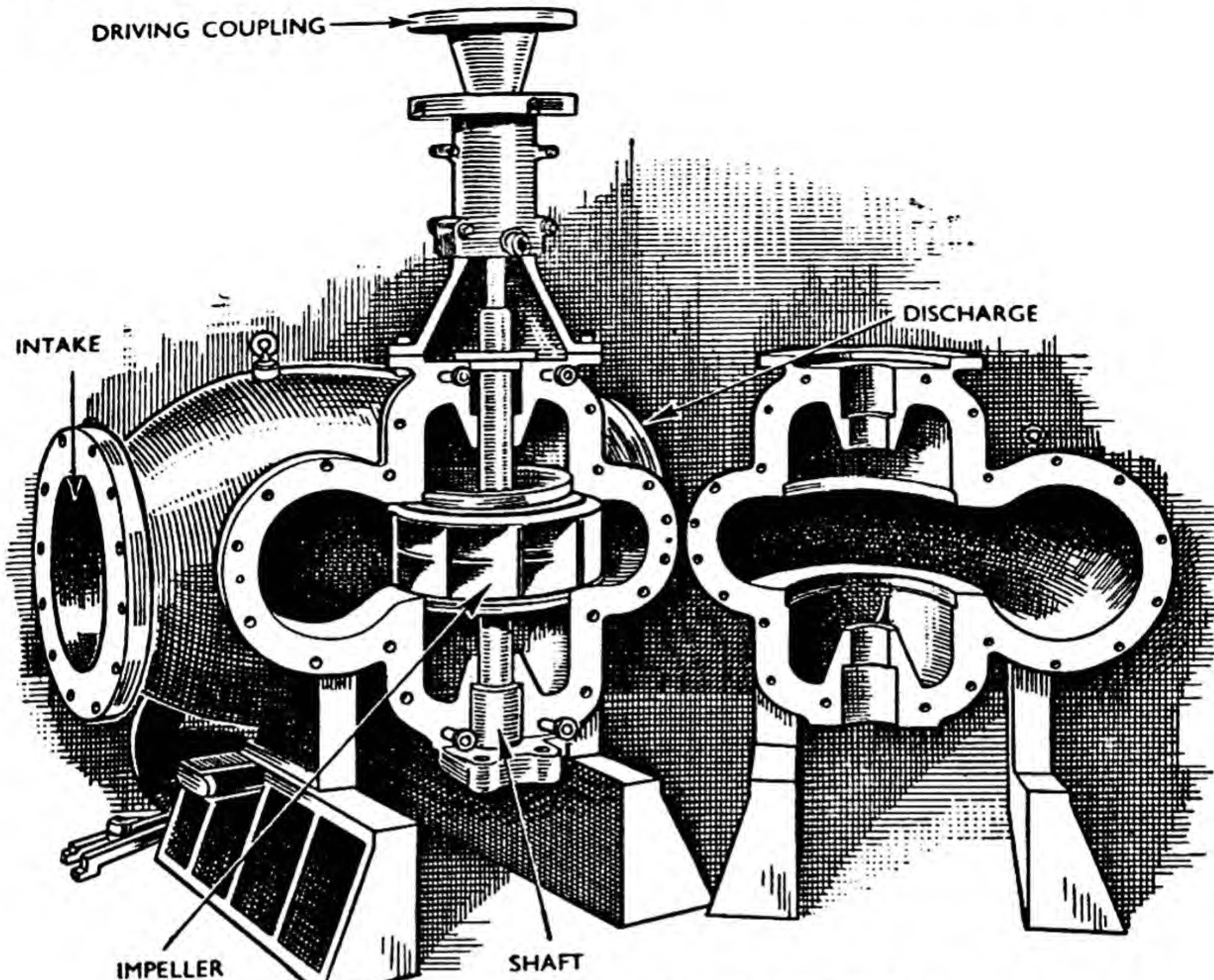
same way as the trumpet of the Venturi meter.

When the water leaves these guides, its velocity is so low that its kinetic energy is small, and it can be led into a simple concentric casing. The pump with the volute casing is slightly more efficient, about 60 per cent of $\frac{S_2^2}{2g}$ being converted into additional pressure head, but the patterns for the casting of the casing are rather expensive. If a manufacturer wishes to standardize his casings and fit various internals into them to suit different conditions of revolutions per minute, rate of discharge and head, he will probably choose the symmetrical form of casing. This latter type, being provided with guide vanes, bears a certain resem-



VOLUTE CASING AND TURBINE PUMP

Fig. 32. (*a*) and (*b*). Volute type of casing around a centrifugal pump. (*c*) and (*d*) illustrate the turbine pump. The spiral casing at (*b*) is more efficient than the concentric casing (*d*), although more expensive in manufacture. In (*b*) the water leaving the impeller all flows in the same general direction, as indicated by the arrow. In (*d*), however, some of the water flows in the normal way as at (1), whereas, unlike (*b*), that at (2) has to turn back on itself. This causes a considerable loss of pressure. The impellers are the same in principle, the essential difference of the two designs being in the method adopted for converting the kinetic energy of water leaving the impeller (*d*) at S_2 ft. per sec. (*e*) into additional pressure head.



DOUBLE-ENTRY CENTRIFUGAL PUMP

Fig. 33. A double-entry centrifugal pump is shown with part of the casing removed and mounted on the right hand side, so that the position of the impeller and its shaft are on view in the left hand diagram.

blance to the reaction turbine, and is known, accordingly, as the turbine pump, to distinguish it from the volute type. The essential difference is in the casing, the impellers being similar. For handling larger quantities of water, a double-entry pump is used, as in Fig. 33.

One great advantage of the rotary over the reciprocating pump is that the flow is steady so that no acceleration head is developed. A weakness of the centrifugal type is that it has no suction effect unless the complete pump is filled with water before it is started up. This means that a non-return valve must be fitted at the foot of the suction pipe, as in Fig. 34(a), and provision made for filling or priming the pump. Fig. 34(b) shows a better arrangement, especially if chemicals, which give off gases freely, are to be handled. Here, the pump is always drowned and so no non-return valve is required.

Syphon System

In the case of the reciprocating pump, the term corresponding to the kinetic energy in the pipe-line can usually be neglected. This is not so, however, in the case of a centrifugal pump operating on a syphon system, as in Fig. 34(b).

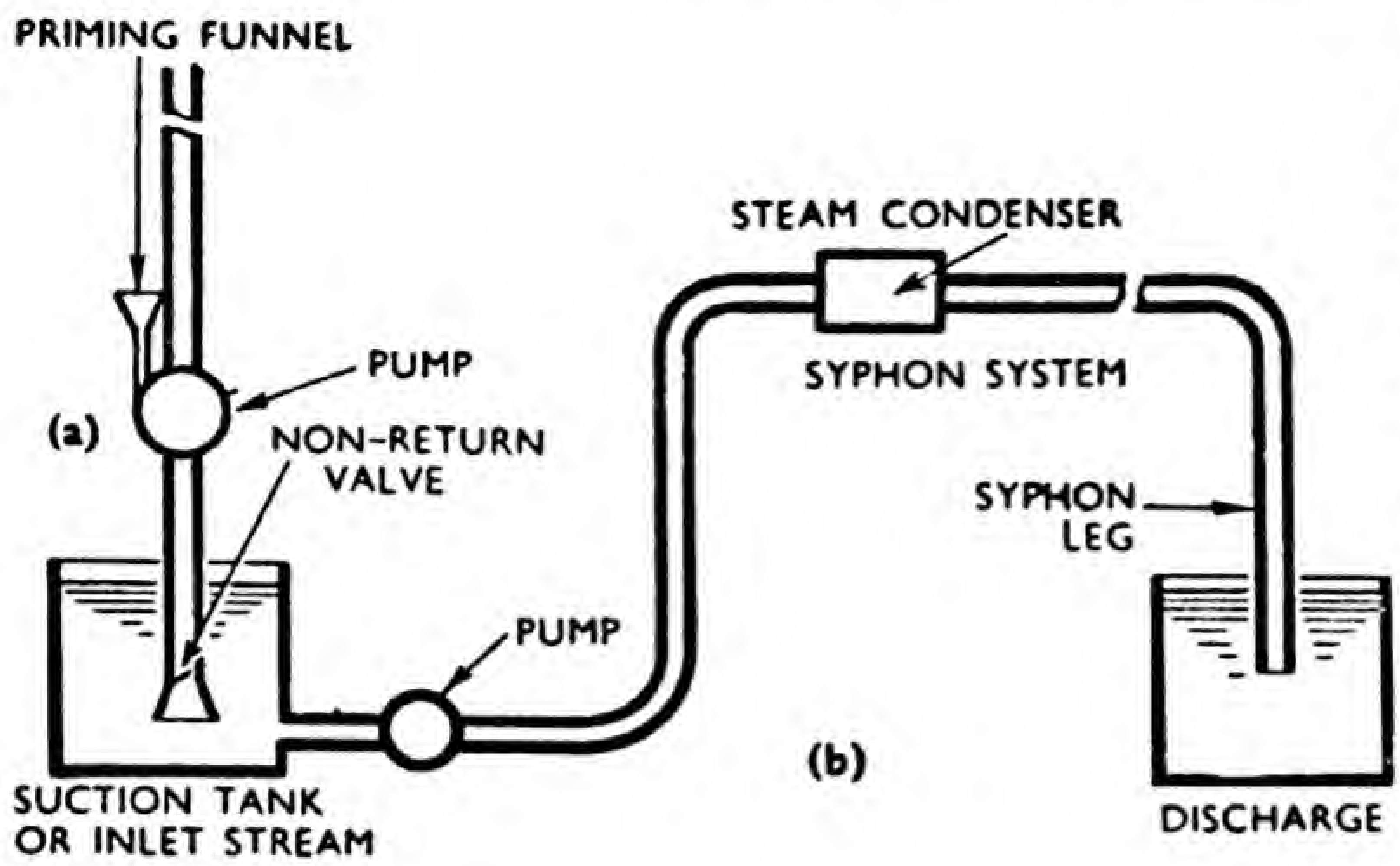


Fig. 34. If suction arrangement is as at (*a*), a funnel and non-return valve must be provided for filling the pump, known as priming, before the driving motor is started. (*b*) Better arrangement of intake pipe. If a syphon system is used, some means must be provided for extracting air, and so flooding the whole pipeline for starting up, after which, syphon leg will assist flow.

This layout is very common in connexion with the steam condensers of electric power stations, where the water is taken from a river and discharged further down-stream at almost the same level, the end of the pipe being taken underwater.

Air Ejector Provided

Provision must be made for filling the whole pipe system. This may take the form of an air ejector, attached to the top of the pipe-line, when, as we have already seen, water will be forced up into the piping by the pressure of the atmosphere acting on the free surface of the water round both the suction and discharge pipes. Another method is to speed up the pump so that it will lift the water to the highest point of the pipe system. When the pipe-line has been filled, the speed may again be reduced, as the vertical discharge leg will act as a syphon, helping to draw the water over.

The total head will now consist of entrance, frictional and exit losses. This last term is important. A common value for the entrance and friction losses is about 12 ft., and as the speed in the pipe is usually about 8 ft. per sec., the exit loss is $\frac{8^2}{2g}$ or 1·0 ft., which cannot be neglected, as it is a large fraction of the total, 12 + 1, or 13 ft.

If no supply of water is available for filling or priming the pump, as in the case of a fire-engine, a separate exhauster must be provided to extract the air from the pump, thus reducing the pressure and drawing up water through the suction pipe.

Another advantage of the centrifugal pump is that there is a definite maximum head which can be developed for any given rotational speed, and if the casing is strong enough to withstand this, no safety-valve is needed.

The pumps which we have discussed so far are suited to heads up to, say, 100 ft. Above this value, two or more single-entry impellers are mounted on the same driving shaft, the outlet of the one being led to the inlet of the next. Each impeller increases the head by the same amount.

Where large quantities of water are to be pumped against relatively small heads, a propeller pump is used. This is very similar to the propeller type of reaction turbine, except that the impeller is now rotated by external power.

CHAPTER 13

TESTING AND DRIVING OF MACHINES

DYNAMOMETER DESIGN AND APPLICATION. MEASURING BRAKE HORSE-POWER. SWINGING-FIELD DYNAMOMETER. WATER BRAKES. NO-LOAD TEST. BALANCING A FLYWHEEL. STATIC AND DYNAMIC BALANCE. MEASUREMENT OF VIBRATION. TESTING MACHINE TOOLS. DRIVING OF MACHINERY. BELT, ROPE, CHAIN AND GEAR DRIVES. MOTOR AND GROUP DRIVES. CHANGE-SPEED GEARBOXES. INFINITELY VARIABLE GEARS.

When we buy an engine, a motor car or an electric motor, it is most important that we should know what horse-power it will develop. But how can we measure this power? If we buy from a maker of repute, his figure is usually accepted without question, because it is known that the maker will have tested the machine to ensure that it is up to specification. For this purpose a dynamometer is used, and, for important machines, it is usual for the purchaser to witness the dynamometer test at the maker's works.

There are many types of dynamometer, and the type we use depends very largely upon the kind of machine that we are testing, but the method of measurement is common to all types, and depends upon the following relationships :—

Work done = Torque × Angle turned through (in radians).

Power = Torque × Speed (in radians per sec. or per min.).

Inserting the appropriate units we can write :—

$$\text{Horse-power} = \text{Torque (in lb.-ft.)} \times \text{Speed (in r.p.m.)} \times \frac{2\pi}{33{,}000}.$$

The dynamometer measures the torque, and a speed counter is used to determine the r.p.m.

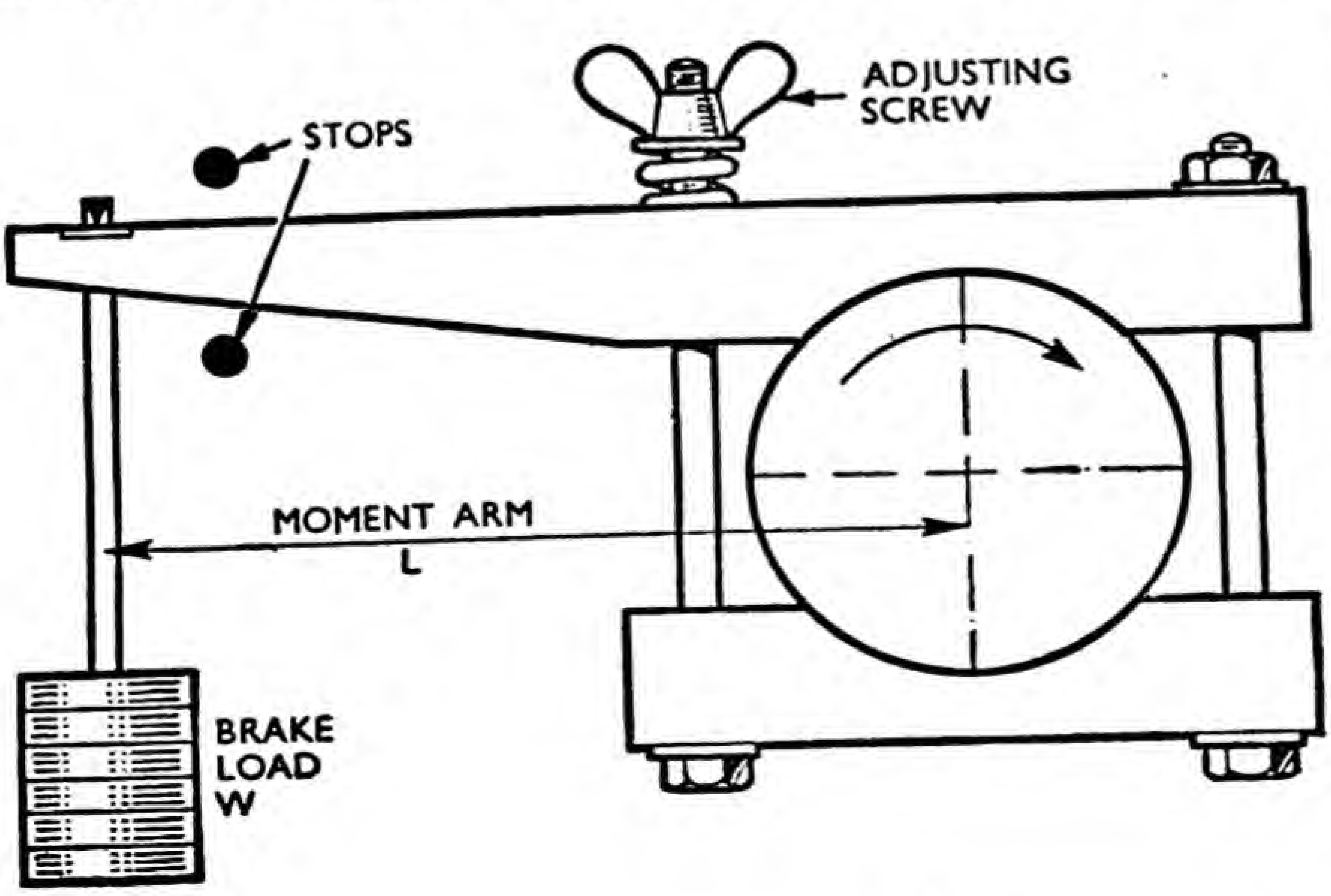


Fig. 1. Horse-power of a machine such as a steam engine, which has a flywheel or driving pulley, may be measured by a simple dynamometer. Resistance is applied to rotation of flywheel by means of a brake made from two shaped wooden members, and torque required to keep brake from rotating with flywheel is measured by weights attached to the end of the moment arm.

If the machine has a flywheel or a driving pulley to which a brake can be applied, it is possible to arrange a very simple form of brake dynamometer as shown in Fig. 1. The idea is to apply a resistance to the rotation of the flywheel, but not sufficient to stop it, and then to measure the torque which is required to keep the brake from rotating with the flywheel, whilst the flywheel slips round within it.

Two wooden members, shaped to fit the rim, are clamped to the flywheel, and one member is extended to form a moment arm, so that weights can be attached in order to measure the torque. As the grip of the clamp is increased by means of the adjusting screw, more weights must be added to the moment arm to prevent its being carried round with the flywheel. When making a horse-power test, the adjusting screw is gradually tightened, and readings of load and speed are taken until the point is reached where the engine starts to lose speed rapidly. Maximum horse-power is developed at that particular speed at which the product of the load and the speed is a maximum. This is usually determined by plotting a curve as shown in Fig. 2.

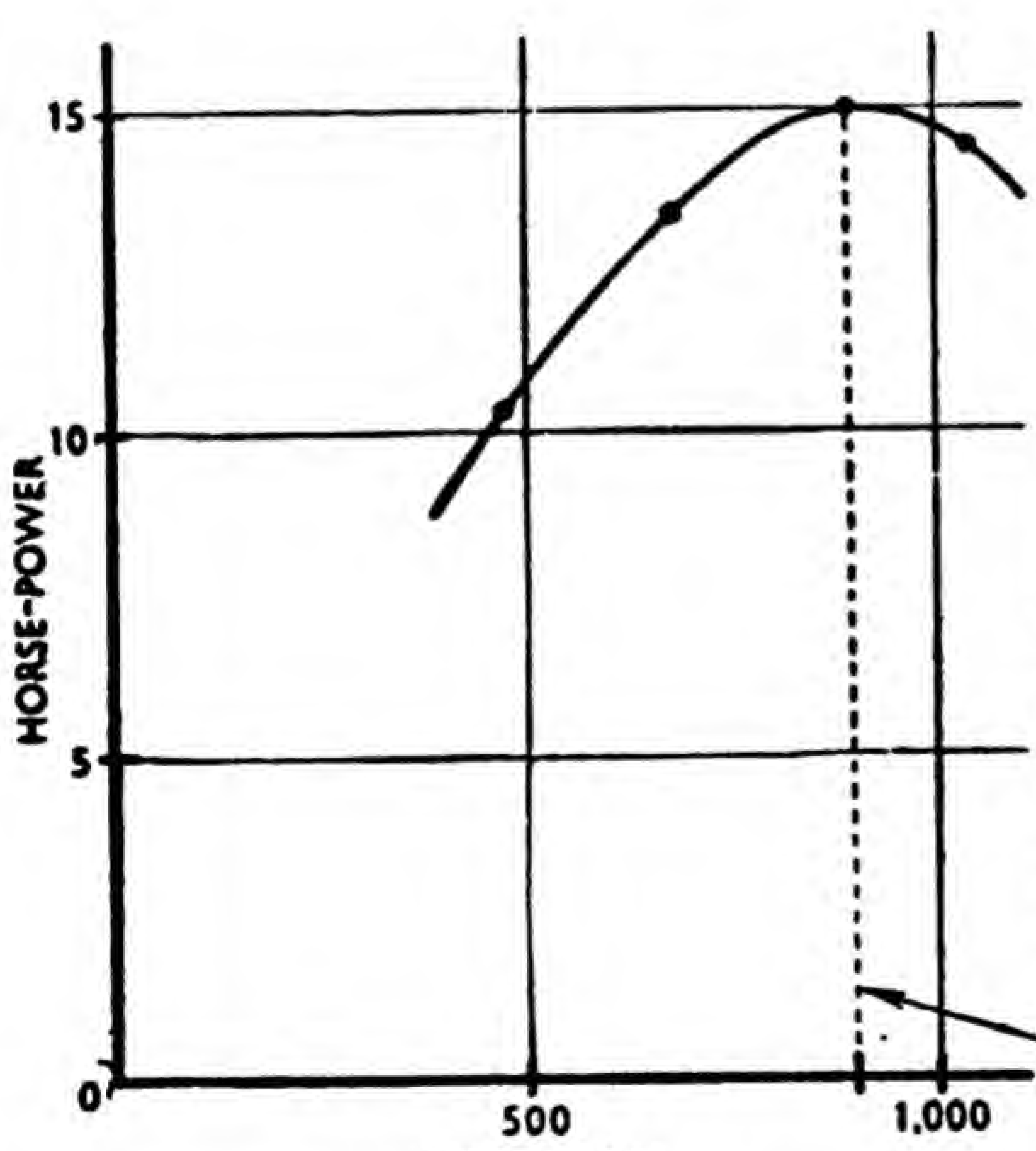
In this test the maximum horse-power of 15 was developed at a speed of 877 r.p.m. The effective length of the moment arm is the distance L, measured from the centre of the flywheel to the line of action of the load, so that :—

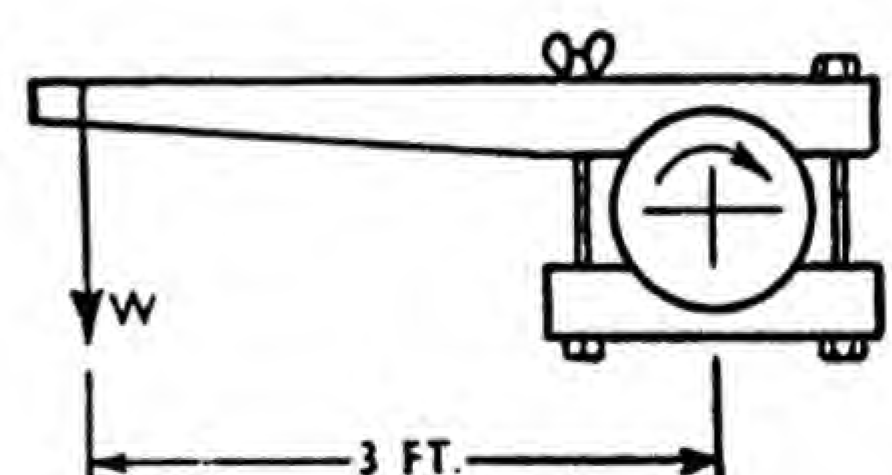
Horse-power

$$= W \times L \times N \times \frac{2\pi}{33{,}000}$$

where N is the speed in r.p.m.

Instead of the clamp, we can use a brake band as shown in Fig. 3. Obviously, we are now applying a braking load to the flywheel in a manner similar to that of a belt on a





BRAKE LOAD W. IN LB.	25	30	35	40
SPEED R.P.M.	1,010	877	670	450
LOAD×SPEED	25,250	26,310	23,450	18,000
TORQUE IN LB.-FT.	75	90	105	120
B.H.P.	14·4	15·0	13·4	10·3

MAXIMUM H.P CONDITIONS

PLOTTING THE RESULTS OF A DYNAMOMETER TEST

Fig. 2. The dynamometer depicted above was used to measure the horse-power of a gas engine. A series of readings of load and speed was obtained by tightening the adjusting screw, seen more clearly in Fig. 1. The graph of horse-power and speed, plotted from these results, showed that the maximum horse-power of 15 occurred at a speed of 877 r.p.m. The product of speed and load was then a maximum.

pulley, except that in this case the pulley is slipping so that the belt remains stationary. In Chapter 10 we saw that with a belt on a pulley, the effective tension was the difference between the tensions in the tight and slack sides. It is just the same here. The tight side is the side opposing motion, and to which the load *W* is attached. The slack side is the one assisting motion, and the tension here is usually measured by means of a spring balance.

The advantage of a spring balance on the slack side instead of a small weight, is that it makes the brake self-adjusting; the brake band slips round if *W* is too great, and so stretches the spring balance until the force *w* exerted by it makes the effective load (W-w) just right. As more weights are added to *W*, to increase the torque, the spring-balance reading automatically increases to the correct figure.

With this belt dynamometer, the brake torque is given by the product of the effective load and the radius of the flywheel rim, viz., $(W\text{-}w) \times R$. Therefore, the horse-power is as follows :—

b.h.p.

$$= (W\text{-}w) \times R \times N \times \frac{2\pi}{33{,}000}.$$

As with belt drives, the ratio of the tensions in the tight and slack sides depends upon the amount of lap of the belt on the pulley. For large powers, therefore, it is an advantage to have the brake band lapping completely round the flywheel rim. This is shown in Fig. 4 with a double loop of rope forming the brake band, and wood blocks to keep the rope in place. We calculate the b.h.p. in the same way as for Fig. 3, but, of course, the load *W* will be much greater for the same spring-balance reading *w*, so that the effective brake load (W-w) will be greater.

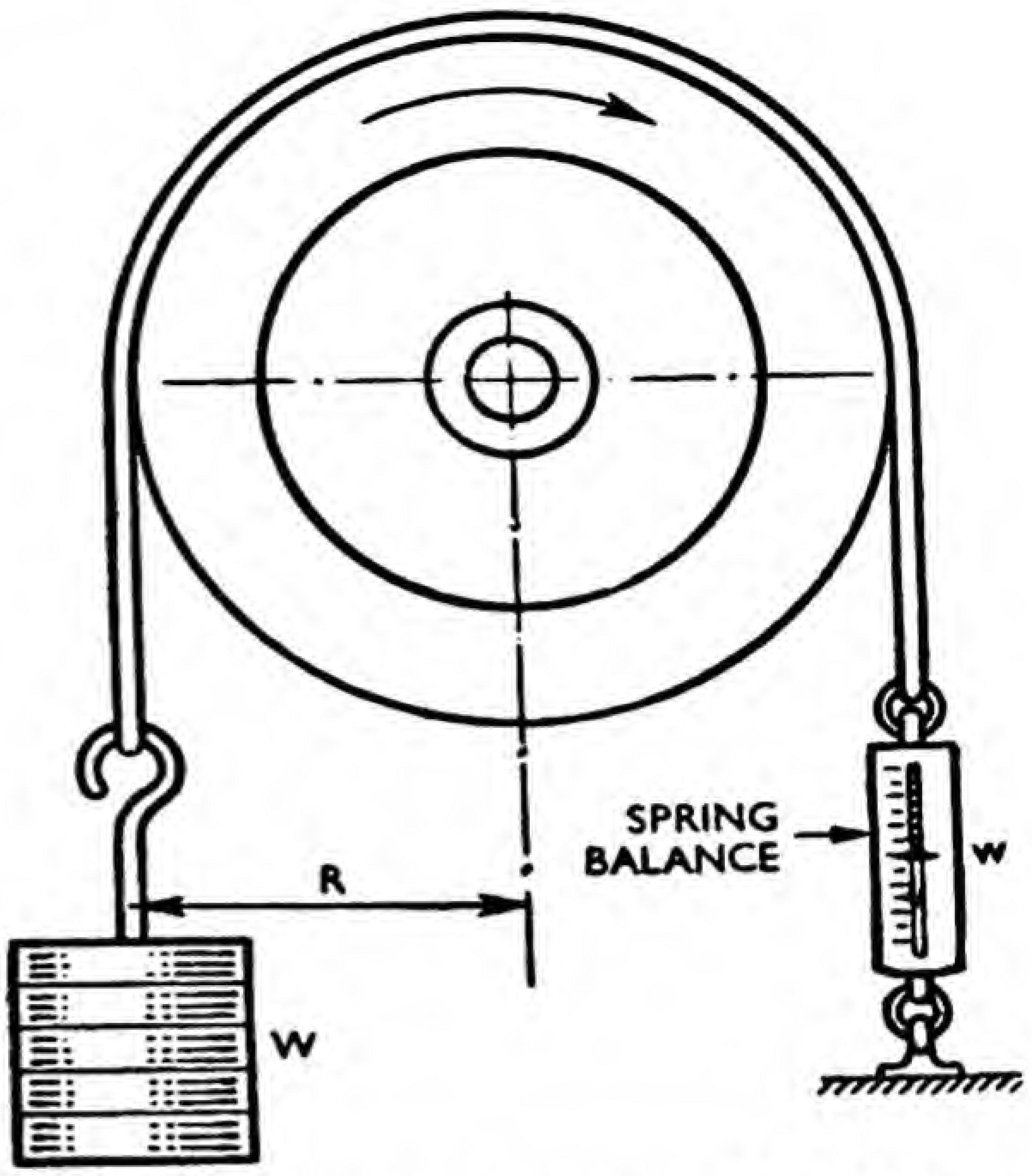


Fig. 3. A convenient way of applying a brake to the flywheel is by means of a belt or ropes. The tension in the side of the belt opposing motion is applied by weights *W*, and the tension *w* in the other side is indicated by a spring balance which makes the brake self-adjusting. The effective tension is (W-w) and the brake torque $R(W\text{-}w)$.

The term absorption dynamometer is used for brake dynamometers of the kind that have been considered, because they absorb the energy given out by the engine, and dissipate it in overcoming the frictional resistance of the brake.

Conducting Heat Away

Except in the case of very small powers, the heat generated is considerable, and special arrangements are necessary to conduct the heat away. The usual method is to circulate water through the brake, and Fig. 4 shows how this can be done very simply. The flywheel rim is made so that it will hold water, and as the wheel revolves, carrying the water round with it,

the centrifugal force keeps the water always in contact with the inside of the rim. The cold incoming water, being denser, is immediately flung against the rim, and the heated water can be skimmed from the inside by means of the scoop pipe.

Brake Horse-power

It can now be seen why we speak of the output of an engine as the brake horse-power, and why, in practice, this measurement is so important. Because the brake horse-power is measured at the output shaft or at the actual driving pulley, it is a true measure of the power that the engine can supply for driving machines, etc. The frictional and other losses in the engine itself absorb some of the power generated, but the b.h.p. measured at the driving pulley is all available for useful work, over and above that lost in internal friction.

In some instances, it is more convenient to measure the b.h.p. of an engine by making it drive some machine like a dynamo or a pump instead of absorbing its energy in a brake. The output of the dynamo or pump can then be measured, and so the b.h.p. of the engine can be estimated. There will be losses in the dynamo or pump, of course, and this means that the power put into the dynamo by the engine will be greater than the electrical power output which is measured.

In other words, if we use a dynamo in this way to make what is known as an electric dynamometer, we must have previously determined the efficiency of the dynamo. For example, if during the test the dynamo is generating 100 amps. at 200 volts, its output is $200 \times 100 = 20{,}000$ watts. This is approximately equal to 27 h.p. To overcome the losses in the

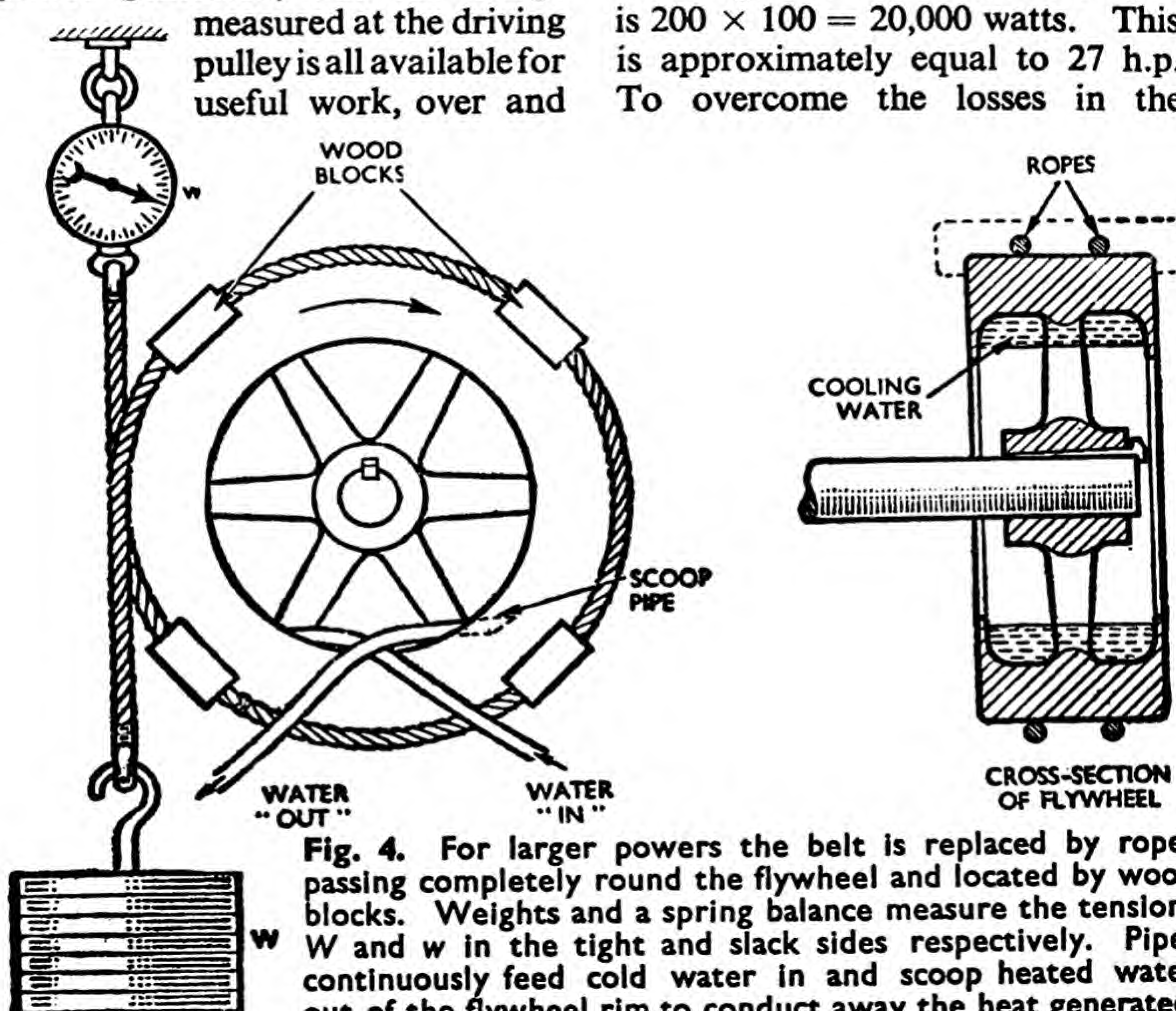


Fig. 4. For larger powers the belt is replaced by ropes passing completely round the flywheel and located by wood blocks. Weights and a spring balance measure the tensions *W* and *w* in the tight and slack sides respectively. Pipes continuously feed cold water in and scoop heated water out of the flywheel rim to conduct away the heat generated.

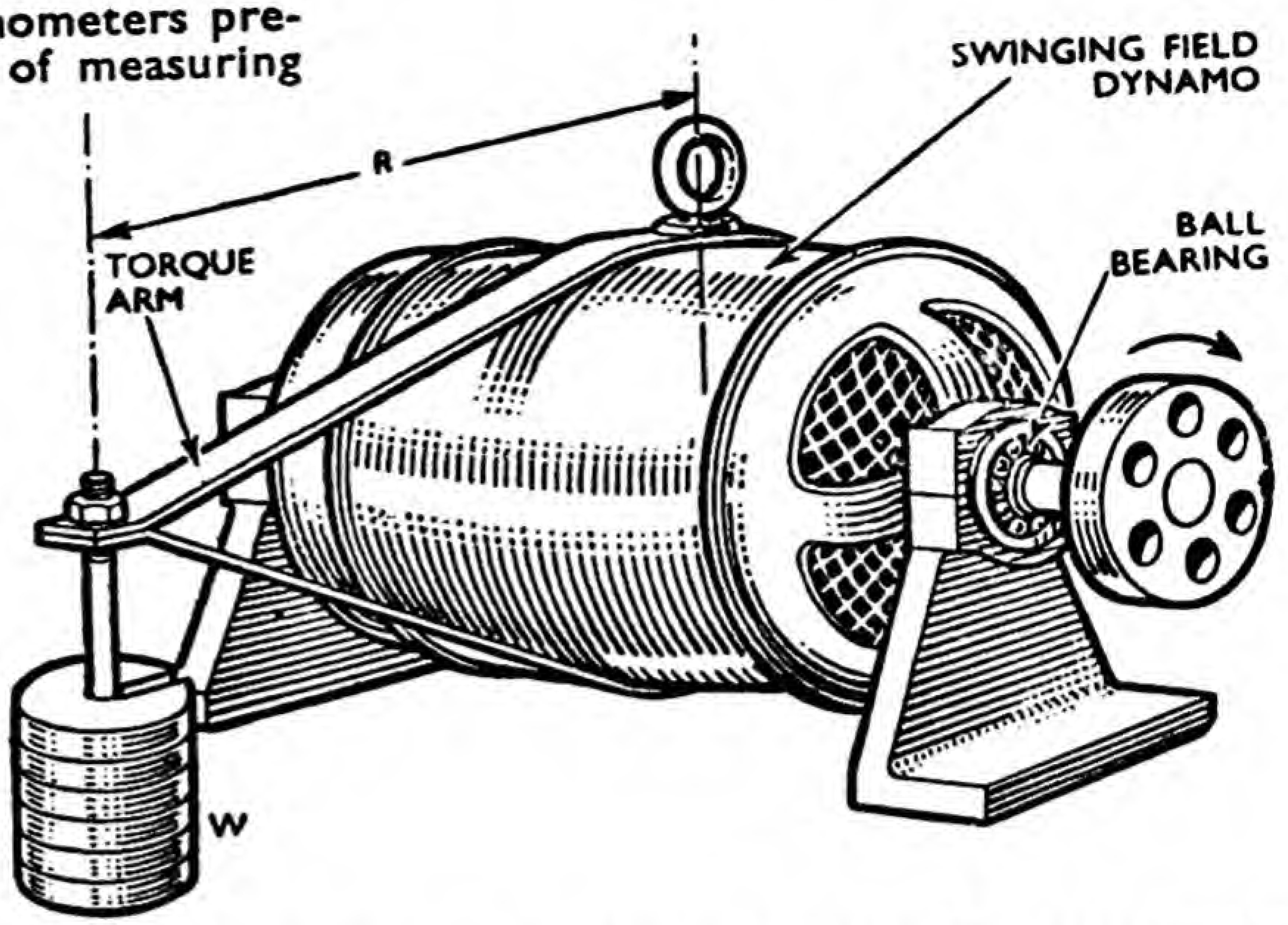


Fig. 5. Electric dynamometers present alternative means of measuring b.h.p. Engine drives a dynamo, output of which can be measured. Swinging field dynamo overcomes difficulty of first having to calculate efficiency of dynamo, for entire machine is mounted on ball bearings. Weight at end of torque arm prevents rotation of field magnets and casing, and from torque *WR* and the speed, b.h.p. can be calculated.

dynamo, the engine will have to supply more power than this, so that if we know that the efficiency of the dynamo is 90 per cent, then the power required from the engine, to drive the dynamo, will be greater than the power obtained from the dynamo in the ratio of 100 to 90.

$\therefore$ b.h.p. of engine

$$= 27 \times \frac{100}{90} = 30$$

Obviously, if we want to determine the maximum power of which the engine is capable, and this is usually what is meant by its b.h.p., we must be quite sure that the dynamo is big enough to load the engine up to its full capacity, that is, up to the point at which, if the load is increased further, the power output decreases owing to the decrease in speed.

Electric Dynamometer

An electric dynamometer is very convenient because very fine adjustment of the load can be obtained, and it is often possible to use the current generated by the dynamo, whereas with the friction brake all the energy is dissipated as heat. For high-speed engines especially, such as motor-car and aero engines, the electric dynamometer is much to be preferred to the friction-brake type.

The main disadvantage of the arrangement that has just been considered is that it is necessary first to test the dynamo, and determine its efficiency, or, as we say, to calibrate it, before it can be used as a dynamometer. As with other machines, the efficiency will be different at different loads, and may also vary from time to time. Fortunately, there is a simple way out of this difficulty.

Swinging Field Type

The entire dynamo can be mounted on ball bearings so that it is free to rotate about the axis of the shaft. As the armature is driven round, the magnetic forces will try to cause the field magnets and the casing of the dynamo to rotate as well. This is prevented by means of a moment arm and suitable weights to measure the torque required to hold the casing stationary. From the torque and the speed, the b.h.p. can be calculated as before. An electric dyna-

mometer of this kind is called a swinging-field dynamometer, and it has the great advantage that, by measuring the torque directly, the result is not affected by the efficiency of the dynamo (Fig. 5).

Froude Dynamometer

The same principle is used in the Froude dynamometer or water brake, as it is often called. Here, instead of the dynamo, there is a rotor with a series of vanes enclosed in an outer casing which is mounted on ball bearings just like the swinging-field dynamo. The casing is filled with water, and as the vanes rotate at high speed, the water thrusts on the casing and tries to turn it. The torque required to hold the casing stationary is then measured by weights and a spring balance at the end of a moment arm. The intense churning of the water inside the casing causes the temperature to rise ; to prevent it boiling and so causing a vapour lock, the water must be changed continuously. Two pipes are arranged so that the flow of cold water into the dynamometer displaces an equivalent quantity of heated water (Fig. 6).

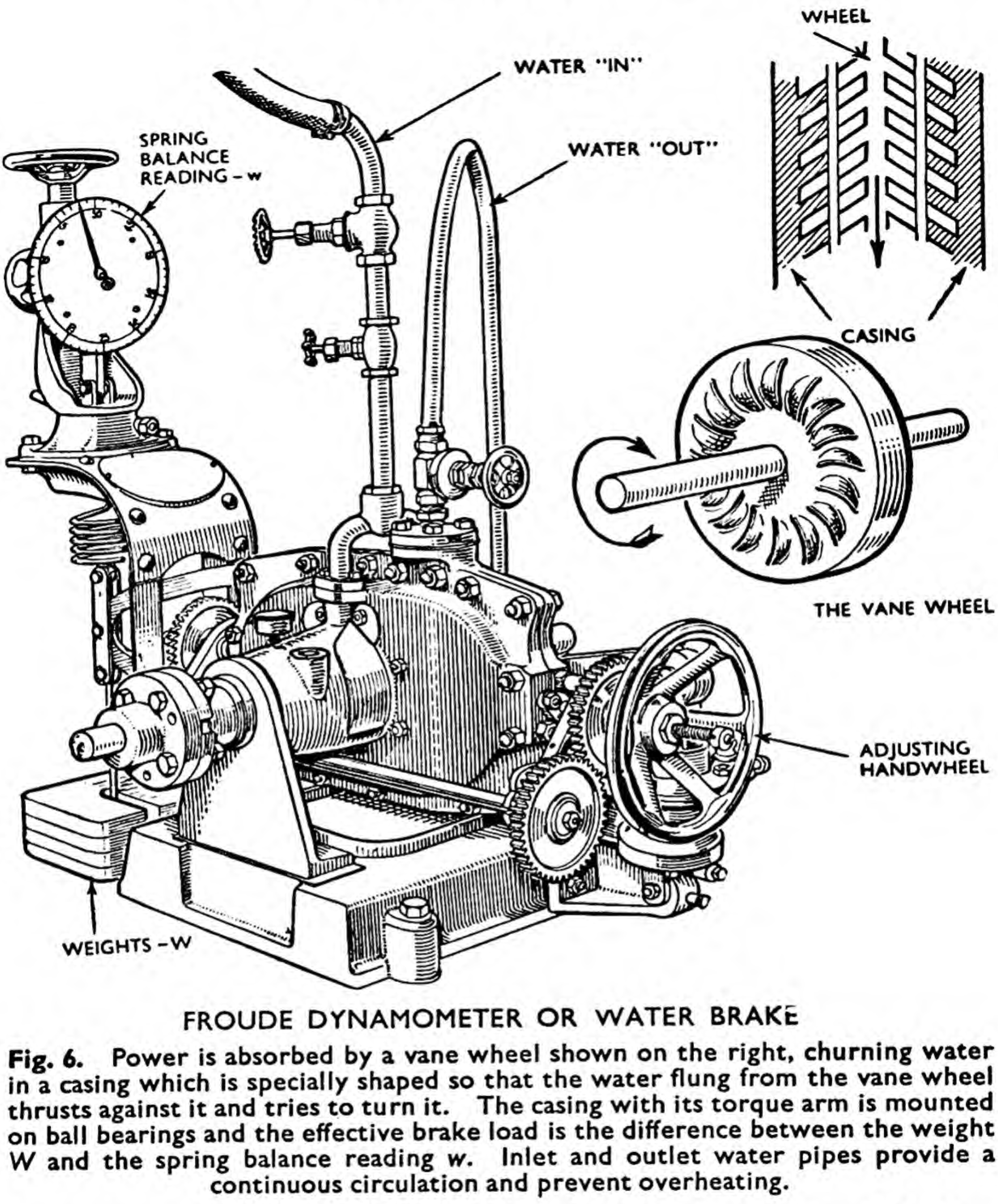
So far we have not said how the load can be varied with the water brake. Inside the casing there is a pair of adjustable shutters which partially screen off the vanes of the rotor. By means of an external hand wheel, these shutters can be moved closer together or farther apart, so reducing or increasing the load on the brake. This kind of brake is suitable for even higher speeds than the electric dynamometer, and as it is self-contained and very compact, it is widely used for engine testing.

Both of the above types depend for their action on the third law of motion, which states that action (torque on rotor) equals reaction (torque on casing).

Often, especially with big machines, it is difficult to test them at full load owing to the large powers involved, and yet it is just as important to know what the b.h.p. and the efficiency are. In such cases it is more convenient to measure the losses, and to deduce the b.h.p. from a measurement of the indicated horse-power (i.h.p.). We have seen already that the b.h.p. which is available at the output shaft or pulley, is less than the i.h.p. owing to the frictional and other losses in the engine itself. This discrepancy is called the friction horse-power (f.h.p.), so that :—

Friction horse-power
= Indicated horse-power
− Brake horse-power.

Therefore, if we know any two of these quantities, we can find the third. The method of finding the i.h.p. was given in Chapter 10. There are a number of ways of estimating the f.h.p. without actually measuring the b.h.p. One of the simplest is by means of the no-load test. In this, the engine is run at full speed but without any power being taken from it. Under these conditions, all the power generated is expended in overcoming the internal friction of the engine itself, and so, by measuring the i.h.p. during the no-load test, we have a measure of the friction horse-power. Alternatively, we can drive the engine at full speed by means of a calibrated electric motor (this is known as motoring the engine) and in this way measure the f.h.p. directly. Actually, the f.h.p. will be greater than this when the engine is working at full



FROUDE DYNAMOMETER OR WATER BRAKE

Fig. 6. Power is absorbed by a vane wheel shown on the right, churning water in a casing which is specially shaped so that the water flung from the vane wheel thrusts against it and tries to turn it. The casing with its torque arm is mounted on ball bearings and the effective brake load is the difference between the weight *W* and the spring balance reading *w*. Inlet and outlet water pipes provide a continuous circulation and prevent overheating.

power, because the forces on the bearings will be greater, and an allowance has to be made for this.

To help in making a suitable allowance it is an advantage to run a quarter-load or half-load test in addition to the no-load test. For example, if the i.h.p. at no-load was 30, then we could say that the no-load f.h.p. was 30. If, in addition, a half-load test gave an i.h.p. of 115 for a b.h.p. of 80, then the half-load f.h.p. would be 115 − 80, which is 35. As the f.h.p. has increased from 30 to 35 as the load has increased from no-load to half-load, we can estimate that the f.h.p. would increase to 40 at full load. This would mean that if the engine were meant to give 160 b.h.p., it would have to develop at least 200 i.h.p. at full load. In

Chapter 10 it was seen that the mechanical efficiency was given by the ratio of the b.h.p. to the i.h.p. So that in the above example the full load efficiency would be 160/200, and this is 80 per cent. Similarly the half-load efficiency would be 80/115, which is approximately 69·5 per cent. When we have to estimate the efficiency in this way from the i.h.p. and the f.h.p., it is useful to write the expression in a different way, thus :

$$\text{Efficiency} = \frac{\text{b.h.p.}}{\text{i.h.p.}}$$
$$= \frac{\text{i.h.p.} - \text{f.h.p.}}{\text{i.h.p.}}$$
$$= 1 - \frac{\text{f.h.p.}}{\text{i.h.p.}}$$

This shows very clearly that the efficiency must always be less than unity, and that as the f.h.p. will be larger in proportion at small values of the i.h.p., the efficiency will decrease as the load is reduced. If it is possible, a full-load dynamometer test is always to be preferred, of course, but it is so much simpler just to disconnect the machine from its drive for a no-load test, when anything seriously wrong will usually be shown up by excessive power consumption.

Balancing Test

During the manufacture of an engine or a machine, the maker carries out numerous tests in addition to the horse-power test, but unless we actually see them carried out, we are usually not aware of them. There is one test, however, to which the machine will immediately draw attention if it has not been carried out correctly. This is the balancing test. Due to lack of balance, machines weighing many tons will develop more or less violent vibrations, depending upon the amount of out-of-balance and the speed.

The speed is very important, because it is not the actual out-of-balance weight that matters, but the centrifugal force on this weight, which, as described in Chapter 4, increases in proportion to the square of the speed. That is why modern machines which run at speeds of 3,000 or 4,000 r.p.m. must be accurately balanced to within the smallest fraction of an ounce. In order to help us to understand this more clearly, let us first see how the balancing is carried out.

For the balance of a part like a flywheel, two accurate straight edges are required which can be set up so that they are dead level. The flywheel, mounted on its shaft or on a special mandrel, is then placed on the knife edges, as shown in Fig. 7. If the wheel is perfectly balanced, it will show no tendency to come to rest in any particular position, if it is rolled slowly along the knife edges. On the other hand, any lack of balance will cause the wheel to roll so that the heaviest part comes to the bottom. This shows where the wheel is out of balance.

The question now is, by how much? This can soon be decided with a few small weights and some modelling clay. The principle is adopted that if the wheel is too heavy on one side, it is too light on the side diametrically opposite, and one of the weights is stuck to the rim with a small piece of modelling clay, and the wheel rolled again.

If it is now too heavy on that side, the weight can either be changed for a smaller one, or the weight removed from the rim and stuck on nearer the centre, in order

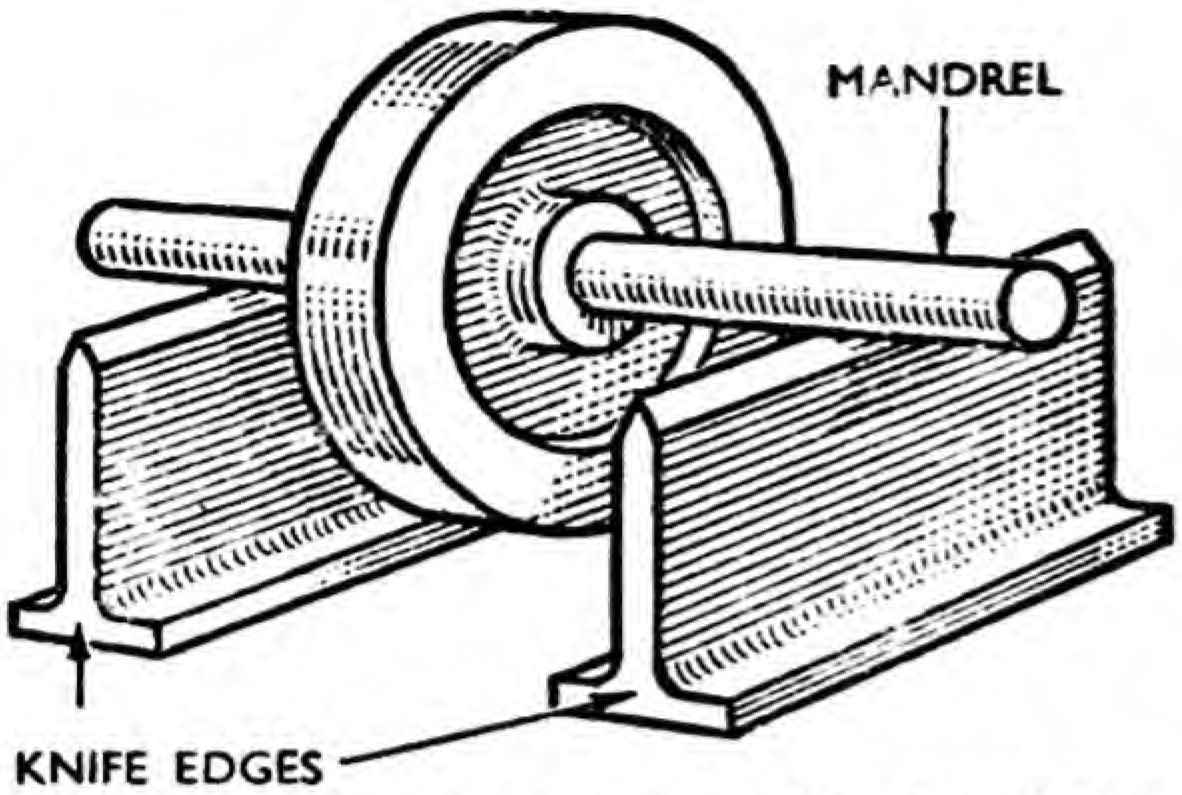


Fig. 7. When testing a flywheel for static balance, the friction of bearings would normally be too great for accurate results to be obtained. A more sensitive method is to mount the flywheel on a mandrel so that it can be rolled on level knife edges as shown.

to make its moment smaller. The latter is usually more convenient in practice, because it enables a very fine adjustment of the balance to be made. And, after all, it is the out-of-balance moment that matters, not the weight. That is, the effect of a ½-oz. weight at the rim is the same as that of a 1-oz. weight half-way between the rim and the centre.

So that if, in balancing a flywheel, we find that a 1-oz. weight at the rim is too heavy, and a ½-oz. weight, also at the rim, is too light, the next step would be to try the 1-oz. weight at three-quarters of the radius from the centre to the rim. The effect of this would be intermediate between that of ½ oz. and 1 oz. at the rim.

When the balance is just right, so that the wheel will come to rest in any position, we have to consider how we will adjust the balance permanently, because, obviously the weight cannot be left stuck on with modelling clay. We first calculate the moment of our temporary balance weight, for example, 1 oz. at 24-in. radius = 1½ lb.-in., and this correction must then be made to the wheel. Usually, the easiest way is to machine off the appropriate weight of metal from the side opposite to that to which we have attached the temporary balance weight, in other words, from the original heavy side. We may take it off either from the boss or the inside of the rim. Thus, we could machine off ¼ lb. from the boss at a radius of 6 in., or we could remove ¾ oz. from the rim at a radius of 32 in. Either would give the required balancing moment of 1½ lb.-in.

Sometimes the metal is removed by drilling shallow holes of a suitable size. For this purpose, we can take it that in a cast-iron wheel a ½-in. diameter hole is equivalent to 0·1 oz. of metal removed for each ⅛ in. of depth. Alternatively, a permanent weight could be added in place of the temporary weight to make the lighter side heavier. This is usually done by drilling holes in the side of the rim and filling them with lead. The actual weight added is the difference between the weight of the lead plugs, and the iron removed in drilling the holes.

Static Balance

The result of this trial on knife edges is to give what is known as static balance, that is, with the wheel stationary, or nearly so, to distinguish it from running or dynamic balance, which is obtained with the wheel running at high speed. In many instances it will be found that if the part has been accurately statically balanced, it will also be balanced when running, but this is by no means always true.

Now why should this be? If the part has been carefully balanced on the knife edges, where does the

out-of-balance come from when it is running? It is caused by the centrifugal forces, and as we have seen, these increase with the speed.

In Fig. 8(a) there are four weights *A*, *B*, *C* and *D* attached to a shaft. *A* and *C* balance each other, and *B* and *D* balance each other, so that if the shaft were tried on the knife edges, we should have static balance. Now suppose that the shaft is rotating at 1,000 r.p.m., there will be centrifugal forces acting on all the weights. On *A*, the centrifugal force will be :—

$$F_a = \frac{2}{g} \times \left(\frac{2\pi \times 1{,}000}{60}\right)^2 \times \frac{6}{12} = 341 \text{ lb.}$$

But this will be exactly counterbalanced by an equal and opposite force on *C*. Similarly, the centrifugal force on *B* will be :—

$$F_b = \frac{3}{g} \times \left(\frac{2\pi \times 1{,}000}{60}\right)^2 \times \frac{12}{12}$$

$= 1{,}023$ lb.

This will be exactly counterbalanced by the centrifugal force on *D*. This arrangement, then, will be dynamically balanced as well as statically balanced, and there will be no vibration. In Fig. 8(b), the four weights *A*, *B*, *C* and *D* are all different. But we shall again obtain static balance because the moment of *C* counterbalances the moment of *A*, and the moment of *D* counterbalances the moment of *B*. Will this also give a running balance? Let us work out the centrifugal forces and see.

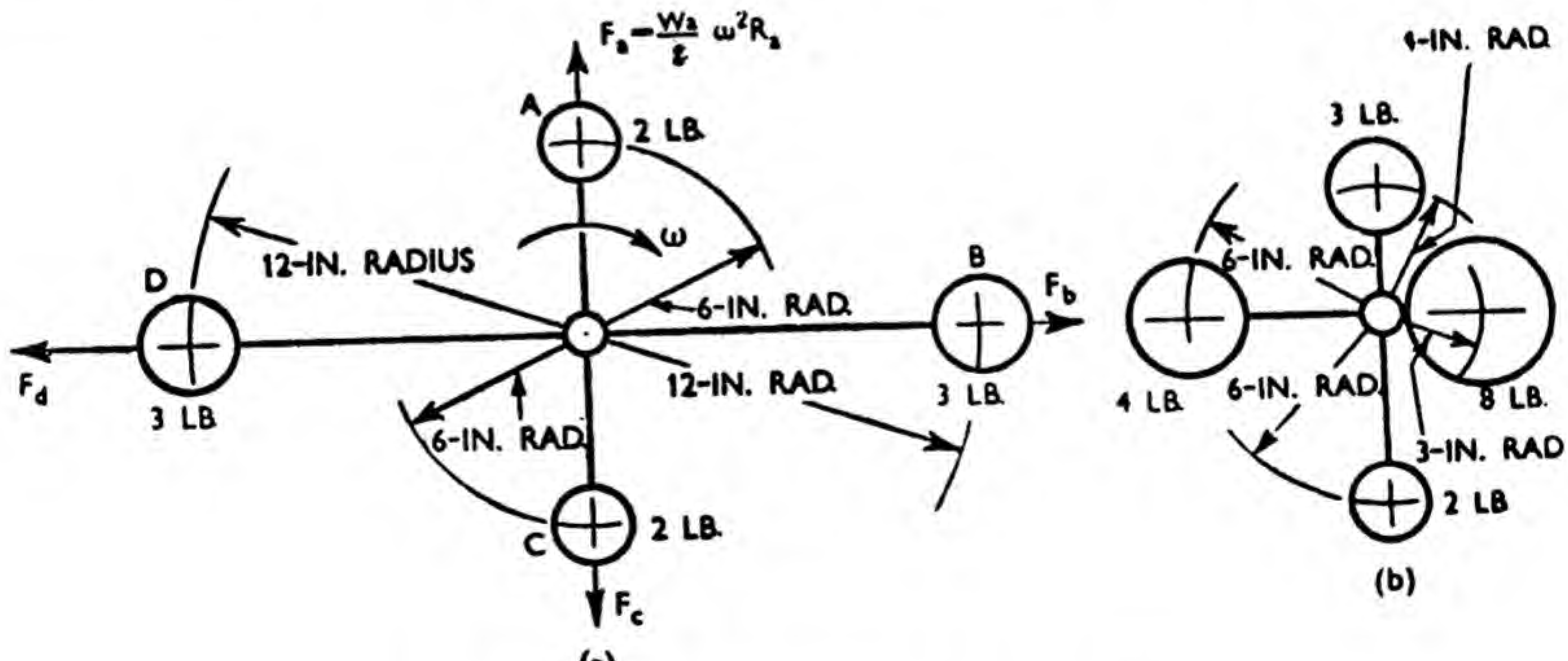
Centrifugal force on $A = 3/g \times \left(\frac{2\pi \times 1{,}000}{60}\right)^2 \times 4/12 = 341$ lb.

Centrifugal force on $C = 2/g \times \left(\frac{2\pi \times 1.000}{60}\right)^2 \times 6/12 = 341$ lb.

Centrifugal force on $B = 8/g \times \left(\frac{2\pi \times 1{,}000}{60}\right)^2 \times 3/12 = 682$ lb

Centrifugal force on $D = 4/g \times \left(\frac{2\pi \times 1{,}000}{60}\right)^2 \times 6/12 = 682$ lb.

Again, it is noticed that there would be running balance, because the centrifugal forces F_a, F_c and F_b,



BALANCING CENTRIFUGAL FORCES

Fig. 8. Elimination of vibration from a machine requires that centrifugal forces acting on rotating parts shall be balanced. The rotating weights shown at (*a*) form a balanced system because they are arranged symmetrically. At (*b*) the weights are arranged unsymmetrically but balance is obtained because the sum of their moments about the centre is zero. Calculation of the centrifugal forces shows that dynamic balance is also obtained.

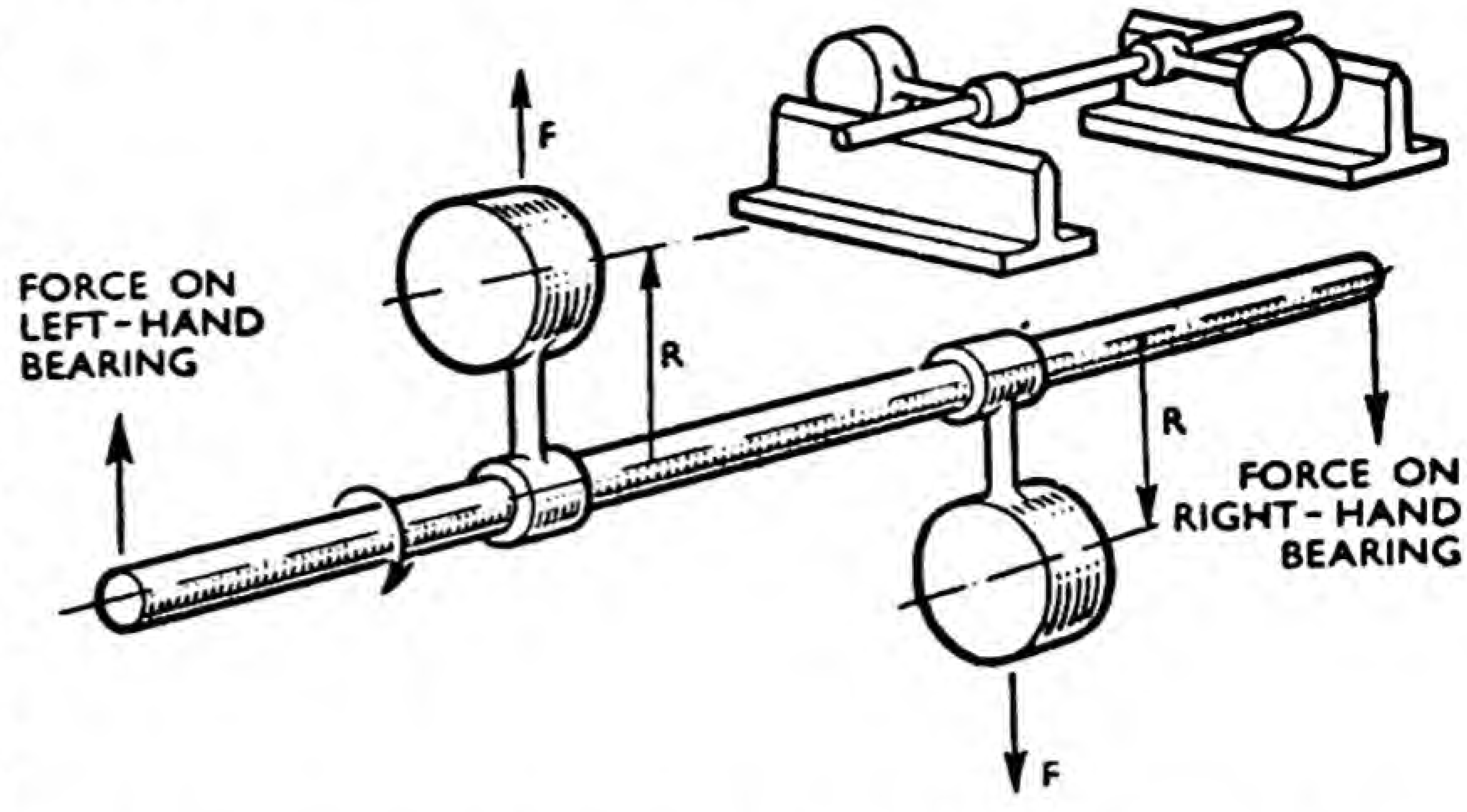


Fig. 9. A part that gives static balance on knife edges will not be balanced when rotating if the centrifugal forces do not act in the same line. Shown here is a shaft carrying two weights which rotate in different planes, and hence the centrifugal forces *F* do not eliminate each other but cause pulsating forces on the bearings.

F_d counterbalance each other exactly.

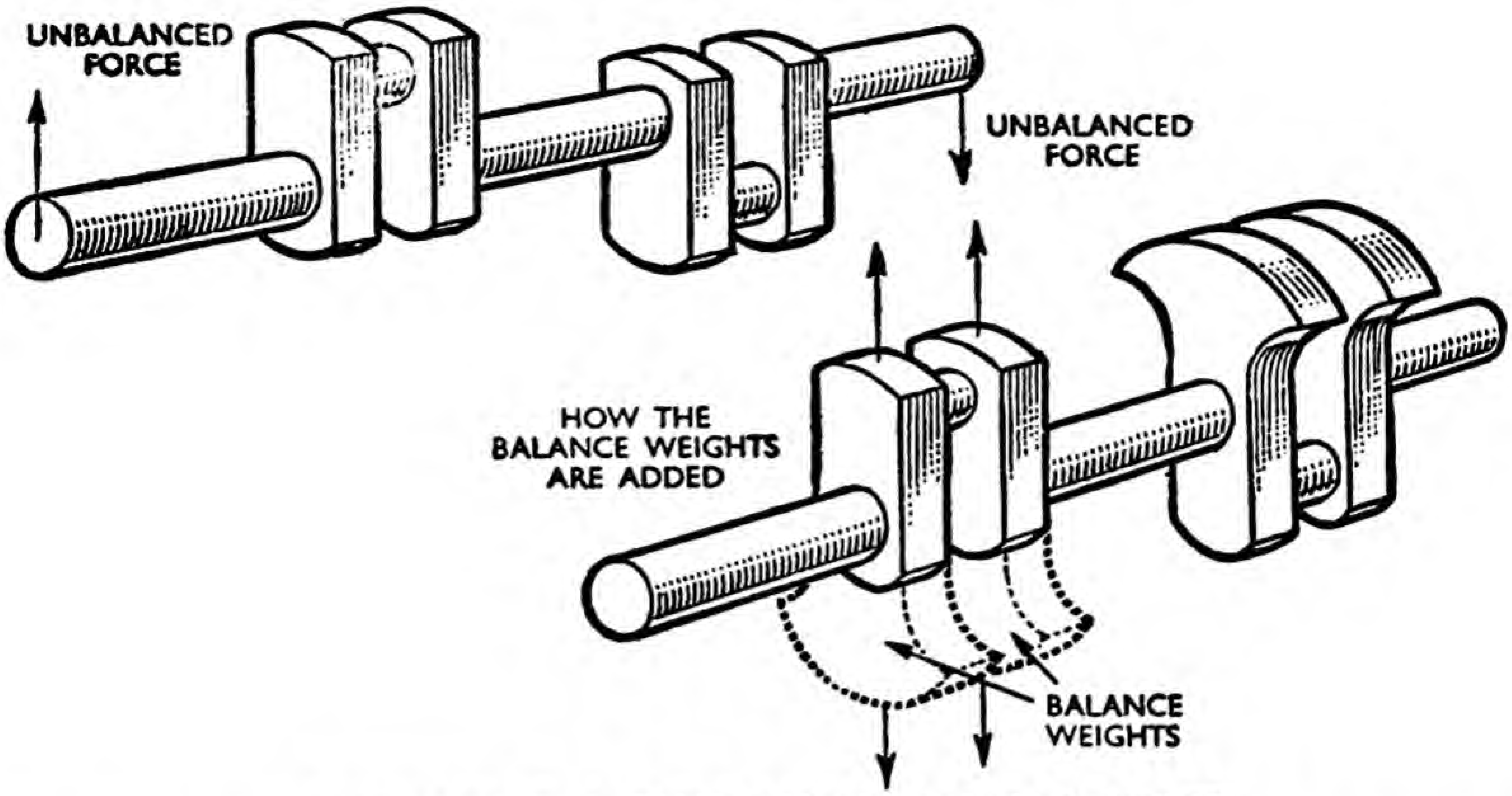
Before leaving these figures, it is worth noticing how large the centrifugal forces are in comparison with the weights that cause them Now suppose that the centrifugal forces did not exactly counterbalance each other. For example, let us assume that *C* weighed 2 lb. 1 oz. instead of 2 lb., so that there would be an out-of-balance of 1 oz. at 6-in. radius when tried on the knife edges. The centrifugal force on *C* would then be 352 lb., and this would be only partially balanced by the force of 341 lb. due to *A*. Therefore, at a speed of 1,000 r.p.m. there would be an unbalanced centrifugal force of 352 – 341 = 11 lb., due to the 1 oz. excess weight of *C*. At a speed of 3,000 r.p.m. this unbalanced centrifugal force would have increased to about 100 lb.

Now look at Fig. 9. This shows a shaft with two weights attached, balanced on knife edges. It will be seen at once that this will not be dynamically balanced because, although the centrifugal forces on the two weights will be equal and opposite, they will not be in line. The difference between this and Fig. 8 is that the two weights do not rotate in the same plane, and whenever this occurs, special steps must be taken to obtain dynamic balance in addition to static balance.

The two-throw crankshaft of Fig. 10 is an example from practice of this type. When placed on the knife edges, the two cranks would balance each other, but when running at high speed, the centrifugal forces would not neutralize each other, because they are not in the same plane, and there would be unbalanced forces on the bearings at the ends of the shaft. The lower figure shows how balance weights are added to the webs of each crank to neutralize the unbalanced forces in the planes in which they arise.

We rarely experience this kind of difficulty in balancing a part like a flywheel, where approximately all the material can be considered to rotate in the same plane, but how is a part like the armature of a dynamo balanced, which is long in relation to its diameter ? The trial on the knife edges may show that it is out-of-balance on one side, but it will not indicate whether the unbalance is at the right-hand end, or the left-hand end, or is part-way between the two.

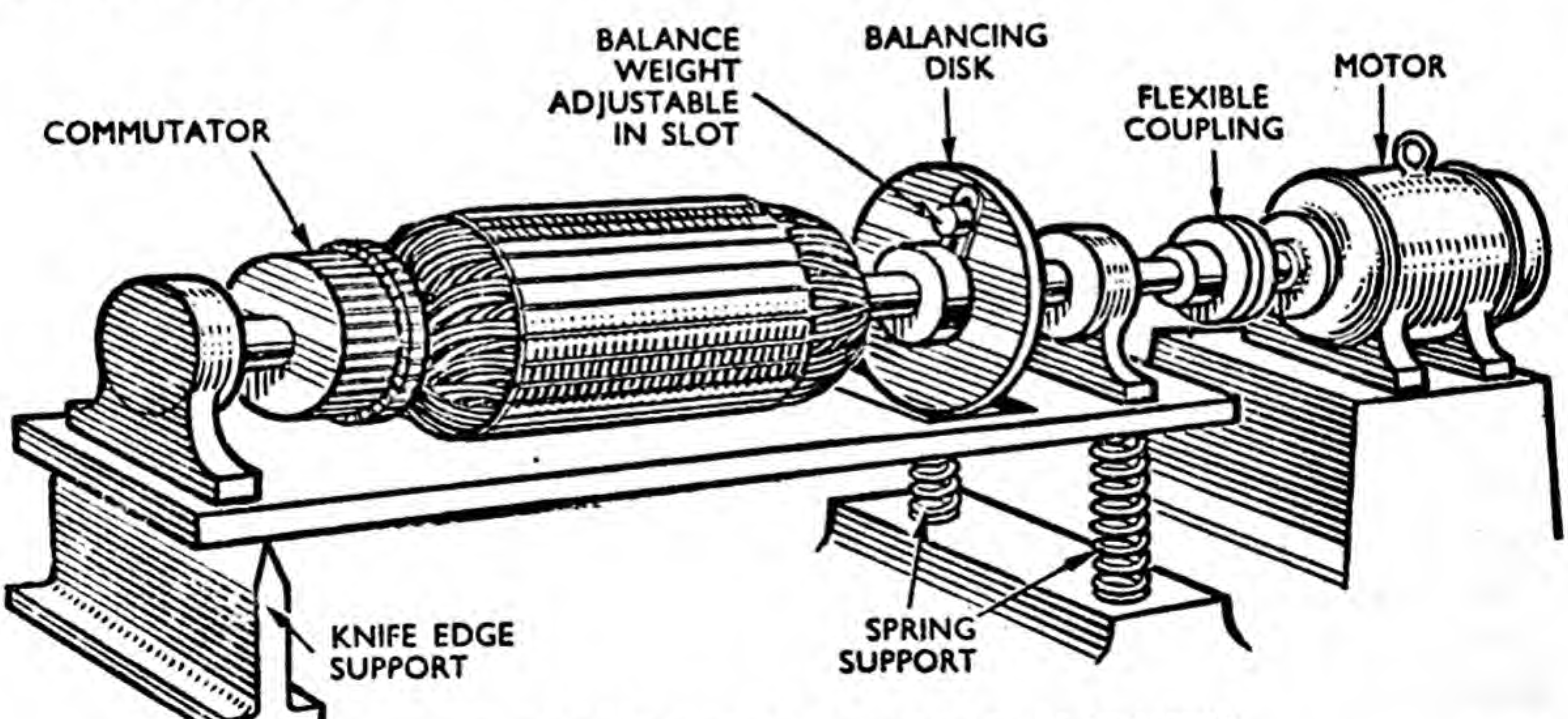
For this a balancing machine is used like that shown in Fig. 11. It



BALANCING A TWO-THROW CRANKSHAFT

Fig. 10. An example of static, but not dynamic balance, is afforded by a crankshaft, in which unbalanced centrifugal forces are produced as all parts do not rotate in the same plane. This is overcome by adding balance weights to the webs of each crank so that the centrifugal forces on the balance weights neutralize the out-of-balance forces due to the crank webs themselves.

will be noticed that the bearing at the right-hand end is mounted upon springs, whereas the other is supported on a fixed knife edge. This is because each end is to be dealt with in turn. Suppose that the unbalance is at the commutator end, the left-hand end in Fig. 11, and that when first tried in the balancing machine, this end is in the bearing which is supported on the knife edge. When the armature is rotated, the unbalanced centrifugal force will act on the knife edge, but there will be no movement, because the knife edge is



SIMPLE DYNAMIC BALANCING MACHINE

Fig. 11. Armature or part to be balanced is supported by two bearings, one resting on a knife edge, the other on springs. If armature is out of balance, spring bearing will vibrate when armature is rotated at high speed. This vibration is eliminated by adjustment of the weight on the balancing disk. Armature is then turned end for end, and process repeated with the commutator end.

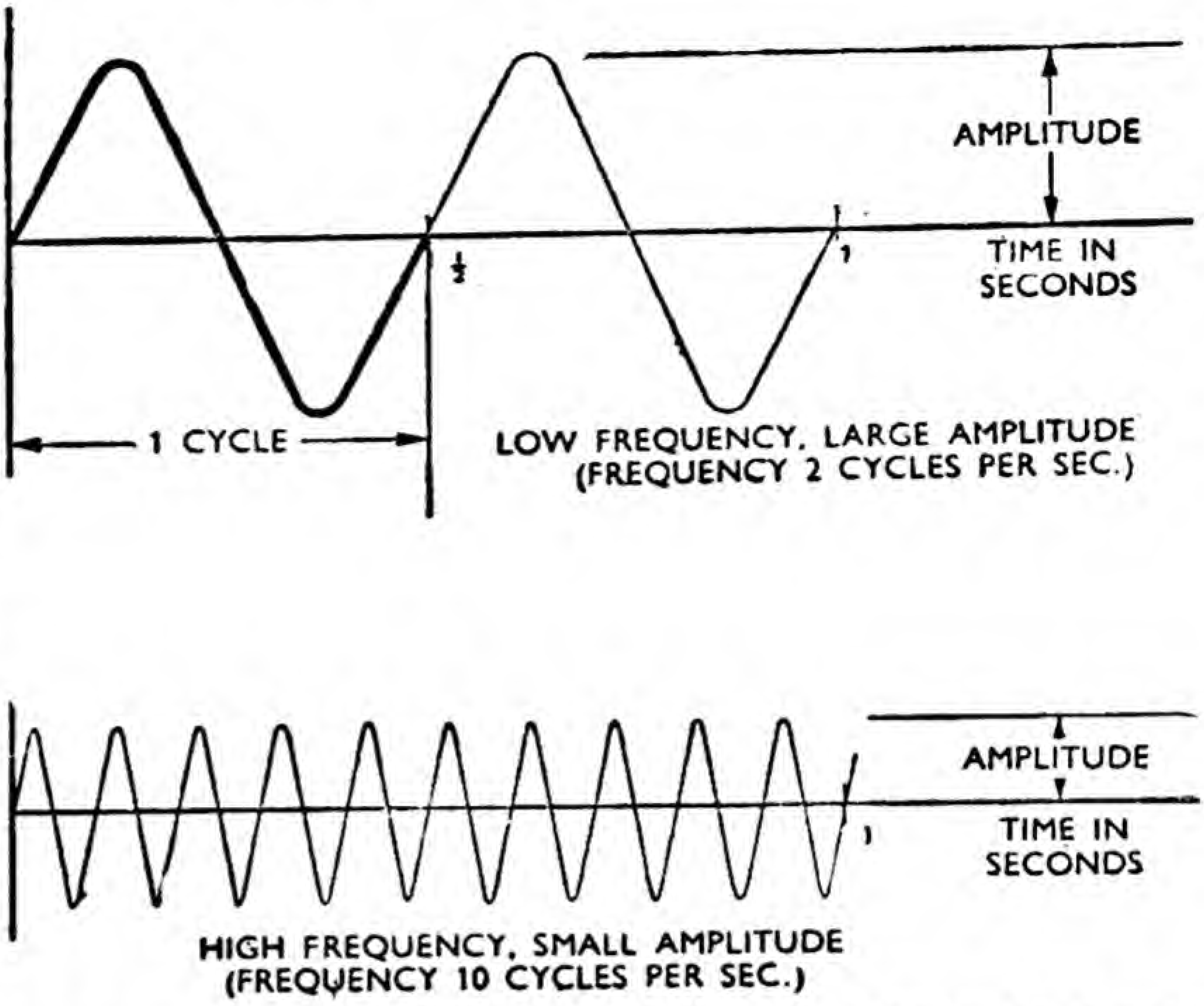


Fig. 12. Vibrations may be represented by graphs of motion on a time base. The motion is alternately positive and negative. Amplitude is maximum displacement from mean position. Complete positive to negative to positive movement is a cycle, and time required for this gives frequency. In upper graph, time for one cycle is half second, and frequency is two cycles per second. Lower graph shows smaller amplitude vibration, but five times the frequency.

fixed. If, however, the armature is turned end for end, and rotated again, the unbalanced force will immediately set the right-hand bearing vibrating, because it is mounted on springs. This tells us at which end the unbalance is, and it can be corrected by placing the balance weight at the same end, so that the centrifugal forces will neutralize each other and not produce an unbalanced couple.

Now what happens if the unbalance is part-way between the two ends? The spring-mounted bearing will vibrate whichever way the armature is placed in the machine. But if, for example, the unbalance is one-third of the length from the right-hand end, and two-thirds of the length from the commutator end, the vibration of the bearing will be twice as violent with the right-hand end at the sprung bearing as with the commutator end. Therefore, we can find out where the unbalance is by comparing the amount of vibration in the two positions.

It has been seen that with high-speed machines, the balancing must be done with great precision, and so, to measure the unbalanced force at each end, a balancing disk is used. This carries a small balancing weight which slides in a radial slot so that its moment can be varied. In practice, this is adjusted until vibration ceases, and the moment of the weight is read off from the balancing disk as so many lb.-in. for each end. In modern balancing machines, this adjustment can be carried out without stopping the machine.

Vibrations

When we realize that the centrifugal forces arising from a lack of balance may amount to hundreds of pounds, it is not difficult to understand how vibrations are set up. In the study of these vibrations and the attempt to eliminate them, we have to distinguish between two properties, viz., intensity or amplitude, which is the amount of movement, and frequency, which is measured by the number of vibrations per sec. (Fig. 12). It is

analogous to sound, which, after all, is a form of vibration.

The sound may vary in intensity (be soft or loud), and it may vary in frequency (be of low or high pitch). Or again, with wireless, there may be a weak or a strong signal, and this may be of low frequency, say, for example, 214·5 kilocycles per sec. (1,400 metres wavelength) or of high frequency, 6,180 kilocycles per sec. (48·54 metres wavelength).

Frequency

The pulsating unbalanced force has a frequency which is related to the speed of rotation of the shaft. So that if the speed is 600 r.p.m., there will be a pulsation in each direction 600 times a minute (10 times a sec.), and this would set up a vibration of frequency 10 cycles per sec. If the balance of the engine is improved, then the pulsating force will be reduced, and the amplitude of the vibration will also be reduced, but the frequency will still remain 10 cycles per sec. On the other hand, if the speed is increased to 900 r.p.m., then the frequency of the vibrations will increase to 15 cycles per sec.

At this higher speed, the amplitude of the vibrations may be greater or it may be less, depending upon the dimensions and properties of the machine. This may seem strange, but the reason is that every machine, and every part, has one particular frequency, called the natural frequency, at which it can be made to vibrate most easily. The more nearly the frequency of the pulsations approaches the natural frequency, the larger will be the amplitude of the vibration. When the two frequencies agree, the amplitude becomes very large indeed, and we have what is known as resonance.

The interesting fact is, that as the speed is increased beyond the resonance point, the amplitude of the vibration dies down again. We are familiar with this in the ordinary motor-car engine. Usually, at all speeds, there is a certain amount of vibration, depending upon how well the engine has been balanced, but

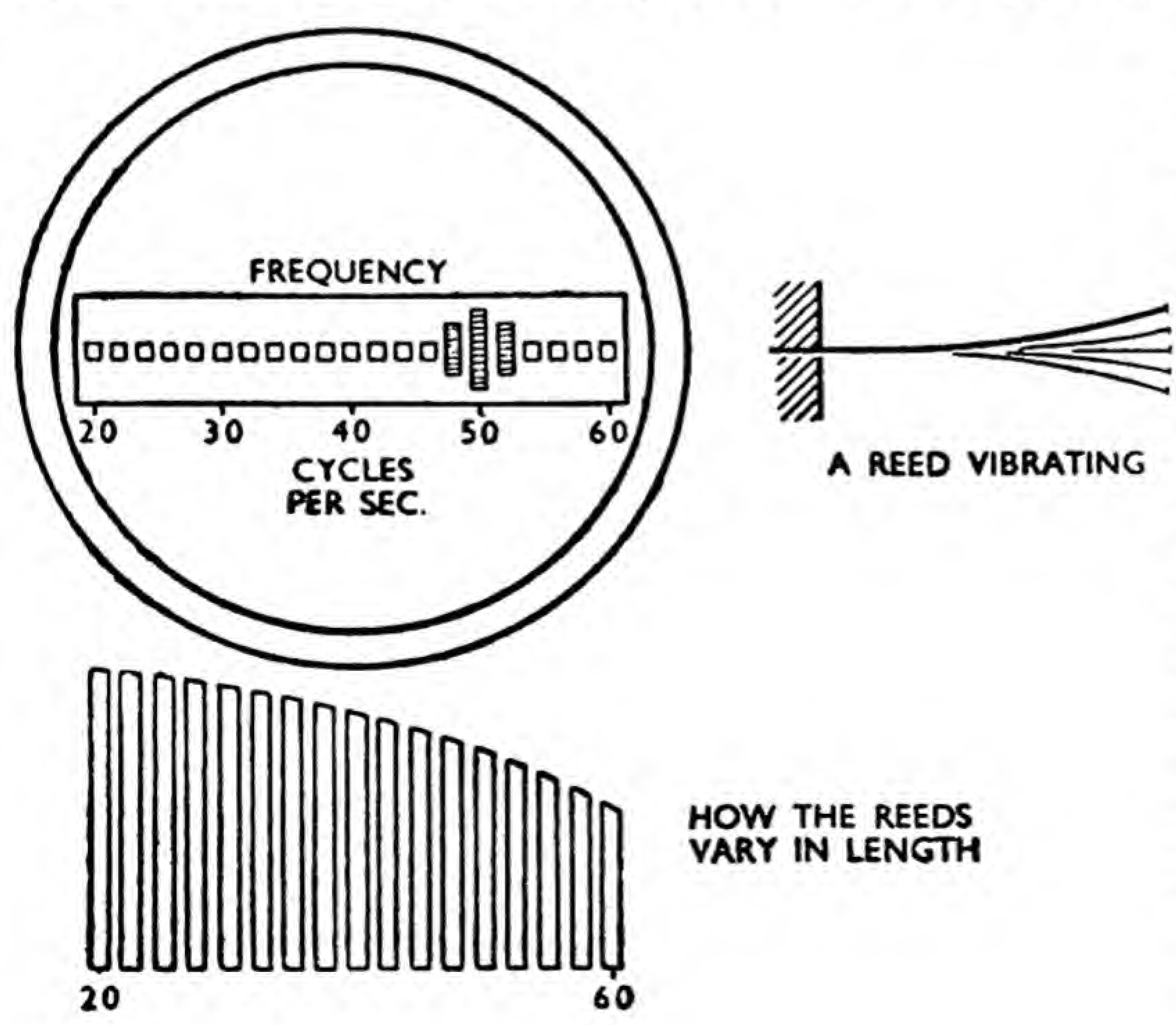


Fig. 13. Any part free to vibrate has a natural frequency at which it can be made to vibrate most easily. This is made use of in the vibration meter shown consisting of a number of spring steel strips or reeds of different lengths, each tuned to vibrate at a particular frequency. Ends of reeds appear as a row of small white squares across the face of the dial. Frequency of any vibration with which the meter is placed in contact, is shown by motion of the particular reed in synchronism with the vibration.

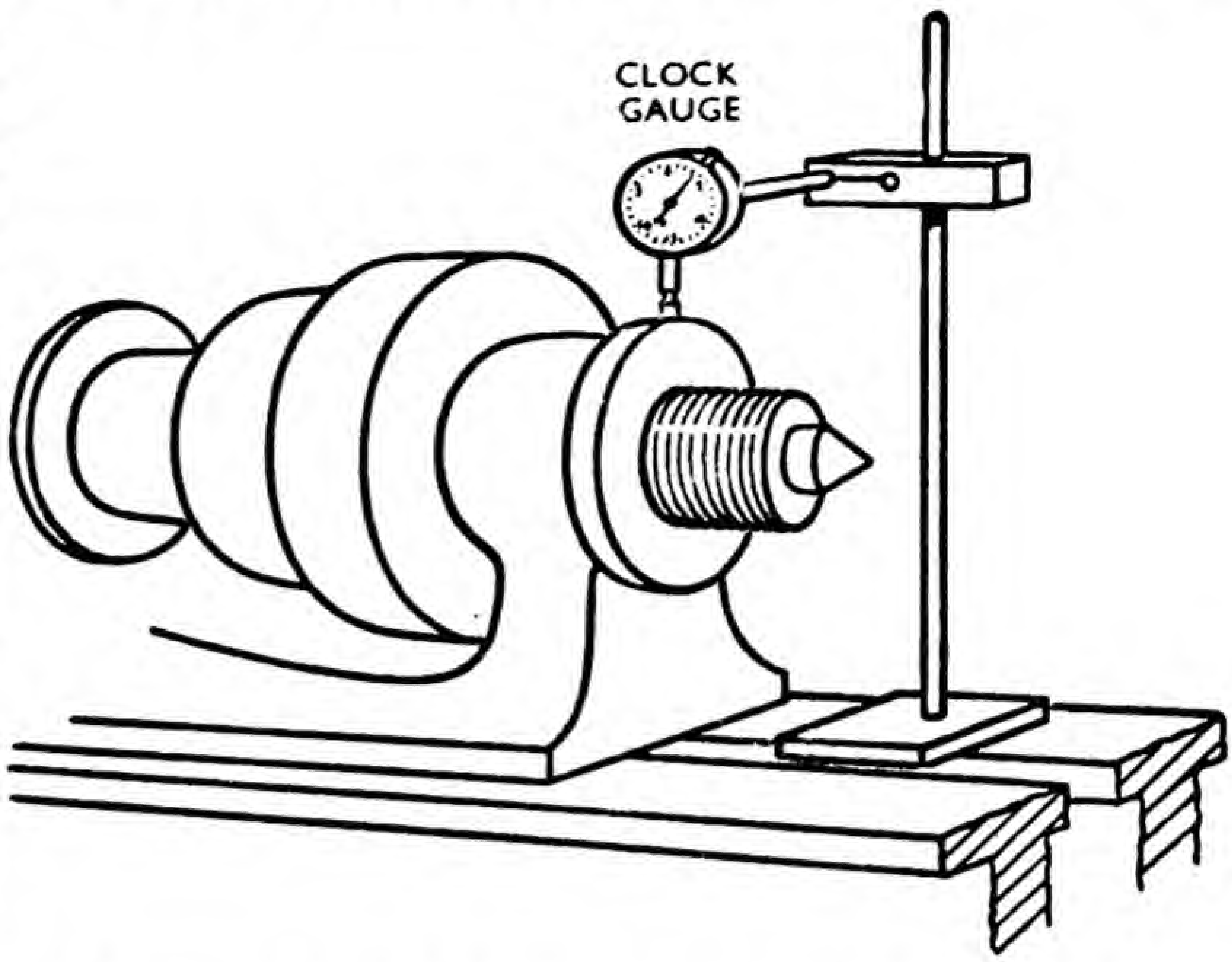


Fig. 14. The spindle of a lathe must be truly concentric with its bearings. This can be checked with the help of a dial indicator or clock gauge graduated in thousandths of an inch. The clock gauge is set up on a fixed surface such as the lathe bed, so that the movable plunger rests on the spindle. Any inaccuracy will cause movement of the pointer as the spindle is rotated.

at one particular speed the vibration increases considerably and we say that the engine has become rough. This is really a case of resonance, caused by the engine speed happening to correspond with the natural frequency of some engine part, usually the crankshaft.

It often happens that something which is a disadvantage under certain conditions is an advantage under others. This is true of resonance, which is made use of in the ingenious vibration meter shown in Fig. 13. This has just to be placed in contact with the frame of a machine, or any part that is vibrating, and it will indicate immediately the frequency of the vibrations. Each of the little white squares, arranged in a row across the face of the instrument, is attached to a spring steel strip or reed. Each reed is adjusted to vibrate at a particular frequency, like the reeds of a mouth organ ; the longer the reeds, the lower the frequency. When placed in contact with a vibrating part, the appropriate reed picks up the vibration and vibrates in sympathy with it ; the resonance occurs with the reed which has the same natural frequency as the vibration.

In Fig. 13 this is the 50-cycle reed. Those on either side of it are also vibrating slightly, but the others are hardly affected at all, and appear to be stationary. We have seen that the frequency of the vibrations is related to the speed, and this instrument, therefore, can be used as a speed indicator.

Machine Tools

To produce accurate machines, there must be accurate machine tools—lathes, shaping, milling, drilling and grinding machines—with which to produce them. The accuracy of modern machine tools, as turned out new by the makers, is of a very high order indeed, but in service, wear occurs and the accuracy deteriorates. In the course of time, excessive play develops, and, although adjustments are provided so that the slackness may be taken up, unfortunately the wear does not occur uniformly, and spindles run out of truth, so that surfaces which should be parallel are machined slightly taper, and turned parts cease to be concentric or even truly

circular. Therefore, machine tools must be checked from time to time, and in testing machine tools the important factor is correct alignment.

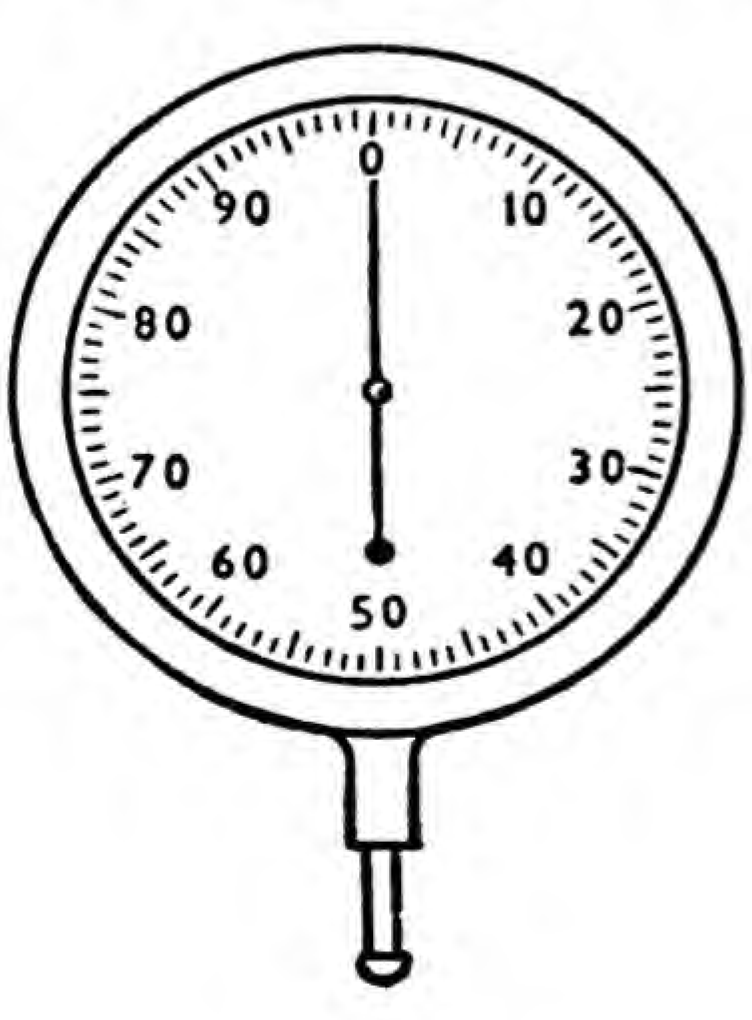
The operation of checking the truth of a lathe spindle is shown in Fig. 14. This is most conveniently done with the help of a dial indicator or clock gauge, which has a dial graduated in thousandths of an inch, that is, a movement of the pointer one division of the scale as the lathe spindle is rotated, would indicate that the spindle is 0·001 of an inch out of truth. The clock gauge merely magnifies the movement of the sliding plunger which rests on the spindle, and the mechanism is clearly shown in Fig. 15.

The plunger moves a toothed sector which meshes with a small pinion. As the pinion rotates, it moves the pointer over the scale. It is usually arranged that the pinion is rotated one revolution by a movement of the plunger of $\frac{1}{10}$ in. As the dial is divided into 100 parts, each division, therefore, corresponds to one-thousandth of an inch of movement. At first we would try the spindle by turning it round slowly by hand and noting any movement of the pointer, but we should also have to check it at speed, as a spindle that is out of balance may appear quite true when turned by hand, but vibrate badly when running at speed. In addition it would be necessary to test the bore of the spindle, in which the centre fits, to make sure that this is concentric with the collar.

The spindle should also be tried for end float, that is longitudinal movement along the axis of the spindle. For this the clock gauge should be mounted with the plunger pressing against the nose of the spindle. Any end float would then be shown by movement of the pointer, as the spindle was pressed backwards and forwards in its bearings. Any movement should then be eliminated by adjustment of the thrust bearing.

Parallelism Test

Having checked the truth of the spindle, we can proceed to test for lack of parallelism. This is quite a common defect in lathes, and makes itself apparent by bars turned between the centres coming out slightly tapered instead of parallel, viz., slightly conical instead of cylindrical. This, in fact, is one way of making the test, by actually turning a bar and measuring the amount of taper by the difference in diameter of the two ends. Alterna-



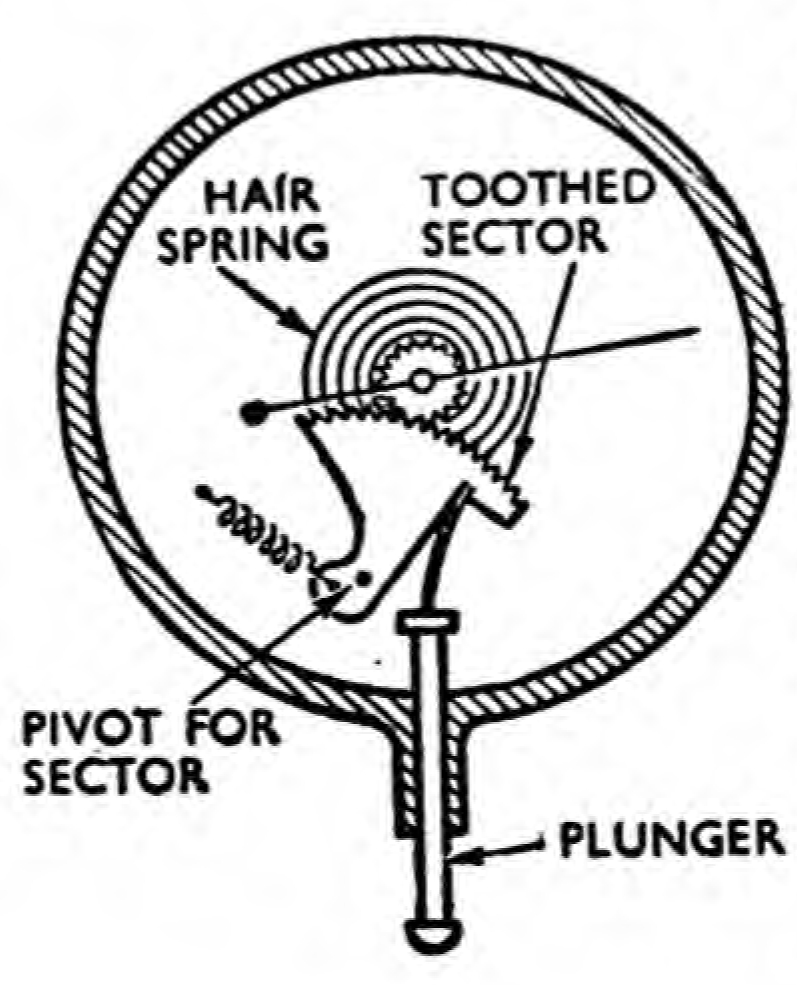
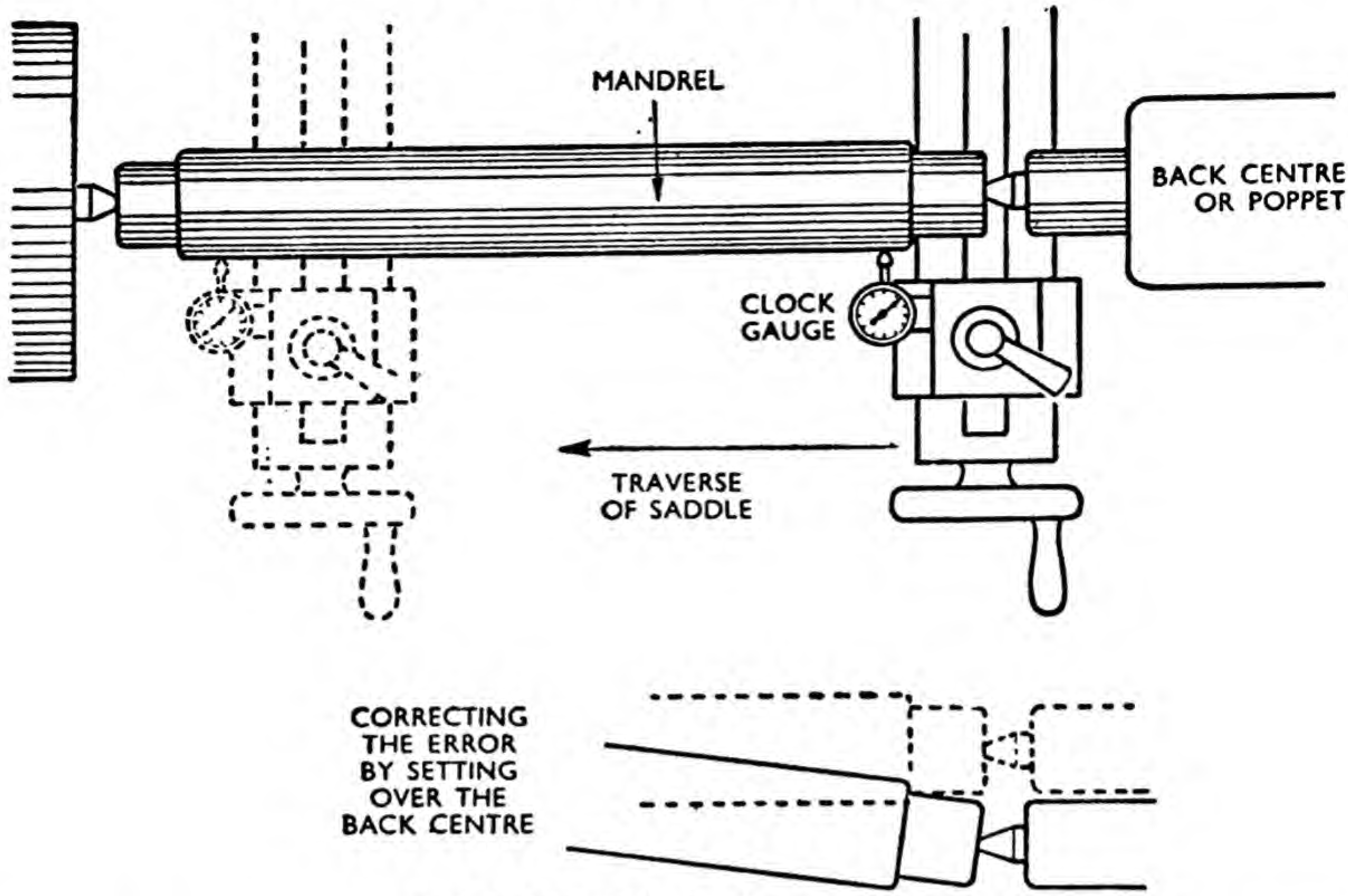


Fig. 15. Function of clock gauge is to magnify small movements of the plunger. Usually one-tenth inch movement causes one revolution of pointer over dial. Plunger moves a toothed sector which rotates a very small pinion attached to the pointer. Springs keep various parts of mechanism always in contact, eliminating back-lash.



TESTING FOR PARALLELISM

Fig. 16. Common error with lathes is that work is turned slightly taper instead of parallel. This can be checked with a clock gauge mounted on the tool post. A parallel mandrel is mounted between the centres and the clock gauge is fed in to touch the mandrel. Movement of pointer as saddle is traversed along the bed, indicates the amount of taper. To correct, back centre is set over by this amount.

tively, the clock gauge may be used again, as shown in Fig. 16. Here a parallel mandrel is mounted between the centres, and the clock gauge is set in the slide rest in place of the tool, so that the plunger presses against the surface of the mandrel. The slide rest is then traversed up and down the bed of the lathe, and any lack of parallelism will be shown by the movement of the pointer.

For a rough test, instead of the clock gauge we can use the point of the tool set so as almost to touch the mandrel, and then check the width of the space between the tool point and the mandrel in different positions, using a feeler gauge or even a piece of stiff paper or thin card. As an additional safeguard, it is wise to repeat the test with the mandrel turned end for end. This will eliminate the possibility of errors due to the mandrel itself being slightly tapered. Fortunately, the back centre of most lathes is made so that it can be moved *across* the bed, on its soleplate, and we can adjust it in this way until parallelism is obtained, as shown, in a greatly exaggerated manner, in Fig. 16.

Use of Clock Gauge

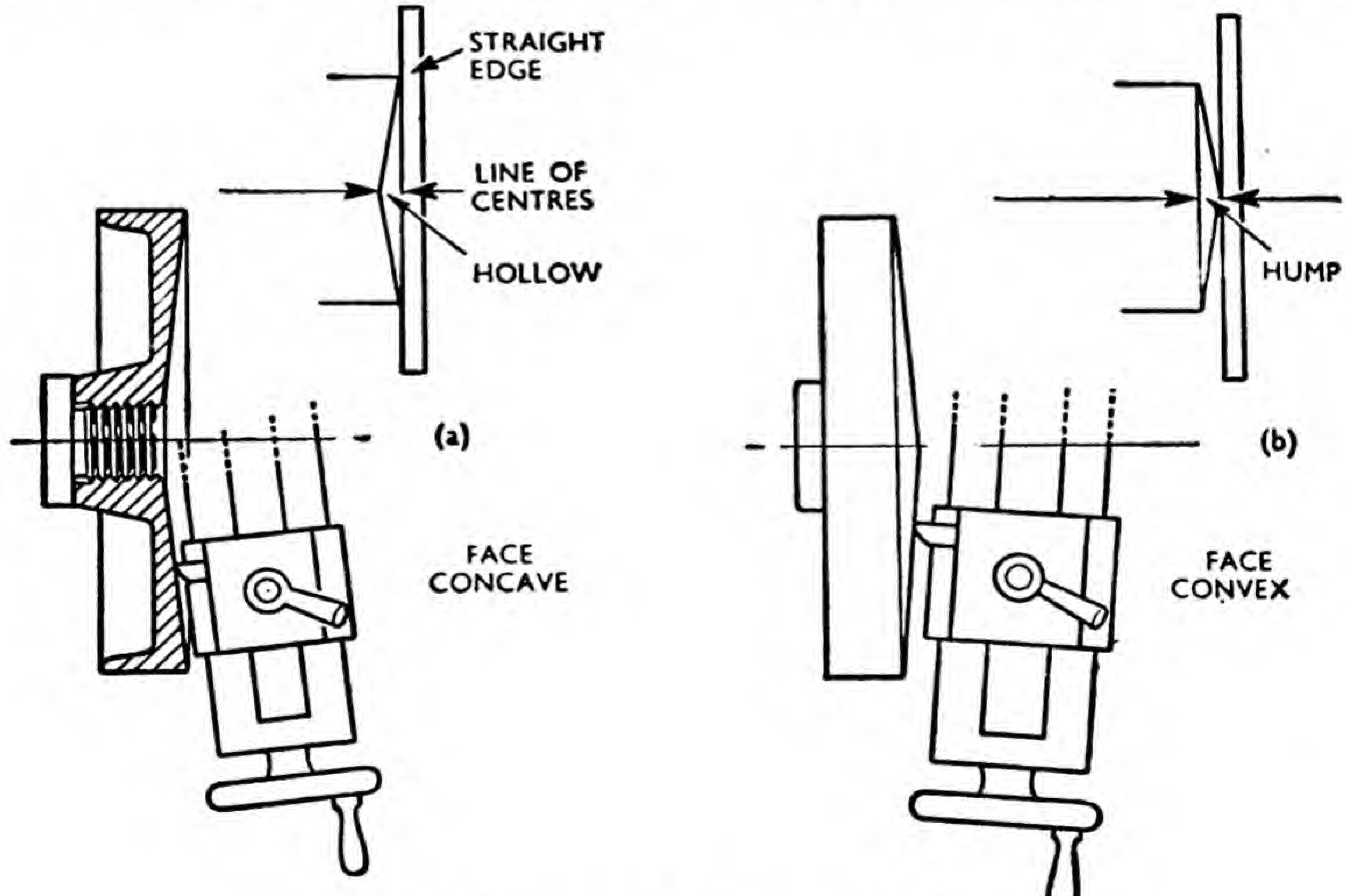
Another defect is that the cross slide may not be square with the bed. This will result in inaccurate facing, the machined surface being either slightly concave or convex instead of truly flat. This is shown in Fig. 17. One way of checking this is by means of the clock gauge and the face plate and at the same time the face plate itself can be checked for being true and square with the spindle.

If the clock gauge is traversed

across the surface of the face plate by means of the cross slide, the amount by which the cross slide is out of square can be detected by the movement of the pointer. We should, of course, repeat this test with the face plate rotated into different positions, to make sure that any error was not in the face plate instead of the cross slide.

When a new face plate is fitted to a lathe, it is skimmed up by a light cut being taken across it, in position on the lathe spindle. This ensures that its surface runs true with the spindle, but it is easy to see that in making this truing cut, the defect of Fig. 17 will arise, unless the cross slide has been set truly square.

The method just described for testing the squareness of the cross slide is a particular example of a general method, the principle of which is to test a thing against itself or testing by reversal; that is comparing a thing with its own mirror image. This is a most useful means of testing for squareness, because its effectiveness is not dependent upon the accuracy of any instrument used for testing. Thus, with the face plate, as the spindle revolves, it is constantly reversing the machined surfaces about the line of centres. Only if the surfaces are truly square with the line of centres, will the two surfaces be exactly in line when tried with the straight edge. Here again, if the straight edge itself is not exactly straight it can be checked by reversal, for if it is hollow on one edge, it is probably humped on the other. Before deciding that the face plate is either concave or convex, it is wise, therefore, to reverse the straight



CROSS-SLIDE OUT OF SQUARE

Fig. 17. Another error in the lathe results from the cross-slide not being square with the bed. When surfacing, the effect is to produce a concave face (*a*) or a convex face (*b*). Amount of correction required can be determined by means of a straight edge as shown, or by using a clock gauge against the face plate.

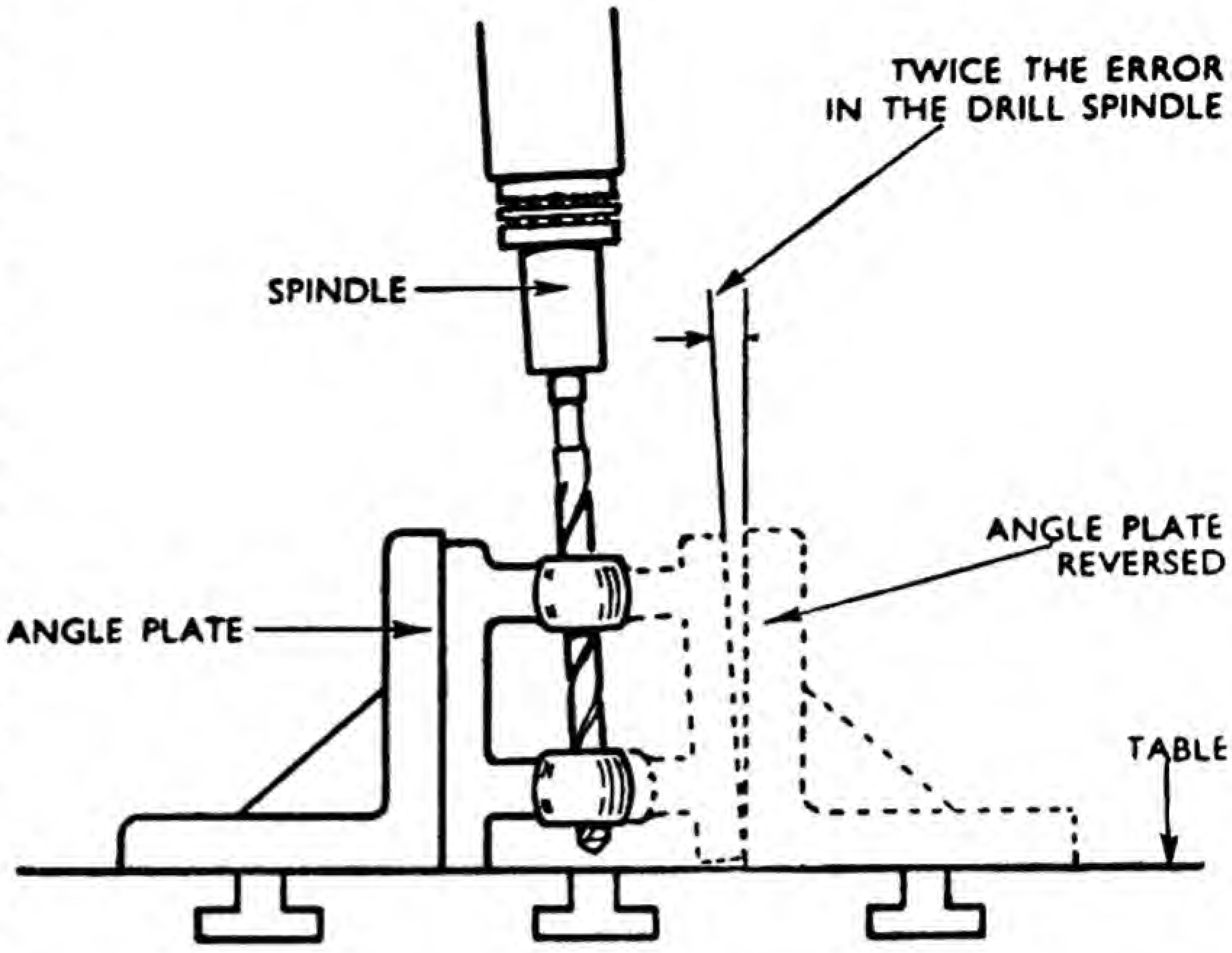


Fig. 18. Useful way of testing for squareness is by the method of reversal. In diagram a bracket has been drilled and it is suspected that drill spindle is out-of-square with the table. This can be verified by reversing the bracket as shown dotted, and checking for parallelism with the angle plate. Any discrepancy represents twice the actual error, and if spindle is adjusted until discrepancy is reduced by one half, spindle will then be square with table.

edge, to make sure that the error is not in the straight edge.

Fig. 18 shows how we apply this method to test a drilling machine spindle for squareness with the table. We drill a hole in the ordinary way, preferably a deep one or two holes in line as shown, and then we reverse the part on the table. If the spindle is truly square with the table, the drill will line-up with the holes in the new position, or, in fact, in any position on the table. If not, the amount of correction required can be determined by threading the part on to the drill, and measuring the gap as shown in Fig. 18. Adjustment of the table or the spindle so as to reduce the gap to half its original value, will then result in the spindle being dead square with the table. The correction must be only one-half of the discrepancy shown on reversal, because this is twice the actual error, a half one way and a half the other.

We must not overlook the possibility of part of the error shown being due to a bent drill, and so we would check this by reversal also, rotating the drill through 180 deg. and noting any variation in the gap.

Machine Drive

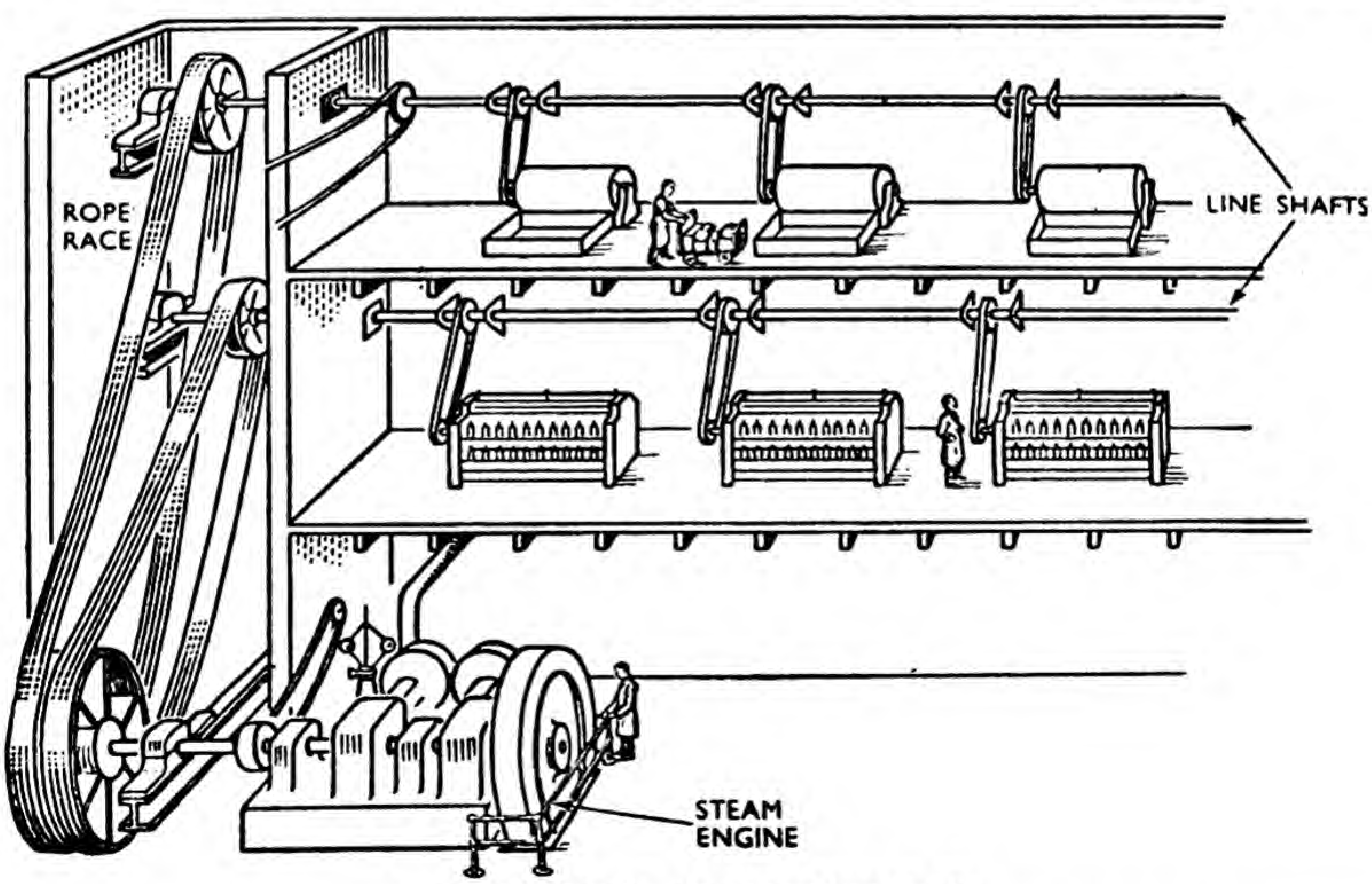
The last twenty years have witnessed a revolutionary change in the methods of driving machines. This is almost entirely due to the development of electricity as the universal source of power in factories. When the water wheel, and later, the steam engine, was the principal prime mover, one large unit usually had to serve the entire factory, and power had to be distributed from this single source to the hundreds of machines spread throughout the factory, in many instances on five, six or more floors. With the coming of electricity, and more particularly the small, efficient and relatively cheap induction motor, the driving of each machine by its own motor, so-called individual drive, became a possibility.

This does not mean that the large steam-engine and its powerful main drive to the various floors is not to be found nowadays. Some of the largest factories, textile,

flour, and paper mills, still rely on their own power supply. One reason is that they require steam in any case for various process purposes and, in these circumstances, the steam-engine is a very cheap source of power, and the scrapping of this machinery in order to change over to individual drive by electric motors would not be justified. On the other hand, when new factories are built, the tendency is to adopt individual motor drive, or at least group drive, which is several larger motors each driving a group of five or six machines.

Fig. 19 shows a typical arrangement of a steam-engine main drive to the machines of a mill arranged on two floors. All the power required by the mill is taken from the steam-engine through the main-rope drive, which is housed in a special shaft, called the rope race, situated either in the centre of the mill, or at one end. This rope race extends from top to bottom of the building, through all the floors on which power is required, and projecting into it at the level of each story is a large rope pulley which takes off the power required on that particular floor.

These pulleys obtain their power from a single pulley mounted on the engine shaft at the bottom of the rope race. This main pulley has grooves for the driving ropes ; there may be thirty, forty, or even more, the total number depending upon the number of floors and the number of ropes required for each floor. The latter will vary from floor to floor in accordance with the power required. Usual sizes for the ropes are from 1-in. to 2-in. diameter, and, taking a 1½-in. rope as an example, each rope of this



MAIN DRIVE BY LINESHAFTS

Fig. 19. Where a single engine provides all the power for a large mill, the usual practice is to distribute the power to various floors by rope drives. A large steam engine on the ground floor drives the main rope pulley, whence ropes pass up the rope race to pulleys at each floor level. These pulleys drive lineshafts which supply power to each machine by individual belt drives.

size can transmit up to 35 h.p. The power transmitted depends upon the speed, of course, and a rope speed of 3,000 to 4,000 ft. per min. is very usual.

As with belt drives, centrifugal force tends to reduce the grip of the rope as it passes round the pulley, and on this account no advantage is gained by running the ropes faster than about 5,000 ft. per min. Above this speed, the loss of rope grip, due to centrifugal action, more than offsets the gain in horse-power, which would otherwise be expected from the increased speed. These high rope speeds are obtained by using large diameter pulleys which have the additional advantage of increasing the life of the ropes.

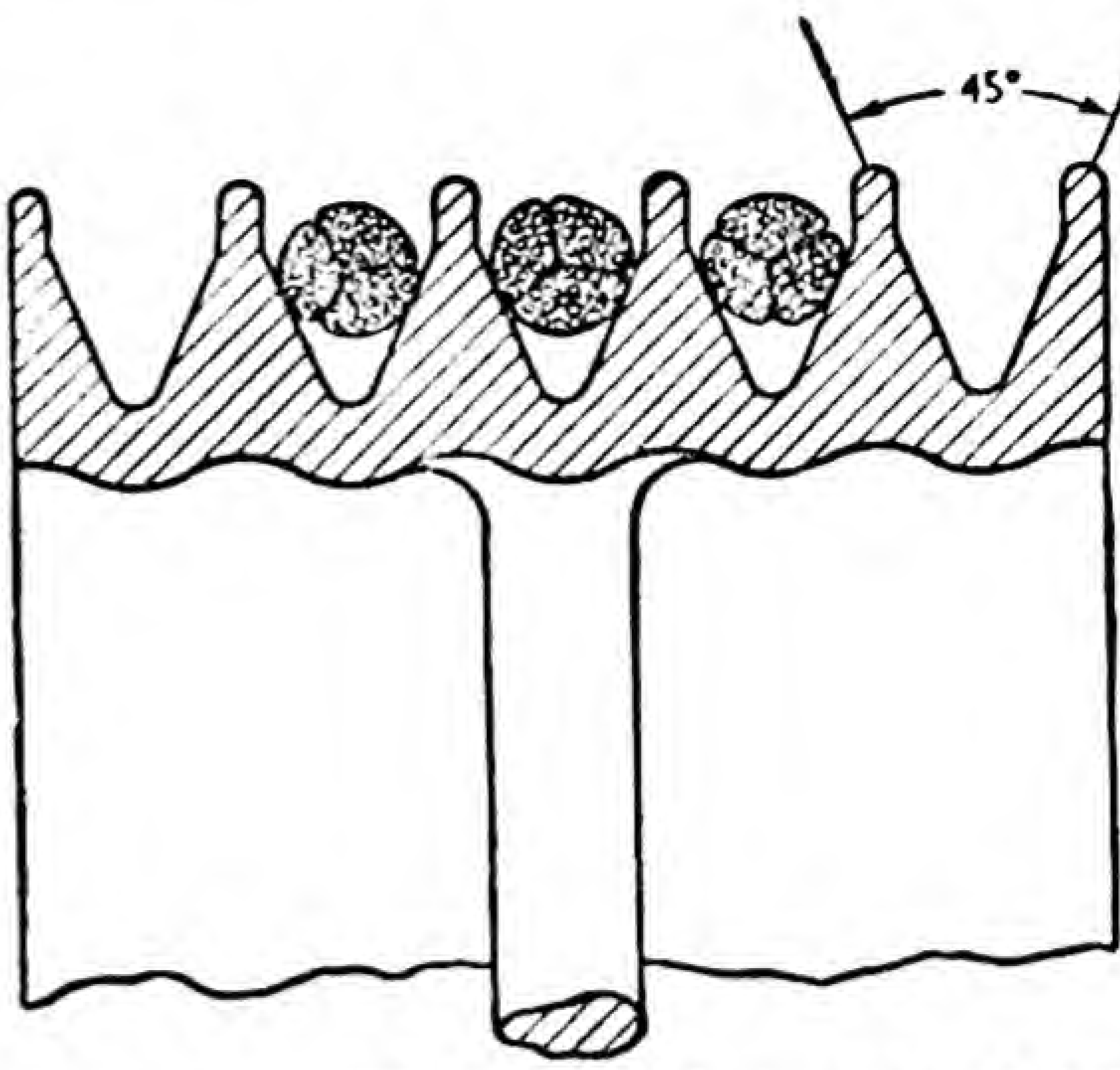


Fig. 20. The rim of a rope driving pulley has separate grooves provided for each rope. The sides of groove taper at 45 deg. so that rope wedges in groove and does not slip. Number of ropes and grooves depends upon power transmitted and may be 30, 40 or more.

Size of Pulley

In any form of drive, belt or rope, it is a great mistake to use too small a pulley, as the repeated bending to a small radius causes premature failure of the fibres of which the belt or rope is composed. The smaller the belt or rope, the more flexible it is, of course, and the smaller may the pulley be. To take an actual example, a 15-ft. diameter pulley running at 100 r.p.m. would give a rope speed of 4,710 ft. per min. Using 20 ropes, 1½-in. diameter, this could transmit about 700 h.p.

A good idea of the shape of a rope-pulley rim can be gained from Fig. 20. This shows that the grooves are V-shaped, so that the rope wedges itself into them and slip is prevented. The two ends of the rope are joined by a special long splice, so that the rope when fitted is virtually endless. Each groove is separated from its neighbours by a flange or division wall. The need for this will be appreciated if it is remembered that rope drives are long-centre drives, and the rope, after leaving the engine pulley at the bottom of the race, may have to travel 100 ft. or more before it reaches the other pulley on the top floor of the mill. For such long drives it is usual to provide intermediate guides, in the form of wooden frames, to prevent the ropes chafing each other.

Now let us return to the rope pulleys which pick up the power from the main drive at the level of each story. Each of these drives a long lineshaft running along one wall, usually for the entire length of the building. Sometimes there is a lineshaft down each side wall, and the second one is driven from the first by a belt drive connecting the two at the rope-pulley end. From the lineshafts, power is taken off to the various machines by means of leather, canvas, or balata belts.

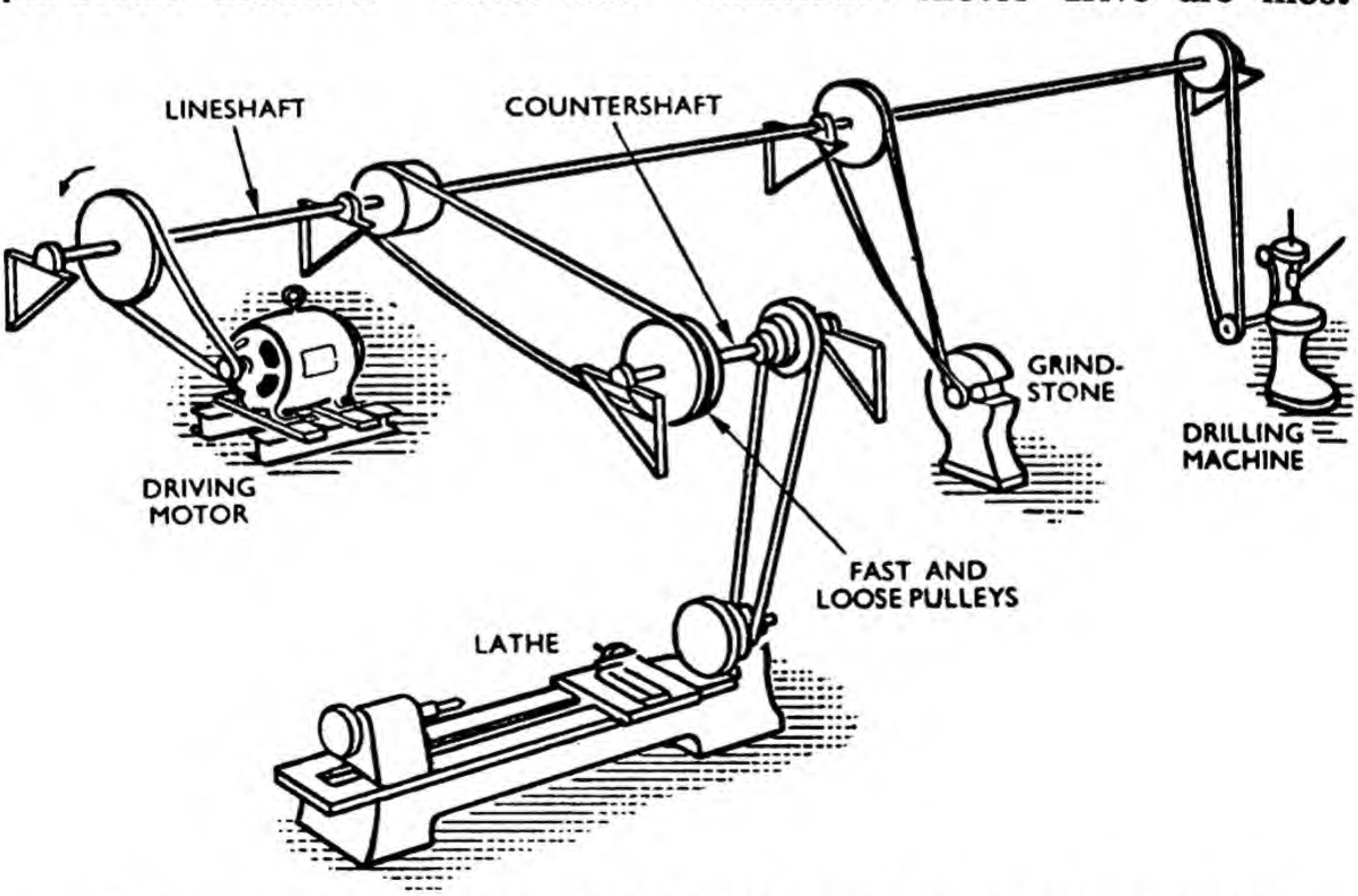
A good rule for leather belts is that they will transmit from 1 to 1¼ h.p. per in. of width at a speed of 1,000 ft. per min. Balata belting

which consists of plies of canvas duck cemented with a rubber-like gum, will withstand a higher tension than leather, and a 5-ply belt will transmit up to 2 h.p. per in. of width at a speed of 1,000 ft. per min. Thicker belts made up of more plies will transmit more power in proportion, but they are less flexible, and can be used only on large-diameter pulleys such as those which are employed for connecting the two lineshafts together.

Lineshaft Speed

The speed of the line shafts will depend upon the sizes of the rope pulleys in relation to the size of the main-engine pulley, and as the line shaft speed will be uniform for the whole of one floor, different size belt pulleys must be fitted to the lineshafts to suit the speed of each particular machine. These lineshaft pulleys are usually double-width pulleys, and drive a pair of fast and loose pulleys on the machine, so that each machine can be started and stopped at will, independently of the others, so long as the lineshaft is running.

This immediately shows up one of the defects of this form of drive; this is that a stoppage of the lineshaft for any reason means a stoppage of all the machines on that floor, or alternatively, that if only one machine is required, the entire mill drive must first be set in motion. In many instances this is not a serious disadvantage, as the majority of the machines will be required during working hours. In others, it is necessary for the machines to be more or less independent of each other, and it is here that the advantages of individual motor drive are most



GROUP DRIVING THROUGH LINESHAFT AND COUNTERSHAFT

Fig. 21. Typical layout of motor drive to a group of machines in a workshop. Along one wall runs a short lineshaft from which belts take the drive to individual machines. For the lathe a countershaft is required, mounted on opposite wall, and carrying stepped pulleys. Fast and loose pulleys permit the countershaft to be stopped without interrupting the drive to other machines.

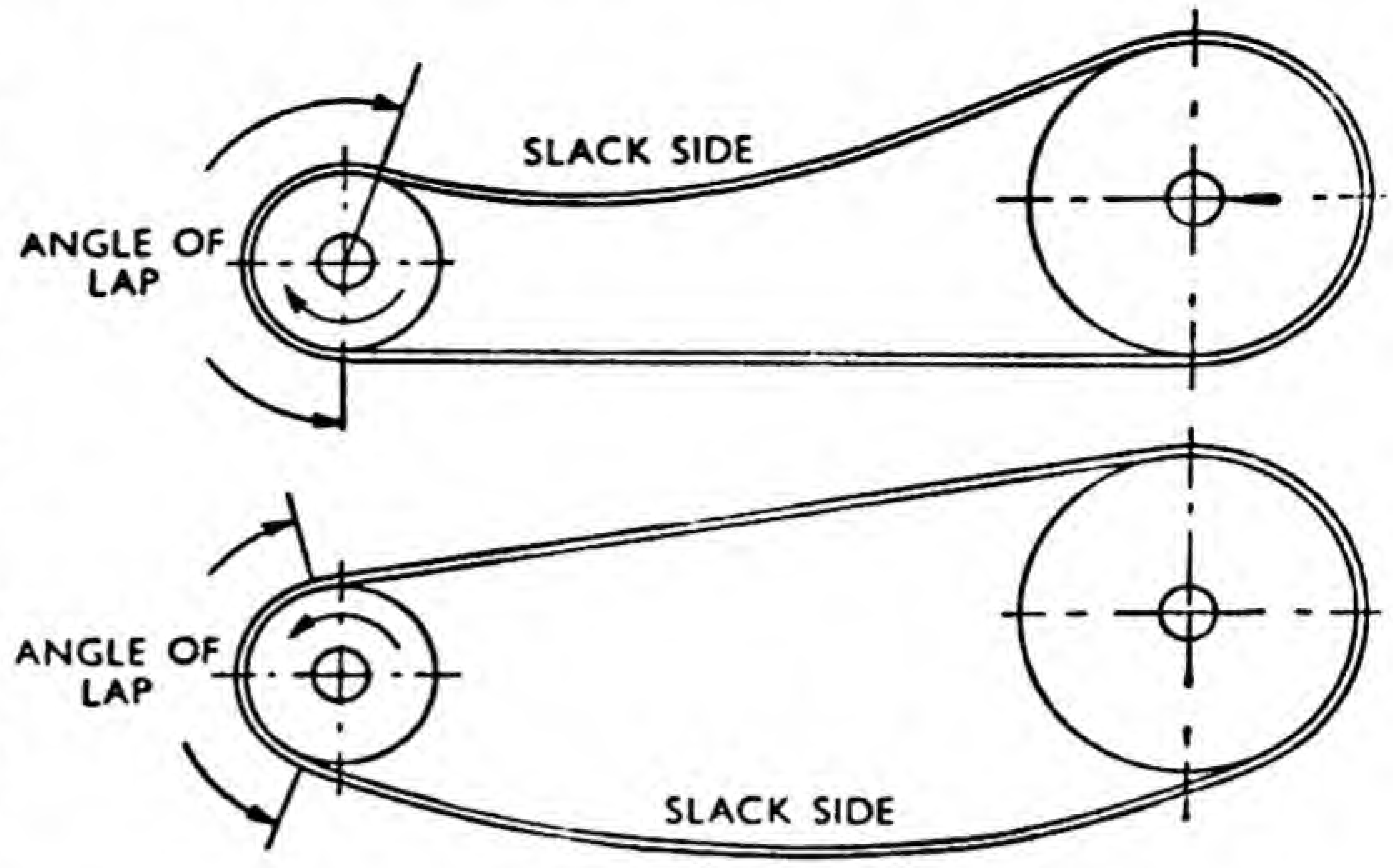


Fig. 22. In arranging a belt drive, direction of rotation and the positions of pulleys should be chosen so that, if possible, slack side of belt is on top. Thus the angle of lap of the belt on the pulleys is increased, enabling greater power to be transmitted without slip. If the slack side is at the bottom, a higher tension must be used.

apparent. It should not be assumed that the individual drive is necessarily more efficient than the tangle of belts and lineshafts, for the single large prime mover is invariably more efficient than a large number of small units.

Provision must always be made for overloads. With most machines the maximum power required for short periods of time may be considerably greater than the normal, 25 per cent or even 50 per cent greater, and in arranging the drive, whether by belt or motor, this reserve of power must be provided. With individual motor drive, this means that each motor must be so much larger than it would otherwise have to be.

Main Drive

In arranging a main drive, however, provided that each belt can take the overload without slip, it is quite safe to assume that all the machines will not be overloaded simultaneously, and the total power required will tend to balance out, the power saved on the machines that are lightly loaded being available for any that are momentarily overloaded.

A good practical compromise is provided by the group drive, such as is shown in Fig. 21. Here we arrange the machines in a number of groups, and use separate motors to drive each group. The motor drives a short lineshaft and, by a suitable choice of pulley sizes, we can obtain the necessary reduction in speed. The various machines are then driven from the lineshaft in the usual way. For the group shown we would require a 5-h.p. motor, of which the lathe would absorb 2 h.p., the grindstone and the drill about ¾ h.p. each, leaving a balance of 1½ h.p. to provide for the losses in the lineshaft bearings and the necessary overload factor.

Fig. 21 also shows an arrangement for driving a lathe which requires a choice of speeds. From the lineshaft, a drive is taken to an auxiliary shaft called a countershaft, on which the stepped pulley giving the required range of speeds is mounted. The drive to the countershaft would be through fast and loose pulleys, so that the countershaft could be brought to rest when it is desired to stop the lathe or to effect a change of speed.

The correct way to arrange a belt drive is shown in Fig. 22, that is, with the slack side on

the top, so that the sag of the belt increases the angle of lap on the pulley. The power that can be transmitted may be reduced appreciably if the slack side is at the bottom. An approximate expression for the necessary initial tension T_o (lb.) in a belt transmitting P h.p. at v ft. per min. if the angle of lapping on the smaller pulley is not less than 150 deg., is

$$T_o = \frac{36{,}000P}{v}$$

Excessive belt tension reduces the life of the belt, and causes high bearing loads which may lead to overheating.

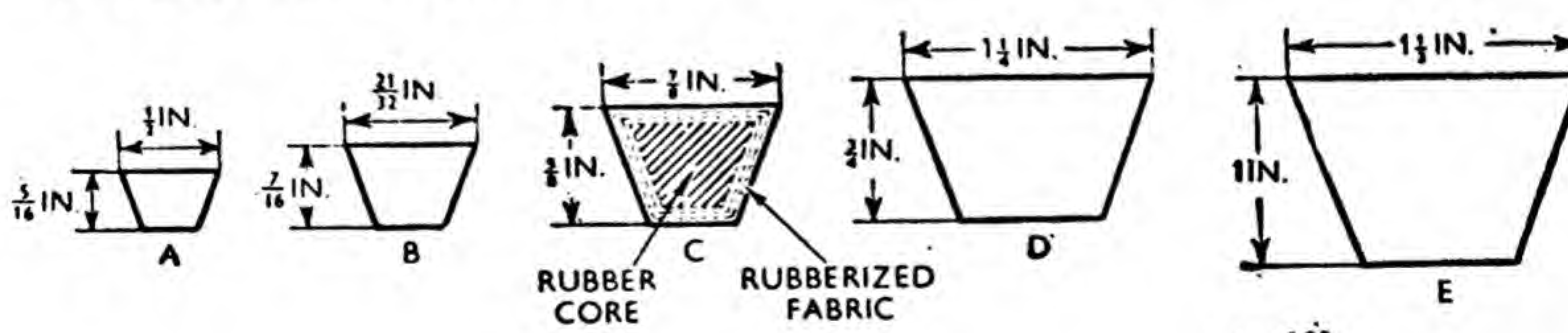
Belt-shifting Fork

Where a belt-shifting fork is used to move the belt from the fast to the loose pulley, this must be arranged to act on the belt as it *passes on to* a pulley. It is useless to attempt to force the belt to move as it comes *off* a pulley. In fact, in this respect the belt is rather like a mule; it can be led but not driven.

The need for a speed reduction when using an electric motor drive usually involves a small pulley on the motor spindle, and it is difficult to obtain sufficient lap on the small pulley, especially if we have a short-centre drive, that is if the two pulleys are only a short distance apart.

For this purpose, the V belt is the ideal form of drive. These are woven in endless form in a series of standard sizes, and are tapered in cross-section to an included angle of 40 deg. By the wedging action of the belt in the groove in the pulley rim, the grip is increased about three times as compared with a plain belt with the same angle of lap. By choosing suitable sizes and numbers of V belts, it is possible to arrange drives from a fraction of a horse-power up to 200 h.p. or more, as with rope driving, only on a smaller scale. These are shown in Fig. 23.

Fig. 24 helps us to find out how many belts are required for any particular drive. The graphs show how the horse-power which each belt can transmit increases with



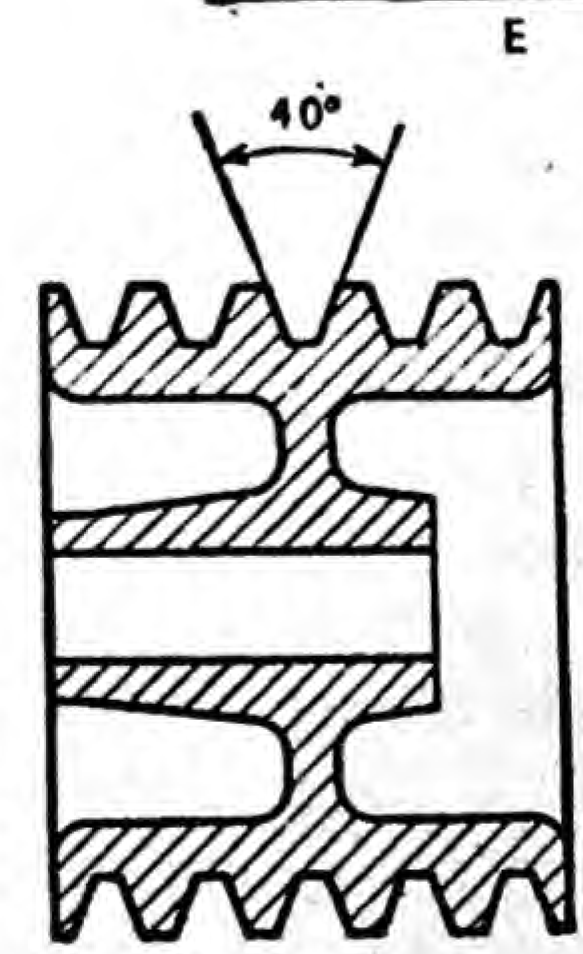


Fig. 23. For short centre drives with small pulleys, standard size V belts with a V-belt pulley are used. These obtain better grip due to wedging action of belts in grooves of pulley. Belts are made of rubber and fabric in five standard sizes as shown, and sides are tapered at 40 deg. Each pulley carries a number of belts depending upon the power to be transmitted.

the speed, and how this is limited by centrifugal force at belt speeds higher than about 4,000 ft. per min. Suppose, for example, it is required to transmit 40 h.p. It can be seen from the graphs that this could be done with two belts of size *E* running at 3,000 ft. per min., or three of size *D*, or six of size *C*.

Which should we choose? This depends upon pulley sizes. The smaller belts can be used on pulleys of smaller diameter, and, in general, it is better to use a number of small belts on a pulley of reasonable size, than to use only one or two belts of large size which would require a correspondingly larger pulley. The minimum pulley diameters for each size of belt are as follows :—

Belt Size	A	B	C	D	E
Pulley dia. in inches	3	5	9	13	21

In using the smaller diameter pulleys, we must not overlook the fact that for the same motor speed in r.p.m., the smaller pulleys will mean a lower belt speed. Also, if the drive is from a small pulley to a much larger one, the angle of lap on the small pulley will be reduced, and the power per belt must be reduced by 10 per cent for each 30 deg. that the angle of lap is less than 180 deg.

Another means of short-centre driving is by means of chains. These may be of the roller type, like the ordinary bicycle chain, or of the so-called inverted tooth or silent type, in which the links are shaped to fit the space between the wheel teeth as the chain passes round the wheel, but which close with a scissors action as the chain passes off the wheel and straightens out (Fig. 25).

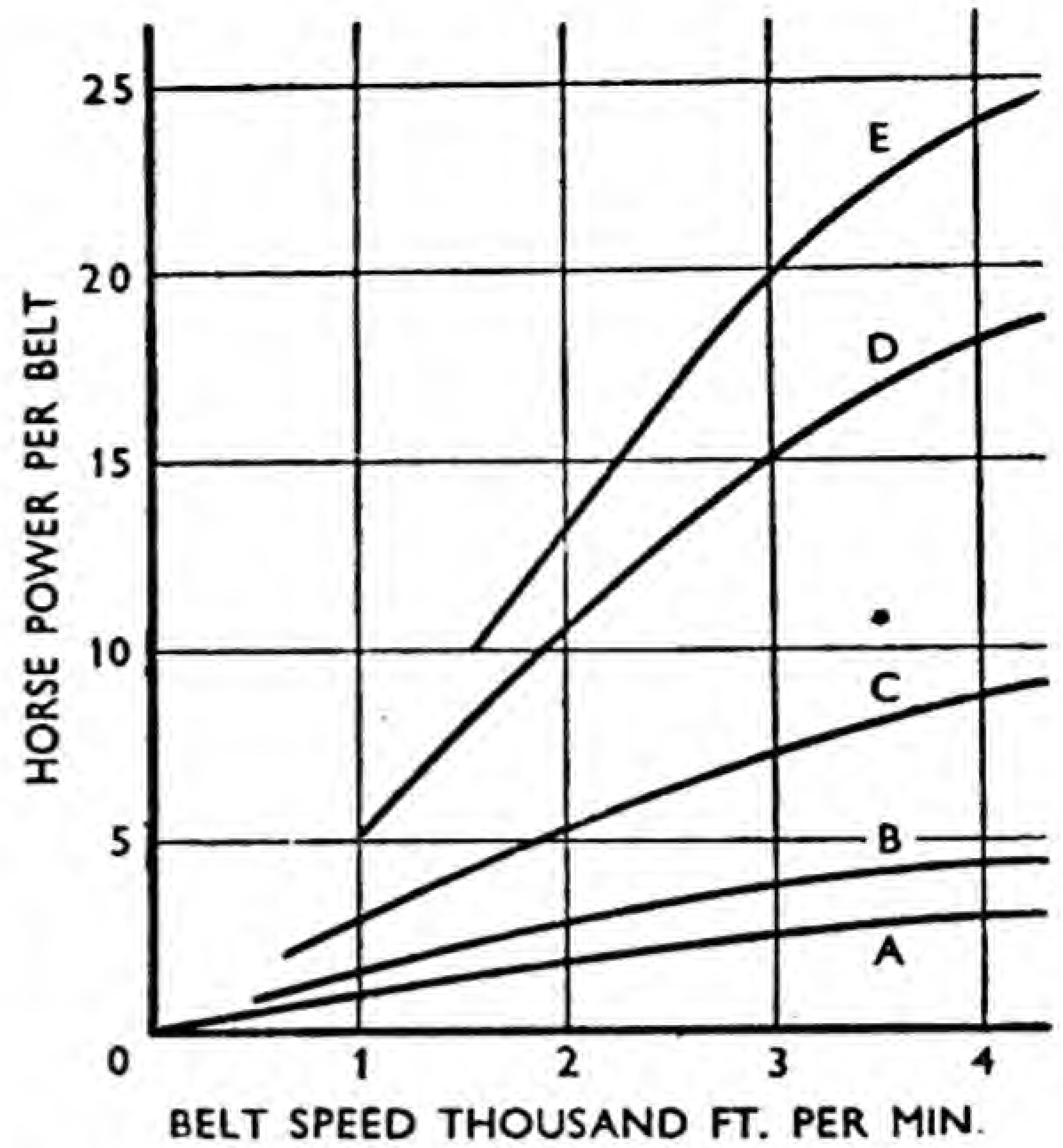


Fig. 24. Here is design data for V belt drives. Graph shows power that can be transmitted by each of five standard sizes of V belt. To transmit 40 h.p. at 3,000 ft. per min., 6 belts size C, 3 size D or 2 size E can be used. The larger sizes of belt require fairly large pulleys.

Chains have a number of advantages as compared with belts.

(*a*) They are stronger and can transmit more power in a given space.

(*b*) They can operate at temperatures which would damage leather or rubber belts.

(*c*) They can operate in oily situations which would cause belts to slip.

(*d*) They are positive, that is, no slip can occur.

On the other hand, they are more expensive, and are not usually used unless either a positive or a short drive is required.

Size of Chain

The size of a chain is specified by its pitch, which determines the spacing of the teeth on the sprocket, and by its width. For larger

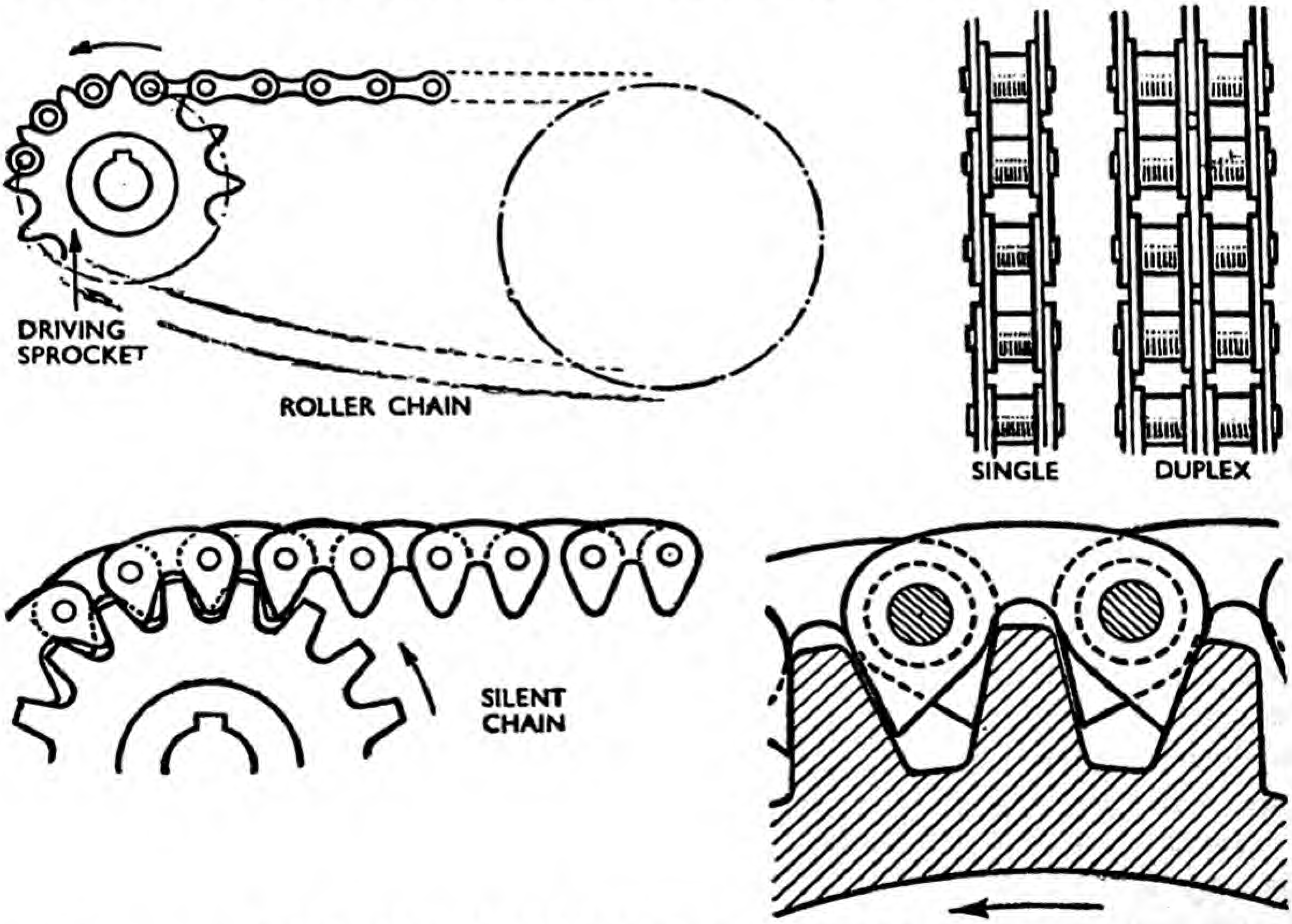
powers, roller chain is made duplex or even treble, and silent chain can be made in any width from 1 in. up to 12 in. or more. Comparing, say, for example, a $\frac{3}{4}$-in. pitch chain with a belt, about three times the power can be transmitted by the chain for the same width at the same speed, and, of course, smaller sprockets can be used. Chains are not usually run at speeds higher than about 1,500 ft. per min.

Speed Variation

Although individual drive gives the convenience of being able to start and stop each machine independently, it does not usually permit variation of speed. In this way, it is like the lineshaft, and separate speed-changing gear must be provided if it is desired to control the speed.

Electric motors also usually run much faster than lineshafts and, in general, the smaller the motor, the higher the speed. For many machines, this speed is inconveniently high, and a speed reduction gear must be fitted. For special purposes it is possible to get slow-speed motors, and even variable-speed motors, but these are always more expensive than the standard machines.

Fig. 26 shows how a choice of three different speeds is possible by means of a simple sliding-mesh



THE TWO MAIN TYPES OF DRIVING CHAIN

Fig. 25. Roller chain gets its name from the series of rollers which actually engage the teeth of the sprocket. These turn freely on the pins between the links of the chain, so reducing friction and wear. Duplex roller chain consists of two sets of rollers side by side. In so-called silent chain, engagement is by tooth-like projections on the links. The action of bending the chain round the sprocket causes the projections on adjacent links to open with scissors-like action and fit tightly between the sprocket teeth. The projections close and release their grip as the chain straightens out again.

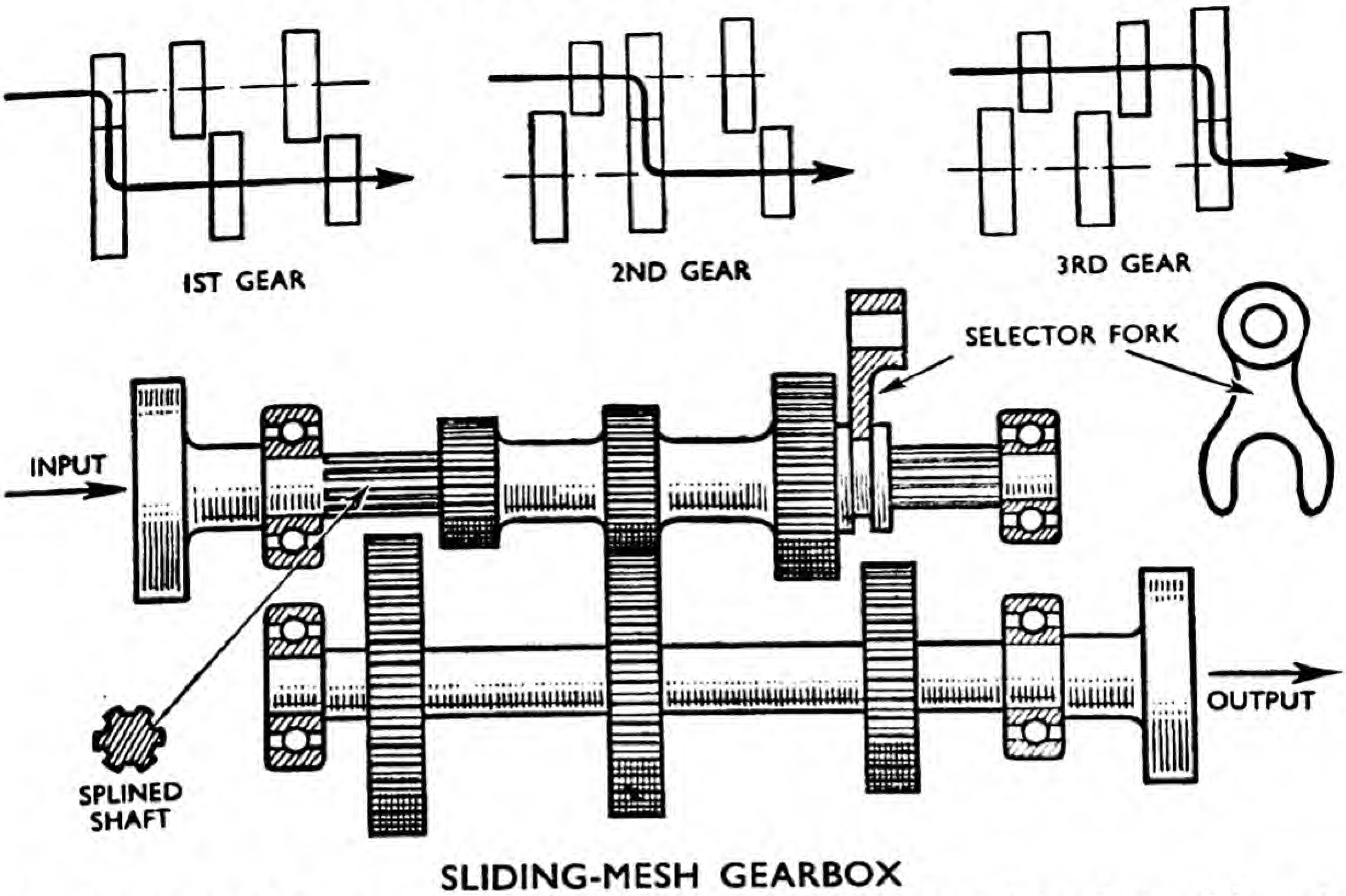


Fig. 26. Upper diagrams show the principle of a three-speed gearbox. There is a separate pair of gears for each speed, and the three diagrams show the lower shaft being driven from the upper by each of the pairs of gears in turn. Lower diagram shows how the upper shaft is splined so that the gear wheels can slide into mesh with the wheels fixed to the output shaft. The sliding wheels are engaged by a selector fork and no two pairs can be in engagement simultaneously.

gearbox. The principle is that there is a pair of gear wheels for each speed, and a selector mechanism ensures that as one pair is engaged to give the speed required, the other two are disengaged. This is accomplished by sliding the selected gear wheel along the input shaft until its teeth mesh with those of the corresponding wheel, which is fixed to the output shaft. To make this possible, the wheels must be mounted on the input shaft so that, although they are driven by it, they can be made to slide along it by means of the selector fork. Therefore, they are mounted on a sliding key or, more usually, on a splined shaft which really consists of a shaft with a number of sliding keys made solid with it. Six are shown in Fig. 26.

The simplest way to ensure that only one pair of wheels is in mesh at any one time, is to make the three sliding gears on the input shaft in one piece so that they form a cluster, as shown in the lower figure. Sliding the cluster along the shaft automatically disengages the teeth of one pair before engaging another pair. In this type of gearbox, the various pairs of gears form simple trains connecting the input and output shafts which, therefore, will rotate in opposite directions. We can obtain as many speeds as we wish by providing a corresponding number of pairs of wheels, and a suitable selector mechanism that will engage the pair required and disengage the others.

The disadvantage of this simple type of gearbox is that it is difficult to slide the teeth into mesh when

the shafts are rotating, and quite impossible if helical teeth are used. Let us see, then, how we can obtain a choice of speeds by means of a gearbox in which all the wheels are always in mesh. There are pairs of wheels as before, as many pairs as speeds required, but whereas one wheel of each pair is fixed to one shaft, the other wheels are entirely free to rotate on the other shaft.

Dog Clutch

Engagement of any desired speed is obtained by clutching one of these free wheels to the shaft. This is shown at the top of Fig. 27, a sliding dog clutch being used to connect either one or other of the wheels to shaft *A*, and so to transmit the drive to shaft *B*. The dog clutch, in this case, is like the sliding-mesh gears, in that it is mounted on splines so that, although it must rotate with shaft *A*, it is free to slide in order to engage the gears.

A slightly different form of constant-mesh gearbox is shown at the bottom of Fig. 27. This has the input and output shafts in line, so that it is possible to get a straight-through drive if desired. This is obtained by sliding the left-hand dog clutch to the left and so connecting the output shaft directly to the input shaft, so that they rotate together.

For the other speeds, the drive is taken through the gears and the other shaft, which is known as the lay shaft. The lay shaft is driven

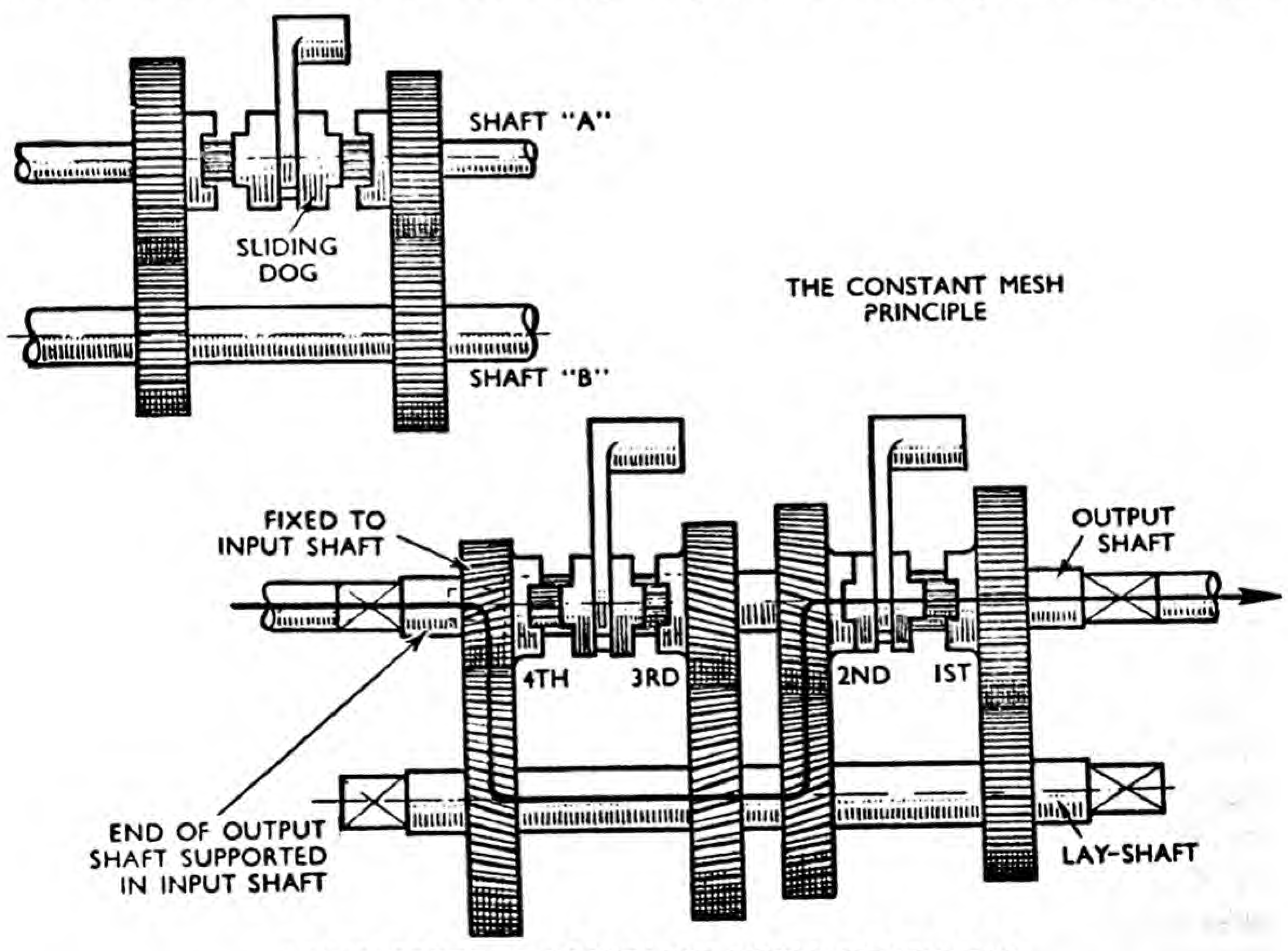
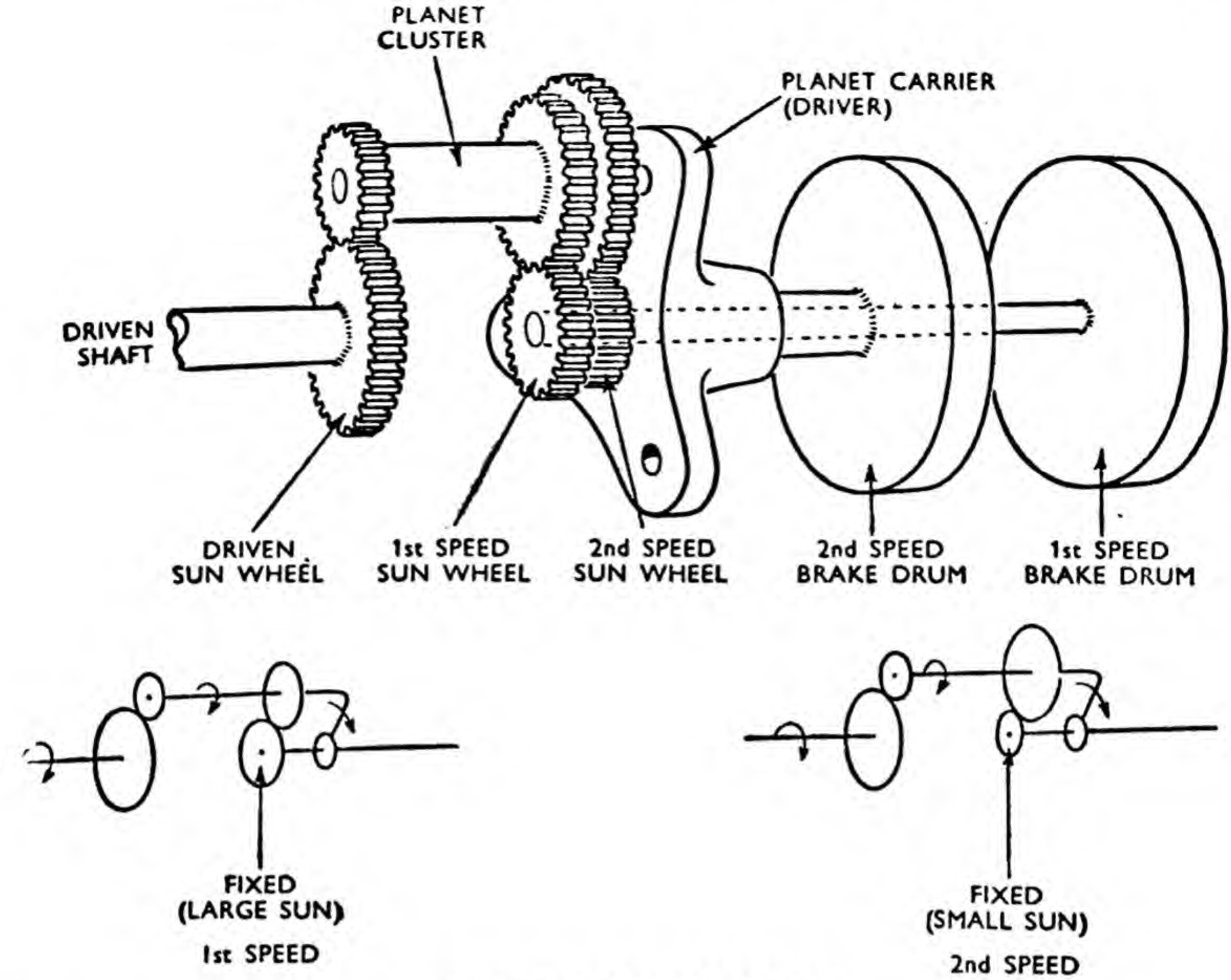


Fig. 27. Gear changing is simplified if gear wheels are constantly in mesh, but can be made to drive shaft when desired by engagement of a clutch or dog. In the diagram, wheels are free to turn on shaft A, but there will be no drive between shafts except when one wheel or other is engaged by the dog which slides on splines. In the gearbox, lay shaft is always driven by pair of gears at left, and this, in turn, drives output shaft only when one or other of two dogs is engaged.



EPICYCLIC GEARBOX

Fig. 28. Another way of simplifying gear changing is to use epicyclic gears. Wheels are always in mesh and gear changing is effected by application of brakes which bring one or other of wheels to rest. Drive is to planet carrier which rotates three planet wheels fixed together to form a cluster. So long as the planet cluster can rotate freely there will be no drive, but if one or other sun wheel is held stationary by means of its brake drum, planet cluster will have to rotate so as to drive the driven sun wheel.

from the input shaft through the pair of gears at the left-hand side which are fixed, one to each shaft. The lay shaft rotates the three gears which are mounted on it, and these, in turn, drive the other wheels, which mesh with them. But these last wheels only drive the output shaft when the appropriate dog clutch connects one of them to it.

The selector mechanism ensures that only one clutch can be engaged at a time. It will be noticed that when the drive is through the gears, the gears form a compound train. That is, the drive is from the input shaft to the lay shaft, and back from the lay shaft to the output shaft. Therefore, the input and output shafts always rotate in the same direction, whether the direct drive or the gears are engaged.

Epicyclic Gears

Although the dog clutches of constant-mesh gears are much easier to engage than the teeth of sliding-mesh gears, some adjustment of the speeds is necessary if the clutches are to be engaged while the shafts are rotating. For automatic operation, this is a disadvantage, and it is here that epicyclic gears come in useful.

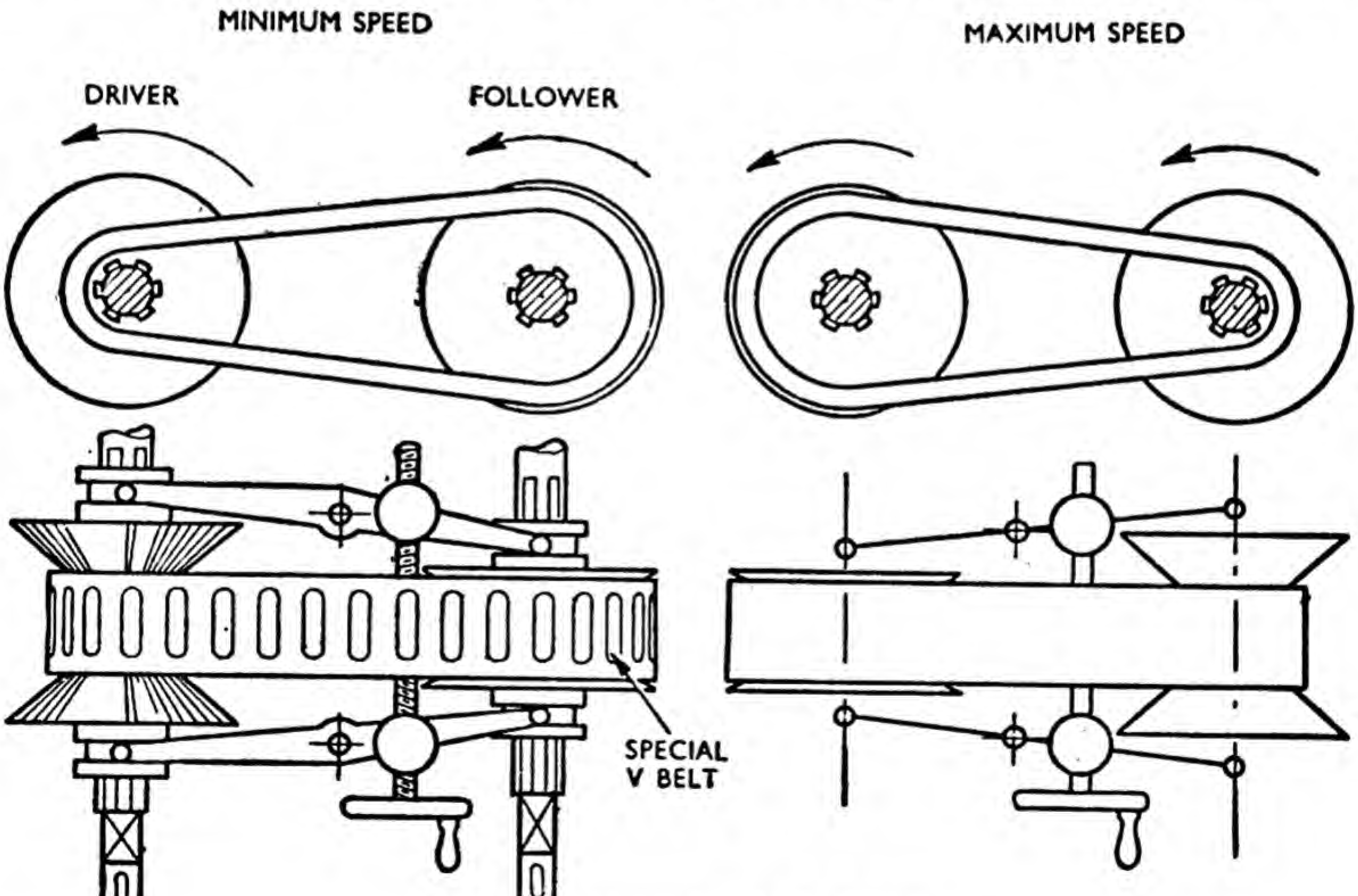
Separate epicyclic trains can be

used for each speed required, and each gear can be brought into action at will simply by tightening a brake band. An arrangement which gives a choice of two speeds is shown in Fig. 28. The drive is to the planet carrier, which, as you can see, would carry three sets of planets, although only one is shown. The planets on each spindle form a cluster, so that they must all turn together. To engage a gear, one of the sun wheels round which the planets revolve must be held stationary, and this is done by applying a brake to one or other of the brake-drums. The speed of the driven shaft will depend on how large its sun wheel is in relation to the sun wheel that is fixed.

By fixing the 1st-speed brake-drum, we hold the larger sun wheel stationary, this giving one speed. Then by releasing that brake and fixing the 2nd-speed brake-drum we hold the smaller sun wheel stationary, and a higher speed is obtained. Just to take an example, suppose the numbers of teeth on the various gear wheels are as follows :—

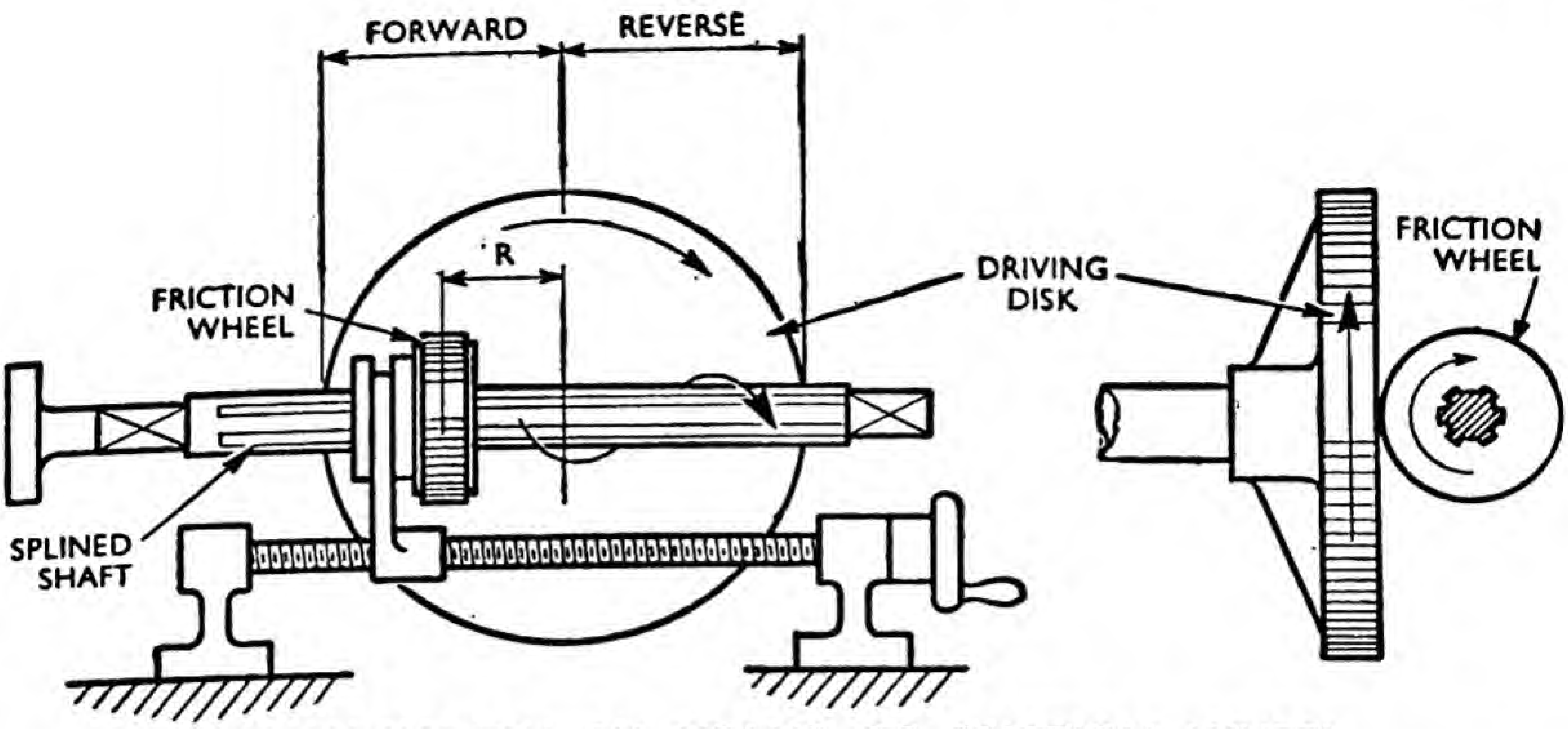
	Driven	1st Speed	2nd Speed
Sun	40	30	20
Planet	20	30	40

By the method shown in Chapter 10 we can readily calculate that the speed of the driven shaft will be half the speed of the driving shaft, or the planet carrier, when the 1st-speed is engaged, and three-quarters



INFINITELY VARIABLE GEAR

Fig. 29. By means of V belts in conjunction with double-cone pulleys, infinitely variable drives can be obtained. The two halves of each pulley slide on splines on their shafts and moving the two halves closer together causes the belt to take up a position at a larger radius. Movement of two pulleys is linked together so that the effective radius of one is increased as the other is reduced.



SPEED VARIATION BY MEANS OF FRICTION WHEEL

Fig. 30. Another infinitely variable gear makes use of a disk driving a friction wheel pressed against it. Friction wheel is mounted on splines so that it can slide across the face of the disk, and its speed will depend upon its distance *R* from the centre of disk. The wheel will be driven clockwise or counter-clockwise according as its distance *R* is to left or right of centre.

of the speed when the 2nd-speed is engaged. Also, both shafts will rotate in the same direction. If desired, we can get a direct straight-through drive by locking the two brake-drums together and allowing them to rotate as one.

Infinitely Variable Gears

With all these gearboxes, changes of speed can be only obtained in a series of steps. For some purposes this may be a disadvantage, as one speed may be too low and yet the next higher speed be too high. In such instances, an infinitely variable gear is required, and one type, using a V belt, is shown in Fig. 29. By means of the adjusting gear, the sides of one pulley can be moved apart whilst the sides of the other are moved closer together. This causes the belt to run at a decreased diameter on one pulley, and at a correspondingly increased diameter on the other, so that the speed ratio between the two shafts is altered.

A different type in which we can get any desired ratio both forward and reverse is shown in Fig. 30. The driving wheel is a large disk, against the face of which is pressed a friction wheel which slides on splines on the driven shaft. The speed of the friction wheel is proportional to its distance *R* from the centre of the driving disk, and it will go forward or backward according as it is to the left or the right of the centre of the driving disk.

Modified Form

In both of these arrangements the drive is by friction only, and with the arrangement of Fig. 30 especially, where it is just the friction of a leather-faced wheel against a smooth disk, slipping will occur if the transmission of more than very small powers is attempted. A modified form of the V belt arrangement has pulleys with radial grooves which prevent slip occurring, but, in general, we find that it is difficult to combine the advantages of infinitely variable ratios with a positive drive.

CHAPTER 14

MECHANICS OF FLIGHT

FORCES ACTING UPON AN AEROPLANE. STREAMLINES AND LIFT. STREAMLINING AND DRAG. MEASUREMENT OF AERODYNAMIC FORCES. AEROFOIL CHARACTERISTICS. SPEED OF FLIGHT. POWER REQUIRED. CLIMBING. GLIDING. HORIZONTAL FLIGHT. TERMINAL VELOCITY DIVE. ACCELERATED FLIGHT. CONTROL. STABILITY. THEORY OF PROPULSION. AIRSCREW PROPULSION. JET PROPULSION.

AERIAL flight is daily becoming a more important factor in our lives. Only a few years ago we looked up out of curiosity if we heard an aeroplane, and very few of us had ever flown in one. Nowadays, in many districts, hardly an hour of the day or night elapses without some aircraft passing within hearing distance, and the day will soon come when the person who has not been up will be an exception, like the person who has never been to the cinema.

We are promised wonderful achievements in the near future, such as immense speed with luxurious comfort and perfect safety, huge air liners, and cheap flying for all. When the jet propelled planes conquer the stratosphere there may be day trips to New York.

It is only natural, then, that we should want to understand something about these achievements. Therefore, it is likely that many readers will turn to this chapter first. Those who do so will find most of it easy to understand, but they are advised to regard it merely as a preliminary reading, and to return to it after having read the rest of the book, when they will reap the reward of a deeper insight.

Basic Principles

This is the last chapter because the basic principles involved in flight are all discussed in the previous chapters, and the problems dealt with will be better comprehended by those with some of the knowledge given earlier. This is not an excuse for complication. In fact, we shall actually find that the mechanics of flight is a simple subject. Take, for example, Fig. 28, which deals with the forces acting during a terminal velocity dive. This diagram is very simple and, with the accompanying text, will give an easily understood explana-

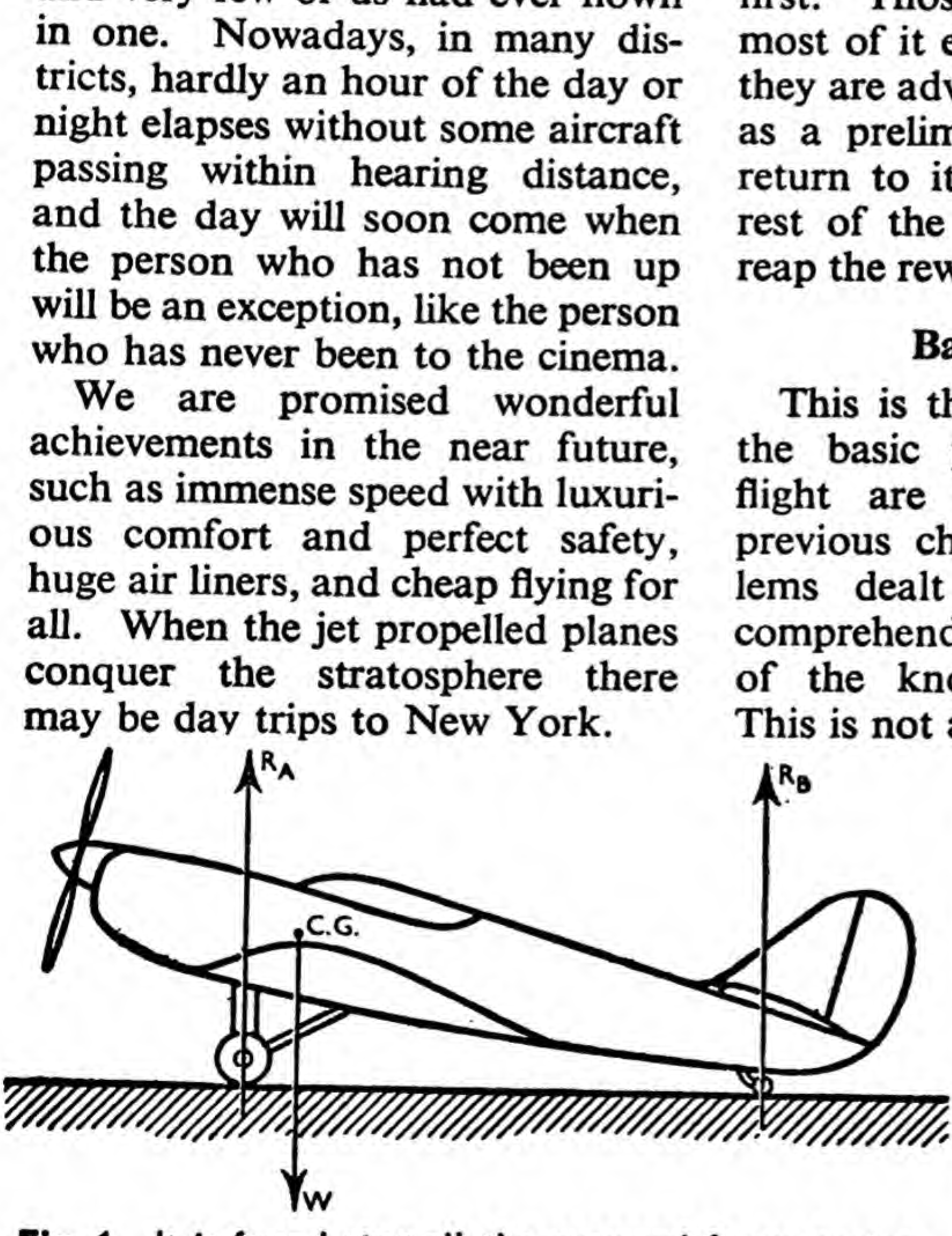


Fig. 1. It is found that all the external forces acting upon an aeroplane at rest on the ground are vertical so that $R_B + R_A = W$.

tion of this manœuvre, about which many of us have heard so much.

It has become fashionable to refer to aeroplanes as *aircraft*. The word aircraft applies to all vehicles which are supported in the air, such as balloons, airships, aeroplanes and helicopters ; but an *aeroplane* is the particular class of air vehicle which is supported by means of aerodynamic forces produced by its wings. We shall deal chiefly with this type of air vehicle, or aircraft, and the word aeroplane will be used whenever the principles discussed apply only to aircraft of the aeroplane type.

Forces Acting on Aeroplane

The solution to most problems in mechanics is obtained by finding equations connecting the forces involved. Therefore, it will be helpful to commence by considering the general relationships between these forces. We should bear in mind that there is no such thing as an unopposed force. It does not matter whether the body under consideration is at rest, or moving with constant velocity, or accelerating ; there are reactions to all the forces acting upon it. The necessary equations are found by applying three conditions to the diagram of forces. The first two are that the algebraic sum of all the forces acting in each of two perpendicular directions must be zero, and the third is that the algebraic sum of the moments about any point must be zero. It is not always necessary to apply all these conditions of equilibrium, as they are called.

Let us now consider the equilibrium of an aeroplane on the ground, as shown in Fig. 1. Its weight W may be regarded as acting vertically downward through the centre of gravity, C.G., and this is balanced by the reactions R_A and R_B, acting vertically upward through the wheel hub and tail skid respectively, so that :—

$$R_A + R_B = W.$$

Now imagine the ground taken away. Something equal to W must be found to replace R_A and R_B. Let us call it the lift L. This quest puzzled man for hundreds of years. The first solution was to use the principle of buoyancy that is, to attach to the structure a large bag, filled with a gas lighter than air, so that the difference between the weight of the air displaced by the bag and that of the gas which it contained, was equal to the weight of the whole structure. This idea gave us the balloon (Fig. 2).

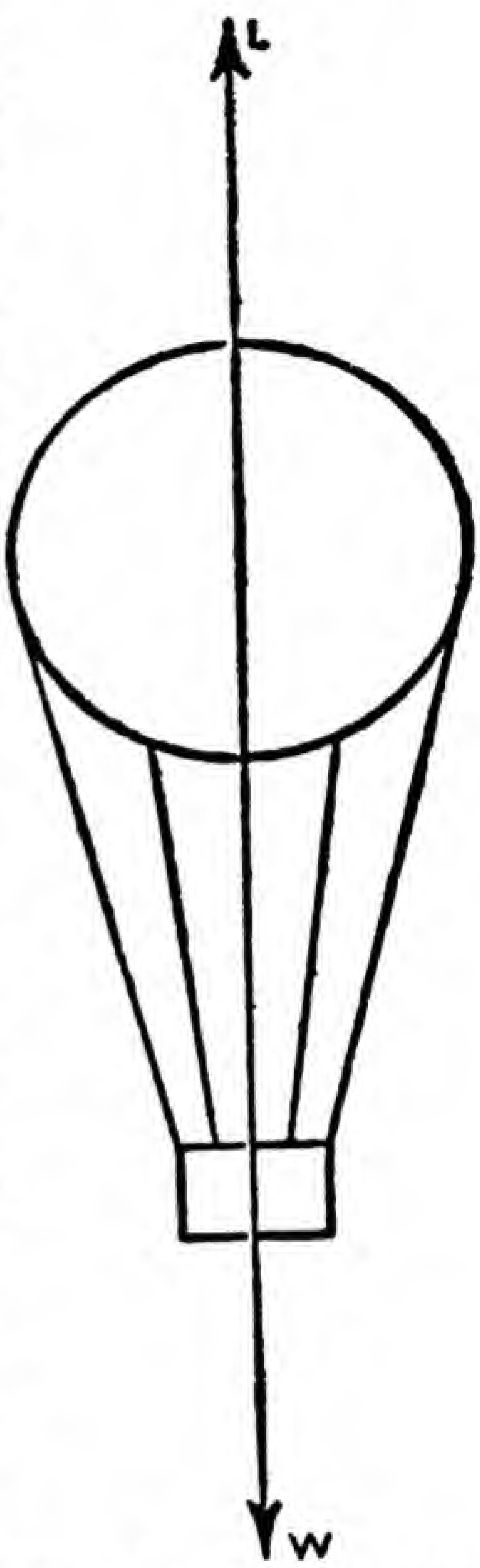
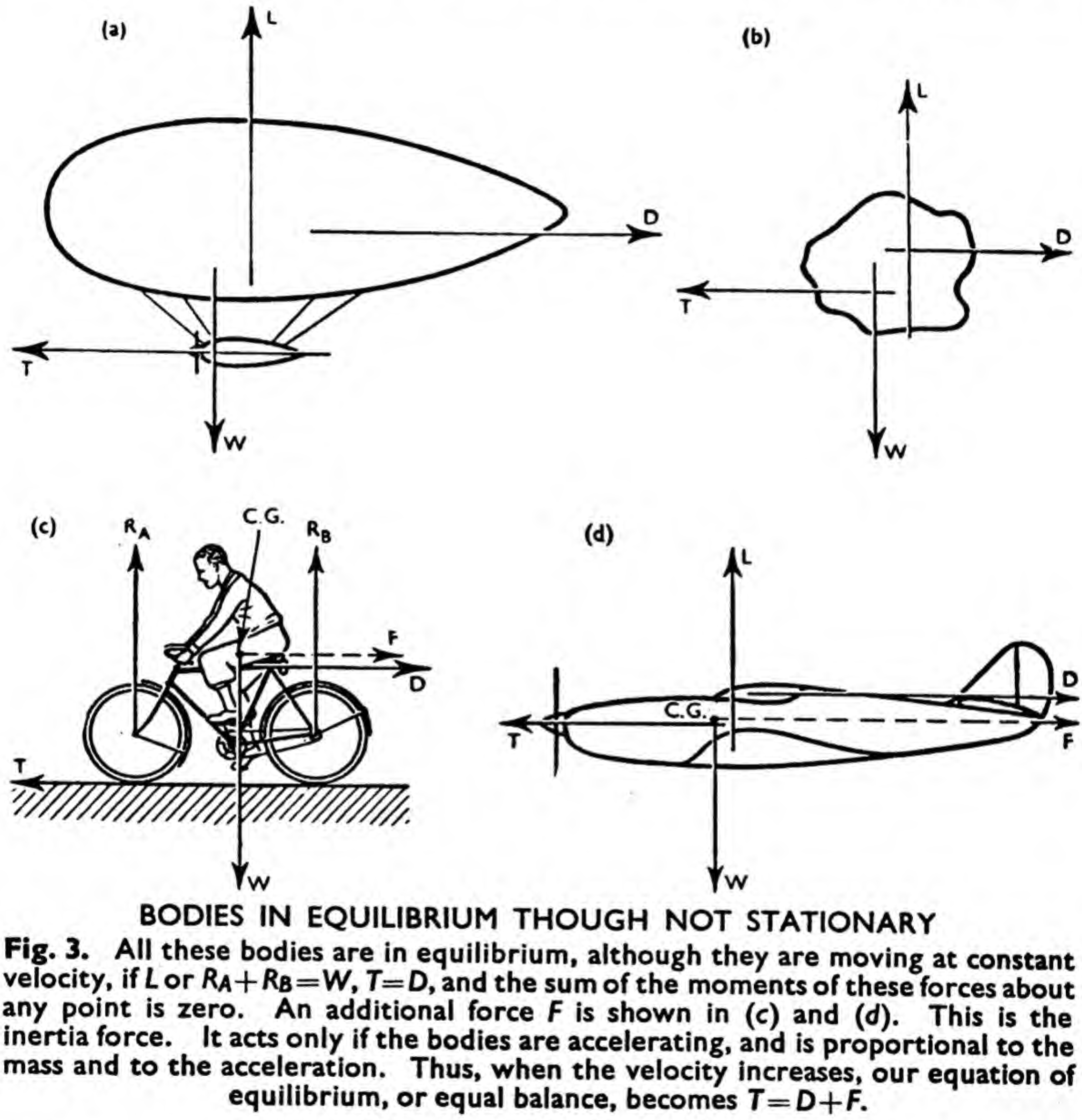


Fig. 2. Considering the external forces acting upon a balloon, the lift L is obtained from the difference between the weight of the air displaced by the envelope and that of the gas which it contains. When $L=W$ the balloon becomes stationary with regard to the air around it.

But man did not want only to rise above the earth's surface into the air ; he wanted to fly through it. So he attached an engine driving an airscrew, and this involved him in two problems.

Firstly, the bag, or envelope, had to be much bigger to support the large weight of the engine, and secondly, the thrust T (Fig. 3(a)), causing the forward motion, was opposed by the drag D of the air passing over the envelope and



BODIES IN EQUILIBRIUM THOUGH NOT STATIONARY

Fig. 3. All these bodies are in equilibrium, although they are moving at constant velocity, if L or $R_A+R_B=W$, $T=D$, and the sum of the moments of these forces about any point is zero. An additional force F is shown in (c) and (d). This is the inertia force. It acts only if the bodies are accelerating, and is proportional to the mass and to the acceleration. Thus, when the velocity increases, our equation of equilibrium, or equal balance, becomes $T=D+F$.

car. He minimized this drag by special shaping of the envelope, viz., streamlining, but it was still excessive.

Note that none of these forces except the thrust are really single forces. The weight, for example, is made up of the separate weights of all the parts, and we have seen in Chapter 2 that all these separate forces may be replaced by one equivalent force acting through a point known as the centre of gravity. Similarly, the drag of each portion may be replaced by a single force acting through the centre of drag, and the lift by one force acting through the centre of lift. The thrust, of course, acts through the thrust line of the engine.

Now Fig. 3(a) may be replaced by Fig. 3(b), which illustrates any body acted upon by two pairs of parallel unlike forces. Applying the conditions of equilibrium, we

obtain :—

$$L = W \ldots\ldots\ldots\ldots \text{(i)}$$
$$T = D \ldots\ldots\ldots\ldots \text{(ii)}$$

and (iii), the turning effect of L and W, viz., the moment, is equal and opposite to that of T and D.

Here is a new state of affairs. A body in equilibrium but not stationary. The following example should make this clearer.

Net Thrust

A man is cycling along a level road at a constant speed. His effort to rotate the rear wheel causes the tyre to tend to push the road backward. The road resists this, and causes an equal and opposite forward push on the bicycle, marked T in Fig. 3(c). Air resistances, marked D, act so as to oppose motion. Road resistance and internal friction do not enter into the problem, because we are considering the net thrust, T, just as, previously, we considered the net thrust of the engine. The weight W, of the man and cycle is opposed by the ground reactions R_A and R_B, which replace the lift L, of Figs. 3(a) and 3(b).

Once again :—

$$R_A + R_B = W \ldots\ldots \text{(i)}$$
$$T = D \ldots\ldots \text{(ii)}$$

and, (iii) the turning effect of R_A, R_B, and W is equal and opposite to that of T and D.

The cyclist is in equilibrium travelling at a constant speed. Note that it is not necessary, as is often thought, for T to be greater than D. Whether he will be in equilibrium if he accelerates depends upon our definition of equilibrium. The usually accepted use of the term applies only to bodies at rest, but we have already extended it.

If the cyclist increases the thrust T, this will be opposed by a force due to the inertia of the mass of himself and the bicycle (Chapter 4). In other words, there will be an inertia force F, shown dotted, acting backward through the centre of gravity, so that $T = F + D$. This will increase the clockwise couple, and equilibrium will be retained by an increase in the anti-clockwise couple due to R_B being increased, and R_A decreased. This can be confirmed by considering the trick of lifting the front wheel by a sudden high pressure on the pedal, when R_A becomes zero.

Newton's Third Law, stating that to every action there is an equal and opposite reaction, really means that everything is always in equilibrium, for the word equilibrium is derived from two Latin words which mean equal balance. At any rate, we can always apply the conditions of equilibrium, as we have done above, to anything in whatever state of rest or motion it may be, provided that we take into account every force which acts upon it. In other words, all the forces acting upon a body always balance each other.

Horizontal Acceleration

These ideas can now be applied to the aeroplane in Fig. 3(d). It is moving horizontally forward with a constant velocity. If T is increased, and horizontal flight maintained, the aeroplane will accelerate horizontally. This will cause an inertia force F to act backward through the C.G., giving $T = D + F$. The increase in velocity will cause D to increase, and, therefore, F to decrease, until F eventually becomes zero. Flight will then be continued at a higher constant velocity with the increased

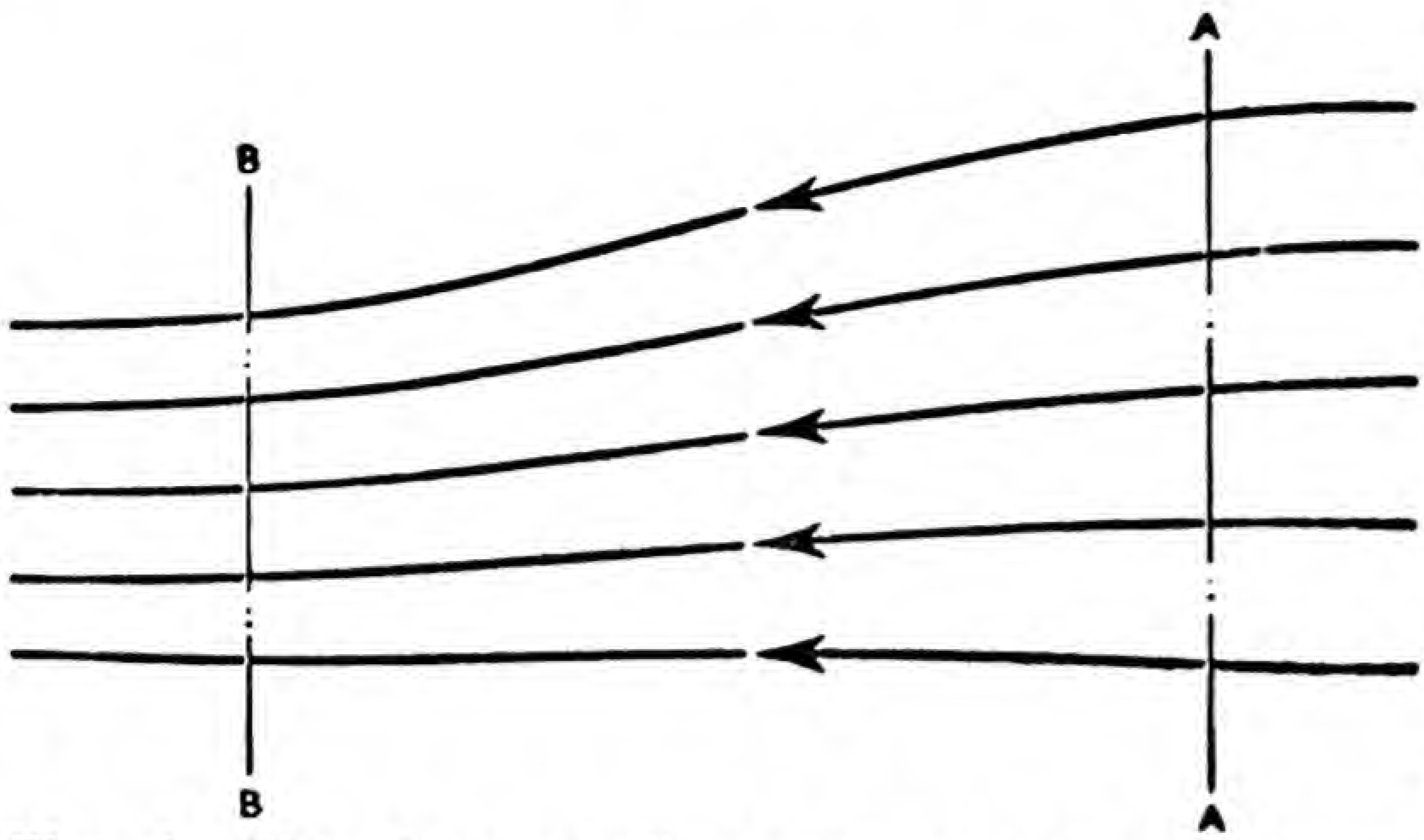


Fig. 4. This diagram illustrates a streamline flow converging from *AA* to *BB*. In accordance with Bernoulli's theorem which has been previously stated, the increased velocity at *BB* is offset by diminished pressure. Thus, crowding together of the streamlines means a reduction in the pressure.

thrust equal to the increased drag.

Similarly, if L is increased, there will be an inertia force acting vertically downward, which will make the aeroplane and its occupants feel heavier. The aeroplane is accelerating, but not in the direction of motion. This will be discussed in more detail later.

To sum up, the aeroplane is always in equilibrium under the combined action of three kinds of forces, viz., (a) gravity forces, (b) aerodynamic forces or forces produced by moving air, and (c) inertia forces. The mechanical forces developed by the engine are converted into aerodynamic forces by the airscrew.

Streamlining

In transferring our attention from Fig. 3(a) to Fig. 3(d) we have replaced the gas envelope as a means of giving lift by a wing. Now a wing is heavier than air, and it can only produce lift if it is moved through the air, but since this is just what we want to do, it is not a serious drawback. We have also eliminated the enormous drag of the large envelope.

Now, why does a wing give lift under these circumstances?

Reconsider for a moment the principle of Bernoulli discussed in Chapter 11. It tells us that, if the velocity of a fluid increases, its pressure decreases, provided that no energy has been added to it. In other words, what it gains in energy due to increased movement (kinetic energy), it loses in ability to exert a pressure (static energy).

This idea is difficult to appreciate because it is contrary to expectations, so a few examples will be given before applying the principle to an aeroplane wing. In the illustrations of these examples, the direction of motion of the particles of a fluid, whether it be a liquid or a gas, will be represented by lines called streamlines.

When these lines get closer

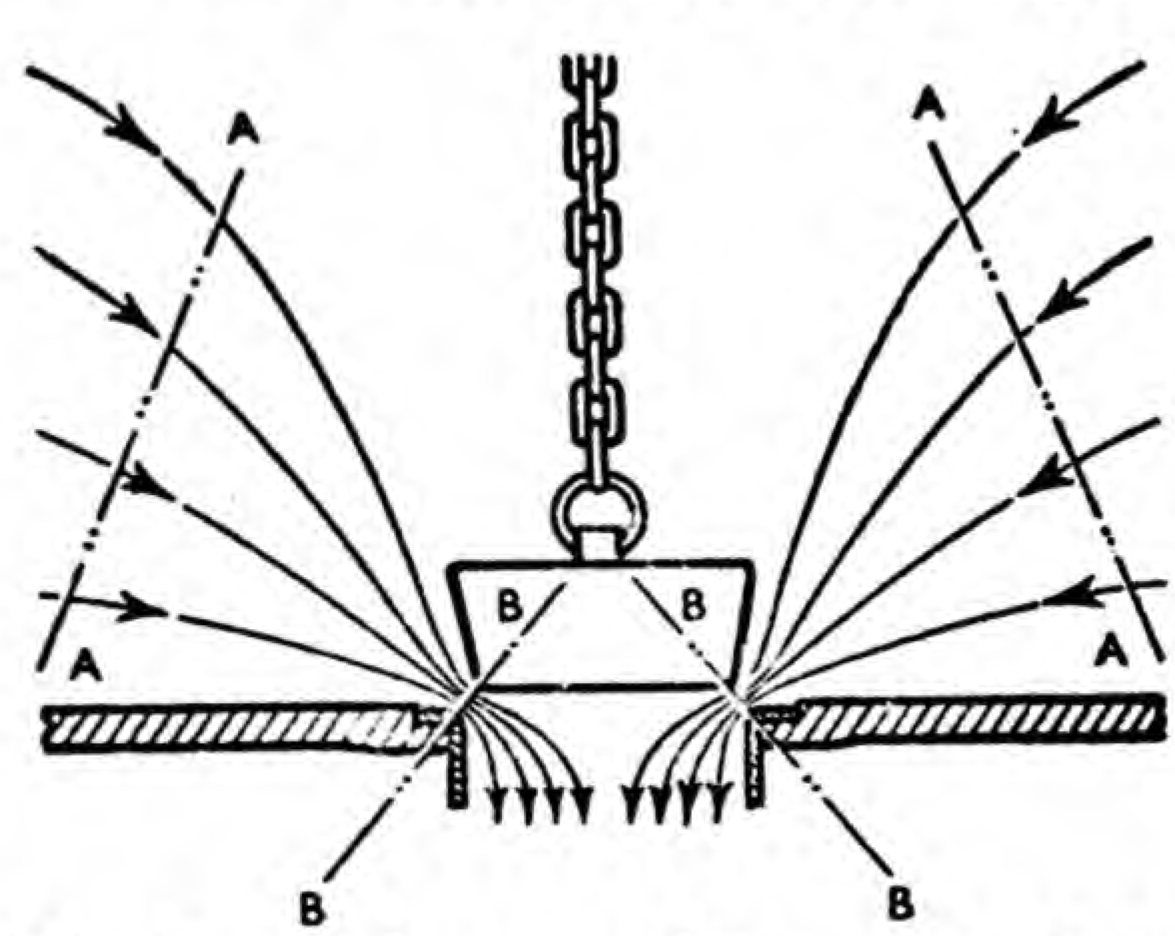


Fig. 5. In the case of flow past a bath plug, the streamlines close up between *AA* and *BB*, indicating increased velocity, and, therefore, decreased pressure. The effect of this is to cause the plug to return to its hole with the familiar *plomp*.

together, the fluid which in Fig. 4 is at one instant flowing through a cross-section of width *AA*, is, an instant later, flowing through a smaller cross-section *BB*. The same volume of fluid passes through each cross-section per second, and, therefore, its velocity increases. If there is an increase in velocity between *AA* and *BB*, there must be a force acting through the fluid from *AA* to *BB* to cause this acceleration (Chapter 4). A force, like this, can only act through a fluid from a region of higher to one of lower pressure. Hence, the pressure at *AA* is higher than that at *BB*.

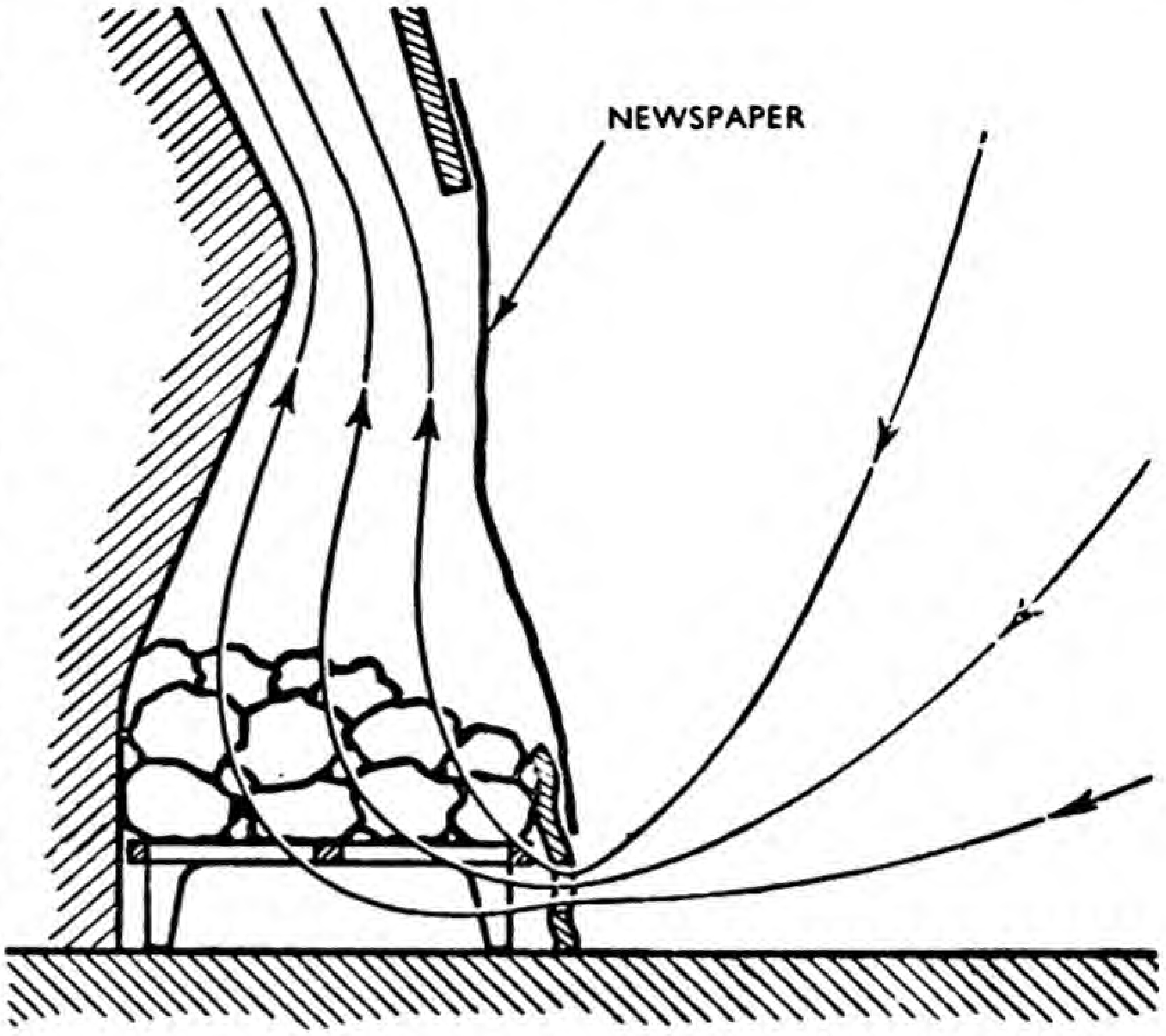


Fig. 6. Domestic fire with a newspaper placed in front to draw it up. As will be seen, the streamlines are closer together above the fire than in the room. Therefore, the pressure is less above the fire, and the newspaper tends to be drawn in.

One practical example of this occurs when the water in a bath is emptied. When the plug is pulled up by the chain it often returns to its hole with a *plomp*. Some force is acting to oppose the pull on the chain. It is not a constant force, but one which increases as the plug gets nearer to the hole. Hence its uncontrollability. If we consult the streamline diagram in Fig. 5, we shall see why.

The convergence of the streamlines causes a decrease in pressure under the plug, and as soon as the plug starts to move downward under the influence of this decreased pressure, or suction, as it would be called in everyday life, the cross-section *BB* decreases in width, and the convergence is intensified, causing a still greater decrease in pressure. The acceleration, and consequent decrease in pressure of the fluid, is further intensified by the fact that the flow is radial, as will be seen by considering a plan view of the bath plug and streamlines.

Another practical example may be quoted. Sometimes, to draw the fire up, a newspaper is held in front of it. As a result, the velocity of the gases passing up the chimney increases, and the paper may be drawn into the fire because of the decreased pressure. The streamline diagram will explain this effect (Fig. 6).

Application to Wing

Now let us apply the principle to an aeroplane wing. Such a surface, which is designed to obtain a reaction from moving air, is called

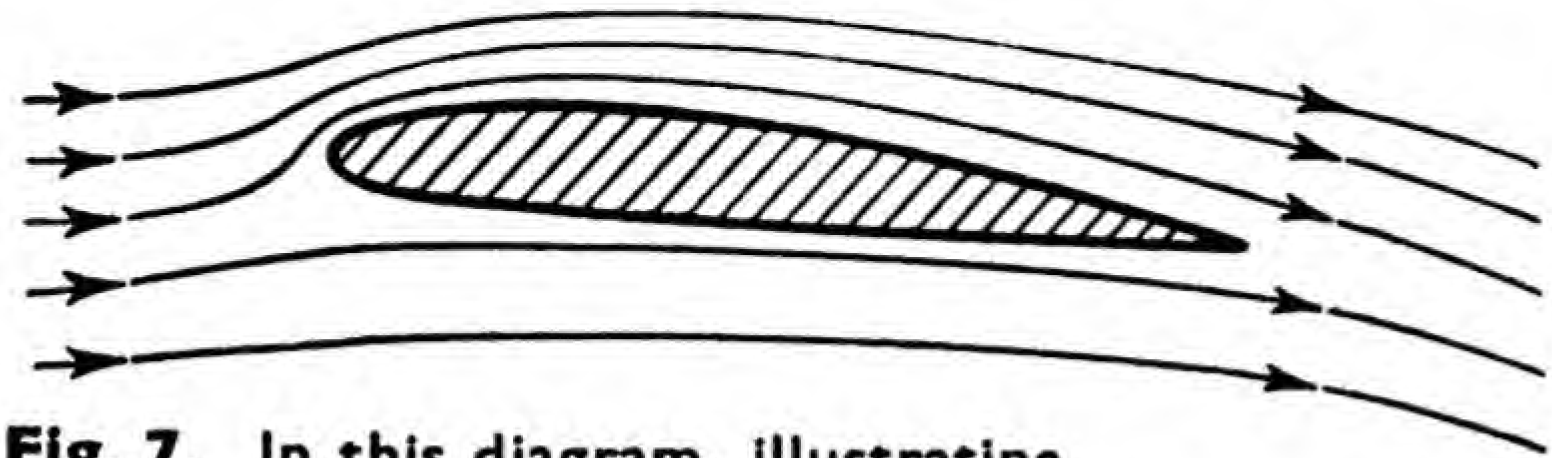


Fig. 7. In this diagram, illustrating the airflow past an aerofoil, it will be noticed that the streamlines converge on the top, and diverge on the bottom. Therefore, we will find the air pressure is reduced above and increased below, giving lift.

an aerofoil. Fig. 7 shows a cross-section of an aerofoil surface in an airflow.

It is now clear from the streamline pattern that the pressure on the upper surface will be reduced, and that on the lower surface will be increased. Hence, lift is obtained.

Effect on Downwash

There are two other good reasons why lift is obtained. (1) Notice the downward inclination of the streamlines behind the aerofoil section, known as downwash. It is obvious that nothing can be pushed downward without there being an equal and opposite upward reaction ; so the more downwash there is, the more lift is obtained. (2) Now notice the slight upward curvature in front of the leading edge, known as upwash. The combined effect of upwash and downwash produces a curved streamline pattern. Since the air has mass, this gives an inertia force acting outward from the centre of curvature, like the force which causes mud to fly off a bicycle tyre. It is this force which gives us lift.

Theories have been advanced explaining why this streamline pattern occurs, and, from them, aerofoil shapes have been calculated. However, most of the aerofoil shapes in use to-day have been evolved by trial and error, and they are quite as good as the mathematical ones.

A glance at Fig. 7, bearing in mind the last reason for obtaining lift, will suggest that the resultant reaction will be inclined backward, and may thus be resolved into horizontal and vertical components. It is the vertical component of this reaction which is called lift, whilst the horizontal component is called drag. This reaction and its components are shown in Fig. 8. It would be more correct to say that lift is the component of the resultant reaction perpendicular to the airflow, and drag is that parallel to the airflow, because, quite often, the airflow is not horizontal. An extreme case occurs at the top of a loop, when the lift acts vertically downward (Fig. 33).

Reduction of Drag

An airflow will, of course, cause drag in passing over any body, irrespective of whether lift is produced. Bodies of the cross-sectional shape shown in Fig. 9, are found to give least drag. The drag

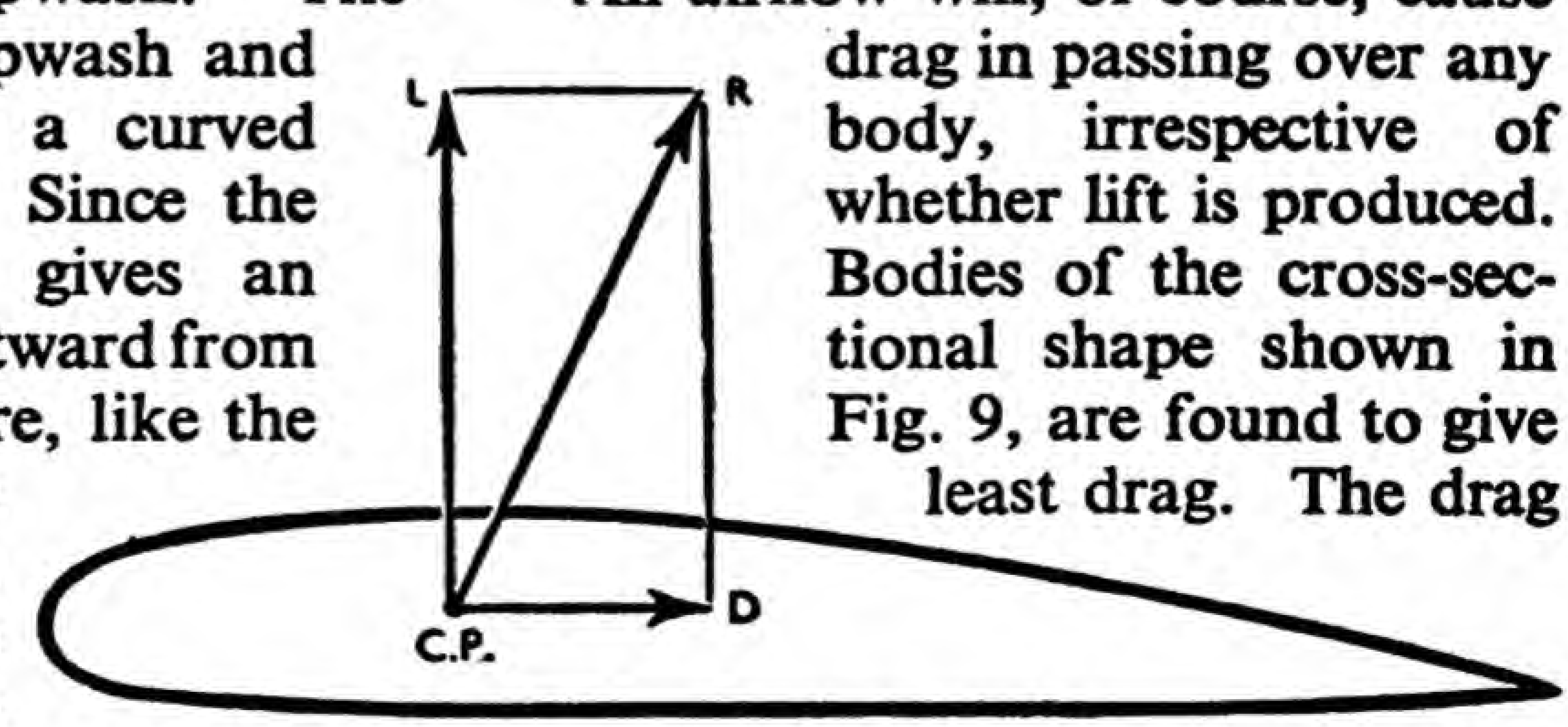


Fig. 8. Airflow past aerofoil causes resultant force to act upon it as shown by inclined line *R*, which acts through the centre of pressure, C.P. Forces *L* and *D* represent the vertical and horizontal components of this force, and are called the lift and drag respectively.

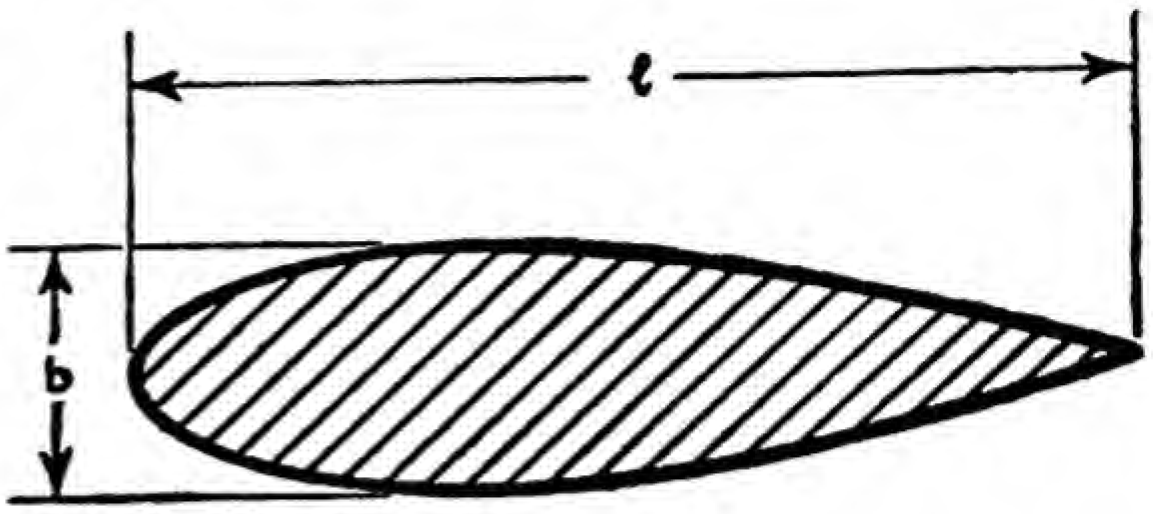


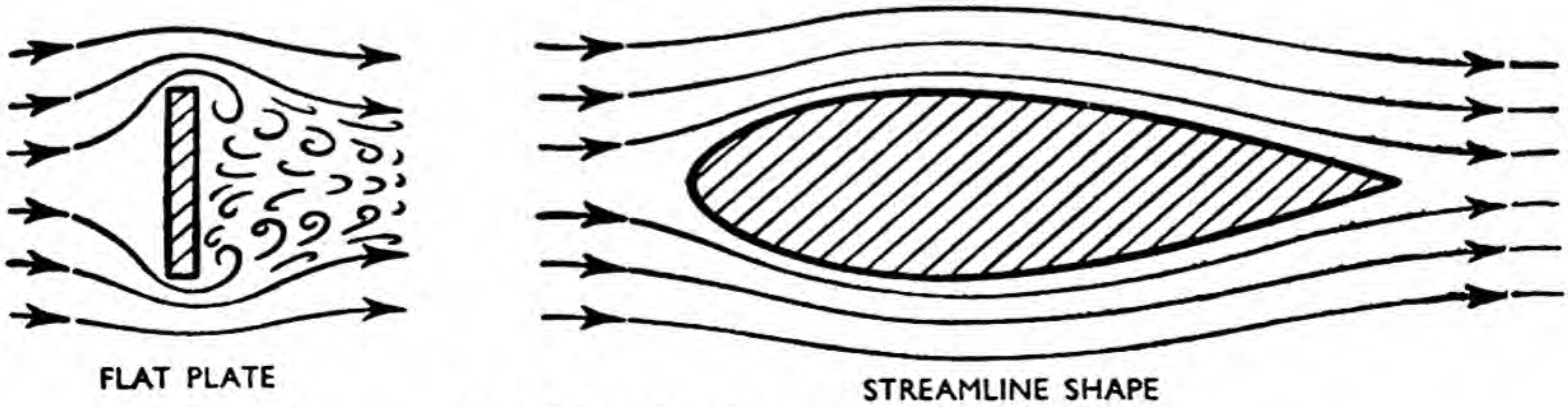
Fig. 9. Here is a cross-section through a *streamline* shape, such as an engine nacelle or a strut. To obtain the least drag the widest part should be about one-third of the way back from the leading edge (the blunt end), and the fineness ratio *l/b* should be nearly 4. Compare this shape with that of a fish or a bird.

of this streamline shape is only about one-twentieth of that of a flat plate of the same breadth, because the air is able to pass over it with practically no eddy formation, or turbulence. The streamlines in each case are shown in Fig. 10.

All parts of an aircraft are made to conform as far as possible to this shape, but there are obvious reasons for departure from it. For instance, the complete streamlining of a fuselage may add more weight than the decrease in drag would warrant. Drag, however, increases with speed, so that streamlining is more worth while for high-speed aircraft. In fact, in modern high-speed machines, the elimination of drag is more important than the elimination of weight. Retractable under-carriages, for example, are the rule today, although only a few years ago their weight was considered excessive.

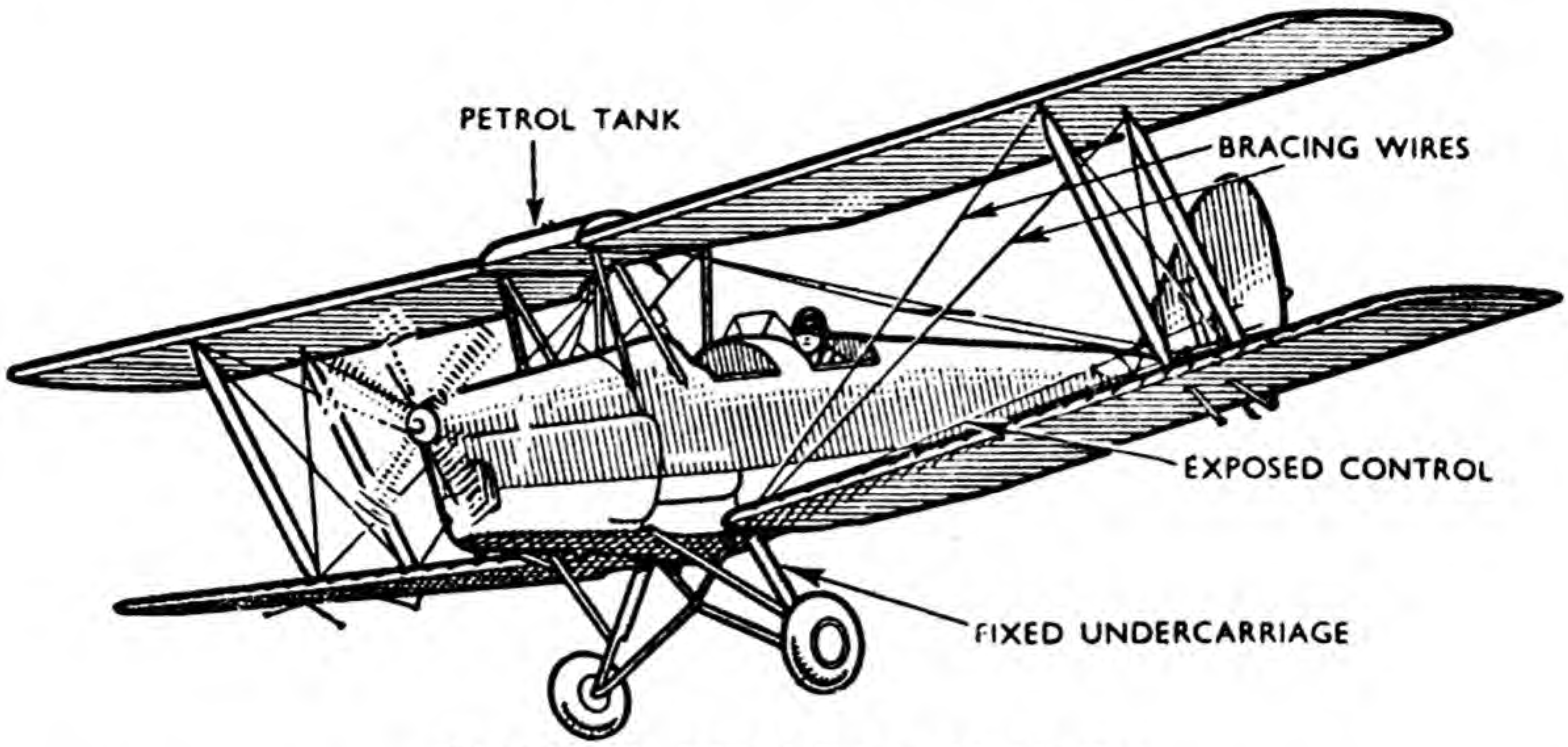
Fig. 11 shows the famous Tiger Moth. It is not fully streamlined, but, if it were, the extra weight would entail a higher flying speed, and that would mean a larger engine, which would in turn, mean more weight, and higher speed still, unless we increased the wing area, but, that too, would mean extra weight. We shall know much more about all this after we have read on. Some details of the Tiger Moth are given here, but they, too, will have much more meaning if we reread them after having read this chapter :

Wing span ..	29 ft. 4 in.
Length ..	23 ft. 11 in.
Wing chord ..	4 ft. 4 in.
Wing area ..	239 sq. ft.
Weight loaded..	1,825 lb.
Wing loading ..	7·64 lb./ft.^{2}.
Maximum speed	109 m.p.h. at 1,000 ft.
Cruising speed..	93 m.p.h. at 1,000 ft.
Landing speed..	46·5 m.p.h.
Maximum rate of climb ..	673 ft. per min.



REDUCING AIR RESISTANCE

Fig. 10. When an airflow passes a flat plate it leaves a turbulent wake, but streamlining prevents this, so the air resistance is lowered. The resistance of the streamline shape is only one-twentieth of that of the flat plate.



FEATURES OF THE TIGER MOTH

Fig. 11. This is an aeroplane built to fly at a low speed, as it is an elementary trainer, so we get exposed control and bracing wires, fixed undercarriage and protruding petrol tank in the centre of the upper wing. These, and other non-streamlined features, do not increase the drag much, especially at the low speed of the machine.

Maximum horse-power	.. 130
Cruising horse-power	.. 120

A careful study of this chapter will enable us to calculate the aspect ratio and check the wing loading. It will also be possible to estimate the maximum lift coefficient, and that at cruising speed and maximum speed. We might then try the drag coefficient, and find the maximum L/D ratio, the gliding angle and the h.p, available for climbing. An explanation will be found on p. 427.

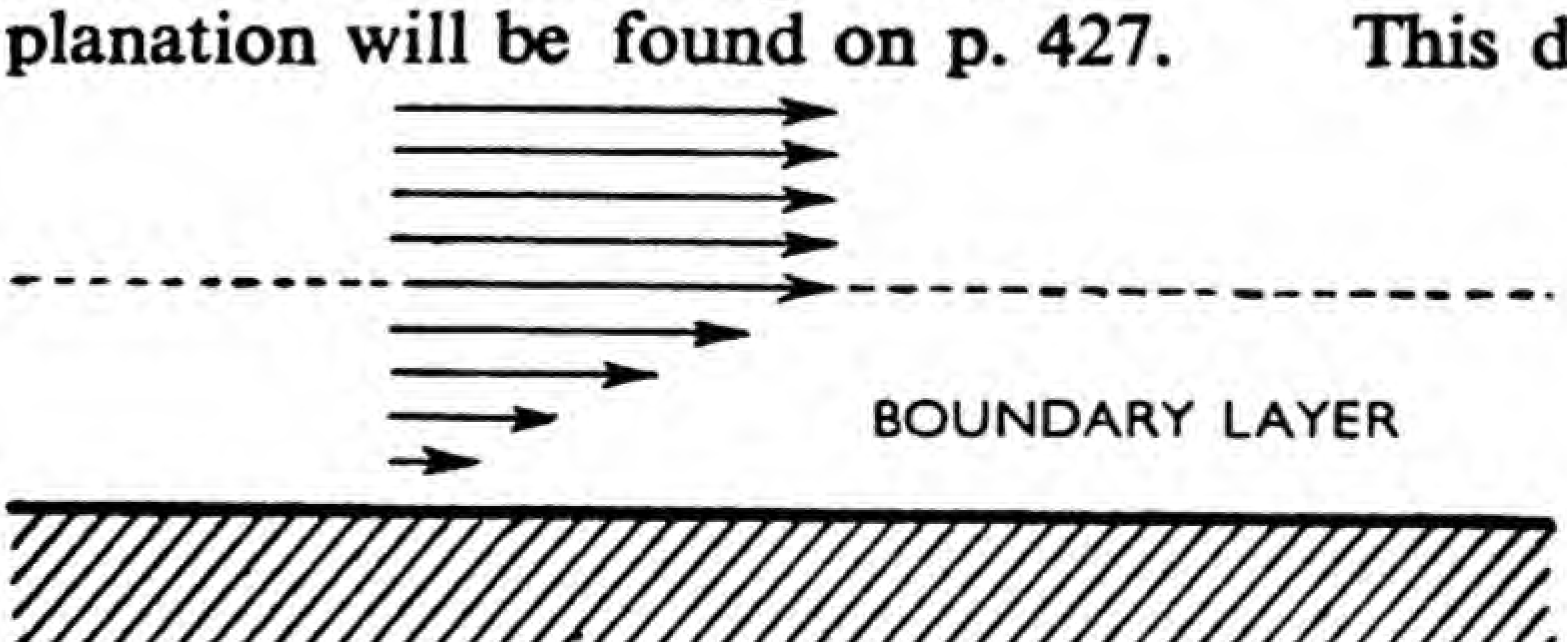


Fig. 12. Much magnified view, showing how velocity of air increases from zero at the surface to that of the airflow at a short distance from it. Average thickness is only about $\frac{1}{10}$th inch. Arrows represent average velocity of particles of air in direction of airflow.

It will be noted that the cruising speed of the Tiger Moth is only 93 m.p.h., weight being kept down at the expense of drag. Compare this with the Dakota (Fig. 42) whose cruising speed is 207 m.p.h. Similarly, the streamlining of a motor car often results in merely making it look more attractive, but the streamlining of express trains designed to work at more than 70 m.p.h. gives a definite fuel saving due to the decreased drag.

Profile Drag

This drag is called profile drag, and is caused by (a) the shape, or form, of the body, and (b) its surface or skin.

(a) Form drag is due to the energy lost in the formation of eddies. The direct object of streamlining is to eliminate this cause of drag, by attempting to obtain a perfect streamline flow.

(b) Skin friction is due to the frictional resistance of the air passing over the surface of the body. It can be reduced by making the surface smooth and eliminating protuberances such as rivet heads. But there is a limit to the results obtainable by continuing to improve the quality of the surface, because the air adjacent to the surface is always at rest, however fast the airflow.

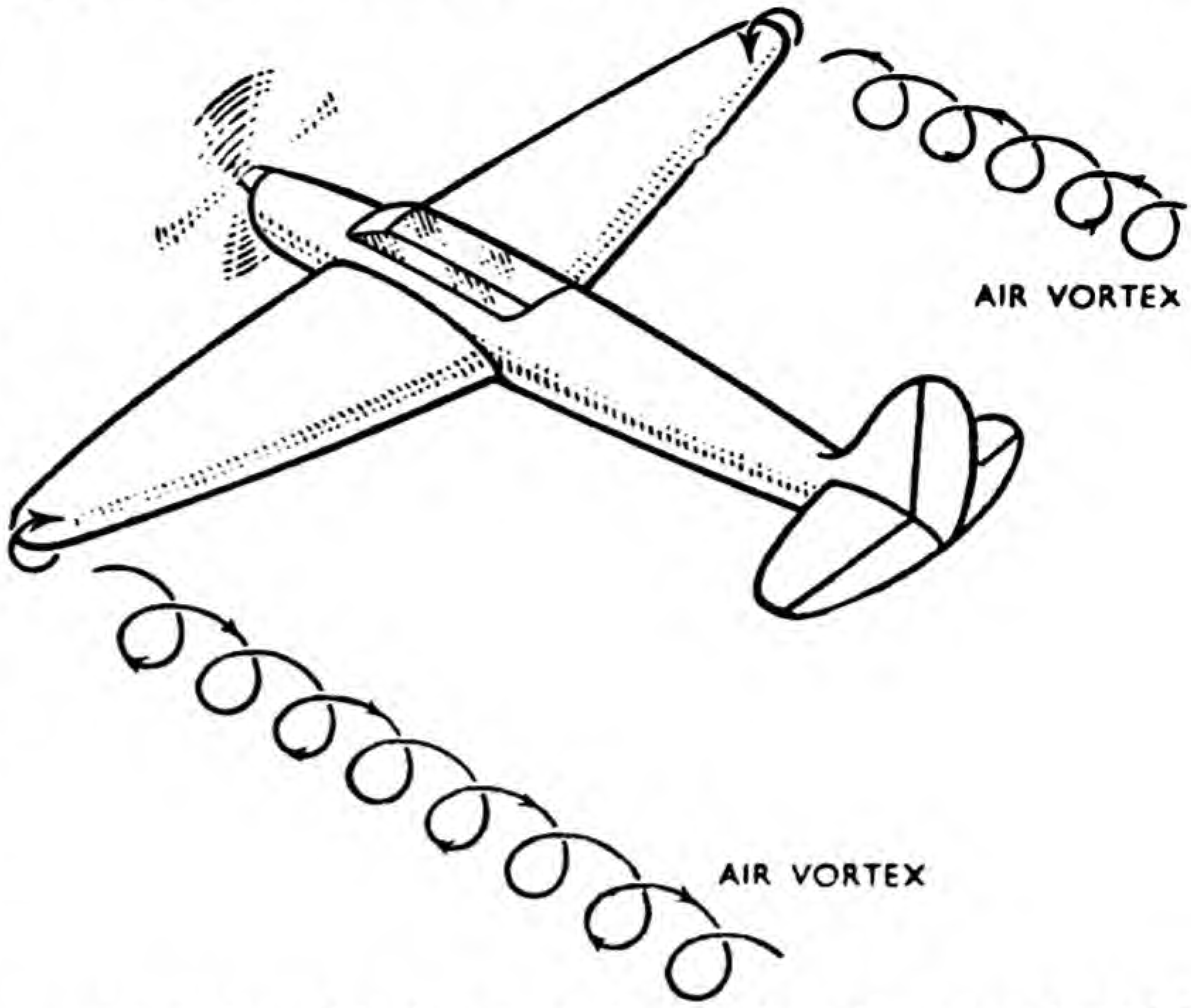


Fig. 13. Spiral movement of the air is produced by air spilling over the wing tips from the bottom to the top, due to the difference in pressure between the lower and upper surfaces of the wing.

The impossibility of cleaning a dusty, but highly polished, car by speeding affords a practical proof of this. Still more dust will settle upon the surface.

Introducing Boundary Layer

The very thin layer of air in which the velocity is gradually increasing from zero to that of the airstream is called the boundary layer. In Fig. 12 the velocity vectors show the average velocity of the particles of air at intervals throughout this layer, which is, of course, magnified in thickness. The relative motion of the particles due to this increasing velocity causes friction, and may cause turbulence. Only if air were an inviscid fluid should we have frictionless motion in the boundary layer. Thus it is the viscosity of air which causes friction in this layer, and hence drag is caused.

The boundary layer affects the streamline flow over an aerofoil, and attempts are being made to control it by suitably placed jets. These will be referred to again when dealing with the theory of propulsion.

Now, the production of lift itself causes drag, apart from that discussed above. This type of drag is called induced drag. Its occurrence is due to the fact that, whenever lift is produced, the pressure on the under surface of the aerofoil is greater than that on the upper surface, and consequently at the tips of the wings air spills over from the bottom to the top, as shown by the heavy arrows in Fig. 13. This sets up rotation of the airflow, causing the trailing vortices shown, which continue for quite a long way behind the aeroplane before their energy is finally dissipated. Thus, the induced drag is caused by the energy used up in rotating the air.

Obviously, more energy will be wasted in this way if the pressure difference between the surfaces is

greater. Therefore, it can be seen that the induced drag increases with the lift. It can be shown that it varies as the square of the lift, other things being equal.

Now, how can this be reduced? Only by reducing the wing tips. That means making the wings longer and narrower, making the ratio of wing span to wing chord, called the aspect ratio (A.R.), as great as possible. If the wing is tapered, the chord is, of course, variable, so we take the ratio of the square of the span to the wing area,

$$\text{viz., A.R.} = \frac{s^2}{S},$$

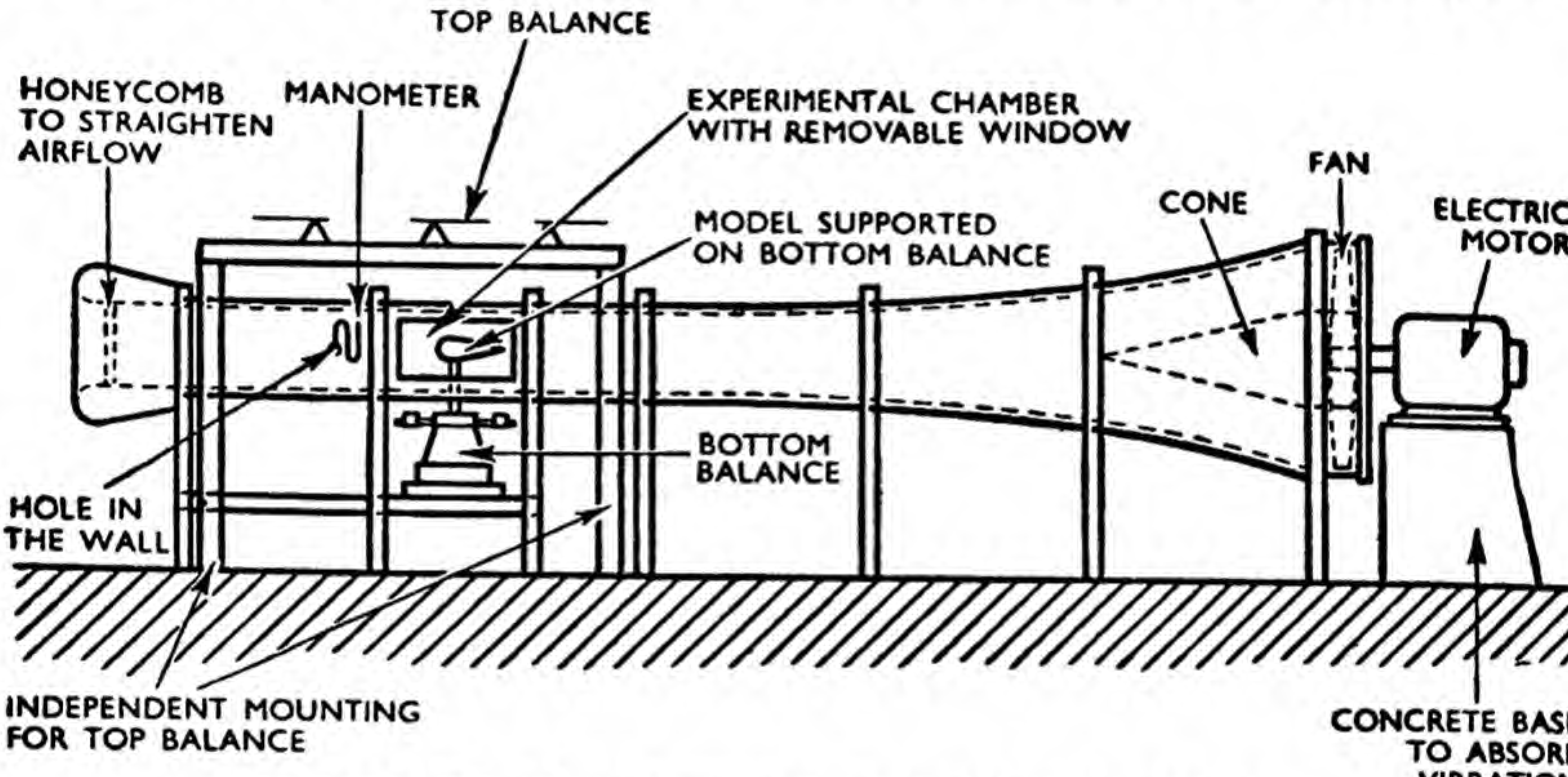
where s is wing span and S is wing area.

If the wing is rectangular this reduces to $\frac{s}{C}$, where C is the chord length. 'As great as possible' implies that for a given wing area, the span increases and the chord decreases until the extra structural weight involved begins to cancel out the aerodynamic advantage obtained. It is the old drag versus weight compromise again.

Measuring Aerodynamic Forces

The aerodynamic forces which have been discussed are difficult to measure during flight. Besides, it is safer to check our calculations by measurement before flight is attempted, so things are reversed, and a model of the part under consideration, such as an aerofoil, or a streamline shape, is suspended in a current of air. Sometimes, a complete, full-size aeroplane is suspended in the air current, but, with the increasing size of modern aircraft, the necessary apparatus becomes too cumbersome and expensive.

The indispensable apparatus is a wind tunnel. One type is shown in Fig. 14, the essential features being (a) a means of drawing a current of air past the model at various speeds, (b) a means of measuring



CLOSED-JET NON-RETURN FLOW WIND TUNNEL

Fig. 14. Streamline body is shown mounted on the bottom balance. Its drag is found by adjusting the positions of the movable weights shown on the horizontal arm, and making a simple calculation based on their turning moment. The three separate balances which comprise the top balance are only shown diagrammatically. They measure drag, lift and turning moment respectively.

the speed of the air, and (c) a means of measuring the forces acting upon the model due to this airflow.

It will be seen from the figure that a large fan, rotated by an electric motor, draws air through a duct containing a glass-sided portion of constant cross-section in which the model is mounted. Balances are provided at the top and bottom of this central portion, or experimental chamber, to measure both the lift and drag forces, and the moments produced by them. Which particular balance is used depends upon the method of suspension, but the turning moment and hence the position of the centre of pressure, can only be found by using the top balance. More accurate work may be done on the top balance, the models of aerofoils, etc., being suspended upside down.

Unfortunately, the results obtained are not quite to scale, because the streamline pattern for similar shapes varies with the air-speed and size. However, the conditions under which the streamline patterns are the same have been studied, and we are able to make suitable corrections. A further trouble arises because the tunnel walls exercise a constraint upon the airflow, which affects the streamline pattern, and, therefore, the forces produced. Again, corrections for this can be made.

Using Bernoulli's Principle

Now, the speed of air cannot be measured directly, like that of a motor car, so we use Bernoulli's principle. This tells us that the pressure of the moving air inside the tunnel will be less than that outside, and enables us to use this pressure difference to find the velocity, because, if no energy is added to or taken away from the air between the mouth of the tunnel and the experimental chamber, the energy it gains due to its velocity will be lost in the drop in static pressure. No energy is added in this portion of the tunnel, because the fan, which imparts energy to the air, is at the other end, and very little energy is taken away because the tunnel is carefully designed to produce streamline flow.

Air Density

It has been learnt in Chapter 7, that the kinetic energy, viz., the velocity energy, of a weight of W lb. moving with a velocity of V ft. per sec, is $\frac{1}{2}\cdot\frac{W}{g}\cdot V^2$ ft.-lb.; but, when considering moving air it is impossible to deal with a definite weight of it, W lb., so we work in terms of density. Thus, if the air weighs w lb. per cu. ft. it can be said that its kinetic energy is $\frac{1}{2}\cdot\frac{w}{g}\cdot V^2$ ft.-lb. per cu. ft.

Now let us look at these units. They may be cancelled, just like figures, thus :— $\frac{\text{ft.-lb.}}{\text{ft.}^3} = \frac{\text{lb.}}{\text{ft}^2}$, viz., lb. per sq. ft., the units of pressure. Also, to avoid the continual use of g in formulæ, the Greek letter ρ is used for $\frac{w}{g}$, thus it may be stated that the kinetic energy per cu. ft. of the air in the tunnel is :—$\frac{1}{2}\rho V^2$ lb. per sq. ft.

The density of air under normal atmospheric conditions is 0·0765 lb. per cu. ft., so that its mass density ρ is $\frac{0{\cdot}0765}{32{\cdot}2} = 0{\cdot}00238$ units of mass per cu. ft. In this chapter ρ will be

used quite a lot and its numerical value will be ignored, but the above value will be useful for those who wish to make the calculations suggested later.

All that has to be done now is to measure the pressure difference between the inside and the outside of the tunnel in lb. per sq. ft.—and the pressure is obviously less inside because the streamlines converge,—and equate it to $\frac{1}{2}\rho V^2$, the gain in kinetic energy per cu. ft. Thus, if this pressure difference is represented by p, we have

$$p = \frac{1}{2}\rho V^2.$$

The units of both sides are lb. per sq. ft.

From this formula we can find the airspeed from the pressure drop, because, if both sides are divided by $\frac{\rho}{2}$, we get

$$\frac{2p}{\rho} = V^2.$$

Therefore, $V = \sqrt{\frac{2p}{\rho}}$.

This will give V in ft. per sec. if p is measured in lb. per sq. ft. If the reader wishes, the units may again be checked.

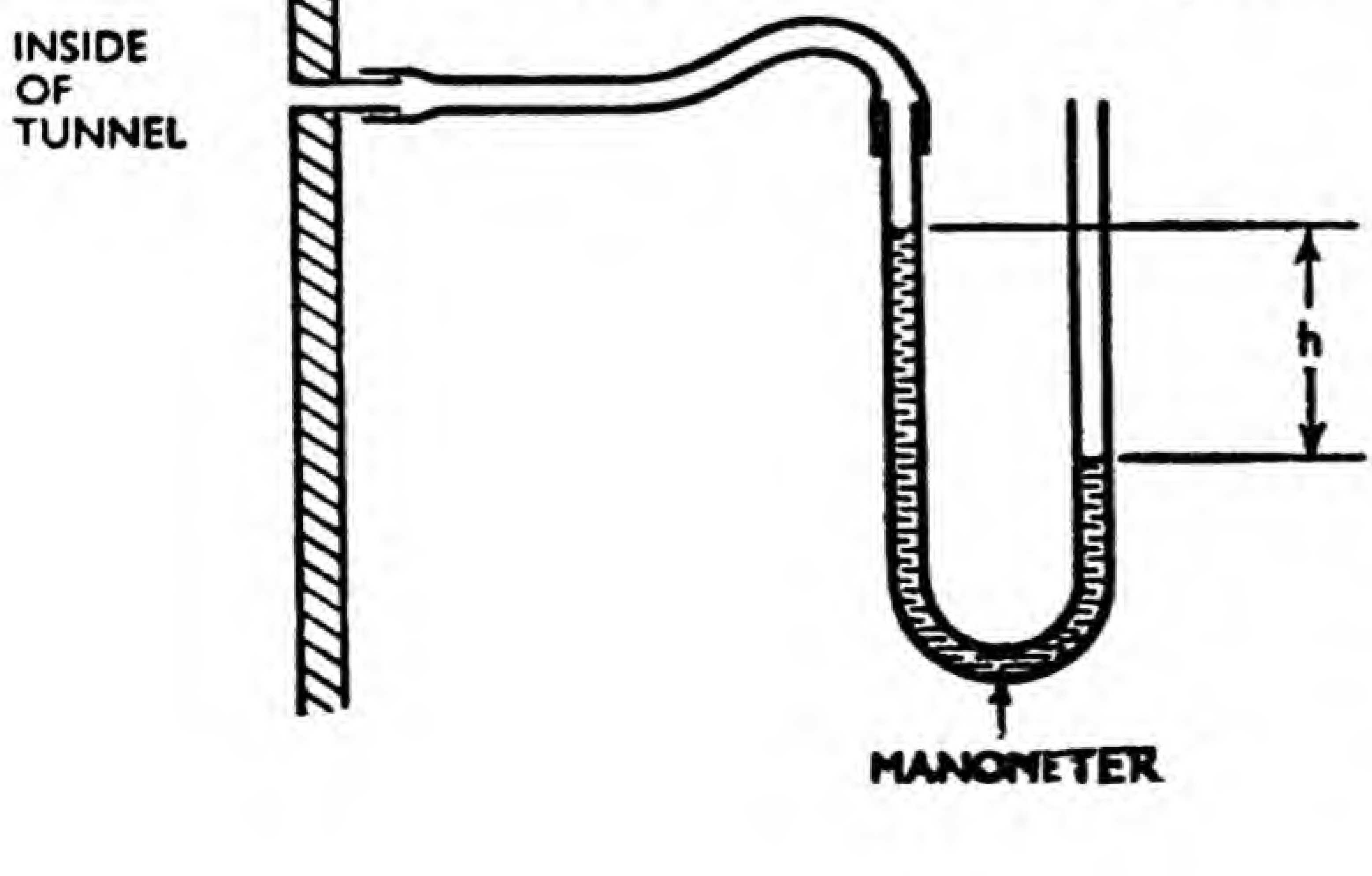
The actual measurement is made by using a U-tube, or manometer, filled with a liquid such as methylated spirit. One limb is connected to a hole in the wall of the tunnel, and the other limb is open to the atmosphere, as shown in Fig. 15. The pressure difference can be calculated from the vertical distance h between the liquid levels in each limb by the method given in Chapter 11.

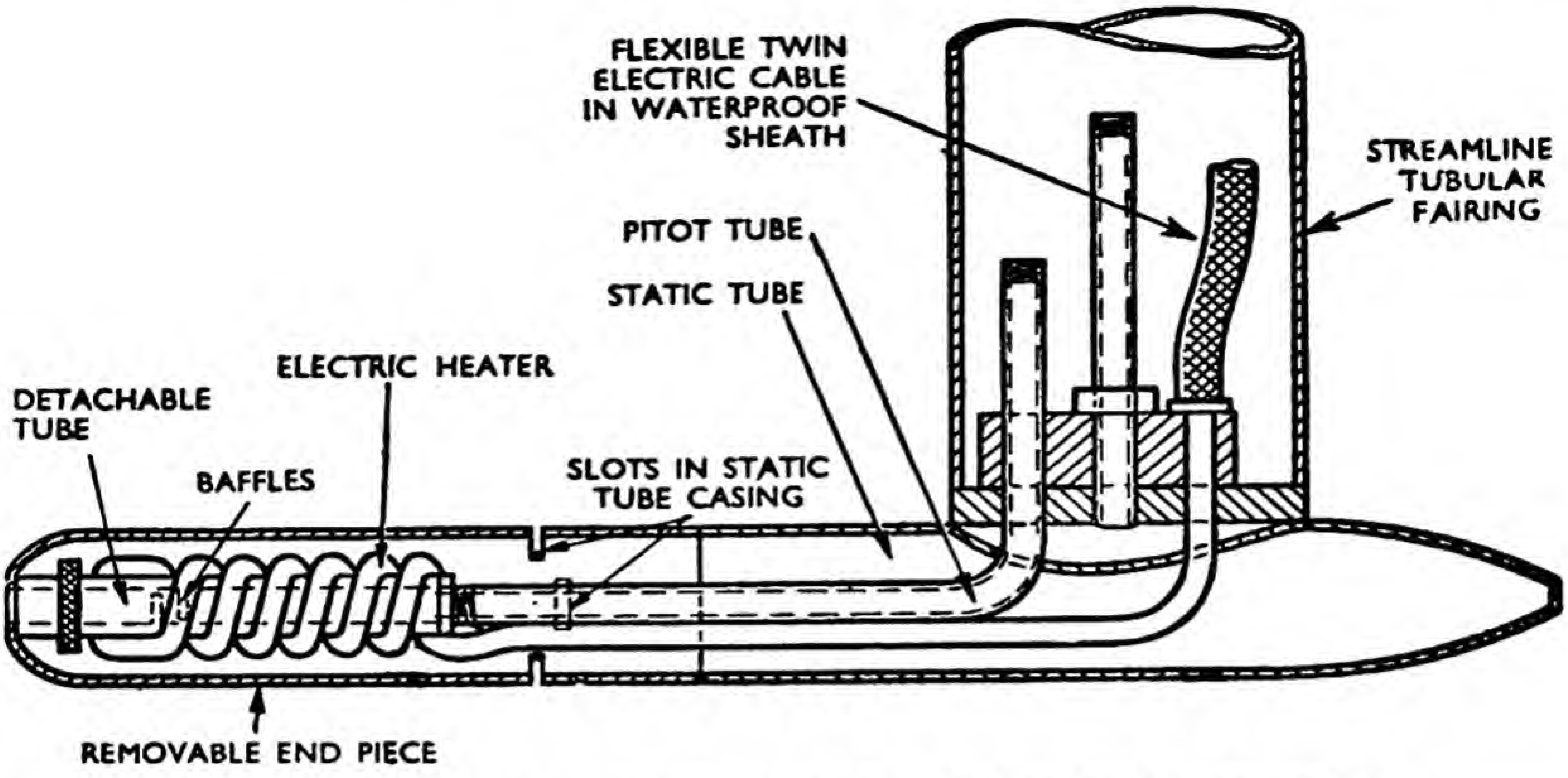
Measuring Airspeed

The same principle is also used to measure the airspeed of an aircraft. Two tubes, facing forward, are fixed to some part of the aircraft which is directly exposed to the airflow, for instance, on an extension near the front of a wing. One of these tubes has a closed end, shaped so as to reduce air resistance, a number of small holes or slots being made in its side. Since the axes of these holes are perpendicular to the airflow, the pressure inside the tube will be unaltered. It will be the static pressure of the atmosphere, so the tube is called the static tube.

The other tube is open at the end. This opening thus faces the airflow, and is subject to an increased pressure due to the movement of the air, just as we experience a force when we hold an umbrella or a sheet of cardboard into the wind. There is no movement of air in the tube, but merely an

Fig. 15. With a wind tunnel manometer we can find the pressure difference due to the movement of the air inside the wind tunnel, by measuring the vertical distance, *h*, between the levels of the liquid. From this we can calculate the wind speed.





MARK VIII PRESSURE HEAD FOR MEASURING AIRSPEED

Fig. 16. In this model, the static tube surrounds the Pitot tube. Communication between the static tube and the atmosphere is by means of the narrow transverse slots shown. The Pitot tube contains two baffles to prevent the admission of foreign matter, and it is heated electrically to evaporate moisture. The instrument is attached to some exposed part of the aircraft by a tube of streamline section which carries the two tubes and the electric cable.

increased pressure. The tube is thus called the pressure tube, or Pitot tube. It is usually mounted inside the static tube, as in Fig. 16, which shows a modern type of Pitot-static head, or Pitot head, as it is often called. The electrically operated heater inside the static tube prevents condensation of water and formation of ice inside it.

Connections from these tubes are brought into the pilot's cockpit, and connected to the airspeed indicator on the instrument panel. For reference to this, take a look at Fig. 34. This instrument works on the principle of the aneroid barometer, the pressure difference being registered by a pointer moving over a dial marked in miles per hour instead of inches of mercury.

Since this pressure difference is due to bringing the air to rest at the open end of the pressure tube, it is, as before, equal to the kinetic energy of the air, viz., $\frac{1}{2}\rho V^2$, so we use the formula, $p = \frac{1}{2}\rho V^2$, to calibrate the instrument. The only difference between this and the hole-in-the-wall method of the wind tunnel is that the dynamic pressure is $\frac{1}{2}\rho V^2$ above that of the atmosphere instead of the static pressure being $\frac{1}{2}\rho V^2$ below it.

Aerofoil Characteristics

Wind tunnel experiments tell us that, as the inclination to the airflow of an aerofoil is altered, there is a variation in the magnitude, direction and position of the resultant force referred to in Fig. 8. A means of measuring this inclination must now be decided upon, so an arbitrary line through the aero-

foil section is chosen, known as the chord line. The angle between this line and the direction of airflow is known as the angle of attack (Fig. 17). It can now be said that the values of the lift and the drag and the position of the centre of pressure will vary with the angle of attack.

Variation of Characteristics

The way in which these characteristics vary with the angle of attack is of great importance because we want to know how to obtain maximum lift and minimum drag; and we must know where these forces act, because this affects the equilibrium of the aeroplane. This information is given by plotting the wind tunnel results in graphical form.

Let us now consider the factors upon which the lift and drag depend.

It has been seen that the pressure exerted by moving air is $\frac{1}{2} \rho V^2$ lb. per sq. ft. in a direction at right angles to the airflow. Therefore, the force in pounds exerted by the air on a flat surface of area S sq. ft. placed perpendicularly to the airflow, will be this pressure multiplied by the area of the surface, i.e.,

$$\frac{1}{2} \rho V^2 \times S.$$

The surfaces, however, are not always perpendicular to the airflow, nor are they always flat, and the forces due to the moving air may not be parallel to the airflow, but they are always proportional to $\frac{1}{2} \rho V^2.S$. We express this mathematically by saying that the forces are equal to some constant multiplied by $\frac{1}{2} \rho V^2.S$. Thus:—

$$L = C_L \cdot \frac{1}{2} \rho V^2 . S \quad .. \quad \text{(i)}$$

and,

$$D = C_D \cdot \frac{1}{2} \rho V^2 . S \quad .. \quad \text{(ii)}$$

for lift and drag respectively, C_L and C_D being the constants of variation, known as the lift and drag coefficients.

C_L and C_D are just numbers without any dimensions such as pounds or feet per second, because the left-hand sides L or D, are forces measured in pounds, and the remainder of the right-hand side $\frac{1}{2} \rho V^2.S$ is also a number of pounds, as we have just seen.

Remember that we only want to *compare* the aerodynamic forces produced by similar surfaces, so we need only measure similar areas, and not necessarily areas perpendicular to the airflow; therefore, S may be any area,

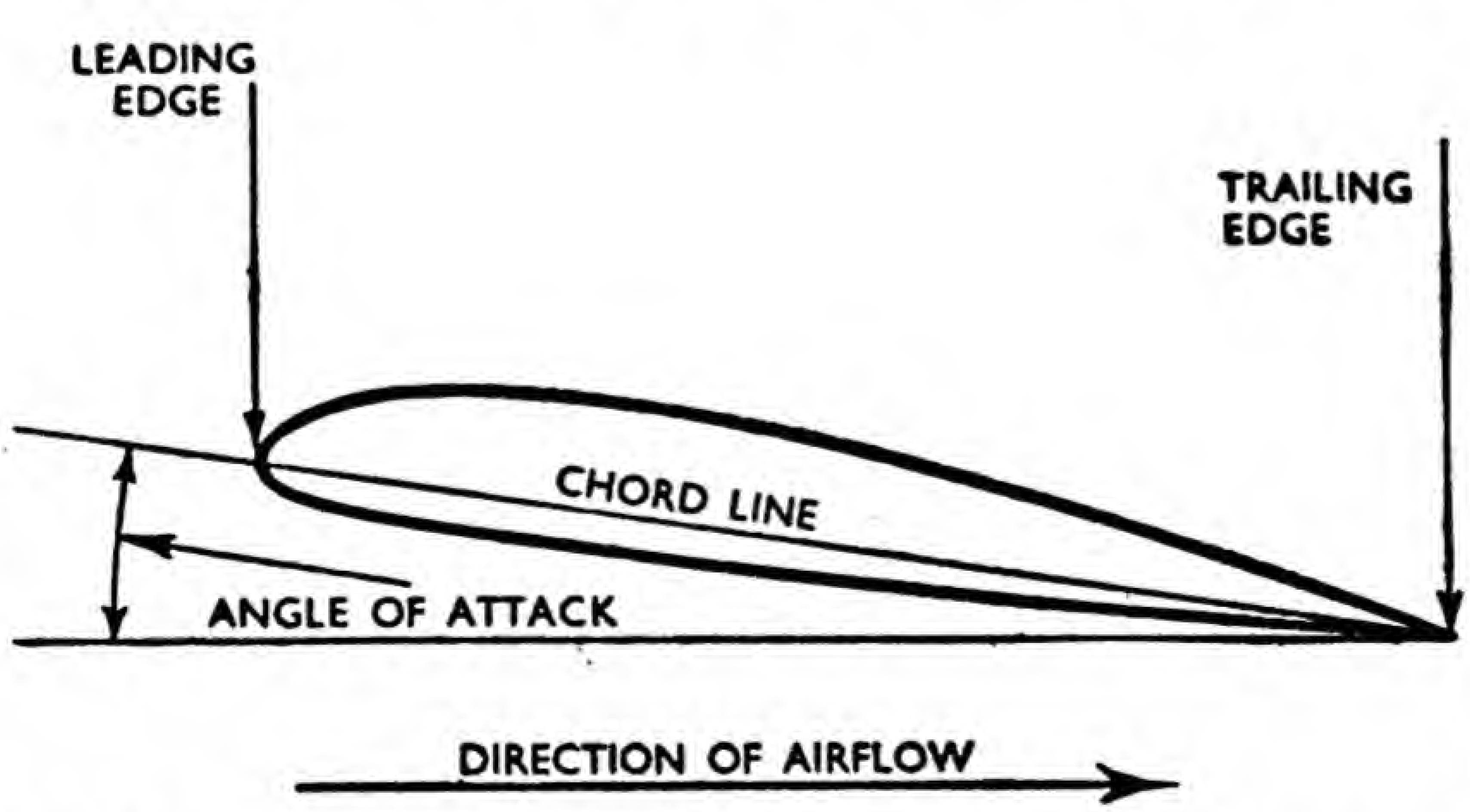
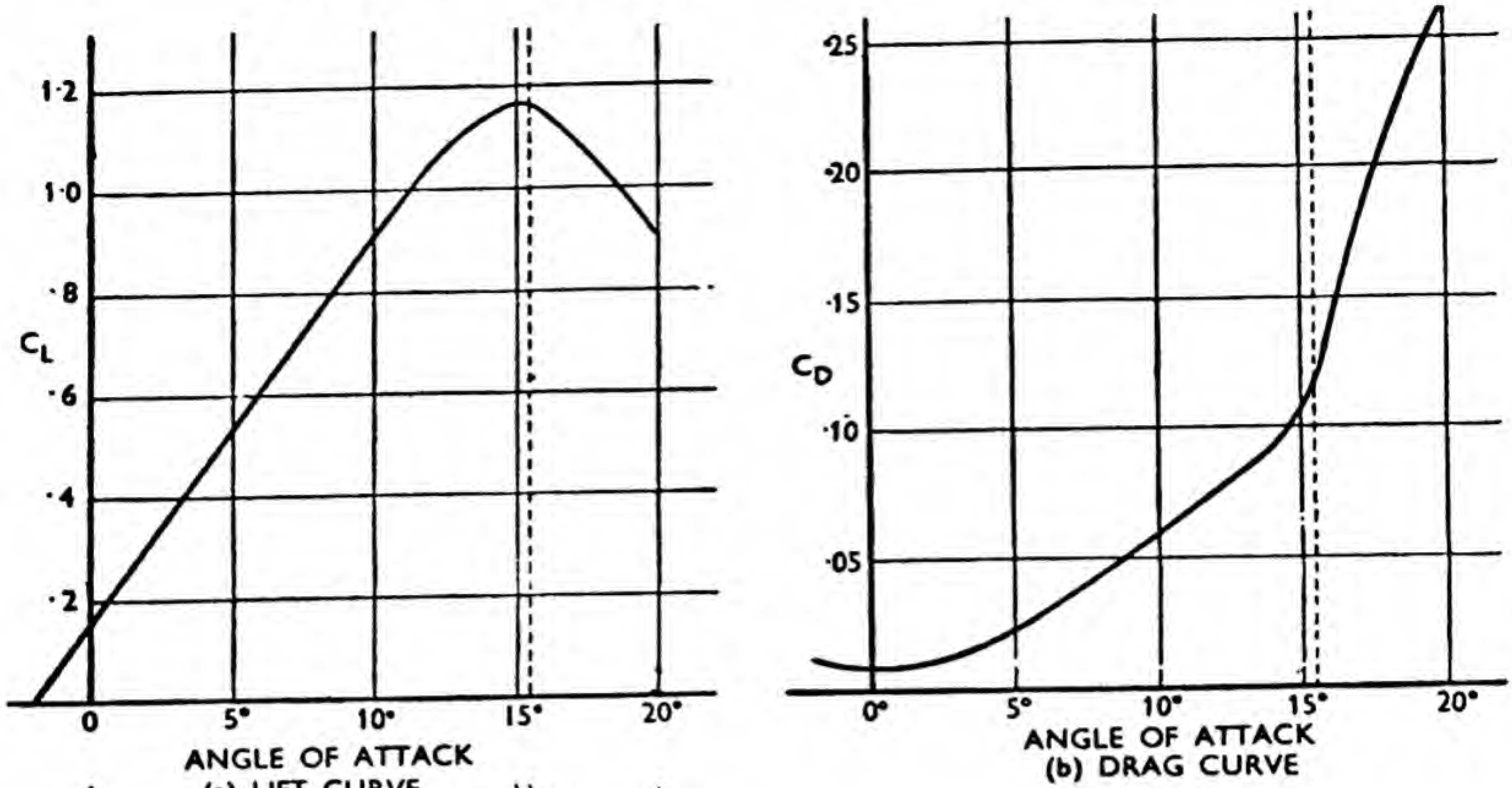


Fig. 17. Above are the terms used when referring to an aerofoil section. The chord line usually passes through the centres of curvature of the leading and trailing edges.



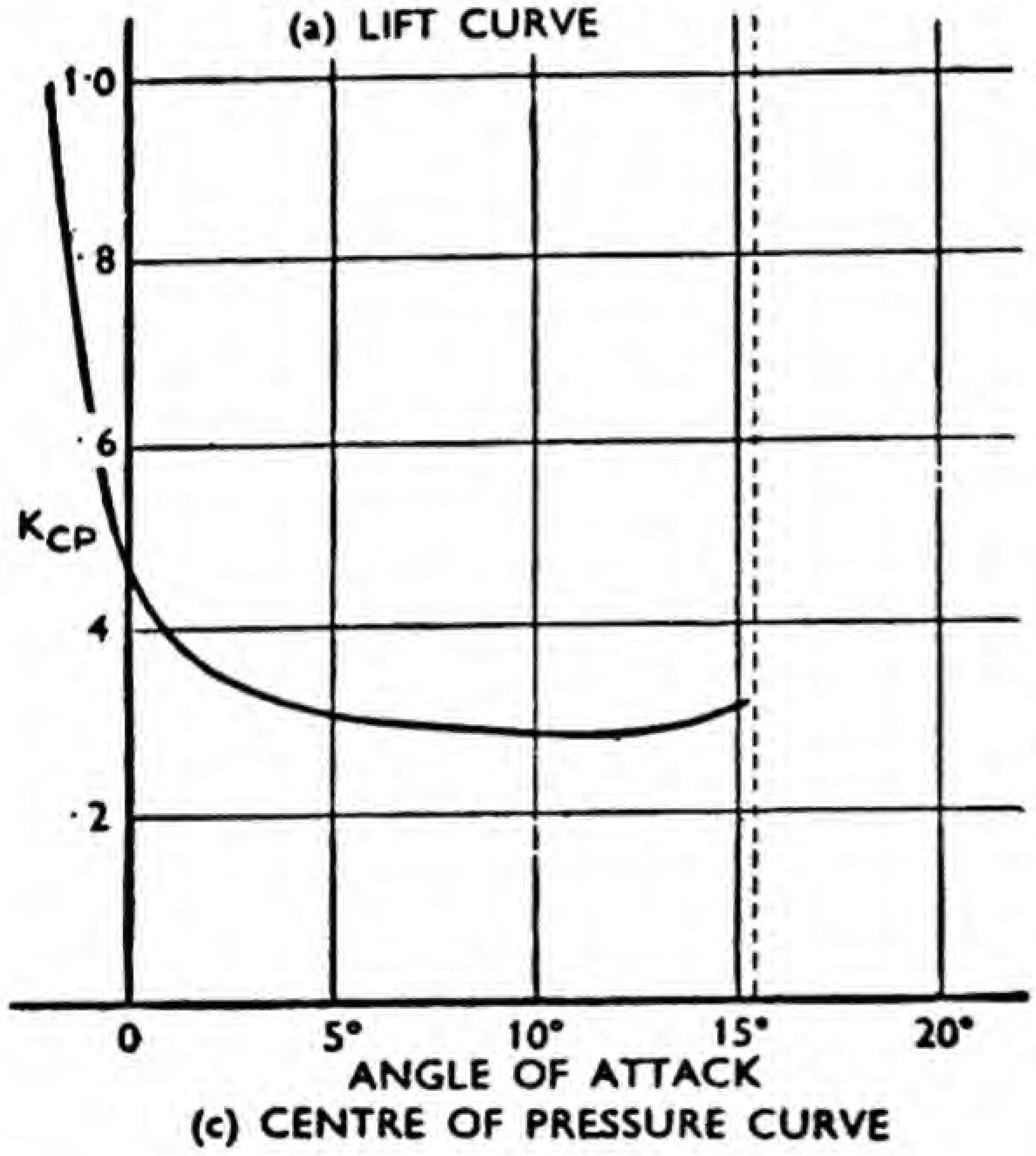


Fig. 18. Stalling angle, 15·6 deg., is marked by a vertical dotted line on each graph. From the lift curve (*a*), we see that the lift increases steadily with the angle of attack until just before the stall ; (*b*) shows that the rate of increase of drag increases just before the stall ; (*c*) shows that the position of the C.P. moves forward slowly until just before the stall, when it begins to move back.

provided that we stick to the same area on each model.

When dealing with bodies which produce drag only, it is usual for S to be the area perpendicular to the airflow, but with aerofoils it is more convenient for S to be the area in the plane of the chord. Thus, in finding the drag of an under-carriage strut, S would be the area of its widest part, taken at right angles to the airflow, but, for a wing, S would be its chord length multiplied by its span.

These two formulæ are very important. Quite a lot of information will be derived from them, sometimes by changing their form, but chiefly by considering the effect of varying certain terms in them. Thus, taking equation (i), if L, C_L, ρ, V and S all varied together, we just could not cope with it. Our minds are best adapted to think of two things only at a time, so we merely let two of the terms vary, and imagine that all the others have some fixed, or constant, values.

If the two terms that are chosen

to vary are on opposite sides of the equation, it can be seen that, if one increases, the other will also increase. Suppose, for example, that we keep ρ, V and S constant, and let C_L increase. Then L must also increase to keep both sides of the equation equal.

On the other hand, if both the variables are on the same side of the equation, one increases as the other decreases. For example, if the variables are V and S, when V is increased, S must be decreased in order to keep the product of V^2 and S constant. L, on the left-hand side, remains fixed, so we must not alter the value of the right-hand side.

The above formulæ may now be rewritten in terms of the lift and drag coefficients. This gives :—

$$C_L = \frac{L}{\frac{1}{2}\rho V^2 . S} \quad \ldots\ldots \quad \text{(iii)}$$

and,

$$C_D = \frac{D}{\frac{1}{2}\rho V^2 . S} \quad \ldots\ldots \quad \text{(iv)}$$

If a model aerofoil is fitted up in a wind tunnel, and the wind set at some constant speed, the lift and drag may be found from the balance readings at various angles of attack from about -4 deg. to 20 deg. But these readings would only apply to this particular experiment, so they are all divided by $\frac{1}{2}\rho V^2 . S$, which is constant, because the air density ρ, wind velocity V, and area of the model S, remain unchanged throughout the experiment. This gives C_L and C_D, the non-dimensional lift and drag coefficients of formulæ (iii) and (iv), which are then plotted against the angle of attack, giving curves like those in Figs. 18(a) and 18(b).

By dividing all the results by $\frac{1}{2}\rho V^2 . S$ we have made them applicable to any similar aerofoil, at any velocity, in air of any density, neglecting the small scale effect mentioned earlier, and the wing-tip effect, which will depend upon the aspect ratio of the model used. Sometimes a calculation is applied to eliminate the induced drag, or wing-tip effect. The resulting curves are then labelled infinite aspect ratio.

The third curve in Fig. 18 refers to the position at which the resultant of L and D acts, viz., the centre of pressure of the aerofoil section, C.P., and this, again, is generalized by quoting its position as a decimal fraction of the chord measured back from the leading edge, instead of giving the actual measurement. This fraction is called K_{CP}.

The curves shown are representative ones, and a lot can be learnt from them about the behaviour of aerofoils. They are extremely important to students of aerodynamics, who are able to derive much more information from them.

We notice from the first curve that, at a small negative angle of attack, there is no lift ; at 0 deg. there is a small lift ; and that the lift increases steadily with angle of attack because the curve is a straight line, until, at about 15 deg., the curve suddenly turns over. This means that there is a sudden decrease in the lift. We find that the aerofoil has stalled.

Stalling

Fig. 19 shows the streamline pattern at three angles of attack.

As the angle of attack increases, the downwash and, therefore, the lift, increases, until the streamlines break away from the upper surface of the aerofoil. This destroys the negative pressure or suction on the upper surface, and so there is a sudden loss of lift. There is still some lift, but it is irregular, and, in general, a stalled aeroplane is out of control.

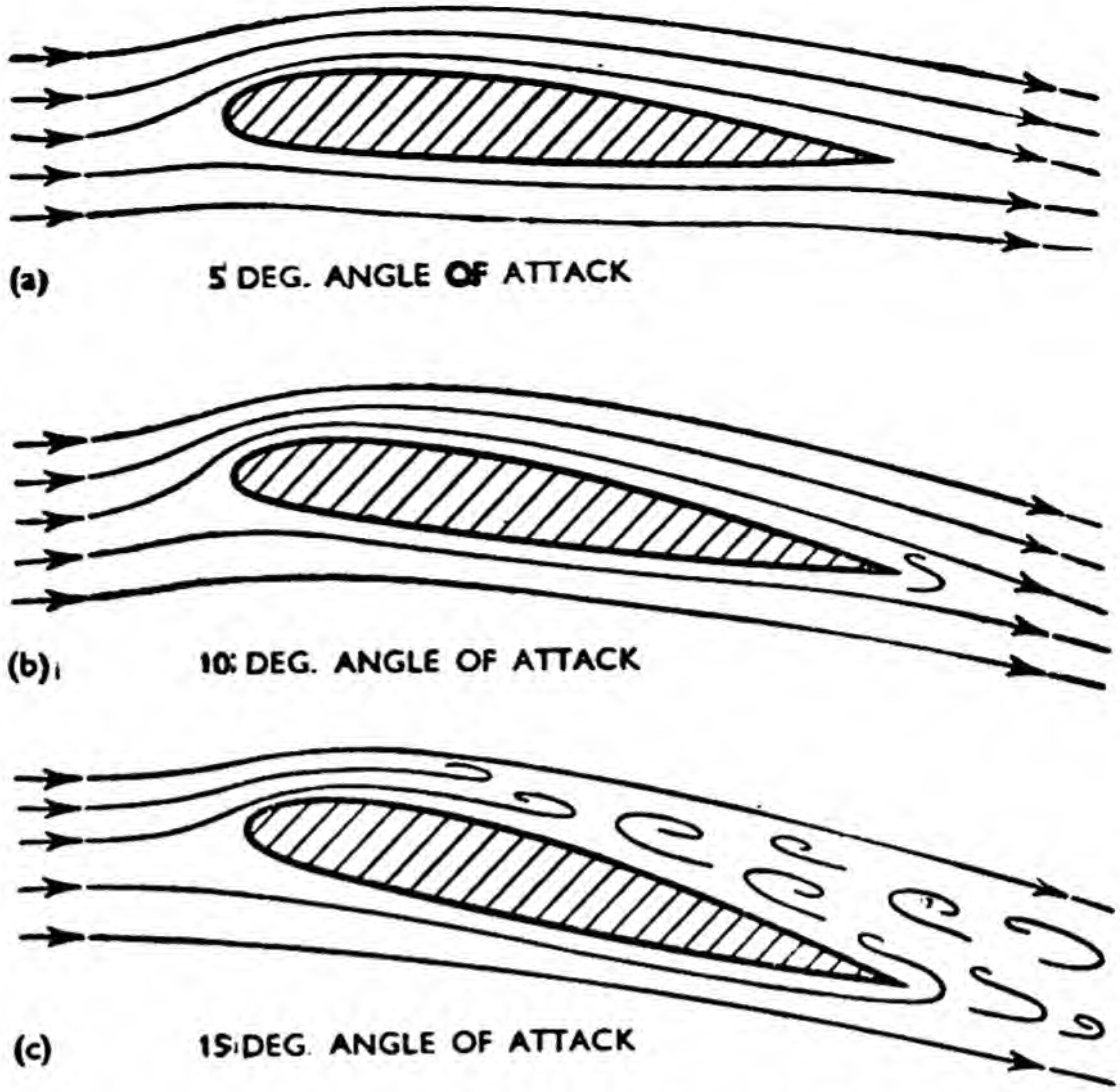


Fig. 19. Effect of angle of attack on airflow is depicted above. (*a*) Here the angle is 5 deg. (*b*) An increase in the angle of attack produces more downwash, and, therefore, more lift, until a critical angle, the stalling angle, as at (*c*) is reached.

Apart from the inconvenience of the sudden decrease in lift, the ability to vary lift by altering the angle of attack is very fortunate. If an aeroplane is to maintain horizontal flight, the lift must always be equal to the weight, but, if the speed is increased without altering the angle of attack, the lift is increased, the lift varying as the square of the speed, because $L = C_L \cdot \frac{1}{2} \rho V^2 . S.$ We simply decrease the angle of attack at higher speeds to compensate this increase in lift.

The second curve tells us that the drag is a minimum at an angle of attack of about 1 deg., and that there is an increase in the rate of increase as the stalling angle is reached.

The third curve tells us about a rather unfortunate fact. It shows that, as the angle of attack increases, the C.P. moves forward. This means that the distance between L and W in Fig. 3(d) decreases, reducing the anti-clockwise moment. Therefore, there will be an excess of clockwise, or stalling moment, which will cause the angle of attack to increase still more, until the aeroplane stalls. This is compensated by designing the tailplane so that it always produces a turning moment to balance this excess. We shall hear more about this later.

Efficient Flight

Now, the drag curve also has its unfortunate fact. Its minimum value does not occur at the same angle of attack as that for maximum lift, so flight can never be attained with both minimum drag and maximum lift. Here we must

compromise, and find the angle at which the *ratio* of lift to drag is the greatest. This is the same as the ratio of C_L to C_D, since these are proportional to L and D respectively.

If the ordinates, viz., vertical measurements, of the lift curve for each angle of attack are taken, divided by the corresponding ordinates of the drag curve, and a new curve plotted (Fig. 20), the peak of this curve will give the angle of attack for maximum L/D, viz., for most efficient flight. The curve is similar in some respects to the lift curve, but its peak occurs, at an angle of attack of about 3 deg., with an L/D of about 27.

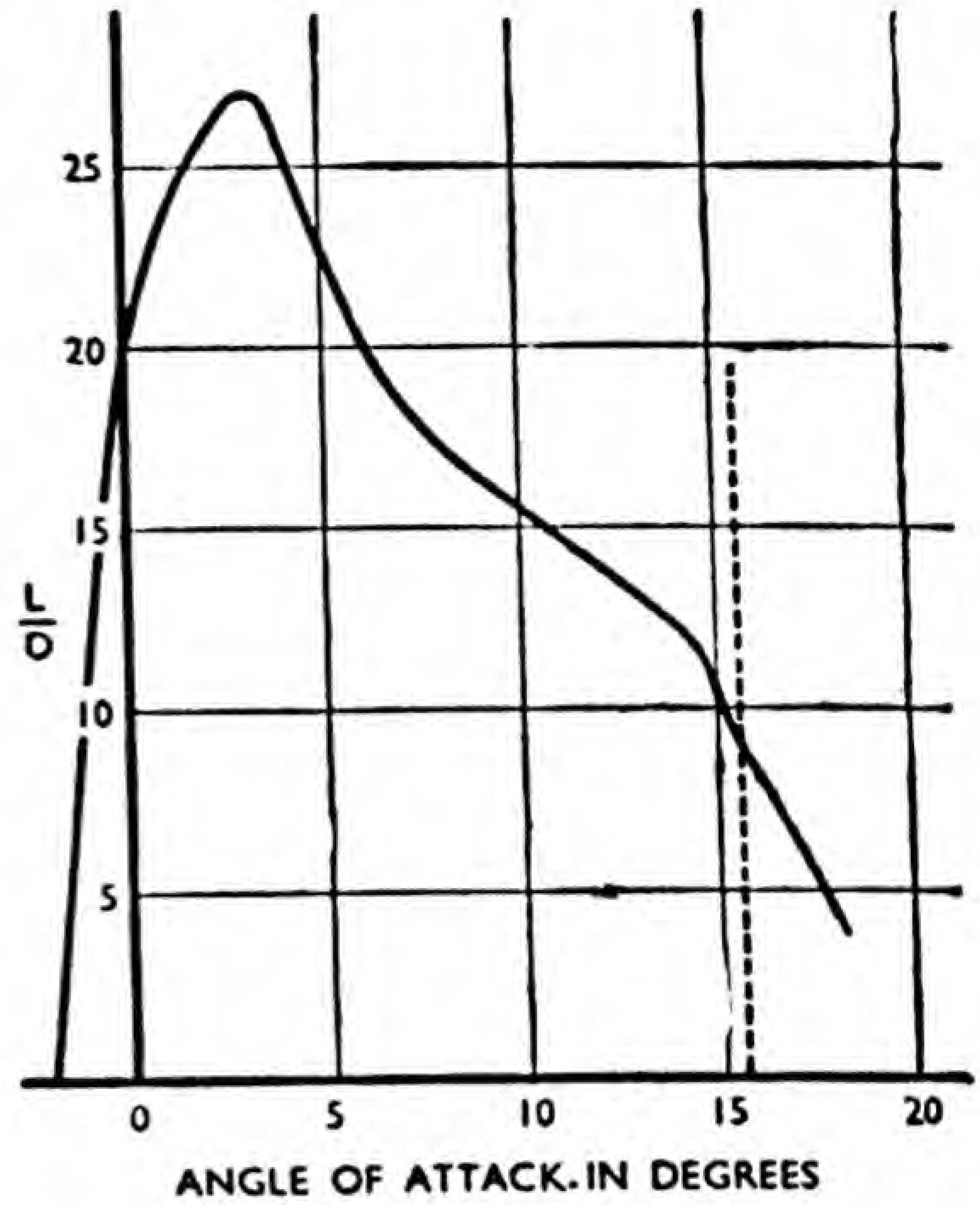


Fig. 20. Here is the *L/D* Curve for R.A.F. 15 Aerofoil, A.R.6. This curve tells us that, with an R.A.F. 15 aerofoil, neglecting the effect of fuselage drag, we can fly most efficiently at an angle of attack of 3 deg.

When considering the aeroplane as a whole, we must take into account the drag of the fuselage, and find the overall L/D ratio. This drag is usually nearly equal to that of the wing, whilst, of course, no extra lift is obtained. The effect of this is approximately to halve the L/D ratio, and the peak of the overall L/D curve will occur at a higher angle of attack than that for the wing alone. It is found in practice that the effect of the drag of the fuselage increases the angle to 4 deg. or a little more.

Speed of Flight

We are now in a position to consider the factors governing the speed of flight.

The condition for steady horizontal flight is that the lift must be equal to the weight, and the thrust equal to the drag, as in Fig. 3(d), provided, of course, that the moments are balanced. Now, in experimental work, the velocity is kept constant and different values obtained for lift, but, in steady flight, the pilot must always keep the lift constant and equal to the weight, whilst the velocity varies. This, as has been seen, is done by altering the angle of attack.

Thus the formula can be written as :—

$$L = W = C_L . \frac{1}{2} \rho V^2 . S \ldots \ldots \text{(i)}$$

In the experimental work just described, the variables were L and the angle of attack (Fig. 18), but L is now constant and equal to W, and V is changing, so the variables in formula (i) become C_L and V. ρ is constant in horizontal flight, and S may be considered constant, although certain devices are in use to vary this slightly. The formula, then, shows us that the product of C_L and V^2 must be constant.

Thus, to fly at half the speed, we must make C_L four times as great by increasing the angle of attack. But this cannot be done indefinitely,

because a time comes when an angle of attack of about 15 deg. is reached, when the lift suddenly decreases. The aeroplane stalls, and control can only be regained by decreasing the angle of attack.

It is not always so simple, however, because one wing may stall before the other, as will be shown later. At any rate, it is obvious that there is a minimum speed of flight, the stalling speed, and that this is the speed corresponding to the maximum angle of attack possible for flight.

It is also obvious that there is one definite speed corresponding to each angle of attack, because, if C_L is fixed, V can be given only one value to make both sides of the formula equal, the other terms being constant. This fact applies whether the aeroplane is ascending or descending, V always being the velocity of the airflow relative to the direction of motion.

The Speed Range

Now, what about the maximum speed? We cannot find it from the above formula, because we can go on decreasing C_L indefinitely with a corresponding very large increase in V. We must consider the formula governing thrust T and drag D :—

$$T = D = C_D . \frac{1}{2} \rho V^2 . S \ldots\ldots \text{(ii)}$$

Unlike the weight, the drag is not constant, but varies with C_D and V^2, viz., D varies as $C_D . V^2$. At high velocities, with the accompanying small angles of attack, C_D is nearly constant (see drag curve, Fig. 18(b)), so the drag, and, therefore, the thrust, are now increasing with the velocity. Now, there is a limit to the amount of thrust that can be produced, so maximum speed depends upon engine power.

The difference between minimum and maximum speeds is known as the speed range.

At large angles of attack, C_D increases, and V^2 decreases, as the angle of attack is increased, and, as the stalling angle is approached, the effect of the increasing C_D outweighs that of the decreasing V^2. Therefore, more thrust will be required at the stalling speed than at higher speeds. The speed at which least thrust is required is that corresponding to the maximum overall $\frac{L}{D}$ ratio. This is the most efficient speed of flight, and is known as the cruising speed.

Now, the statement that there is one definite speed for each angle of attack requires qualification. If we divide both sides of formula (i) by S, we obtain the relationship :—

$$\frac{W}{S} = C_L . \frac{1}{2} \rho V^2 .$$

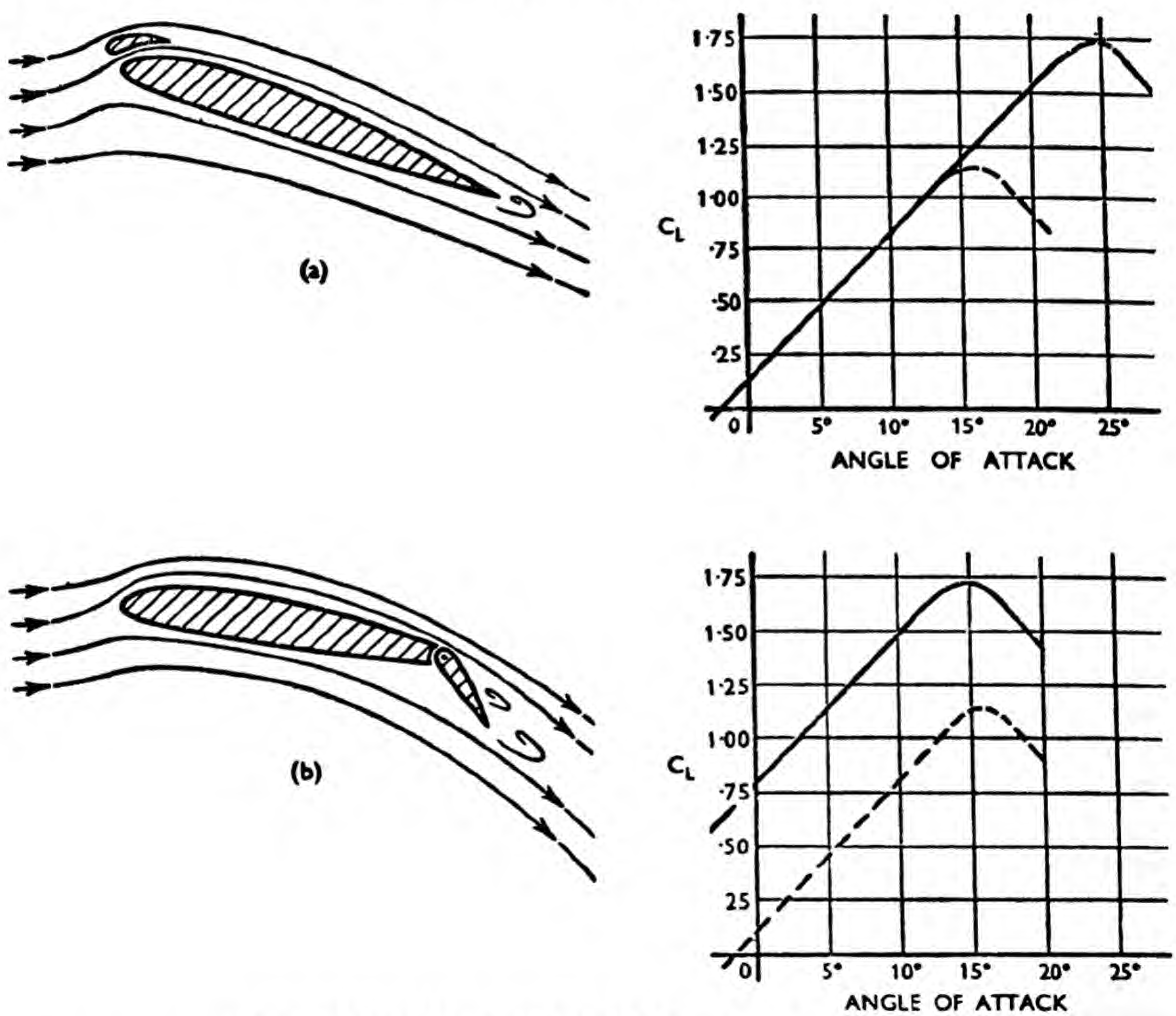
Here, $\frac{W}{S}$ is the wing loading in lb. per sq. ft., and if this is altered, the product of C_L and V^2 must change with it. The wing loading will only be constant for a given aeroplane, provided that the weight of the aeroplane does not change. By regarding $\frac{W}{S}$ and V as the variables, the formula tells us that if the wing loading is increased, either by design, or by the carrying of additional weight, the speed of flight for each angle of attack will be increased.

Further, the air density ρ decreases with altitude. At 10,000 ft. it is about two-thirds of that at sea level, and at 25,000 ft. it is less than half. It will be evident from the right-hand side of formula (i) that

this decrease in ρ will again require a higher V for each angle of attack, ρ and V being the variables whilst the other terms remain constant. Thus, the higher the altitude, the higher the speed for a given angle of attack.

These speed increases for both weight and altitude apply throughout the speed range, thus both the stalling speed and the cruising speed will be increased. The maximum speed, however, depends upon engine power, and may at first be increased, but will then be decreased until the speed range becomes zero.

It is now clear that any statement about speed should be qualified by stating the weight and altitude. In practice, if stalling speed is mentioned without qualification, it means the stalling speed of the aeroplane at sea level when nor-



COMPARISON BETWEEN SLOTS AND FLAPS

Fig. 21. (*a*) Left-hand diagram of this section illustrates the airflow over a slotted aerofoil at an angle of attack of 20 deg., which is greater than the normal stalling angle. The full line on the graph gives the lift curve, whilst the dotted line indicates the stall at the normal angle, as it would occur with an unslotted aerofoil. (*b*) The left-hand portion here shows the airflow over an aerofoil fitted with a simple flap, the chord of the aerofoil being inclined at an angle of attack of 10 deg. The graph shows the corresponding lift curve along with that for the plain aerofoil (shown dotted) for comparison. It will be noticed that in both cases a lift coefficient of 1·75 is obtained, but that the slotted aerofoil requires an additional 10 deg. inclination to produce this lift.

mally loaded. Cruising speed means the most efficient speed when normally loaded and flying at a stated altitude. This altitude is the one at which the aeroplane is designed to fly, and is called the rated altitude.

Both engines and airscrews behave differently at different air densities, so the designer plans to fit the engine and airscrew, viz., the power unit, which will be working under its most efficient conditions when providing just sufficient thrust to overcome the drag when flying at maximum L/D at the rated altitude.

Economical Flying

It is worth noting from formula (ii) that, for a given thrust the velocity at which we fly increases with altitude because ρ decreases whilst the remaining terms are constant. It is thus economical to fly at a high altitude. We are, however, up against the difficulty that engine power falls off with decreasing air density, but this can be overcome, up to a point, by supercharging, and by jet propulsion.

It is an obvious advantage for an aeroplane to have as great a speed range as possible. The top speed can be increased by fitting a larger power unit, but this increases the weight, both directly and indirectly, because the structure must be stronger. Consequently the wing loading, and, therefore, the stalling speed, goes up. We can overcome this by fitting a larger wing, but this lowers the top speed by increasing the drag. It is a vicious circle. Our remaining hope is to increase the maximum value of C_L. This can be done by (a) fitting slots to the leading edges of the wings, and (b) fitting flaps under the trailing edges.

These are shown in Fig. 21. The slots, which are normally closed, but open automatically at high angles of attack, delay the stall for about 10 deg., with a corresponding increase in C_L. The disadvantage is that the high angle of attack involves a large undercarriage for take-off and landing, and the high angle adversely affects the pilot's view of the ground. The slot idea is more often used nowadays as a cowling around an air-cooled engine—the Townend ring. The cross-section of the ring is of aerofoil shape, and it improves the airflow over the engine, thus reducing the drag.

There are many kinds of flaps, of which only the simplest, the plain flap, is shown. Flaps increase the lift at normal angles of attack by increasing the effective camber of the wing, but they also increase the drag. However, since their angle is adjustable they give the pilot control over the L/D ratio, and, therefore, as we shall see later, control over the gliding angle. These advantages favour their adoption in preference to slots on modern aeroplanes.

Power Required for Flight

We have mentioned power several times during our discussion of speed. Now, power, as described in Chapter 4, is the rate of doing work, and is measured in foot-pounds per second. That is the same as saying pounds multiplied by feet per second, viz., force multiplied by velocity. The force is the thrust produced by the power unit, and, at constant speed, is equal to the drag. Thus, power is the product of D and V.

Now, since 1 horse-power = 550 ft.-lb. per sec., we can write

$$\text{h.p.} = \frac{DV}{550}$$

and, since drag varies as $C_D V^2$ (formula ii, page 399)

$$\text{h.p. varies as } C_D V^2 \times V,$$

viz., h.p. varies as $C_D V^3$.

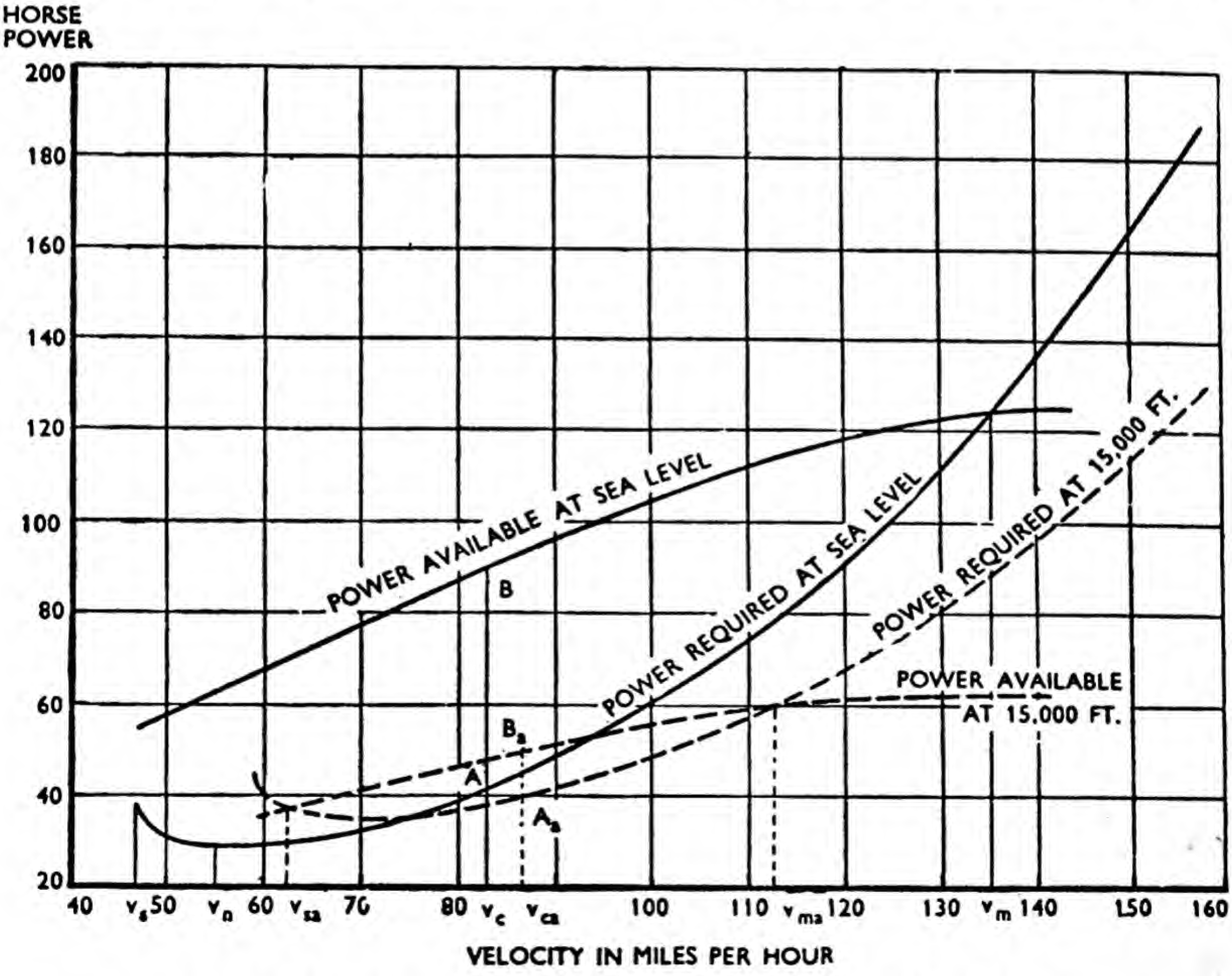
This variation is similar to that of the thrust, so the power required will be high at both low and high velocities, and lower for intermediate velocities, as shown by the lower full curve in Fig. 22.

Its minimum value occurs at v_n, a velocity not much in excess of the stalling speed, and below that for minimum thrust. As we have just seen, power is thrust multiplied by velocity, and, as we increase the velocity through that for minimum power, the effect of the thrust decrease is just equal to that of the velocity increase.

Now, the power available at full throttle is not constant. It varies with the speed, as shown by the upper full curve in Fig. 22. The intersection of the two curves gives the maximum speed for horizontal flight, v_m.

If AB is the longest vertical line which it is possible to draw between the two curves, its length will give the maximum amount of power available for climbing, and the base of the line, v_c, will give the velocity at which the rate of climb will be greatest.

It is possible to fly at a greater



PERFORMANCE CURVES FOR A LIGHT AEROPLANE

Fig. 22. These curves tell us how fast and how slow we can fly, the speeds for minimum power and maximum rate of climb, and how much power is left over for climbing at different altitudes.

speed than v_m, but the power available will not be sufficient to maintain height.

Changing Weight and Altitude

Let us now consider the effect of changing (a) weight and (b) air density.

(a) It has been pointed out that increase in weight increases the angle of attack for a given velocity, therefore, throughout most of the speed range, the drag will increase. It follows that the power required at most velocities will be increased, and the power-required curve will be displaced upward. This will result in a smaller speed range and a reduction in the length of AB, so the power available for climbing will be reduced. The rate of climb will be affected by both the increased weight and the reduced power available. When the additional weight is sufficient to displace the power-required curve so far upward that it becomes tangential to the power-available curve, there will be no power for climbing. Of course, this loading would never be reached in practice because it would be impossible to take off.

(b) This time both curves will be affected. The upper one will be depressed because of the decrease in the power obtainable from the engine at high altitudes. The lower one will be (1) raised, because, from formula (i), page 398, at a given velocity C_L must be increased to compensate decrease in ρ, and since this generally involves an increase in C_D, from formula (ii), page 399, T, the thrust required at a given velocity, will be increased ; (2) displaced to the right as the decrease in ρ in formula (ii) will give a greater velocity for a given thrust.

The dotted curves in Fig. 22, show the effect of altitude. This will usually result at first in an increase in the maximum speed of flight, but this speed will eventually be lowered. We shall get a decrease in the power available for climbing from AB to A_aB_a, and an increase in the velocity for maximum rate of climb from v_c to v_{ca}. The curves will now intersect at the lower end, showing that, at altitude, the power available, not stalling, determines the minimum speed of horizontal flight, which is now v_{sa}.

It will be impossible to reach the stalling angle whilst flying level, through lack of engine power, and there will be a big increase in the minimum speed of flight. Eventually, the curves will be tangential, and the ceiling will be reached. There will then be no power available for climbing, and the point of tangency will give the only possible velocity at which it will be found possible to fly.

A Short Revision

It will clarify this argument to follow it again in close consultation with the graph in Fig. 22, noting the actual figures, which apply to a light aeroplane.

The position of the left-hand extremity of the power-required at sea level curve gives the stalling speed at sea level v_s, which is 47 m.p.h. : v_n, 55 m.p.h., is the speed of flight at which minimum power is required, and v_m, 135 m.p.h., is the maximum speed of horizontal flight.

At all speeds between v_s and v_m the vertical distance between the two full curves will give the power available for climbing. This is greatest at v_c, 83 m.p.h., where the excess power AB is 48 h.p. Thus 83 m.p.h. is the speed for maximum

rate of climb. This value is nearly the same as the most efficient speed for horizontal flight.

These curves become displaced to the dotted positions at an altitude of 15,000 ft., when the minimum speed for horizontal flight becomes v_{sa}, which is 62½ m.p.h. Note that the stalling speed is now 59 m.p.h. ; but there is not sufficient power available at this altitude to maintain horizontal flight at this speed, so we could only stall our aeroplane whilst losing height.

The maximum speed v_{ma}, is now reduced to 113 m.p.h. The speed for maximum rate of climb v_{ca}, is increased to 87 m.p.h., but the length of the line A_aB_a shows that there is only 10 h.p. available for climbing, so that the rate of climb will be reduced.

Flight at Constant Velocity

It has now been seen that questions of speed and power are intimately connected with those of change of attitude, or change of the direction in which the nose is pointing relative to the direction of flight. In the following, it will be shown how vertical changes in the direction of flight, as well as change of attitude, affect the forces involved.

For the sake of simplicity we will first of all regard all the forces as acting through one point, and will then consider the effects due to the centres of gravity, pressure, and drag, and the thrust line not coinciding. That is, we shall then apply our third condition of equilibrium, which deals with moments.

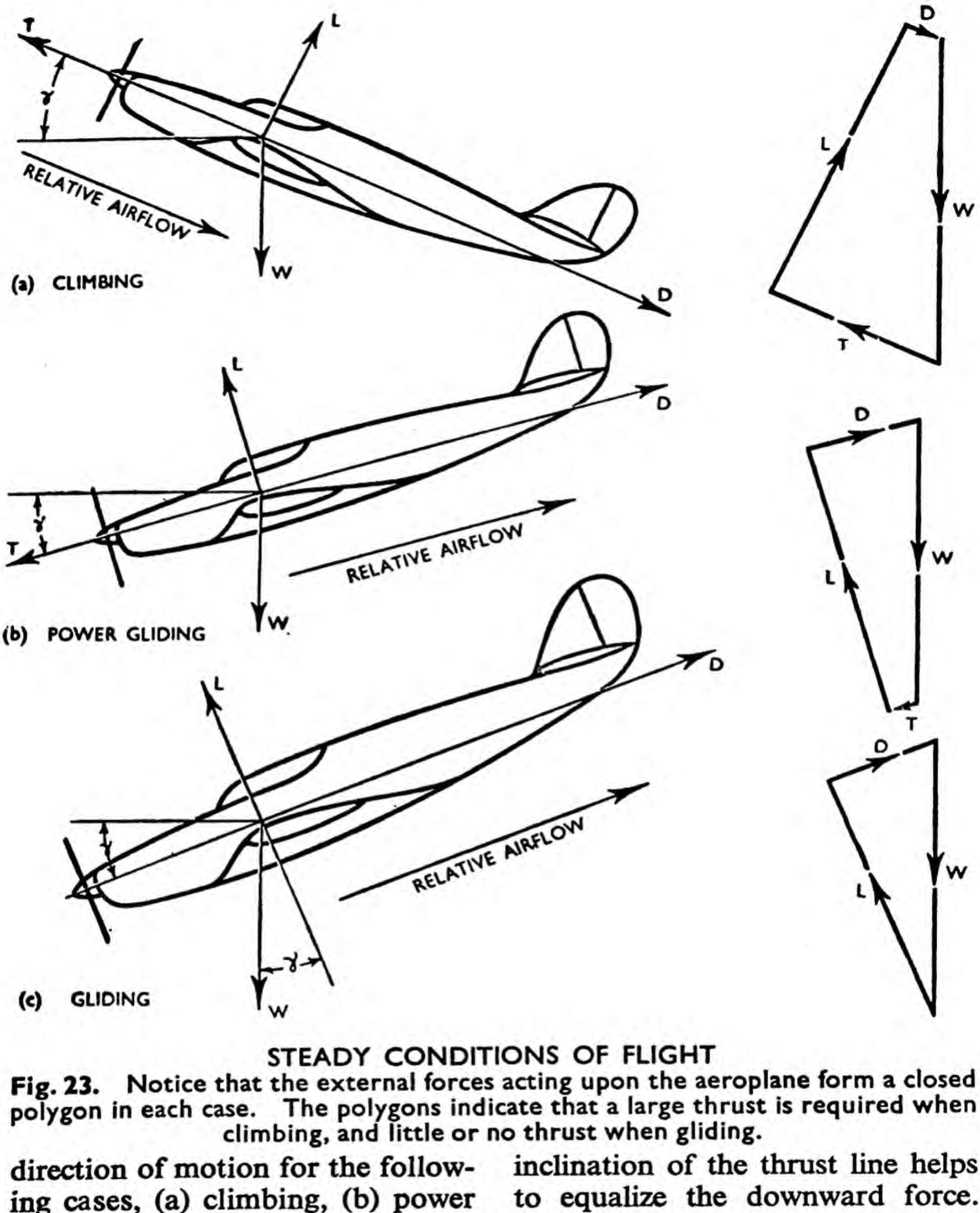
Flight at constant velocity is called steady, and we have already dealt with steady horizontal flight. The simplified force diagrams for steady flight other than horizontal, are shown in Fig. 23. The conditions shown are (a) climbing, and (b) and (c) gliding with and without the use of the engine. If the angle of descent is large, the aeroplane is said to be diving.

Now, we should note carefully that the direction of the airflow *relative to the aeroplane* is exactly opposite to the direction of motion of the aeroplane, so that, if we are going upward, the air comes downward, and going downward, the air is coming upward. Remember that the air itself is not moving, but it has a velocity relative to the aeroplane. After all, if we are cycling down a hill, the air still meets us in a direction parallel to the road, viz., its velocity relative to us is *up* the hill.

The airflow direction is important because, as we have seen, it defines the direction of action of lift and drag.

Since we are first of all regarding all the forces as acting at one point, we may apply the condition that, if each force be drawn to scale, and placed end to end in the same direction, a closed polygon will be formed. These polygons are shown in the figure, and they give us a good idea of the relative values of the forces involved. The second polygon, for example, shows how little thrust is required during a descent.

However, more precise information can be obtained by applying the first two conditions of equilibrium analytically, viz., by resolving in two perpendicular directions. Now, it is found much more convenient to choose these directions relative to the airflow, instead of to the horizontal. We shall resolve perpendicular and parallel to the



STEADY CONDITIONS OF FLIGHT

Fig. 23. Notice that the external forces acting upon the aeroplane form a closed polygon in each case. The polygons indicate that a large thrust is required when climbing, and little or no thrust when gliding.

direction of motion for the following cases, (a) climbing, (b) power gliding, (c) gliding, (d) horizontal flight, and (e) terminal velocity dive.

(a) From Fig. 23(a)

$$L = W \cos \gamma \quad \ldots\ldots \quad \text{(i)}$$

$$T = D + \mathrm{W} \sin \gamma \quad \ldots \quad \text{(ii)}$$

by resolving (Chapter 2, page 30).

From (i) it will be seen that the lift is actually less than the weight because $\cos\ \gamma$ is less than 1, although, of course, the upward inclination of the thrust line helps to equalize the downward force. Sometimes an aeroplane is designed so that the thrust line is inclined upward in horizontal flight.

The total upward force must be equal to the total downward force, and this could be analysed by resolving vertically, viz.,

$T \sin \gamma + L \cos \gamma = W + D \sin\gamma$, but the more complicated nature of this equation shows why we

normally resolve relative to the air-flow direction.

Note that the total upward force is not greater than the total downward force. If it were we should be accelerating upward, a condition of flight which will be discussed later.

From (ii) it will be noticed that the thrust is greater than the drag. Now, it has been shown that, if the angle of attack is unaltered, the drag will be unaltered, so it can now be seen that if the throttle is opened without changing the angle of attack, the extra thrust will cause the aeroplane to climb without change of speed. To clarify this, regard D and W as constant in equation (ii). Then, if T is increased, $\sin\gamma$, and, therefore, γ, the climbing angle, must increase.

It will be observed now that more information can be obtained from the analytical treatment than from the graphical treatment, so the former method will be adhered to in this discussion. Most problems can be tackled by both methods ; in fact, graphical solutions can always be replaced by analytical ones, but it is not always convenient to do so. In practice the method is chosen which gives the result with the least amount of work, or in the clearest way.

(b) By resolving forces in Fig. 23(b), the equations of equilibrium become

$$L = W\cos\gamma \quad \ldots\ldots\ldots \quad \text{(i)}$$
$$D = T + W\sin\gamma \quad \ldots\ldots \quad \text{(ii)}.$$

Small Thrust Confirmed

The small thrust previously referred to is confirmed by rewriting equation (ii) in the form :—

$$T = D - W\sin\gamma.$$

Now, as the angle of descent γ, and, therefore, $\sin\gamma$, increases, the value of the term $W\sin\gamma$ increases, thus decreasing the right-hand side of the equation. Therefore, the left-hand side, the thrust, will decrease.

This condition of flight applies to that referred to when performance at velocities above v_m, the point of intersection of the power curves, was considered. Sufficient thrust could not be obtained at this high

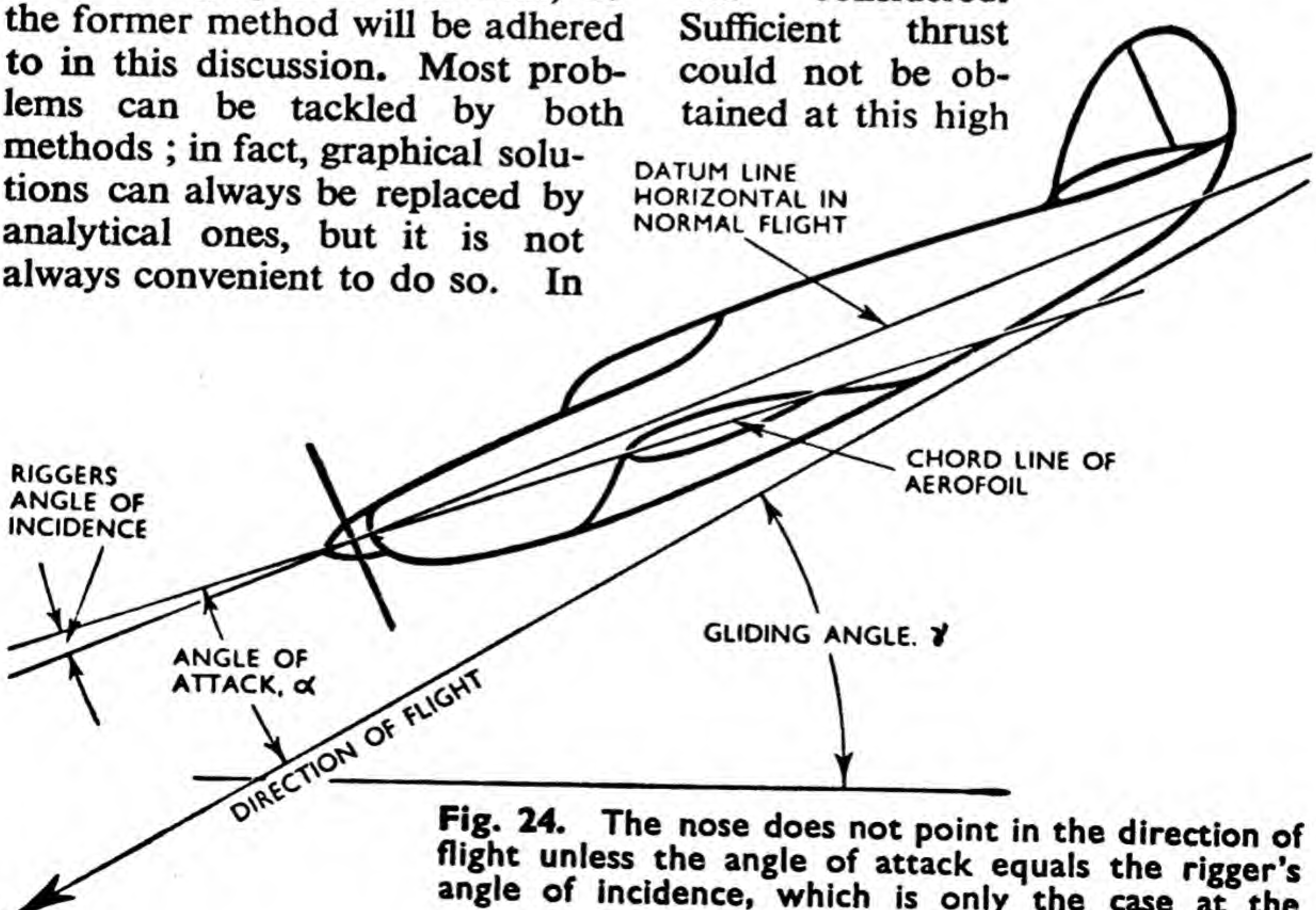
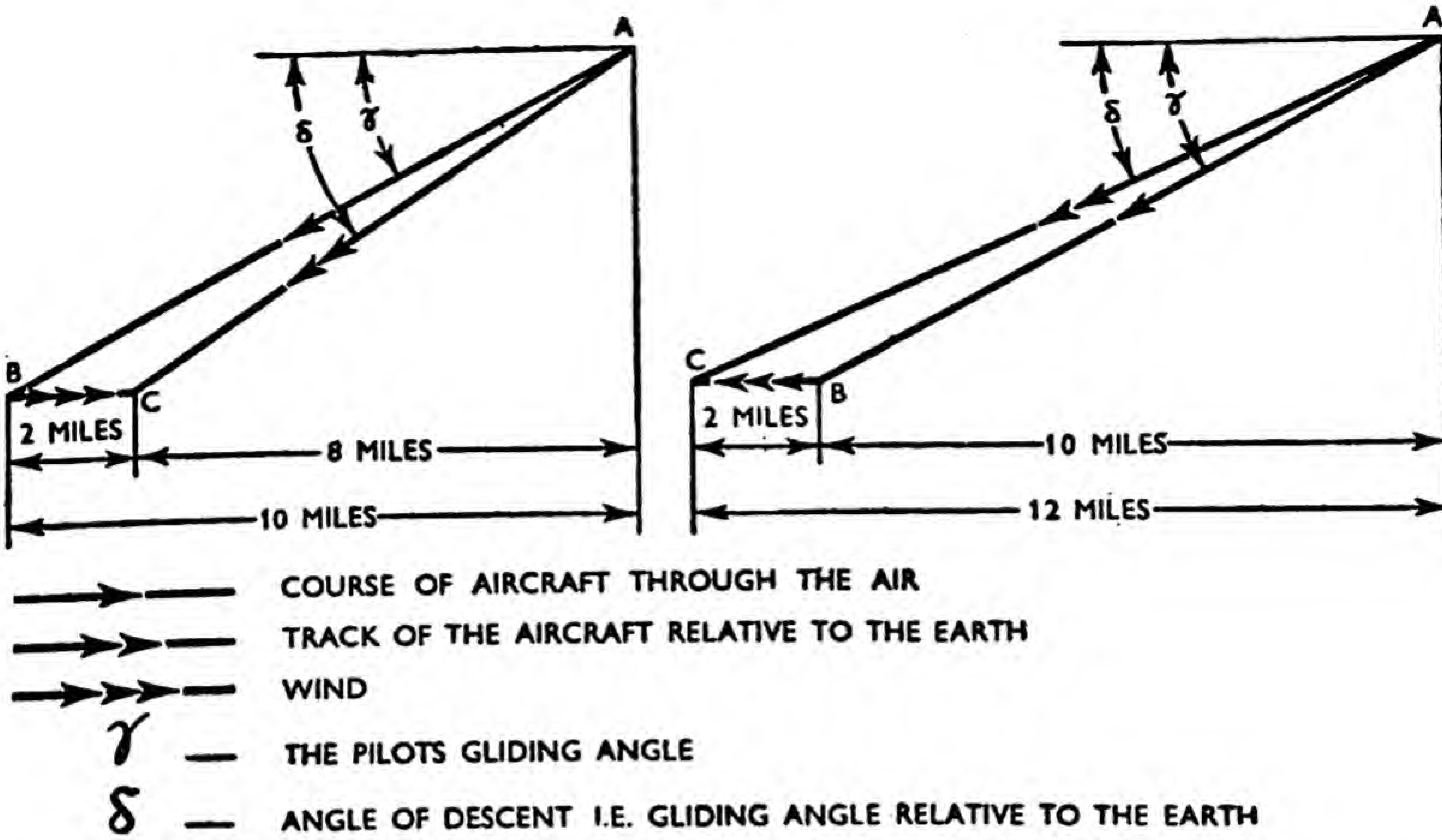


Fig. 24. The nose does not point in the direction of flight unless the angle of attack equals the rigger's angle of incidence, which is only the case at the cruising speed.



EFFECT OF WIND ON GLIDING ANGLE

Fig. 25. All the heavy lines with arrows are velocity vectors. In other words, they represent a velocity in magnitude and direction.

velocity to overcome the drag, but it can be made equal to the drag minus $W \sin \gamma$.

(c) If the engine is cut out, T will be zero, so, substituting this value for T in the above equations :—

$$L = W \cos \gamma \quad \ldots\ldots\ldots \text{(i)},$$
$$D = W \sin \gamma \quad \ldots\ldots\ldots \text{(ii)}.$$

Dividing (i) by (ii),

$$\frac{L}{D} = \cot \gamma,$$

or the same result can be found from the force triangle in Fig. 23(c).

Measure of Efficiency

Thus, the gliding angle depends upon the $\frac{L}{D}$ ratio : therefore, we can read off the cotangent of the gliding angle for any angle of attack from an overall $\frac{L}{D}$ curve. This is not quite the same as the $\frac{L}{D}$ curve for an aerofoil (Fig. 18(c)), as explained previously, because of the drag of the fuselage, etc. It follows that we can glide furthest at the cruising speed when the $\frac{L}{D}$ ratio is a maximum, and, thus, minimum gliding angle is a measure of the aerodynamic efficiency of an aeroplane.

Care is required here to appreciate that the direction in which the aeroplane is gliding is not the direction in which the nose is pointing. In other words, the gliding angle γ must not be confused with the angle of attack α. This will be clear by reference to Fig. 24, which also shows the rigger's angle of incidence, viz., the angle between the fore and aft line of the aeroplane at cruising speed and the chord of the aerofoil. Thus, the nose will only be pointing in the direction of flight when flying at cruising speed, or when gliding at the minimum angle, viz., flying at cruising speed with the engine shut off. At higher speeds the nose will be down, and at lower speeds it will be up relative

to the flight path, whether flying level, ascending or descending. We may also reason out that we can glide steeply and slowly at a large angle of attack, nose up (Fig. 24) or steeply and quickly at a small angle of attack, nose down.

It can now be seen how the use of flaps, by altering the $\frac{L}{D}$ ratio, can alter the gliding angle, and enable a pilot to come in steeply to an airfield if he wishes to clear obstacles near the boundary.

It was previously pointed out that the average value of the maximum overall $\frac{L}{D}$ ratio was about 10. This means that a machine could glide 10 miles at cruising speed whilst losing 1 mile of height. But this is 10 miles of air. If it takes 5 minutes to do this, and there happens to be a head wind of 24 m.p.h., only 8 miles of the earth's surface will be covered because wind of this velocity will blow the machine back 2 miles in the 5 minutes. Similarly, with a following wind, 12 miles will be covered.

The velocity triangles illustrated in Fig. 25, show that the effect of this wind will alter the gliding angle relative to the earth. Now, a pilot can only tell his gliding angle from his airspeed indicator reading (note relationship between gliding angle and airspeed), but, if he is landing, he must judge his angle of descent from a knowledge of the wind velocity. He must also avoid the temptation to pull the nose up to endeavour to glide further.

(d) Now consider the effect of the moments of the forces acting upon the aeroplane.

Tail Plane Design

Fig. 26 is similar to Fig. 3(d), but we have added dimensions to enable us to calculate moments, and there is a force on the tail. Neglecting this tail load for the time being, the third condition for equilibrium gives :—

$$La = Db,$$

by equating moments. But, L acts through the C.P. of the wing, and it was seen earlier that the position of the C.P. changes with the angle of attack. Therefore, a is variable, and we can only have equilibrium at one angle of attack ! It sounds serious, but this is where the tail plane comes in. It can be designed to produce the missing turning moment at all angles of attack by consideration of its area, aerofoil section and distance behind C.G.

Now Fig. 18(c) tells us that as the angle of attack increases, K_{OP} decreases, until just before the stall, therefore, a (Fig. 26) will get less. This will reduce the diving moment, the anti-clockwise arrow, and must be balanced by a down load on the tail. Similarly, in high-speed flight, at small angles of attack, there must be an up load on the tail. Tail planes are often made of symmetri-

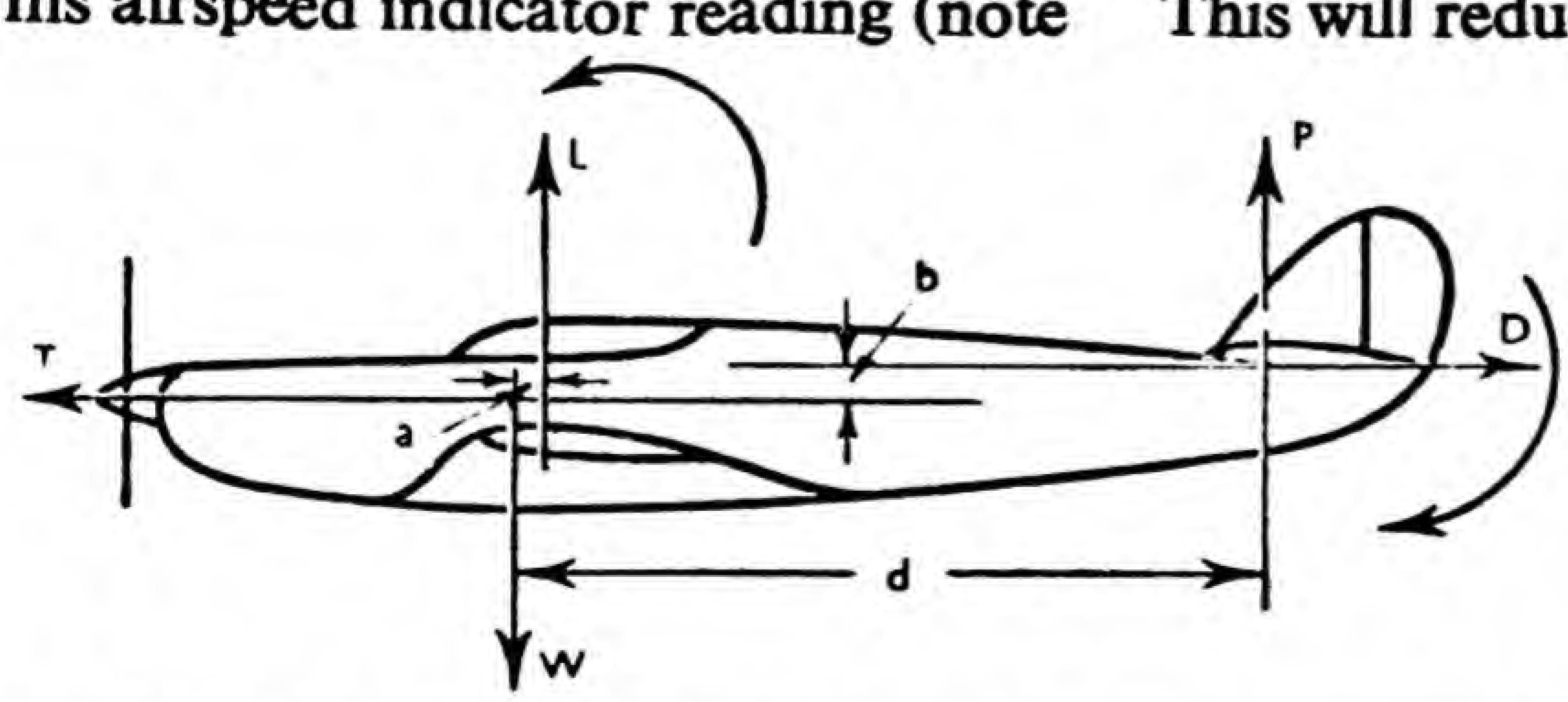


Fig. 26. This illustration shows steady horizontal flight with an up load on the tail.

cal section, which is the same camber top and bottom, to facilitate this reversal in the required direction of the load.

If the C.P. of the tail plane is d feet behind the C.G. of the aeroplane, as in the figure, there is :—

$$Db - La = Pd,$$

by taking moments about the C.G. P may be up or down, according as $Db - La$ is positive or negative. If we always draw P upward, we can rewrite the lift equation as

$$L + P = W.$$

This equation will suffice for all cases, because if P happens to be downward, a negative value can be substituted for it. We would like P to act upward on all occasions, because it would then add to the lift, but this cannot be managed at all speeds of flight. There are, in fact, cases in which it always acts downward. The flying boat is an interesting example of this.

In the aeroplane diagrams so far, D has been shown above T, and L behind W. This is the normal arrangement, because, if the engine cuts out, there will be an unbalanced diving moment, causing the nose to drop, until equilibrium is regained at the gliding angle corresponding to the particular angle of attack.

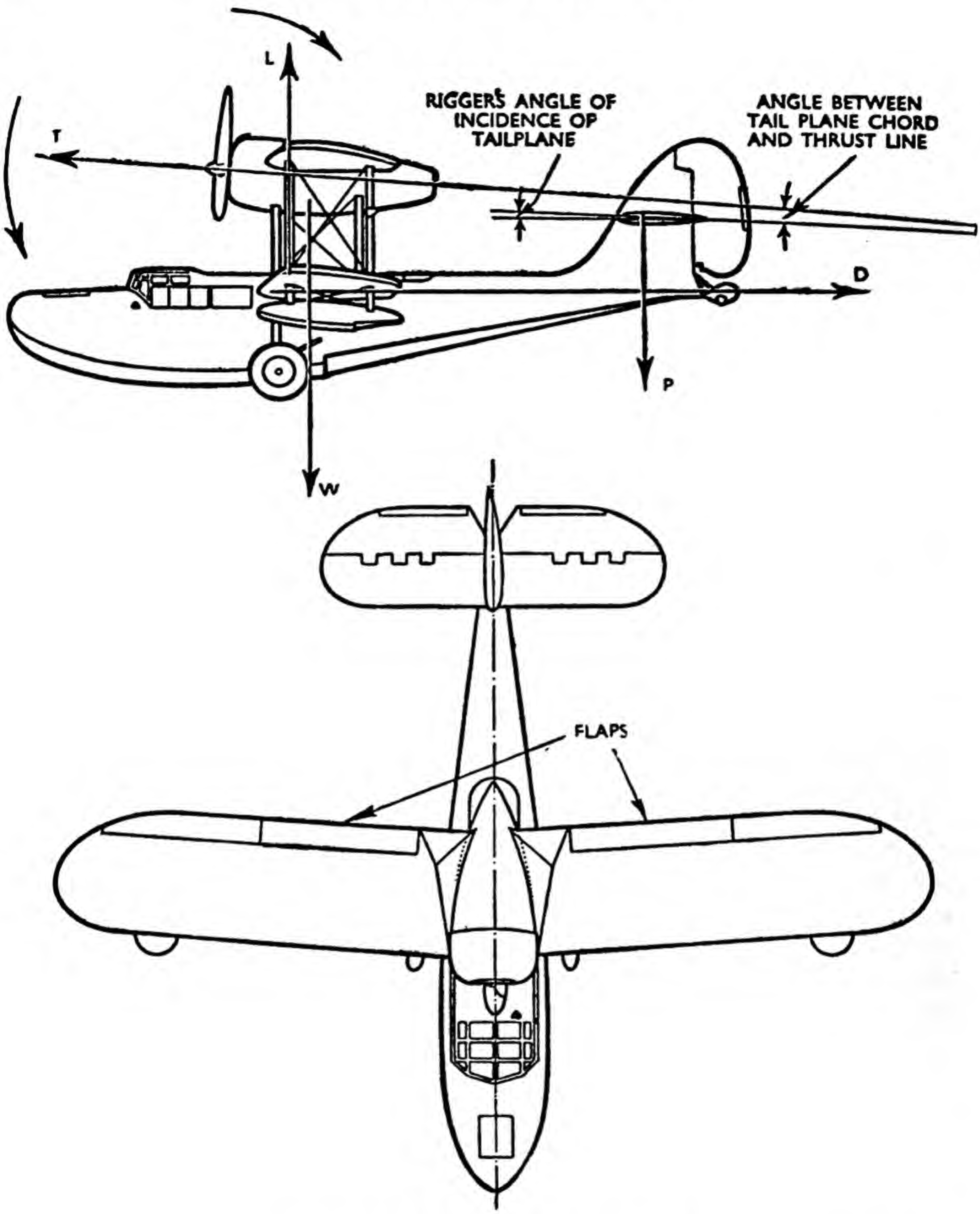
Now, with the high thrust line and low drag line necessary in flying boat design, the positions of L and W must be reversed to balance the moments (Fig. 27). Therefore, an increase in the angle of attack will now increase the stalling moment (the clockwise arrow in Fig. 27), and we shall require an up load on the tail at low speeds and a down load at high speeds.

But that is not all. If the engine cuts out, there will be an excess of stalling moment, and the flying boat will stall instead of assuming a gliding position. We overcome this difficulty by setting the tail plane (a) at a negative angle of attack, and, if necessary, inverting it, and (b) high up, so that it is in the slipstream from the airscrew. Condition (a) produces a down load at all angles of attack of the main planes, and condition (b) causes a big reduction in this down load if the engine cuts out. That this is so can be seen by supposing the velocity of the slipstream to be twice that of the flying boat. Then, if the slipstream disappears, the lift of the tail plane (the down load) will be quartered, since L varies as V^2. If the stalling moment thus lost is greater than the diving moment lost by the engine cutting out, a net diving moment will be left, causing the nose to go down as in the previous case. The downward tail load reduces somewhat the efficiency of the machine, but it is the best solution to an awkward problem.

Sea Otter's Tail Plane

The flying boat depicted in Fig. 27 is the Sea Otter. In this case the tail plane is set at a positive angle of attack, but the thrust line is inclined at a greater angle. Thus, as long as the engine is running, the slipstream meets the tailplane at a negative angle of attack, producing a down load; but, if the engine cuts out, the tailplane then meets the airflow at a positive angle of attack, so producing an upload, giving the diving moment necessary to replace that due to the thrust.

In the plan view, we see that the cross-section of the fin is unsymmetrical. This is to produce a



FEATURES OF THE SEA OTTER

Fig. 27. This amphibian flying boat exhibits several unusual features in order to balance the external forces. The thrust line is high because the engine must be kept clear of the water, and the line of drag is low because of the hull and floats. The C.G. must therefore be behind the C.P. of the wings, so the engine, which constitutes the major weight concentration, is placed well back. In this aircraft's predecessor, the Walrus, the engine was behind the main planes, driving a pusher airscrew. This was a further help in getting the C.G. further back. The tail plane is set high up to bring it into the slipstream. The bottom of the hull, as in all flying boats, is designed like that of a speed boat, to enable it to ride on the surface of the water when the speed attains a certain value. The step shown behind the undercarriage assists in this.

lift force to the right, to balance the leftward turning effect due to the rotation of the slipstream and the engine torque reaction.

Another balancing effect is indicated by the extensions of the elevators and rudder fore of their hinge lines. When the elevator or rudder is moved, the wind effect on these extensions reduces the pull on the control cable, making control easier for the pilot. The narrow strips on the rear of the rudder and elevators are trimming tabs. They can be adjusted from the cockpit to prevent pull on the control cables when flying normally, enabling the pilot to fly 'hands off'. Note also the flaps, which are discussed on page 400, and the sweepback of the wings which is discussed on page 426.

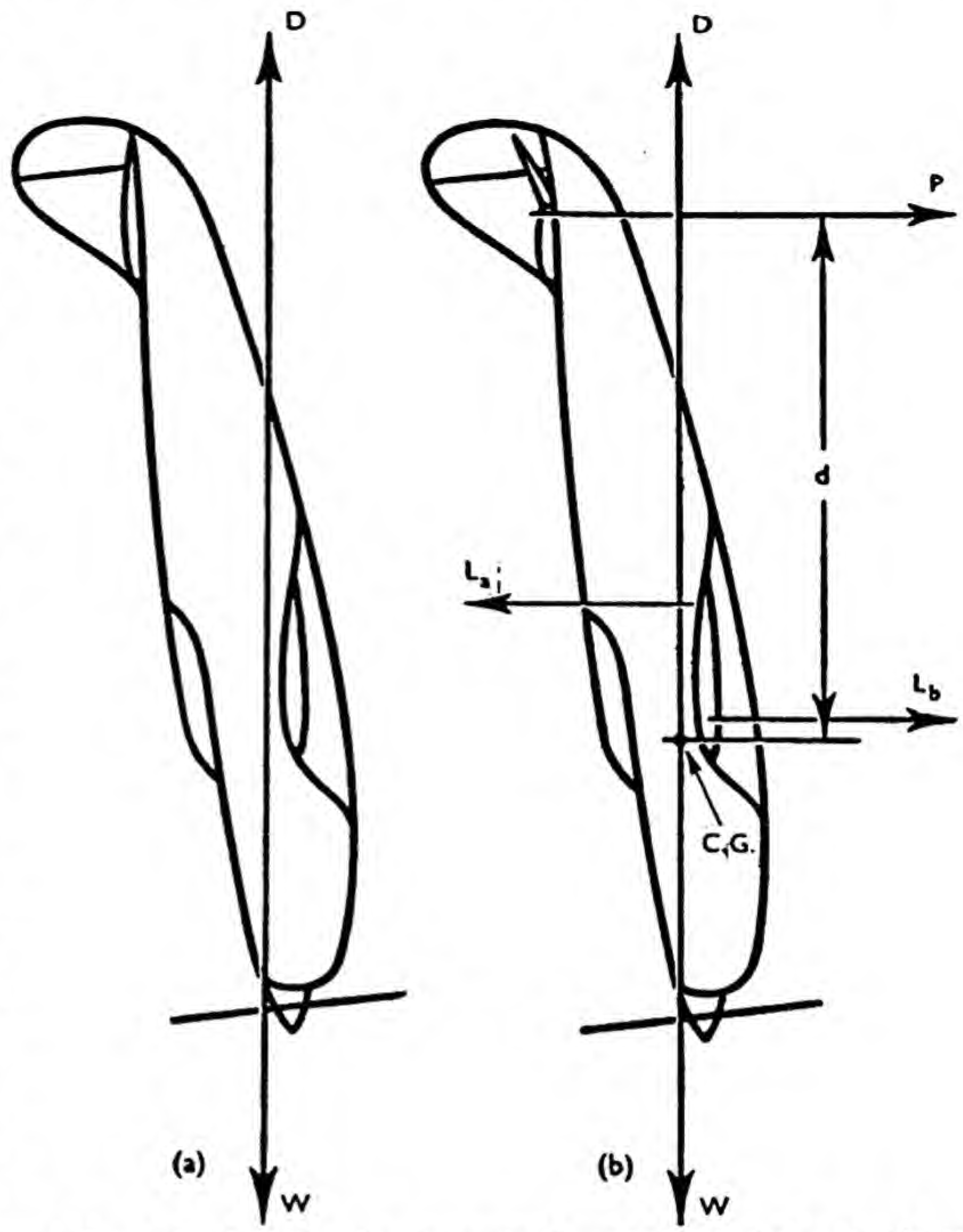


Fig. 28. With the terminal velocity dive, a down load on the tail is necessary to counteract the turning moment of the wing. Neglecting any moment which the drag may have about the centre of gravity, the following equations may be obtained :—

$$D = W \ldots\ldots (i) \qquad P = L_a - L_b \ldots\ldots (ii)$$

The wings are braced and strutted, since drag is not of paramount importance. However all the struts are streamlined, and so is the engine nacelle.

The Sea Otter is used largely for reconnaissance, its usefulness being enhanced by the provision of a retractable undercarriage, enabling it to operate over land or sea. Therefore, it is an amphibian flying boat.

The Sea Otter is a modernized version of the well-tried Walrus, and in view of its duties, its cruising and landing speeds are kept to the low values of 100 m.p.h. and 55 m.p.h. respectively. The wing loading is only 15 lb. per sq. ft. It is designed to operate at a height of 3,500 ft., when its Bristol Hercules radial air-cooled engine developes 740 h.p. at its cruising speed, and 855 h.p. at its maximum speed of 150 m.p.h. The maximum rate of climb is 870 ft. per min. It has a wing span of 46 ft. and a wing area of 610 sq. ft. Its weight, loaded, is 9,150 lb.

(e) It has been seen that, if the angle of attack while gliding is steadily decreased, the velocity will be increased and the $\frac{L}{D}$ ratio will be decreased, thus increasing the gliding angle. Imagine this continued until the small negative angle of attack is reached at which the lift is zero. $\frac{L}{D}$ will be zero, and we shall be descending vertically, as in Fig. 28 (a), with the drag equal to the weight. The velocity, known as the terminal velocity, may be found by equating the drag to the weight, viz.,

$$D = W = C_D . \frac{1}{2} \rho V^2 . S$$

Therefore, $V = \sqrt{\frac{2W}{C_D \rho S}}$

Since W is much greater than the thrust obtained from the airscrew, it would not make much difference if the throttle were open. It might even reduce the velocity, since the airscrew would usually be inefficient at this high speed, and might actually cause more drag than if merely windmilling.

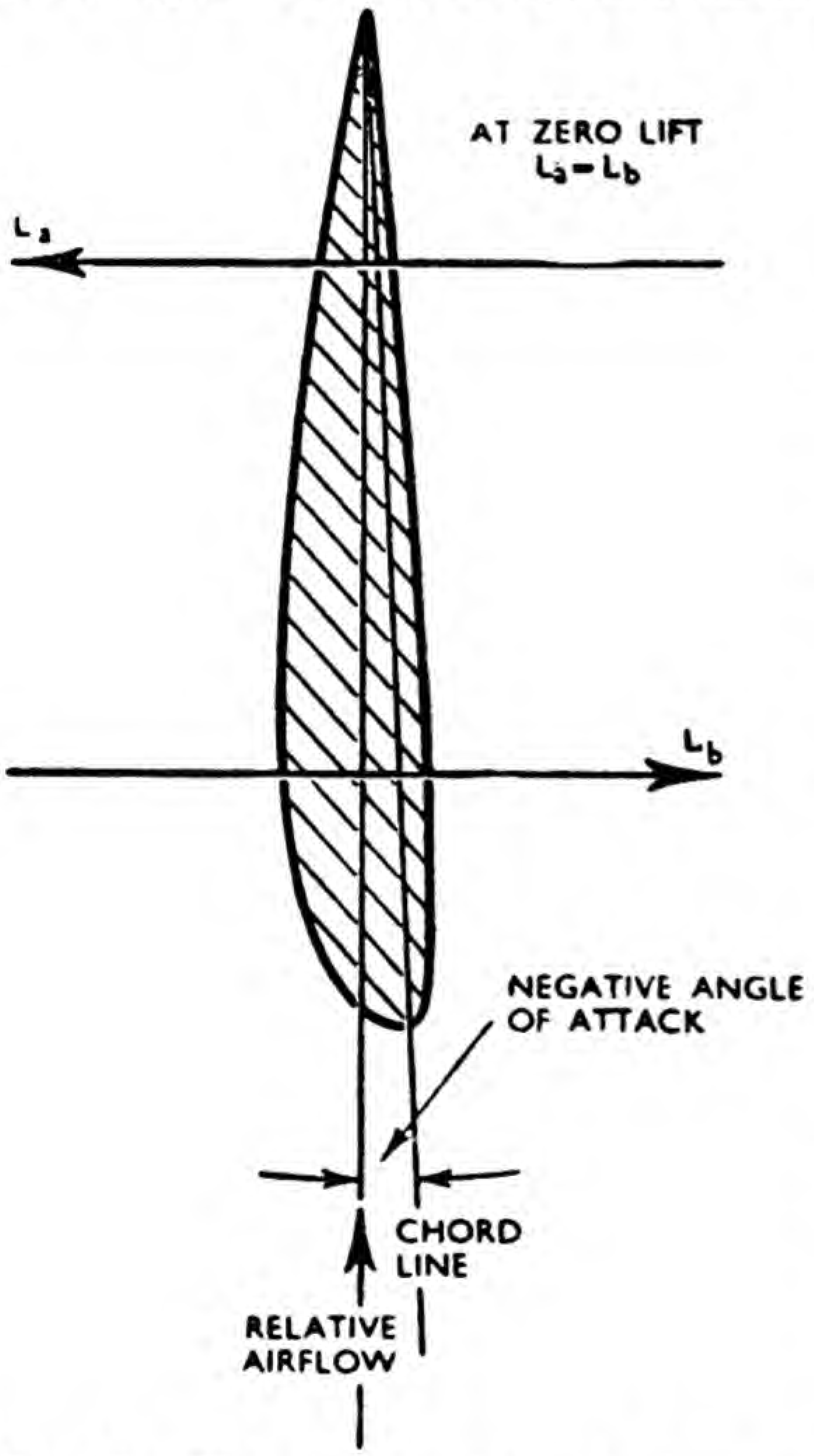


Fig. 29. This diagram shows an aerofoil section which is giving no lift. The angle of attack is negative (—2 deg. in the case of R.A.F. 15, as shown in Fig. 18(*a*)). There is, however, an aerodynamic reaction, which is equivalent to two equal and opposite parallel forces L_a and L_b, acting perpendicularly to the air flow. These form a couple tending to turn the nose over.

Effect of C.P. Movement

But, this is not the whole problem. We have omitted one important point, the movement of the C.P. Fig. 18(c) tells us that, as the angle of attack is decreased, the C.P. moves back. It actually goes back beyond the trailing edge, and, as we approach zero lift, it rushes off to infinity. This is a mathematical conception of two equal and opposite forces acting along parallel lines, viz., a couple.

What has really happened is that much of the negative pressure on the upper surface of the aerofoil has been wiped out, particularly near the leading edge, and that the positive pressure on the underside has become more effective towards the trailing edge, resulting in the conditions shown in Fig. 29, when L_a is made equal to L_b. Thus, although there is no lift, we cannot say that the aerofoil produces no effect. It produces a diving couple which would turn the nose over. Once again, the tail plane affords the solution. A down load is applied on the tail, by raising the elevators, as shown in Fig. 28(b), so that its moment, *Pd*, is equal and

opposite to the diving couple of the wing.

We shall not then be flying quite at the angle of zero lift, but at some smaller negative angle of attack at which $L = L_a - L_b = P$. This becomes obvious if we resolve horizontally, giving,

$$L_a = L_b + P,$$

Therefore,

$$L_a - L_b = P.$$

We are still in equilibrium, although we are approaching the earth at several hundred miles per hour. If the conception of equilibrium outlined previously is adopted we shall remain in equilibrium whether a recovery is made from the terminal velocity dive or not, but whatever it is called, all the forces involved will be balanced, and we find that it is still possible to apply the conditions of equilibrium.

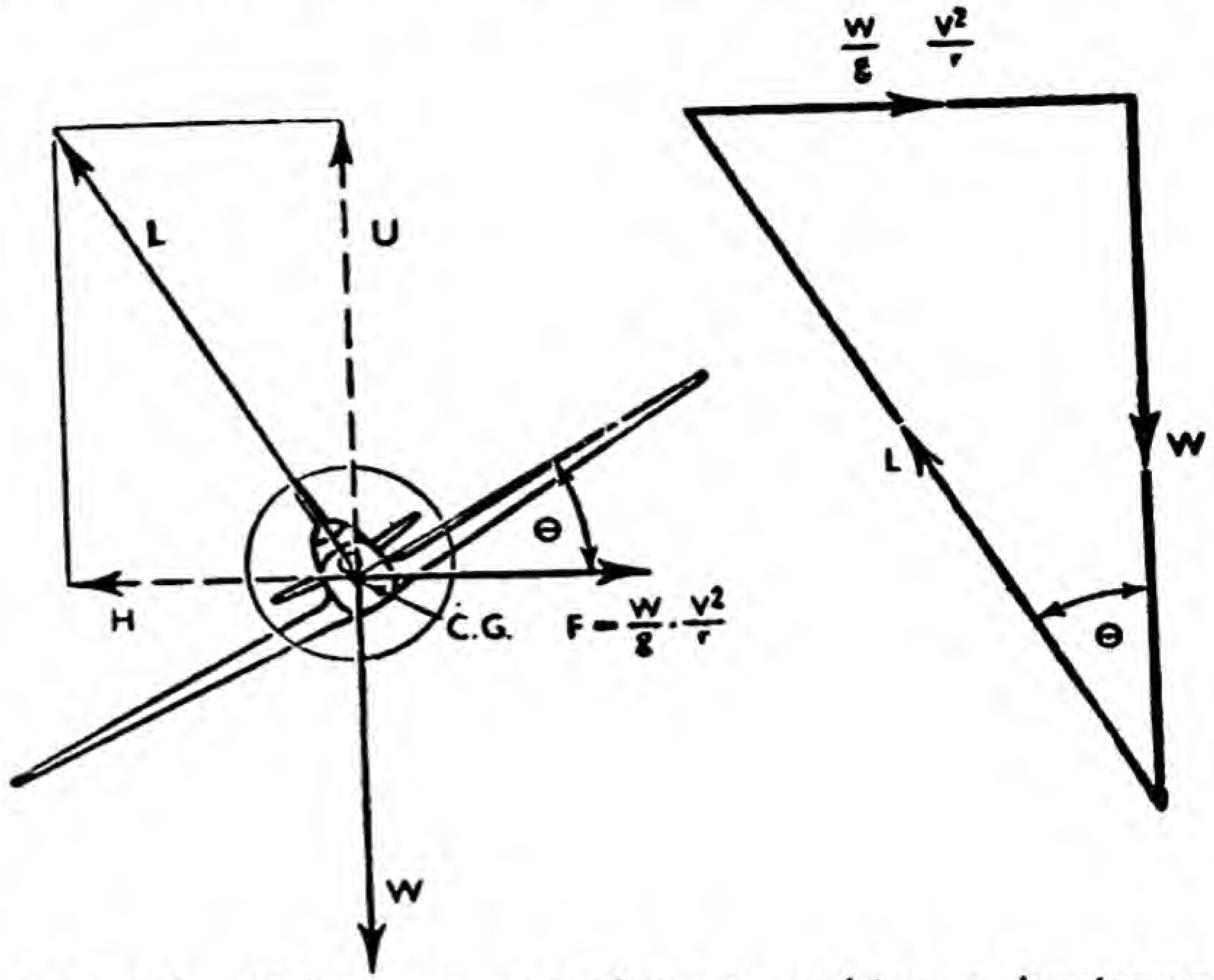


Fig. 30. When an aeroplane is making a horizontal turn, the lift is inclined inward, and an inertia force F, the centrifugal force, acts outward. The condition of equilibrium is that F, L and W make a closed triangle. By resolving the lift into its vertical and horizontal components, we see that $U = W$, and $H = F$. H is the centripetal force, and is the inward force, exerted by the wings to counteract the outward force due to dislike of the mass of the aeroplane, to change its direction of motion. From the force diagram we see that the angle of bank is given by $\tan \theta = \frac{V^2}{rg}$.

Changing the Velocity

The accelerations with which we are most familiar take place in the direction of motion, for example, the acceleration or deceleration of a car, by pressing either the accelerator or the brake pedal; but with aeroplanes, these accelerations are relatively unimportant. This arises from the fact that the lift forces are usually about ten times as great as the drag forces, and it is thus easier to produce greater changes in the force perpendicular to the flight path. Again it is the $\frac{L}{D}$ ratio.

In the next few pages accelerations along the path of flight will be neglected by regarding the speed as constant whilst changing the velocity. The term velocity includes direction as well as speed, so we are going to change our direction of motion. In other words, we shall accelerate in a direction perpendicular to the path of flight, either horizontally, as in turning, or vertically, as in pulling out of a dive. In each case there will be an inertia force, proportional to the mass, acting in the opposite direction to that of the acceleration.

Now, when a body moves along a circular path, there is an acceleration-

tion towards the centre of the circle (Chapter 4). There will thus be an inertia force F, the centrifugal force, acting radially outward. This was shown to be equal to $\frac{W}{g} \times \frac{V^2}{r}$, r being the radius of the turning circle. In horizontal flight this force is balanced by inclining the wings inward so that the lift force is inclined inward, as in Fig. 30. The manœuvre is called banking, and the inclination of the wings to the horizontal θ, is known as the angle of bank. An expression for the angle of bank may be obtained from the force diagram in Fig. 30. Thus,

$$\tan \theta = \frac{\frac{W}{g} \cdot \frac{V^2}{r}}{W}$$

W cancels out, simplifying the expression to,

$$\tan \theta = \frac{V^2}{rg}.$$

This formula shows that angle of bank is independent of the weight, and depends only upon the speed of flight, and the radius of turn. This fact, of course, also applies to cycling, and the banking of roads, racing and railway tracks. A railway track is banked for a particular speed. If a train exceeds this speed we feel an outward pull, and if the speed is too low, or zero, as when stopping in a station on a curve, the resultant force will be inward.

Effect on Lift

If we now resolve the lift into the dotted horizontal and vertical components H and U, shown in the figure, it is found (a) that the horizontal component is equal to the inertia force, $\frac{W}{g} \times \frac{V^2}{r}$, viz., the horizontal component of the lift provides the centripetal force to balance the centrifugal force, or

$$L \sin \theta = \frac{W}{g} \cdot \frac{V^2}{r},$$

and (b) that the vertical component of the lift is equal to the weight, or,

$$L \cos \theta = W$$

Thus, $$L = \frac{W}{\cos \theta},$$

showing that the lift is greater than the weight.

Now,

$$L = C_L \cdot \frac{1}{2} \rho V^2 . S,$$

and, since the velocity is being kept constant, C_L must be increased and, therefore, the angle of attack, to make the right-hand side of the equation equal to the increased value of L on the left-hand side. At the same time the throttle must be opened a little, because there will be a small increase in C_D in the formula,

$$T = D = C_D \cdot \frac{1}{2} \rho V^2 . S.$$

Conversely, if we retain the original angle of attack, the lift formula shows that we must increase V, and must, of course, open the throttle more because of the increase in V in the drag formula. This is similar to the previous argument about weight and speed, viz., all speeds, including the stalling speed, corresponding to a given angle of attack will go up. Remember that this will apply to all manœuvres which increase the loading, viz., if L is greater than W.

If, during normal flight, the angle of attack, and therefore C_L, is suddenly increased there will be a sudden increase in lift, because the aeroplane will temporarily retain its original velocity due to its inertia, leaving us with C_L and L

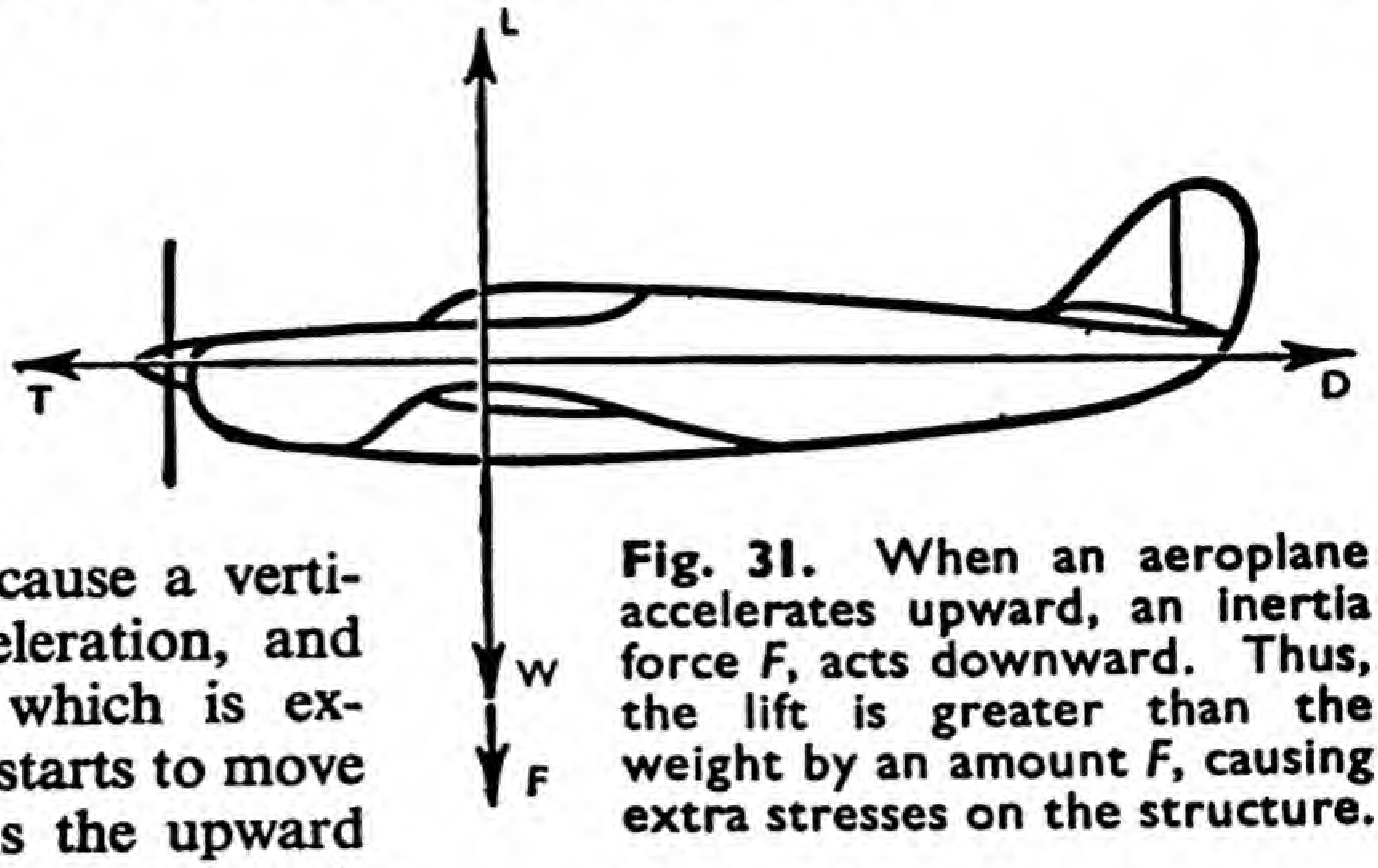


Fig. 31. When an aeroplane accelerates upward, an inertia force F, acts downward. Thus, the lift is greater than the weight by an amount F, causing extra stresses on the structure.

as the only variables in the lift formula. This will give rise to a vertically upward acceleration, because L will be greater than W. Similarly, a sudden decrease in the angle of attack will cause a vertically downward acceleration, and *that sinking feeling* which is experienced when a lift starts to move downward. But it is the upward accelerations which are most usual, and most important, because the resulting inertia forces act in the same direction as the weight, and they impose high stresses, both on the pilot and the machine.

This case is exemplified in Fig. 31, which shows an aeroplane accelerating upward. In this and the succeeding diagrams, for the sake of simplicity, all forces are shown acting through one point. This makes no difference to the principles illustrated.

The lift L is opposed by the weight W, and the inertia force F. Hence,

$$F = L - W.$$

The accelerating force is $L - W$, and the resulting acceleration in its direction of action (upward) is proportional to it. Thus, if $L = 2W$, the accelerating force is $2W - W$, which equals W. Now, if a mass of weight W lb. is acted upon by a force of W lb., there will be an acceleration of 32 ft. per sec. per sec., viz., g ft. per sec. per sec., so the upward acceleration will be the same as the downward acceleration produced in a body falling freely. But, the total force acting on each particle of mass will be the sum of that due to its weight, and that caused by its inertia. Thus, in the case considered, each particle of mass of the aeroplane will weigh twice its normal weight, whilst its upward acceleration will be 32 ft. per sec. per sec.

Informative Experiment

The following experiment will clarify this, and is well worth carrying out.

Suspend a 7-lb. weight from a spring balance held in the hand, and jerk the hand upward, noting the balance reading. It is easily possible to obtain a reading of 28 lb. By comparison with Fig. 31, this is equivalent to a lift of $4W$, which is opposed by the weight W, and an inertia force $F = 3W$. Hence the upward acceleration is $3 \times g = 3 \times 32 = 96$ ft. per sec. per sec.

It is more convenient to think of these upward accelerations in terms of the unit g ft. per sec. per sec., instead of the usual unit of 1 ft. per sec. per sec. By doing this we can refer to the acceleration given to the weight in the above experiment as $3g$. The question is sometimes asked :—How much g can a pilot stand? The answer is that he will black out at 4 to $5g$, depending on his physique, age and state of health.

Blacking out is caused by the

inability of the heart to keep blood of this additional weight supplied to the optic nerve, because at $4g$, each part of the pilot's body will weigh five times its normal weight. The plane must be designed to withstand the stresses due to this apparent extra weight of all its parts, but it is obviously futile to design it to withstand much more g than the pilot can endure. The ratio of the loading during manœuvre to the normal loading, viz., $\frac{L}{W}$ in this case, is termed the load factor, a very important factor in aircraft design.

Zooming and Diving

When this manœuvre is performed from horizontal flight it is known as a zoom. Height continues to be gained until the velocity in the direction of flight is reduced to that corresponding to the new angle of attack, during which the vertical component of the inertia force does work against gravity.

The most important case, however, occurs when pulling out of a dive. Here, the flight path may be considered as an arc of a vertical circle. There will be an inertia force F, acting radially outward (downward, to the pilot), as shown in Fig. 32(a), and this will be balanced by the difference between the lift L, and the component of W acting in the direction of F. It is obviously simpler to consider the problem at the end of the pull out, as in Fig. 32(b), which is the same as Fig. 31. Since the inertia force and the weight will then be acting in the same direction, the stresses in the structure will be a maximum, which is the condition that is required for design purposes.

Loads During Loops

Similar loads occur during a loop, shown in Fig. 33. If we assume the flight path to be a vertical circular arc, the inertia (centrifugal) force F, acting radially outward, will be constant throughout, but the actual flight path is usually similar to that drawn, giving a variable inertia force. In the extreme cases shown in the figure, the weight either directly helps or opposes this force.

The accelerating (centripetal) force is provided, in case (a), by $L - W$, and, in case (b), by $L + W$. If F were less than W at the top of the loop, case (b), it would be necessary to fly at a negative angle of attack, giving negative lift. It is only in this eventuality that the pilot would fall out if not strapped

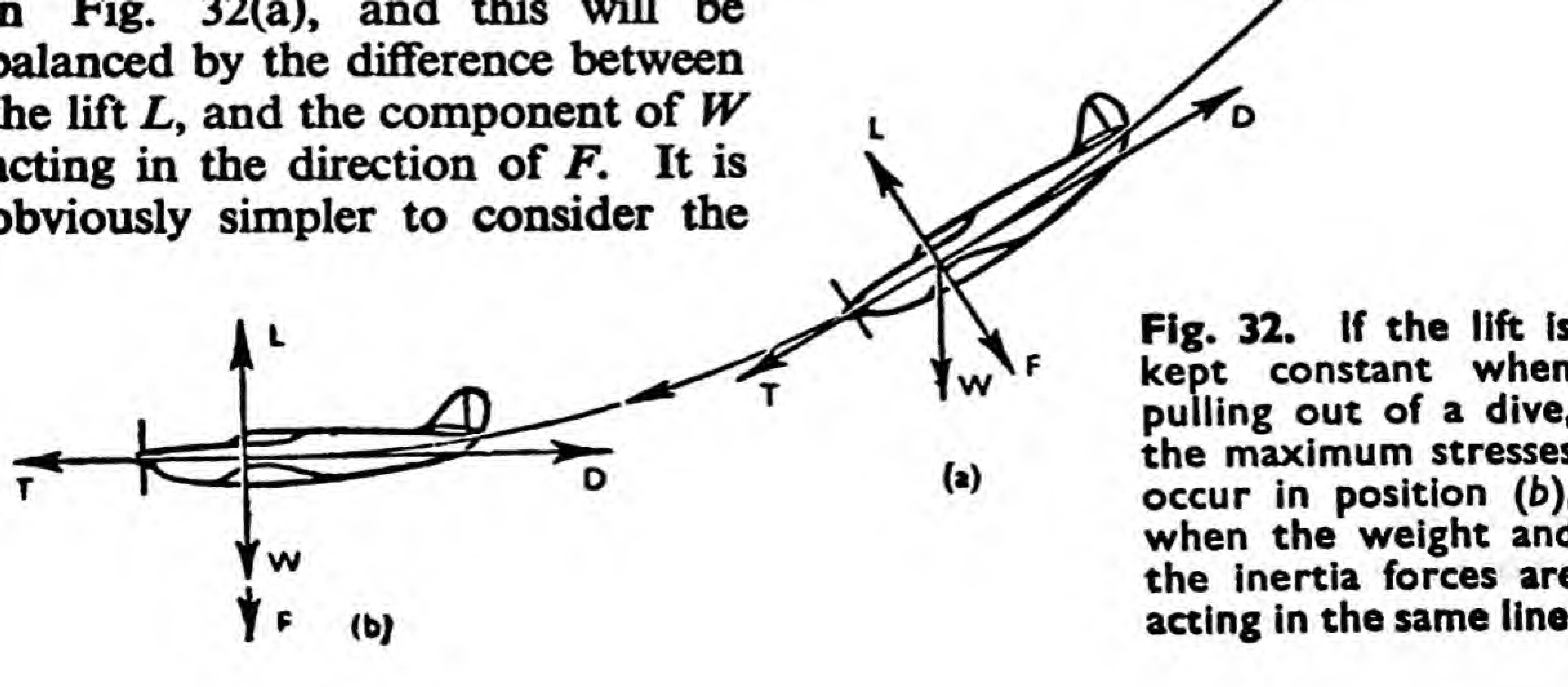
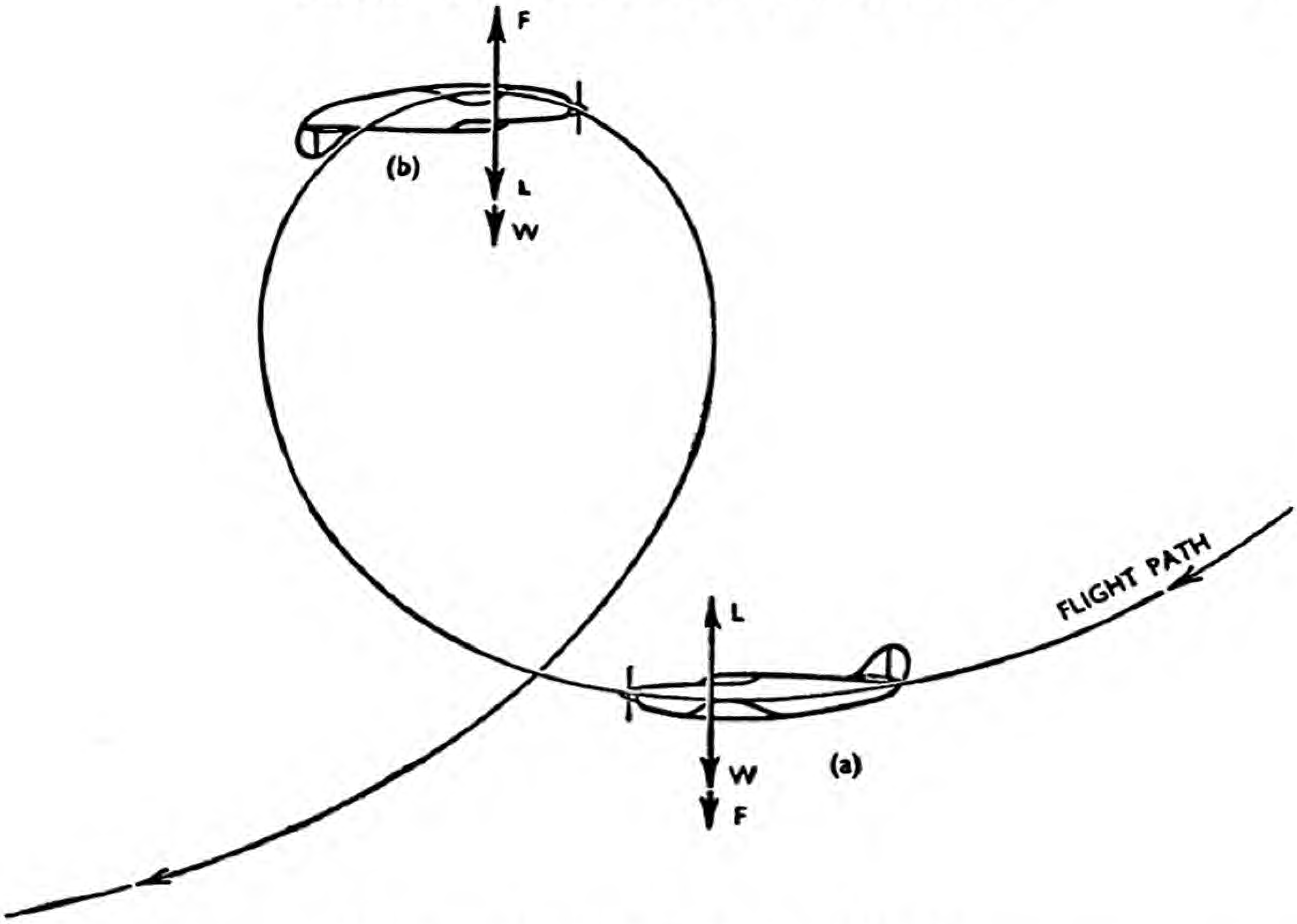


Fig. 32. If the lift is kept constant when pulling out of a dive, the maximum stresses occur in position (*b*), when the weight and the inertia forces are acting in the same line.



FORCES ACTING DURING A LOOP

Fig. 33. (*a*) The aeroplane is shown at the bottom of the loop, and (*b*) at the top. In each case the forces on the aeroplane are balanced in all directions. Considering the forces perpendicular to the flight path, we obtain (*a*) $L = F + W$ and (*b*) $F = L + W$, which may be written $L = F - W$. If F is less than W at the top, the lift will be negative, that is, acting downward relative to the pilot. The pilot would then fall out if he were not strapped in.

in. Of course, he usually sees that the velocity is great enough, and the vertical radius of turn small enough to prevent this, even if he does not know that,

$$F = \frac{W}{g} \cdot \frac{V^2}{r}.$$

Shetland Flying Boat

The following particulars regarding the Short Shetland Flying Boat shown in Fig. 35 will form an interesting contrast with those of the Tiger Moth and the Sea Otter, especially if we attempt the calculations suggested on page 427.

Wing span ..	150 ft.
Length ..	110 ft.
Wing area ..	2,636 sq. ft.
Weight loaded	130,000 lb.
Wing Loading	49·3 lb. per sq. ft.
Maximum speed ..	267 m.p.h. at 4,000 ft.
Stalling speed	80 m.p.h.
Maximum rate of climb	660 ft. per min. from sea level.
Landing speed	91 m.p.h. with all up weight of 85,000 lb.
Take-off power per engine	2,400 b.h.p. at 2,700 rev. per min.
B.h.p. per engine ..	1,250 at 52 per cent take-off power with 130,000 lb.

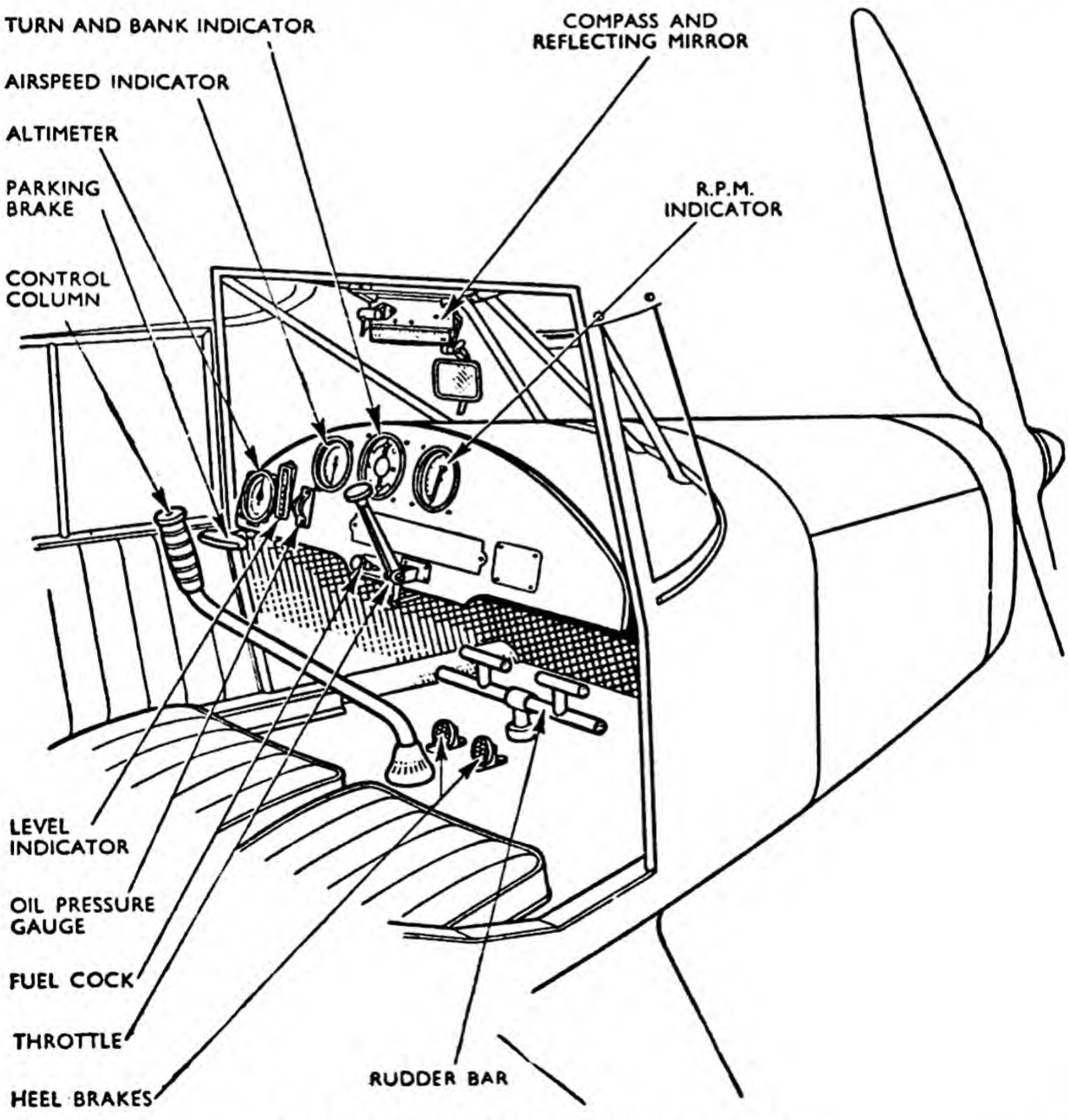
This machine is able to carry 70

passengers 4,650 miles in one hop at a cruising speed of 184 m.p.h.

Control

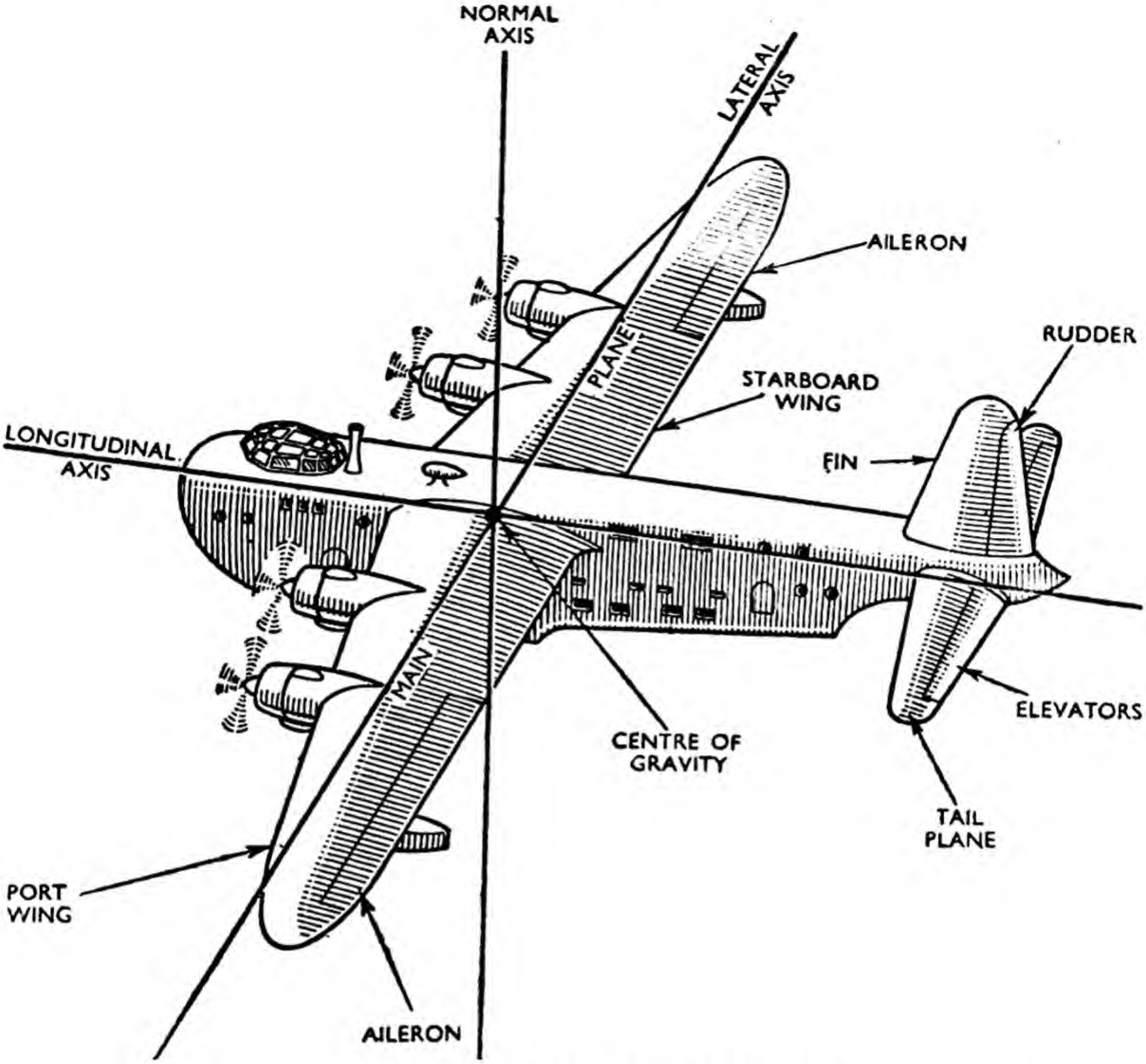
A good deal has been said about altering the angle of attack, and inclining the wings, and so on, but we have not discussed how the pilot does it. Now, he has three main controls in his cockpit, the throttle lever, the control column, and the rudder bar. These are shown in Fig. 34. The throttle lever, of course, merely alters engine power, but the other two affect the controlling surfaces, shown in Fig. 35, viz., the ailerons, the elevators and the rudder.

The instruments shown in the figure are also indispensable to the pilot; in fact, pilots are now trained to fly by instruments alone. The r.p.m. indicator tells him just how the engine is responding to



COCKPIT OF A LIGHT AEROPLANE

Fig. 34. Chief controls illustrated are the control column which moves the ailerons and elevators, the rudder bar which moves the rudder, and the throttle lever which controls the engine power.



CONTROL SURFACES AND AXES

Fig. 35. Movement of the ailerons, elevators and rudder produces rotation about the longitudinal, lateral and normal axes respectively, and these motions are respectively called rolling, pitching and yawing. The machine depicted is the Short Shetland Flying Boat, the first post-war machine to be designed specially for commercial purposes.

the throttle lever, and the oil-pressure gauge indicates its condition. The upper needle of the turn-and-bank indicator is deflected if the aeroplane sideslips, so the pilot's aim is usually to keep this central. The lower needle shows him the rate of turn. If this is deflected as far as the figure 3 on the dial, a rate 3 turn, it is practically impossible to stand up in the aeroplane owing to the centrifugal force. Normal turns would not exceed rate 2. We find that the most important instrument is the air-speed indicator, because the pilot uses it to judge the attitude of the aeroplane. The level indicator is merely a spirit level, and is used to judge angle of climb or descent, or to maintain level flight. The altimeter is really a barometer, suitably marked in feet, because pressure decreases at a fixed rate as we go up. The heel brakes are used to pull up the aeroplane on the landing run, or to turn it when taxying, and the parking brake

acts like the hand brake of a motor car. Lastly, the use of the compass is obvious. The readings are laterally inverted, so that they can be read in the mirror.

Larger aeroplanes have more instruments still, but familiarity breeds contempt, or, at least, it prevents bewilderment.

The simplest type of control column is a vertical stick mounted on a universal joint, and connected near its base to the ailerons and elevators by means of control cables. A movement to the right raises the right-hand or starboard aileron, and lowers the left-hand or port aileron, and *vice versa*, whilst a forward motion lowers the elevators and a backward motion raises them. The rudder bar is a transverse, centrally pivoted lever, with stirrups at its extremities for the feet. If the left foot is pushed forward, the rudder is turned to the left, and *vice versa*.

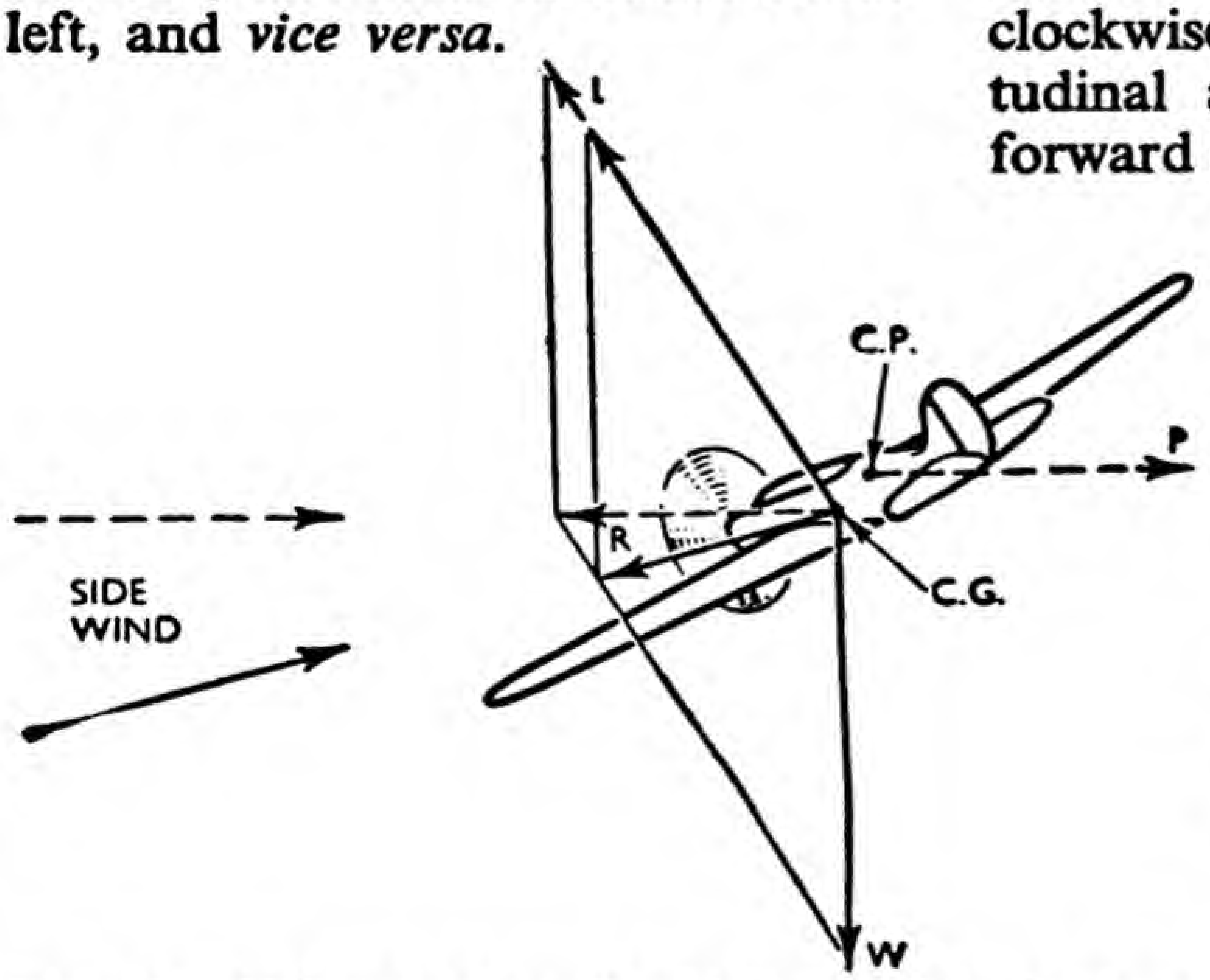


Fig. 36. When an aeroplane is banked by the use of the ailerons alone, the resultant of the inclined lift and the weight causes a sideslip coupled with loss of height. If the lift is increased, the sideslip may be made horizontal. The dotted arrow *R* is the force causing this sideslip, and it is counteracted by an equal and opposite force *P*—the force due to the sideslip wind. This acts behind the C.G., thereby turning the aeroplane towards the direction of sideslip.

Reference to Fig. 35 will reveal that each control surface is hinged behind a fixed aerofoil surface. Thus, a movement of a control surface will change the effective camber of the respective aerofoil, and alter its lift, so producing an aerodynamic force perpendicular to that surface.

Control Surfaces

Now, flight is in three dimensions, so it will be best to consider the effects produced by reference to three mutually perpendicular axes. These axes of reference, the body axes of an aeroplane, are marked on Fig. 35. They all pass through the C.G., and each control surface causes rotation about one of them.

We can now reason out that a movement of the control column to the right will decrease the lift on the starboard wing, and increase that on the port wing, producing clockwise rotation about the longitudinal axis, and *vice versa*. A forward movement will give an up load on the tail, causing the nose to go down, viz., rotation about the lateral axis, and *vice versa*. And a push with the left foot will produce a force on the tail towards the right, causing anticlockwise rotation about the normal axis, and *vice versa*. These movements are termed rolling, pitching, and yawing respectively. It is considered that the directions of movement of the controls are the instinctive ones to produce the given movements of the aeroplane, although a beginner is

sometimes inclined to think that his instinct is not instinctive.

Each control surface should be placed as far from the C.G. as possible (Fig. 35) so that the force produced by its alteration in position will give the greatest rotational effect. If the C.G. is kept well forward, the leverage exerted by the tail plane and rudder will be increased. When considering stability, it will be seen that there are other reasons for a forward position of the C.G.

Now, the immediate effect of moving any control surface is merely to rotate the aeroplane about the C.G. on one of the three body axes, but the C.G. will continue to move along its original path until aerodynamic forces, brought about by the rotation, come into play to alter its direction of motion.

First, consider the effect of moving the elevators alone. Suppose that the movement is gradual, so that inertia forces can be neglected. As pointed out previously, the up or down tail load produced pushes the nose down or up, altering the angle of attack. Now, this does not directly cause the aeroplane to ascend or descend. As we know, it merely alters the speed, but we can only overcome the increased drag associated with increased speed by opening the throttle, or by descending so that a component of the weight acts in the direction of flight, as in Fig. 23(b).

Thus, supposing we are flying at cruising speed, a forward movement of the control column will cause descent at increased speed, because C_L will be reduced ($L = W = C_L . \frac{1}{2} \rho V^2 . S$, with C_L and V^2 as the variables). A backward movement of the stick, by increasing C_L, will reduce the velocity, and will eventually cause descent, because of the large increase in C_D at high angles of attack ($P = DV = C_D . \frac{1}{2} V^3 . S$—power, not thrust, is constant). We could, however, maintain level flight, or even ascend, by opening the throttle.

It will be observed that we are merely applying the arguments of our discussion on speed and power; therefore, we will sum up by saying that although the elevators and throttle may be used in conjunction with each other, elevator control is primarily connected with speed, and throttle control with angle of flight. It is one of the paradoxes of flight.

Making a Level Turn

Now try the effect of moving the stick sideways. If moved to the left the resulting anticlockwise roll will incline the lift to the left. Fig. 36 shows that this will produce a resultant force towards the left, and inclined downward, giving movement in that direction. We can make this resultant force horizontal, and thereby prevent loss of height, by increasing the lift as the dotted arrows show.

This is, of course, done by opening the throttle, when the turn will be executed without change of speed, or by pulling the stick back, when it will be executed at a lower speed, although, if we were originally flying at cruising speed, the throttle would have to be opened in any case, to maintain level flight. Note that the increased lift required confirms the findings about turning in a horizontal circle.

We shall now be sideslipping

inward, because nothing has happened to change the direction in which the nose is pointing. Thus there will be a component of the wind, *P* in Fig. 36, acting through the centre of pressure of the side surfaces in the opposite direction to the sideslip. Now, the designer ensures that the C.P. of the side surfaces is behind the C.G., by providing a large fin as far back as he can get it. Therefore the side wind will cause the aeroplane to yaw to port, because of the turning effect of the force *P*.

Next, return to normal flight, and push the left foot forward. This produces a force on the rudder which will rotate the aeroplane about its normal axis in an anticlockwise direction, a yaw; but no force has yet come into play to change the direction of motion of the aeroplane. We are simply moving along crabwise with the nose pointing to the left as in Fig. 37. The wind will thus strike us from the right; we are sideslipping outward, or skidding.

To correct this, the designer ensures that the centre of pressure of the side surfaces is above the C.G. by providing a high fin, so that the relative wind due to the skid will roll the aeroplane in an anticlockwise direction. This will incline the lift to the left, giving the resultant force necessary to turn the aeroplane in that direction. Fig. 37 will make this clear.

The motions discussed above are complementary, and the last few paragraphs should be reread to drive home the conclusions, first, that a correctly banked turn, one without sideslip, can only be executed by using both rudder and ailerons, and second, that, in a correctly designed aeroplane, a roll produces a yaw, and a yaw produces a roll. However, it should be added that modern aeroplanes require very little rudder and can be turned successfully on the ailerons only.

New Factors

We may now think that we know how to turn; but we have only just started, because new factors

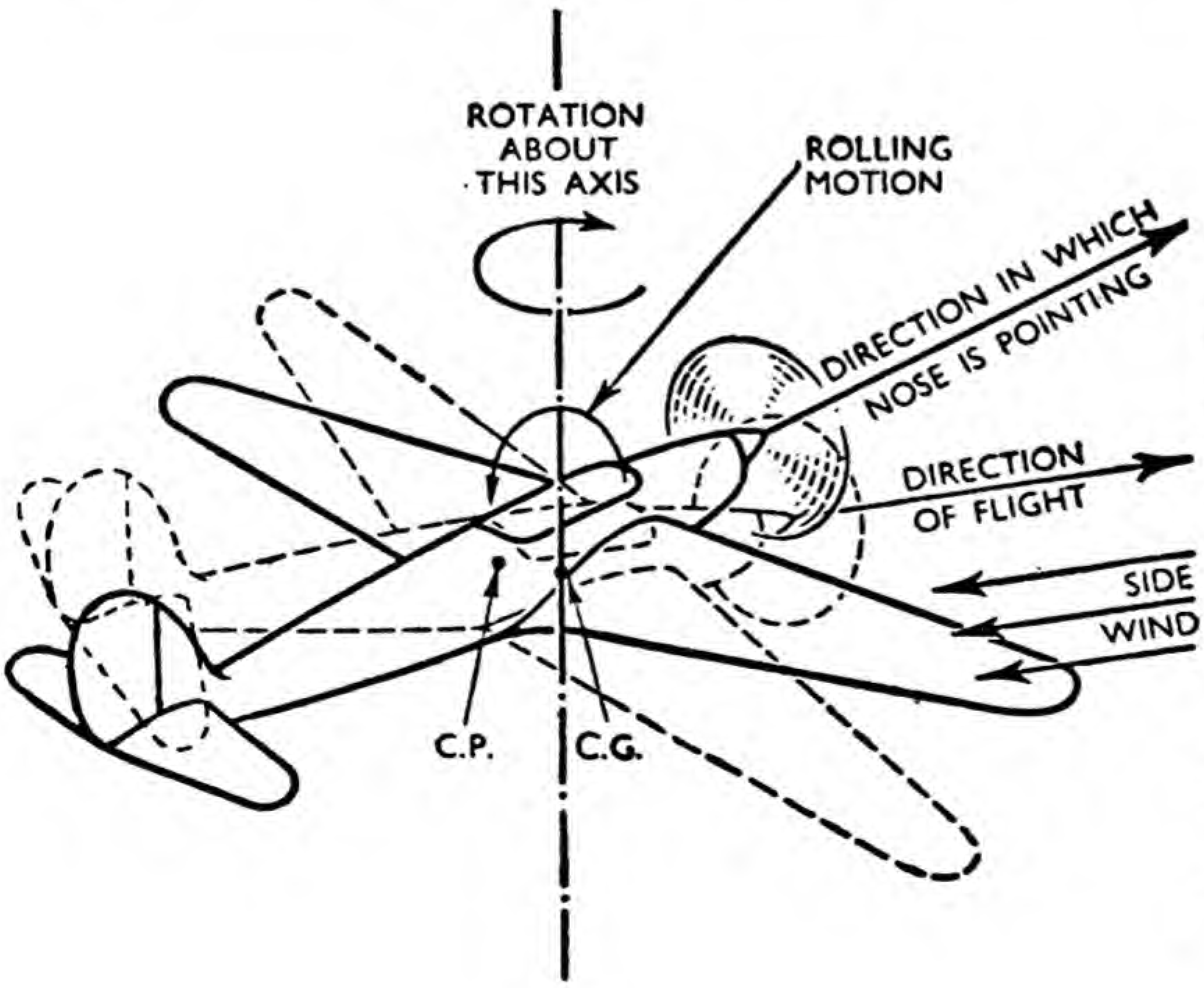
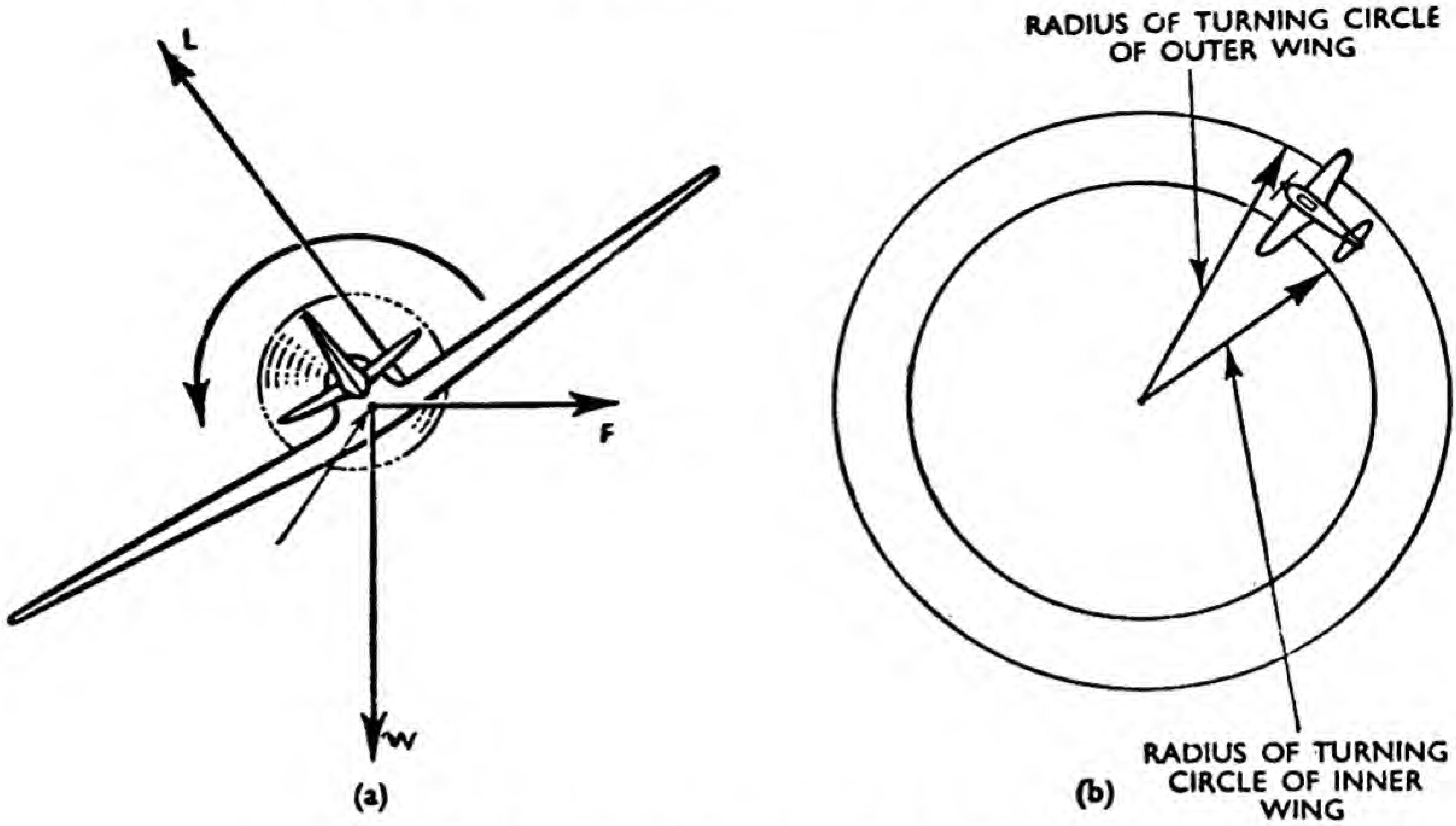


Fig. 37. Dotted lines show the aeroplane in its original position. When left rudder is applied there is an immediate left yaw, but the direction of flight continues as before, so there is a side wind. This strikes the side of the fuselage, and since the C.P. of the side surface is above the C.G., a roll is caused in the direction of the lower curved arrow.



EFFECT OF MOVING THE STICK

Fig. 38. (*a*) When the stick is held to the left, the right wing gives more lift than the left one, displacing the resultant lift force to the right. This causes a turning moment which will continue to rotate the aeroplane about its longitudinal axis until the stick is centralized. (*b*) Even after the stick has been centralized, the right wing continues to give more lift because it is moving faster, so we must now hold the stick slightly to the right.

arise owing to the differential action of the air on the wings.

Fig. 36 shows the line of action of the lift force passing through the C.G. ; but the outer wing has been given more lift than the inner one by means of the ailerons, so the line of action of the resultant lift of both wings will be displaced outward, causing a turning moment about the C.G. as shown in Fig. 38(a). Therefore, the angle of bank will continue to increase, under this turning tendency, until the lift force is again centralized by returning the stick to its middle position.

But that is not all. The outer wing is now travelling along the circumference of a circle of larger radius than the inner one (Fig. 38(b)), and, therefore, is travelling faster, and so is still producing more lift. This is neutralized by actually pushing the stick outward, known as holding off bank.

Climbing and Gliding Turns

But this is not yet finished! We have only executed a level turn. Let us suppose, then, that height is being lost during the turn. We shall then be travelling along a curve known as a helix, like going down a helter-skelter. Now, thinking of the spiral staircase of an old church tower, it will be recollected that the steps are much steeper on the inside than on the outside. In just the same way, the inner wing of the aeroplane will be taking a steeper course than the outer one, as Fig. 39 indicates. Therefore, the air will meet it at a greater angle of attack, and it will produce more lift than the outer one, unless it stalls. In this case, the wing will drop, and the aero-

plane will go into a spin! We shall, of course, keep the speed high enough to prevent a stall, but this extra lift on the inner wing must be reduced by holding the stick inward, holding on bank, as it is known.

In a climbing turn the reverse will be the case, and it will be doubly necessary to hold off bank.

The alteration in the angle of attack of one wing relative to the other may, at first, seem impossible, but a close study of Fig. 41, which shows the differential alteration in angle of attack during a roll, should make this point clear.

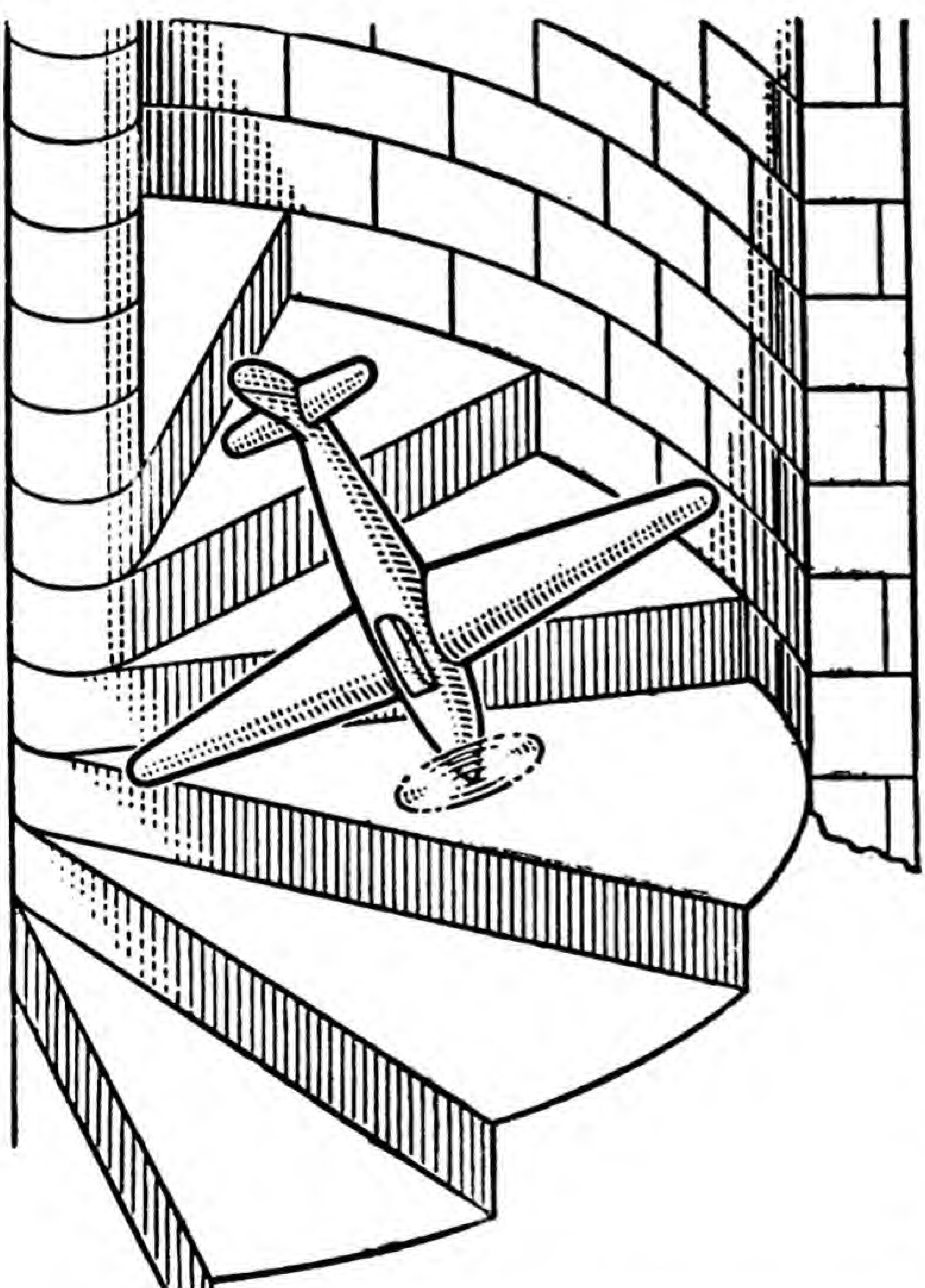


Fig. 39. Just as the steps of a spiral staircase are steeper on the inside, so the inner wing of an aeroplane descends a steeper spiral on a gliding turn, thus increasing its angle of attack.

And finally, low speed brings another problem besides that of the stalling of one wing. If we try to turn to port while flying near the stalling speed, the immediate effect will be to increase the effective angle of attack of the starboard wing, because the lowering of the aileron on that wing may be likened to twisting that wing. Similarly, the effective angle of attack of the port wing will be lessened. Now, the drag curve (Fig. 18(b)) tells us that, as the stalling angle is approached, the drag rises sharply. Therefore, provided we succeed in preventing a stall of the starboard wing, and its consequences, we shall increase the drag so much that our aeroplane will yaw to starboard, just the opposite motion to that desired.

Differential Controls

One solution to this awkward problem is differential aileron control. That means arranging the control mechanism so that the travel of the aileron which goes down is less than that of the one which goes up. Another solution, the Frise aileron, is to shape the ailerons so that there is a piece in front of the hinge which protrudes when the aileron goes up, as shown in Fig. 40, causing additional drag on the inner wing. Both these solutions, and others, aim at causing more drag on the inner wing than the outer wing, thus tending to

turn the aeroplane in the desired direction by means of drag forces.

We have been considering movements of the aeroplane produced intentionally, viz., control. Let us now consider the correction of unintentional movements, viz., stability.

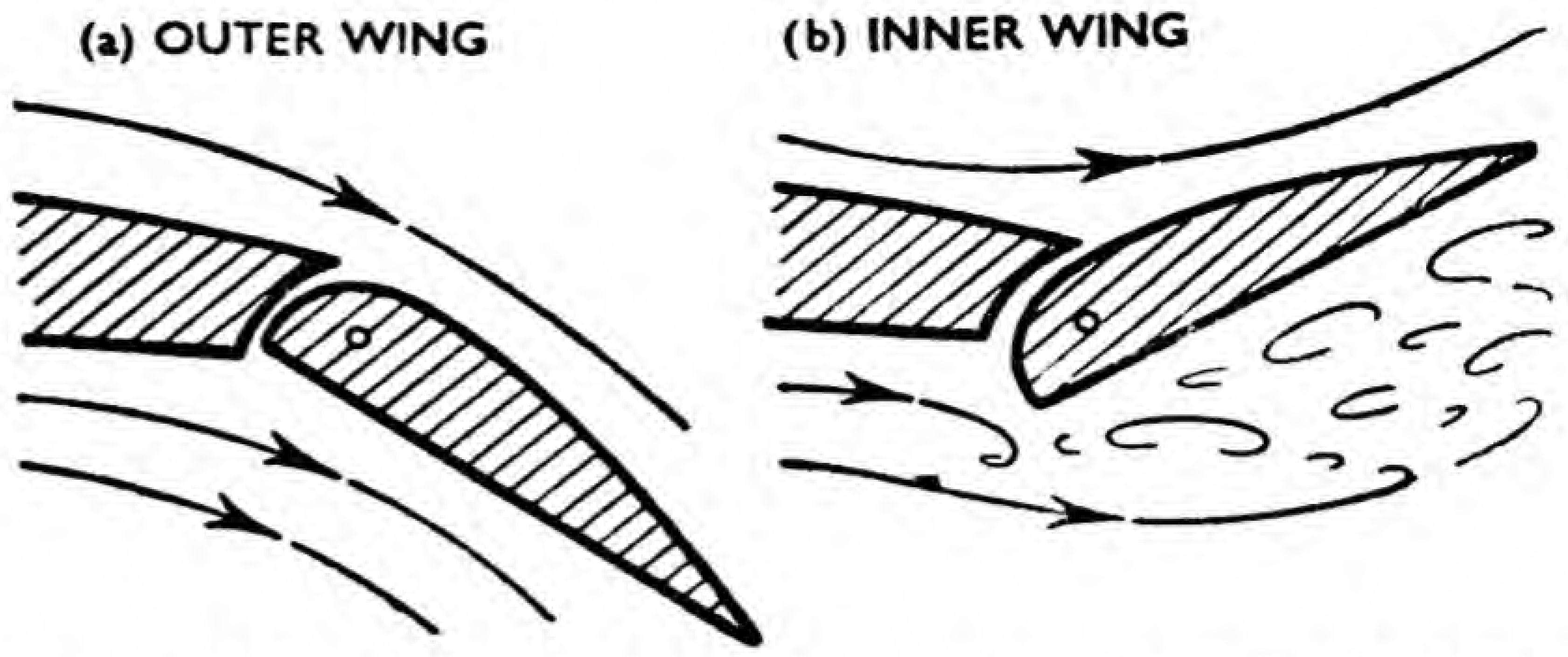


Fig. 40. Action of Frise ailerons is illustrated above. (*a*) The aileron on the outer wing goes down—little extra drag. (*b*) The aileron on the inner wing goes up—a lot of extra drag.

We can define a stable aeroplane as one which will return automatically to its original condition of flight if some external force, such as a sudden gust of wind, momentarily upsets it.

Very little need be said about this, because the problems are similar to those of control. Besides, we were really considering stability when flight at constant velocity was discussed.

As with control, stability is considered with respect to the three body axes. It may be summarized thus:—

(1) Longitudinal stability is stability about the lateral axis.

(2) Lateral stability is stability about the longitudinal axis.

(3) Directional stability is stability about the normal axis.

This will be clear if reference is made to Fig. 26, which indicates that the forces shown on a longitudinal diagram effect turning moments about the lateral axis. Fig. 35 will then help us to see the meaning of statements (2) and (3).

It will be remembered that, when horizontal flight was discussed, we dealt with the problem of balancing moments about the lateral axis at all angles of attack. Now, if we ensure that, when the angle of attack is altered by some external agent, a moment is produced which is greater than, and opposite to, the disturbing moment, the aeroplane will return to its original attitude—it will be longitudinally stable. It all depends upon the relative positions of the C.P.s of the wings and tail plane, and the C.G., and the forces exerted by them. The problem is complicated by the fact that the positions of the C.P.s vary in accordance with the curve of Fig. 18(c).

Now, the slope of the lift curve is constant, except near the stall, so an increase of, say 1 deg. in the angle of attack, will produce the same increase in lift whether the angle of attack is small or large. But this fixed increase in lift is a greater proportion of the total lift at small angles of attack, when the total lift is small. This may be checked on the graph of Fig. 18(a). Therefore, if the tail plane is rigged at a smaller angle of incidence than the main plane, a sudden increase in the attitude of the aeroplane will create a greater *proportionate* increase in the lift of the tail plane, giving an excess diving moment, which will restore the aeroplane to its original attitude. The angle between the chords of these two planes is called longitudinal dihedral.

An aeroplane will have lateral stability if a roll produced by external forces will bring a restoring

couple into play. This means that the wing moving downward must produce more lift than the wing moving upward.

Now, the actual movement will give this effect, because the downward-moving wing will meet the oncoming air at a greater angle of attack than the other wing. This problem was met when dealing with control, but we must be quite sure about it, so the resultant wind velocity will now be split into its horizontal and vertical components. If we are descending by parachute, the relative direction of the wind will be upward. In the same way, there will be an upward flow of air relative to the wing moving downward. There is also a backward flow relative to the wing, due to its forward velocity. The resultant of these two velocities will be inclined upward, as the right-hand side of Fig. 41 shows.

Similarly the airflow will be inclined downward relative to the wing moving upward. The angles of attack will thus be changed, giving the downward-moving wing more lift, and the upward-moving wing less lift. The combined effect will be a couple acting in the opposite direction to the arrow shown, unless we are near the stalling speed. In that case, the downward-moving wing will stall.

Correction of Roll

But this cannot correct the roll, because, as soon as it stops the roll, the correcting force disappears. A force is needed which will operate all the time that the aeroplane is flying in a banked attitude.

Now, such a force cannot be produced without sideslipping, but since a banked aeroplane will sideslip, the means is at hand. There are three methods of using this means to produce the desired force. These are illustrated in Fig. 42.

(1) If the position of the resultant reaction of the sideslip wind is above the C.G., a restoring moment will be produced. A high fin will help here, and so will a high wing; but care has been exercised not to refer to high position of centre of pressure, because that part of the restoring force due to the high wing is the profile drag caused by the sideslip wind moving across the wing, as Fig. 42(a) shows. The necessarily low position of the C.P. of the side area in a seaplane or flying boat is a source of trouble which can only be overcome by fitting a large, high fin.

(2) If the wings are inclined upward (Fig. 42(b)) giving what is known as lateral dihedral, the wing which is uppermost, due to bank, will present a greater surface area to the sideslip wind, resulting in the appearance of a restoring moment.

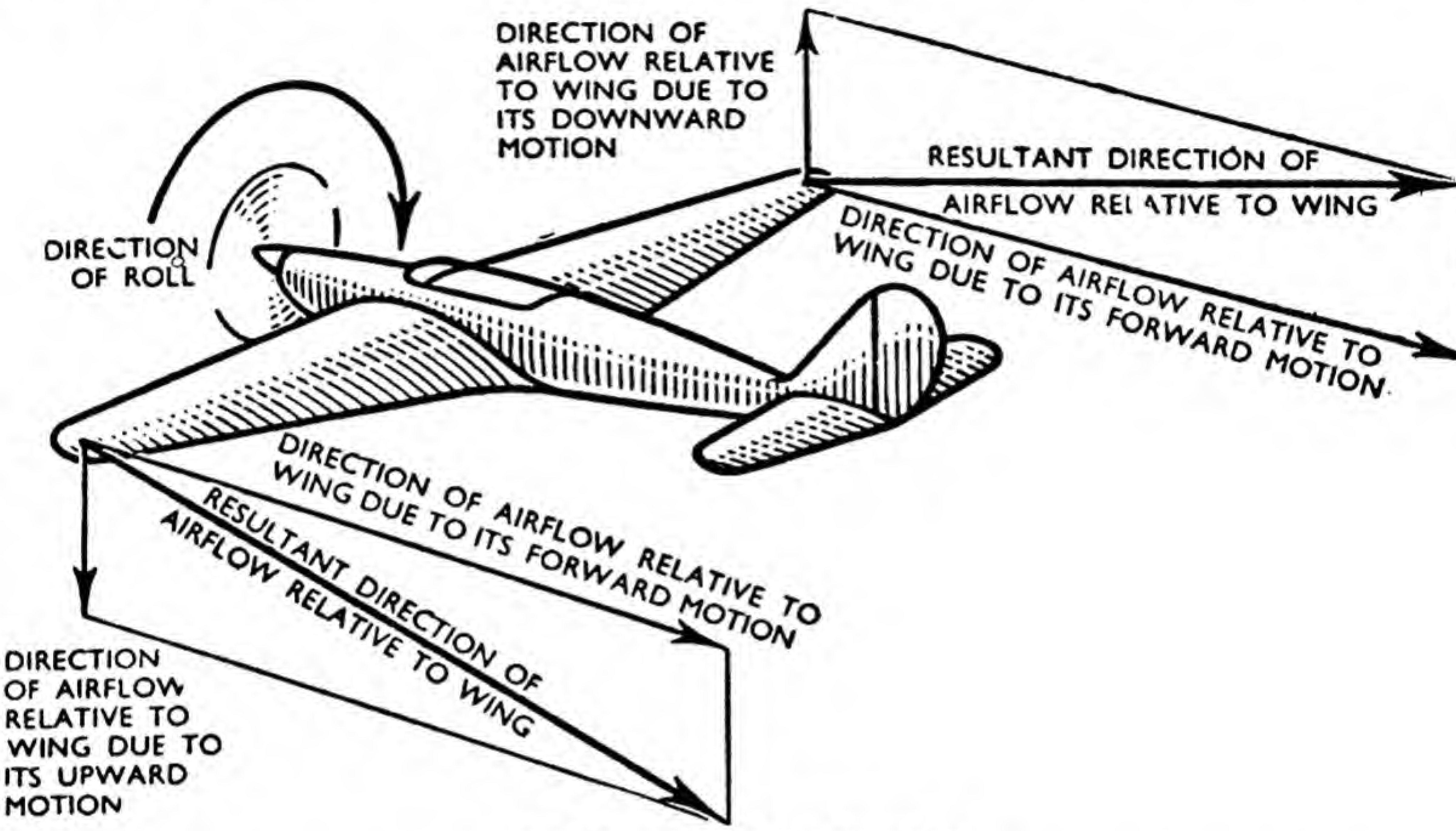
(3) If the leading edges of the wings are swept back, the sideslip causes a decrease in the effective aspect ratio of the upper wing, and an increase in that of the lower wing, giving greater aerodynamic efficiency to the lower wing than to the upper one, and thus producing a restoring moment. In Fig. 42(c) the starboard wing has dropped, causing a sideslip to starboard, and, consequently, the relative wind shown. It is obvious from the figure that a is greater than b, and, therefore, the starboard wing will be more efficient.

Yawing and Rolling

So much for attempts to correct a lateral upset, but, recollecting the necessity of placing the C.P. of the side surfaces behind the C.G., we

shall remember that a roll produces a yaw. This tendency, of course, goes on at the same time as those enumerated above, so that by the time that the correction has taken place, the aeroplane will have changed its course. It could not be otherwise, though, because, if the side C.P. were in front of the C.G., a sideslip, once started, would interesting application of the mechanics of flight.

And, lastly, directional stability. If the C.P. is behind the C.G., as it must be, for reasons previously stated, a yaw will be immediately corrected in the same way as a weathercock turns into the wind (Fig. 43), although, since the C.P. is also above the C.G., there will



ROLLING AEROPLANE

Fig. 41. The aeroplane shown is rolling to starboard, the port wing is going up, and the starboard wing going down. The parallelograms of velocities show that the resultant airflow is inclined downward to the port wing, and upward to the starboard wing. Thus, the angle of attack, and the lift, of the port wing is decreased, and that of the starboard wing is increased. This tends to stop the roll, but it will not correct it.

increase until the aeroplane had turned broadside to the wind, resulting in complete loss of lift.

Such behaviour is reminiscent of the flight of some model aeroplanes, but, if we watch correctly designed models, it will be observed that the course is always changed by a gust of wind. The roll has produced the yaw. Model aeroplanes must have a high degree of stability because there is no control during flight. Model aeronautics, in fact, provides us with a simple and be a roll. But the old argument is starting all over again. The fact is, that lateral and directional stabilities are so closely interrelated, that they cannot be considered separately ; they are two aspects of the same problem.

Specimen Calculations

Data have also been given relating to the Tiger Moth (page 387), the Sea Otter (page 411), and the Shetland (page 419). It would be interesting for the reader to try

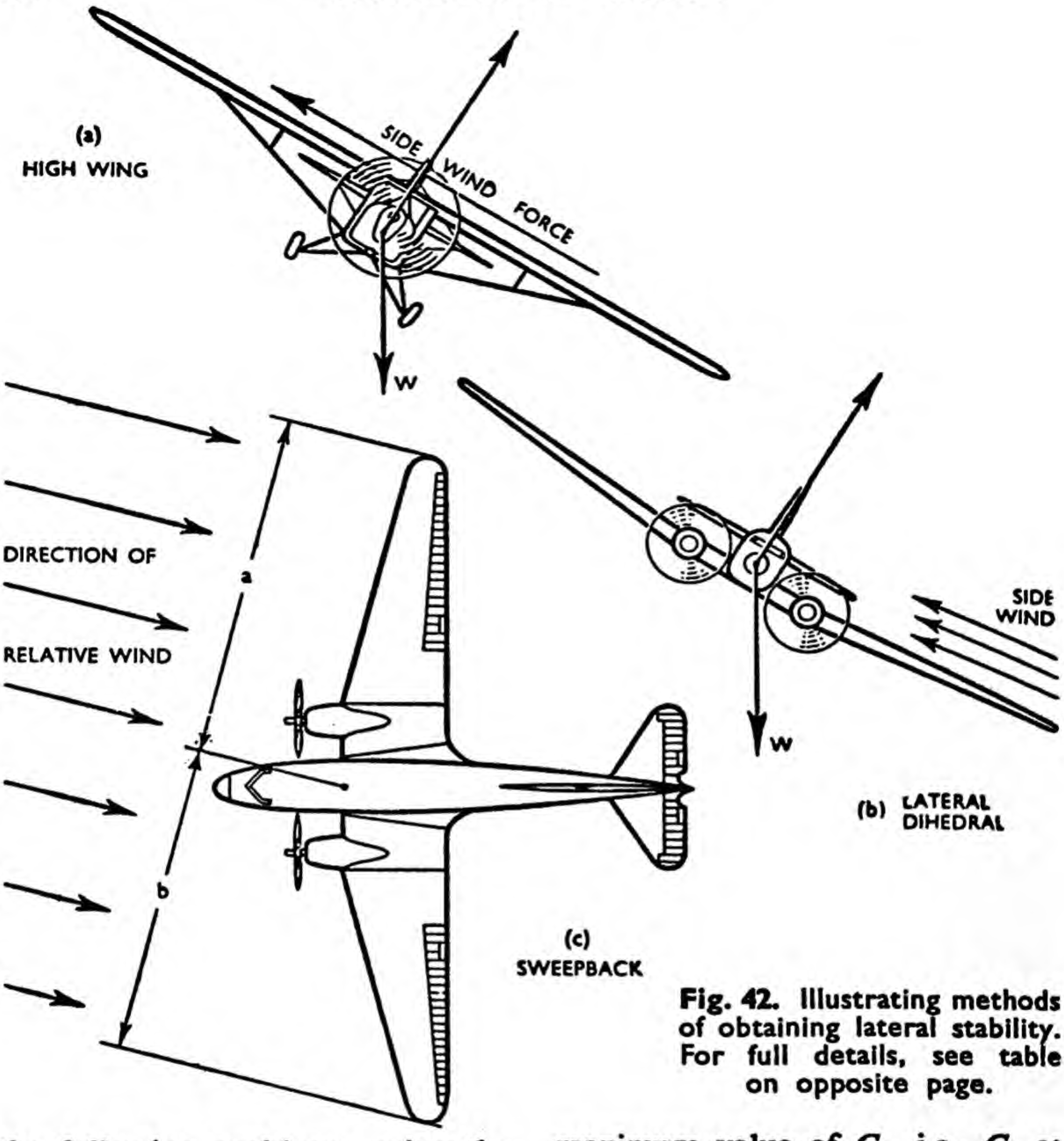


Fig. 42. Illustrating methods of obtaining lateral stability. For full details, see table on opposite page.

the following problems, using the figures given for each aircraft in turn.

The first is to find the aspect ratio from A.R. $= \frac{s}{C}$ or $\frac{s^2}{S}$. The second problem is to check the wing-loading figures given for each aircraft with the wing loading $= \frac{W}{S}$. Then we could find the C_L at cruising speed. To help in this, $L = W = C_L.\frac{1}{2}\rho V^2.S$. Therefore, $C_L = \frac{\frac{W}{S}}{\frac{1}{2}\rho V^2} = \frac{2W}{\rho V^2 S}$.

For the fourth problem, find the maximum value of C_L, i.e., C_L at landing speed, taking the landing weight as 80 per cent of the loaded weight.

For the next, find the drag at cruising speed, assuming that the airscrew is 80 per cent efficient, and bearing in mind the following: h.p. $= \frac{DV}{550}$, therefore,

$$D = \frac{550 \times \text{h.p.}}{V}$$

We might then find C_D at cruising speed ($D = C_D \cdot \frac{1}{2}\rho V^2 . S$, therefore, $C_D = \frac{D}{\frac{1}{2}\rho V^2 S}$). The seventh problem

METHODS OF OBTAINING LATERAL STABILITY

Reference to Fig. 42 (opposite) shows that in (*a*) and (*b*) the port wing has dropped, and in (*c*) the starboard wing has dropped, and that in each case there is a sideslip towards the lower wing, so the wind which meets the aeroplane has a lateral component. This component is used to restore the aeroplane to a level keel. In (*a*) the wing is above the C.G. and the restoring moment is formed by the frictional force between the air and the wing. In (*b*) the dihedral causes a greater area to be presented to the side wind on the starboard side. In (*c*) the effective wing span perpendicular to the relative wind is greater on the starboard side. This increases the aspect ratio, and therefore the lift. Sweepback, however, is often given for another reason—longitudinal stability. It enables the designer to keep the centre of lift well back relative to the centre of gravity, and thus to increase the diving moment. The Sea Otter (Fig. 27, page 410), and the Tiger Moth (Fig. 11, page 388), have swept-back wings, but these wings are rectangular, and therefore both leading and trailing edges are swept back. We have chosen two contrasting types for this figure, so that we can amuse ourselves by comparing and contrasting the details which follow. The differences between wing loadings and their effect upon speed are particularly illuminating. (*a*) is the Taylorcraft Auster, the 'plane for the man in the street. It can fly very slowly (note the low wing loading). It is very manœuvrable and can turn on a circle of small radius. (The clue to this is on pages 399 and 414). A further impression of this aeroplane may be gained from Fig. 34, page 418, which shows the Auster's cockpit. (*b*) and (*c*) show the Douglas Dakota, a transport aeroplane. In this respect it is similar to the Short Shetland (Fig. 35, page 419), but the latter is of more recent design. Compare the data, and note the increase in speed and wing loading. Wing loading has increased much more than speed, indicating that the wing is more efficient.

	Auster IV	*Dakota*
Wing Span	36 ft.	95 ft.
Length	22 ft. 5 in.	64 ft. 6 in.
Wing Area	185 sq. ft.	987 sq. ft.
Weight Loaded ...	1,700 lb.	25,200 lb.
Wing Loading	9.2 lb./ft.^{2}	25.5 lb./ft.^{2}
Maximum Speed ...	128 m.p.h. at 1,000 ft.	230 m.p.h. at 8,500 ft.
Cruising Speed ...	104 m.p.h. at 1,000 ft.	207 m.p.h. at 15,000 ft.
Landing Speed	40 m.p.h.	67 m.p.h.
Maximum Horse Power	130	2,100
Cruising Horse Power	110	1,400
Maximum Rate of Climb	784 ft. per min.	1,070 ft. per min.

Landing speeds are a little above stalling speeds for safety's sake, so our calculated maximum C_L's will be a little less than the actual values.

is to find the maximum $\frac{L}{D}$ ratio, that is, the $\frac{L}{D}$ ratio at cruising speed.

The next is to find the gliding angle by drawing a triangle with a horizontal side proportional to L, and a vertical side proportional to D. The last problem suggested is to find the maximum h.p. available for climbing, using

$$\text{h.p.} = \frac{\text{work done per min.}}{33{,}000}$$

$$= \frac{W \times \text{maximum rate of climb}}{33{,}000}.$$

The results of these problems will be found on page 444.

Useful Hints

Whenever problems are going to be attempted, and it is immaterial what subject is being studied, it is always advisable to

spend a short time in thinking about the method by which these problems should be approached. It will be found that this time is seldom wasted, in fact, usually it is more than saved as more often than not it is more difficult to find the correct method by which any problem should be approached, than it is to perform the actual mathematical calculations. Here are a few useful hints.

It must be remembered that the Tiger Moth and Sea Otter are biplanes, and, therefore, the formula for the aspect ratio must be modified to suit these particular aircraft.

Another interesting point is that velocities are given in miles per hour, but this figure must be

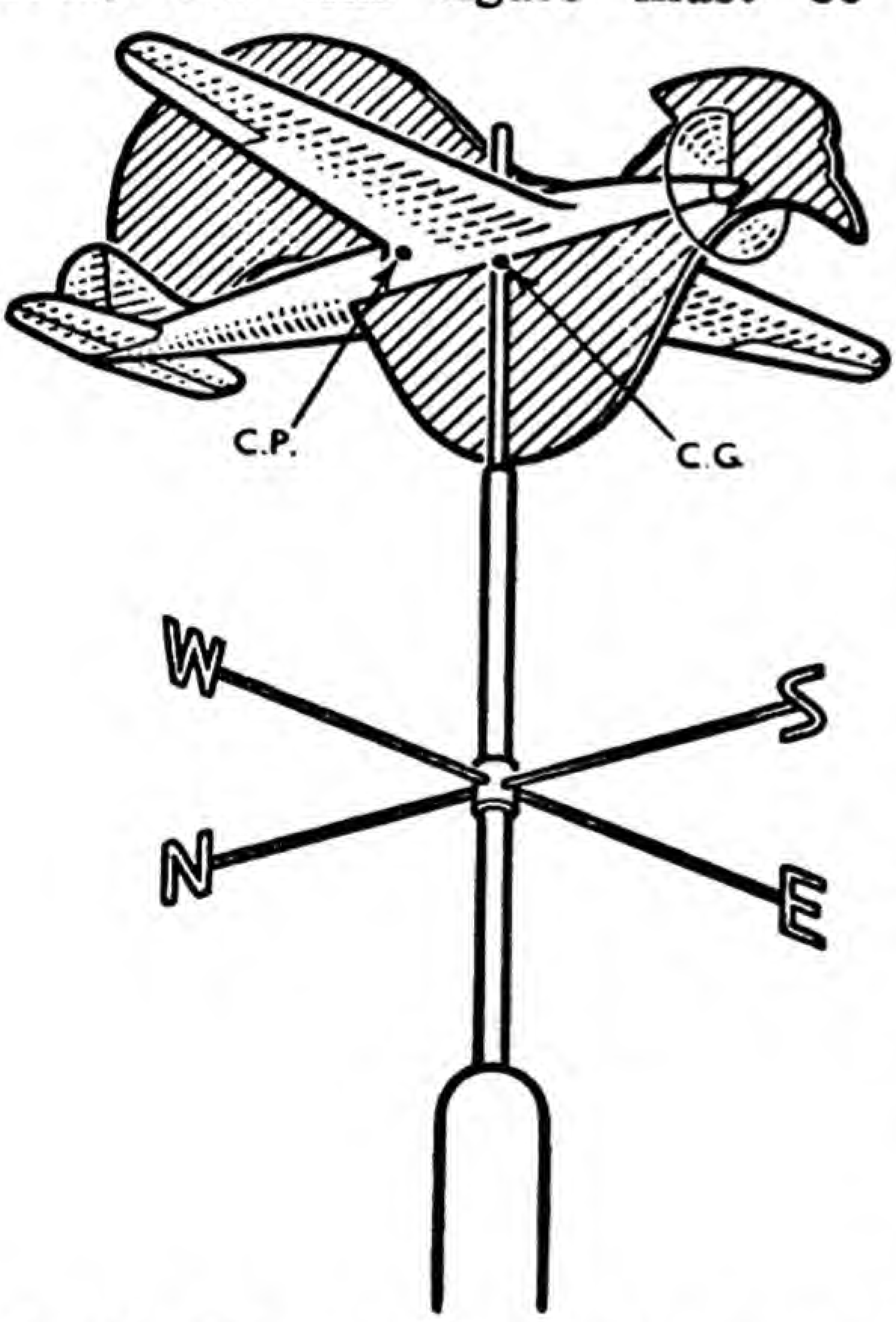


Fig. 43. Directional stability is often called weathercock stability, because any side wind will turn the nose of the aeroplane into the wind if, like the weathercock, the centre of pressure of the side surfaces is behind the centre of gravity, or turning axis.

replaced in the formula by feet per second. Now it is known that 60 m.p.h. is just the same as 88 ft. per sec., so that velocities in miles per hour must be multiplied by $\frac{88}{60}$ to obtain velocities in feet per second.

When substituting the formula $\frac{1}{2}\rho V^2$, if all constants are multiplied together first, thus

$$\frac{0{\cdot}00238}{2} \times \left(\frac{88}{60}\right)^2 = 0{\cdot}00256,$$

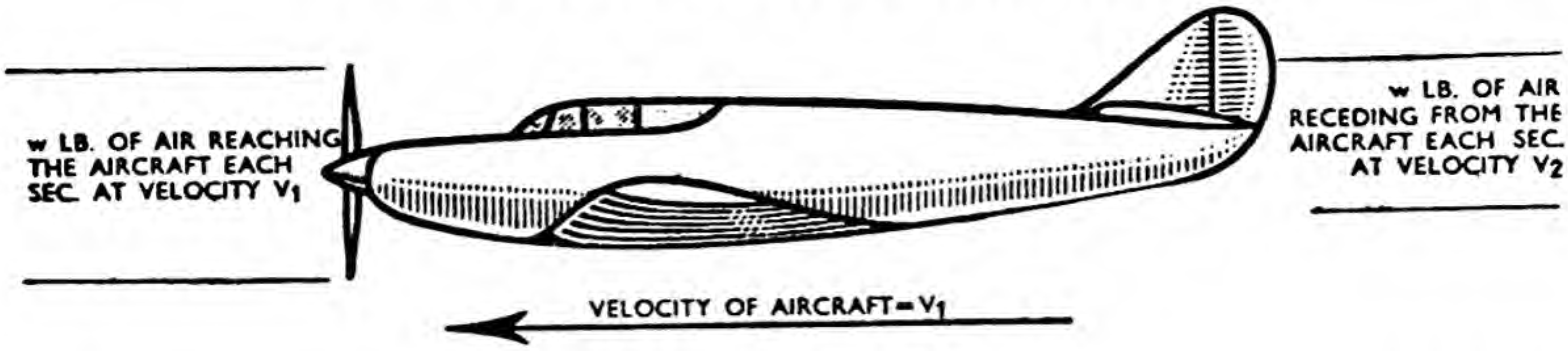
valuable time and a considerable amount of work will be saved.

Production of Thrust

Whenever we have wanted a thrust, an arrow has been drawn and marked T, but the time has now come to find out how this thrust T is produced. Very few difficult problems will be met, but we shall need to apply the ideas of force, momentum, work done, kinetic energy and efficiency which were discussed in earlier chapters, and a quick revision of the relevant parts of those chapters might be helpful at this stage.

Now, it has been seen that an upward force cannot be obtained without pushing air downward. Similarly, a forward force cannot be obtained without pushing air backward, and this is the only possible way of propelling an aircraft. The usual method of pushing the air backward is that of rotating an airscrew, although the idea of directly producing a rearward jet, viz., jet propulsion, is now beyond the experimental stage.

Let us now consider the momentum imparted to the air by the propulsive unit, because the rate at which the momentum is changed will be equal to the forward force



PROPULSIVE FORCE

Fig. 44. Increase in velocity of w lb. of air is $V_2 - V_1$. Momentum given to it each second is $w(V_2 - V_1)/g$. This change of momentum provides the propulsive force.

produced (Chapter 4, page 93). We must be careful here, always to consider our air velocities relative to the aircraft ; so, if we are sitting in an aircraft which is moving through the air with velocity V_1 ft. per sec., the air will be coming towards us with velocity V_1 (Fig. 44). If, after acceleration by the propulsive unit, the air is leaving us with velocity V_2 ft. per sec., its change of velocity will be $(V_2 - V_1)$ ft. per sec. Now let us suppose that w pounds of air are dealt with each second. Then, the momentum imparted to the jet per sec., viz., the rate of change of momentum, will be,

$$\frac{w}{g}(V_2 - V_1).$$

$\frac{w}{g}$ is called the mass flow.

Note that the units of w are really lb. per sec. Hence the units of this expression are,

$$\frac{\text{lb. per sec.}}{\text{ft. per sec.}^2}(\text{ft. per sec.})$$
$$= \frac{\text{lb. }\cancel{\text{sec.}}^2\,\cancel{\text{ft.}}}{\cancel{\text{sec.}}\,\cancel{\text{ft.}}\,\cancel{\text{sec.}}} = \text{lb.}$$

So the rate of change of momentum is a force in pounds. This is the thrust T, hence

$$T = \frac{w}{g}(V_2 - V_1) \ldots\ldots\ldots \text{(i)}$$

Now, efficiency =

$$\frac{\text{Work done per sec.}}{\text{Energy expended in doing this work}}$$

(Chapter 7, page 160), so that, if we find (1) the work done per sec. and (2) the energy expended in doing this work, we can obtain an expression for the propulsive efficiency by dividing (1) by (2).

(1) The work done per sec. will be the thrust multiplied by the distance through which the thrust is moved in a sec., viz.,

Work done per sec. =

$$TV_1 = \frac{w}{g}(V_2 - V_1)V_1 \ldots\ldots \text{(ii)}$$

This is the power absorbed in propulsion.

(2) The energy expended in doing this work will be the kinetic energy given to the air per sec., viz., $\frac{1}{2}$ mass multiplied by (final velocity of air relative to aircraft)^2 minus $\frac{1}{2}$ mass multiplied by (initial velocity of air relative to aircraft)^2.

$$= \frac{1}{2}\cdot\frac{w}{g}\cdot V_2^2 - \frac{1}{2}\cdot\frac{w}{g}\cdot V_1^2$$
$$= \frac{1}{2}\cdot\frac{w}{g}(V_2^2 - V_1^2)$$
$$= \frac{1}{2}\cdot\frac{w}{g}(V_2 + V_1)(V_2 - V_1) \ldots\text{(iii)}$$

Therefore, propulsive efficiency

$$= \frac{\text{(ii)}}{\text{(iii)}}$$

$$= \frac{\frac{w}{g}(V_2 - V_1)V_1}{\frac{1}{2}\cdot\frac{w}{g}(V_2 + V_1)(V_2 - V_1)}$$

$$= \frac{2 V_1}{V_2 + V_1} \quad \ldots\ldots\ldots\ldots \text{(iv)}$$

If we put $V_2 = V_1$ in this formula, we obtain,

$$\text{Propulsive efficiency} = \frac{2 V_1}{2 V_1} = 1.$$

This is our ideal, but it can never be realized in practice, because, if $V_2 = V_1$, formula (i) tells us that there would be no thrust, which could be the case only if there were no drag. Similarly, formula (ii) tells us that there would be no work done, and formula (iii) that there would be no energy expended, so the limiting value of the ideal propulsive efficiency is really $\frac{0}{0}$, which is not necessarily unity.

No ! We have not proved that one equals any number you like, but we have done the same thing as the mathematical conjurers do when they try to deceive us. Equation (iv) was obtained by cancelling by $V_2 - V_1$, and this is zero in the case that has been considered. In mathematics we must never cancel by nought if we want to keep out of trouble.

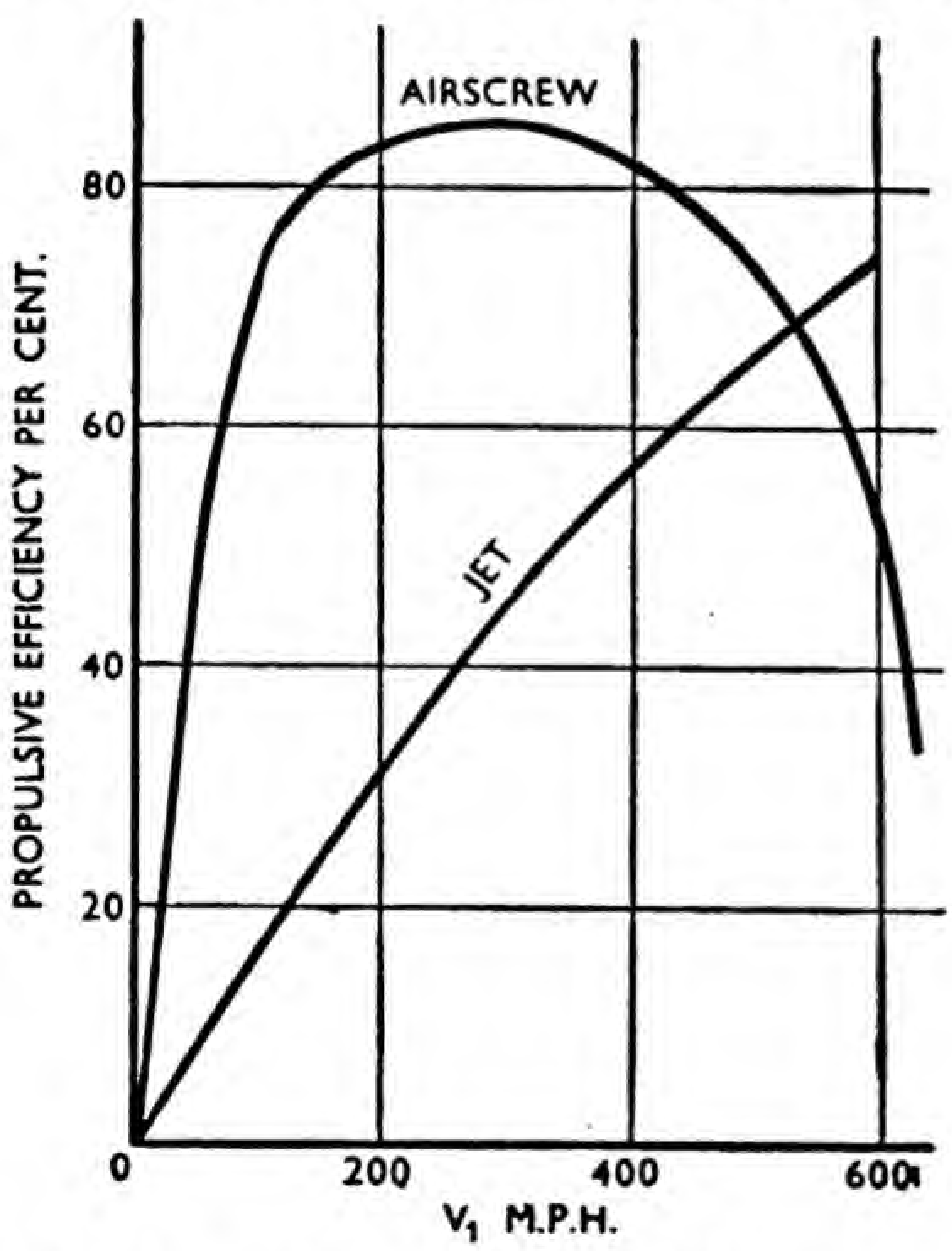


Fig. 45. Comparison of propulsive efficiencies of airscrew and jet at an altitude of 20,000 ft. It will be noted that the efficiency of the airscrew begins to fall off at 300 m.p.h. and at 550 m.p.h., it is actually lower than that of the jet. Now that operating speeds are coming into this region, it is only natural that much thought should be given to the alternative of jet propulsion.

Rotation of Slipstream

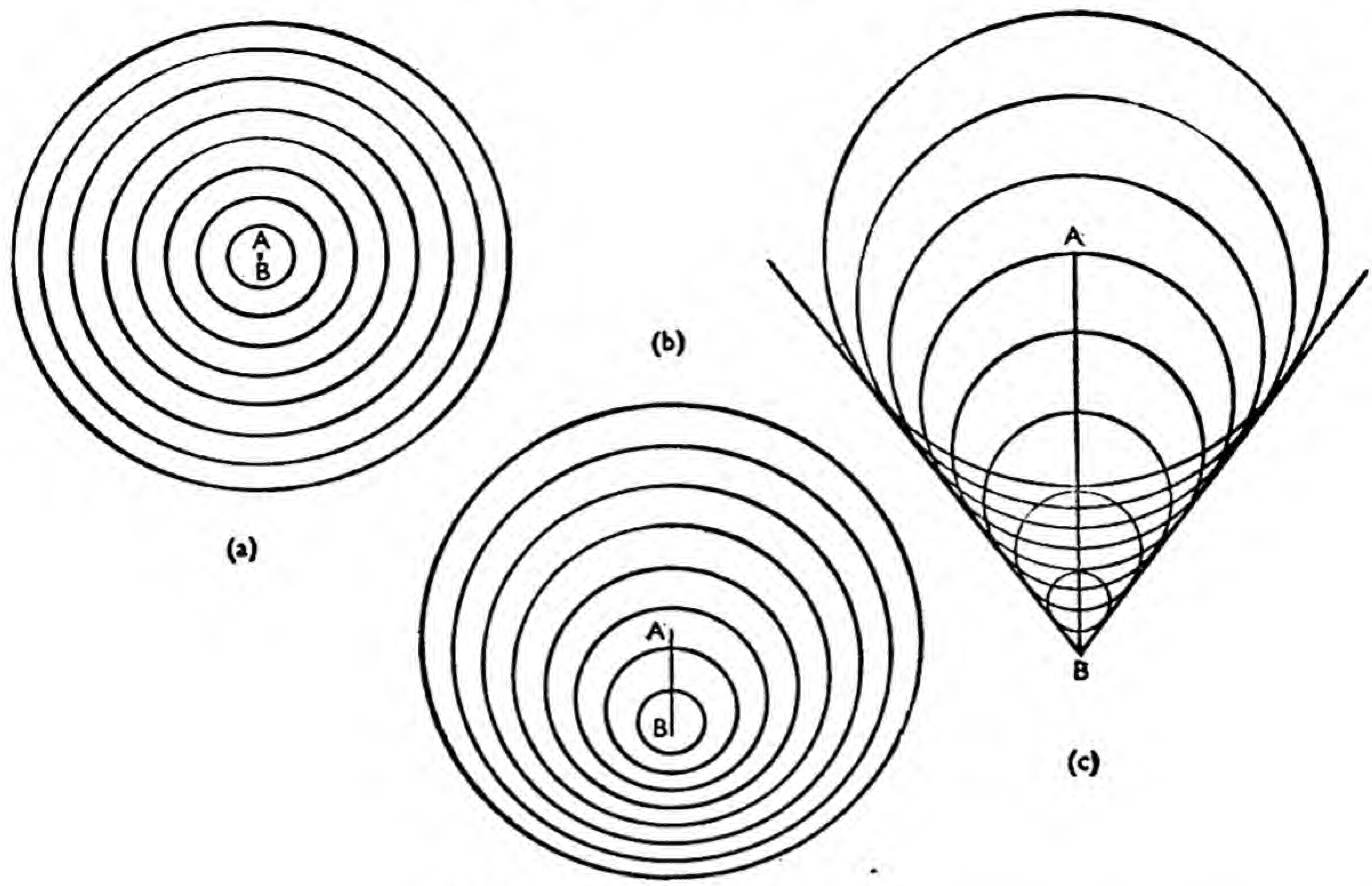
Now, a step further. Actually more energy is expended than that given in formula (iii) because the means of imparting this energy to the air has not been considered. We may, for example, be giving additional energy to the slipstream or jet by rotating it, as with an airscrew, so that the energy thus wasted must be added to the bottom of the fraction. The actual efficiency will thus tend to zero as V_2 tends to V_1, because the limiting value of $\frac{\text{(ii)}}{\text{(iii)}}$ has become

$$\frac{0}{0 + \text{energy wasted in producing slipstream}}$$

which is obviously zero. It will, of course, also be zero if V_1 is zero, viz., when the aircraft is stationary on the ground. In the first case T is zero, and, in the second case V_1 is zero, so that, in each case the work done, the product of T and V_1, is zero.

Consequently, if a curve of efficiency versus velocity is drawn, it will be humped, with zero values of efficiency for $V_1 = 0$ and $V_1 = V_2$, although it will be impossible to reach V_2.

Fig. 45 shows approximate efficiency-speed curves for airscrew and jet propulsion. In the latter curve, the maximum, viz., the top of the hump, is not reached because the jet accelerates a smaller mass of air than the airscrew, and



FORMATION OF WAVES

Fig. 46. All the figures show waves which are radiating at the same velocity. *A* indicates the source of the wave motion at the beginning, and *B*, at the end of the time during which the waves shown are formed. Thus the line *AB* is the distance that the source has moved during this time. In (*a*) the source is stationary, in (*b*) it is moving at a lower velocity than the waves, and, at (*c*) faster than the waves. (*c*) Shows how these waves combine to form a bow wave.

since the thrust is equal to the product of the mass flow and the increase in velocity of the jet or slipstream (formula (i)), it is necessary for V_2 to be high. Thus, at low values of V_1, the ideal propulsive efficiency of the jet will be low, because V_2 will be large compared with V_1, making the denominator in formula (iv) high compared with the numerator.

Fig. 45 indicates that, whilst it is at present unlikely that we shall be able to reach speeds at which we can use the maximum propulsive efficiency of a jet, it will, at least, be more efficient than an airscrew at speeds in excess of 550 m.p.h.

Speed of Sound

It may be asked, what happens to this energy given to the air? It is frittered away in eddy motion between the jet and the surrounding air, until the jet is eventually brought to rest. Now, the energy used in doing useful work to move the aircraft through the air, is also frittered away in forming the eddies associated with drag, so, for all the turbulence caused by drag, there is a complementary turbulence associated with propulsion. It all happens because we cannot overcome drag unless V_2 is greater than V_1, but there is a suggestion to neutralize some of these eddies by using jets to reduce drag.

When a body passes through air, it disturbs the air, or compresses it, setting up waves—sound waves —which travel out from it at constant speed in ever-widening circles, just like those caused by a stone

when thrown into calm water (Fig. 46(a)). But, since the body is moving, the correct wave pattern will be as shown in Fig. 46(b). A model boat, moving slowly, makes waves like this. If its speed is increased until it is moving faster than the waves are radiating, a bow wave will be formed. Fig. 46(c) shows that this is due to the formation of a straight wave front, composed of the crests of all the radiating waves. Behind this wave front the crests and troughs interfere with one another, tending to cancel each other out, but a stern wave, and sometimes one amidships, may also be formed in the same way.

Now, the formation of these V waves absorbs a lot of energy, so designers try to reduce them by suitable shaping of the body. The shape on page 387 is obtained from a consideration of the prevention of eddy formation, but the prevention of wave formation is obviously quite a different problem. We have just seen that V wave formation starts when the speed of the body reaches that at which waves are propagated through the medium, although the effects of compressibility, as it is called, are actually felt at lower speeds, becoming stronger as the critical speed is reached. Thus, in air, a different shape is required as we reach the speed of sound, just as the shape of an airship differs from that of a surface ship. The former moves through its medium, air, at speeds below that of the waves formed, and its shape approximates to Fig. 9, whilst the latter moves faster than the waves which it forms on the surface of the water.

Remember that the drag of a body increases with C_D and V^2. The increase with the square of the velocity is serious enough at high speeds, but the formation of these V waves makes matters worse by increasing C_D. And when we also consider that power must be increased with the *cube* of the velocity it can be realized that the production of sufficient power may become an uneconomical proposition. Of course, this may be offset by flying very high where the air density, ρ, is less, but we shall then meet the problem sooner, because the speed at which these waves are formed becomes lower.

It is thus obvious that present developments in speed are reaching a critical point, the speed of sound, so that, at speeds in excess of about 600 m.p.h., not only does the drag increase rapidly, but new aerodynamic considerations arise.

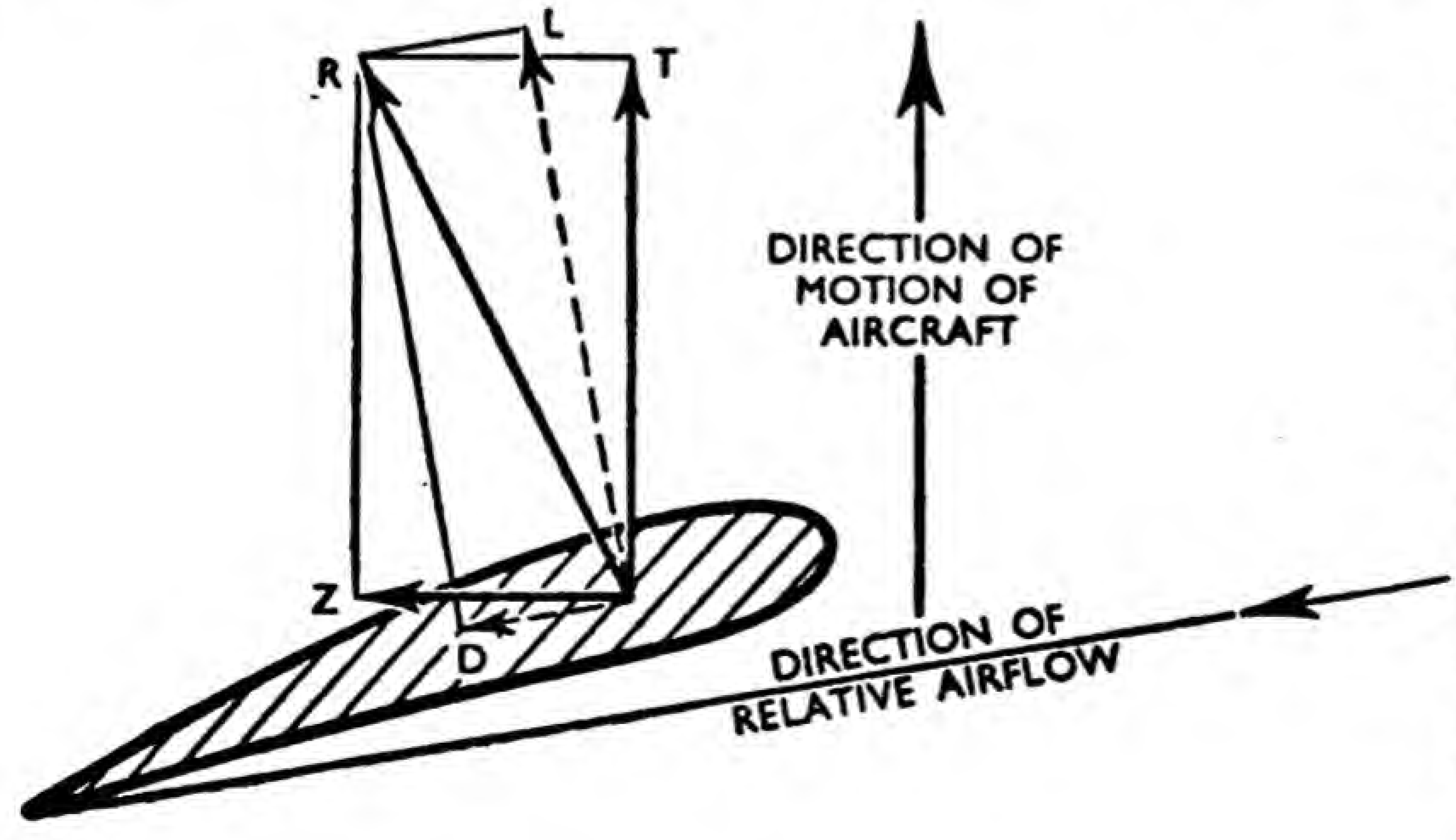
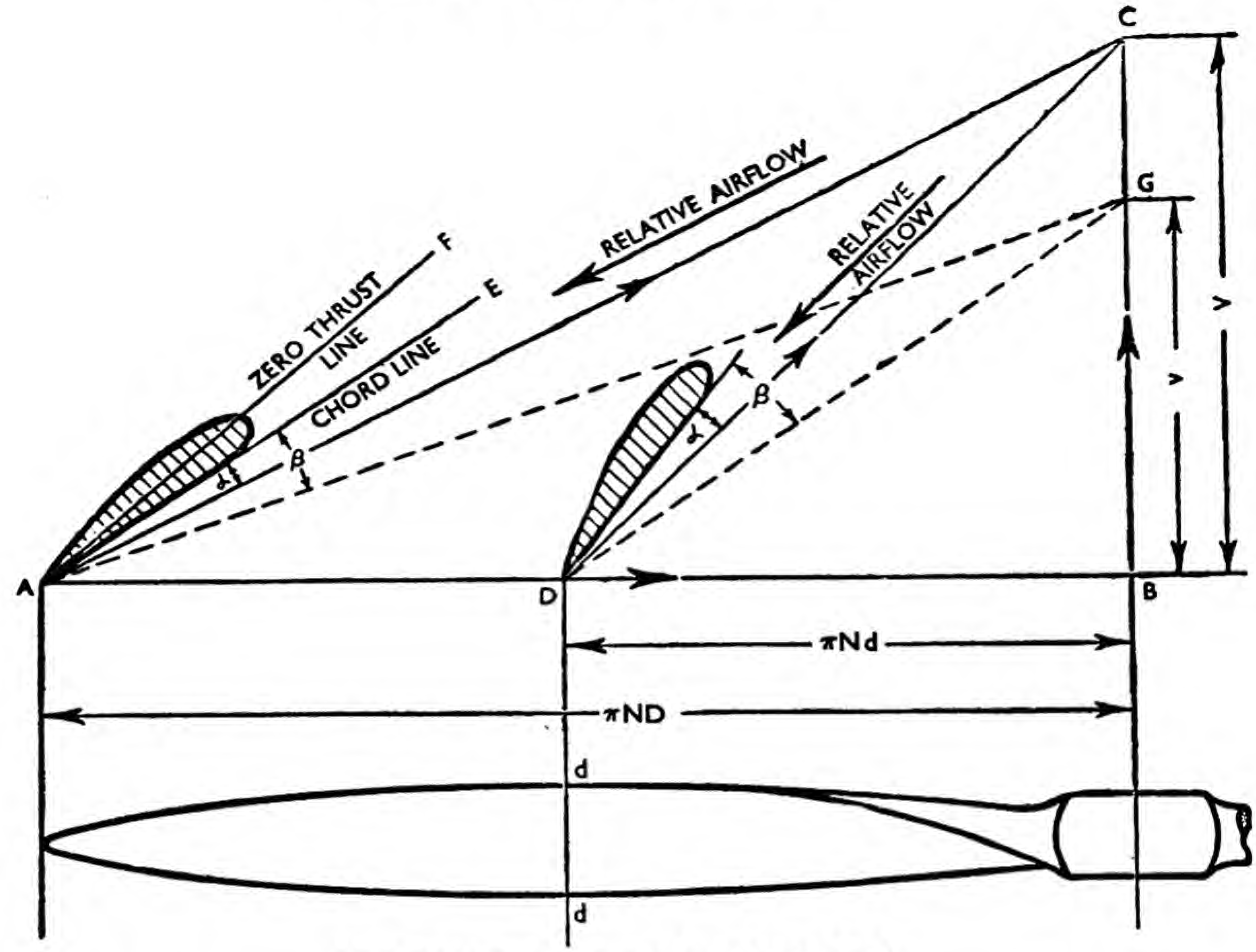


Fig. 47. Forces acting upon an airscrew blade element are resolved into components T and Z acting along and perpendicular to the direction of motion of the aircraft, instead of L and D, acting perpendicular to and along the direction of airflow, the same as in the case of an ordinary aerofoil section.



VELOCITY OF TIP OF PROPELLER

Fig. 48. *AB* represents the velocity of the blade tip if the aircraft is stationary. *BC* represents the forward velocity of the aircraft. Therefore, *AC* represents the resultant velocity of the blade tip. Similarly the velocity of a section of the blade at *dd* is represented by *DC*. If the forward velocity of the aircraft is reduced to *V*, these velocities become *AG* and *DG* respectively, and the angle of the blade is increased from α to β.

It has been seen from Fig. 45 that it is just in this region that jet propulsion becomes more efficient than airscrew propulsion, so the full development of jet propulsion can only be implicated when we have found out how to reduce these waves to such an extent that resistances due to them are not abnormally high.

Airscrew Design

When considering the design of an airscrew, it is necessary to think of it as a screw winding its way through the air. The velocity of the air relative to each part of the blade will thus be inclined like a screw thread. The blade is made of aerofoil section inclined at the appropriate angle of attack to this airflow. The sum of all the lift thus produced will give the thrust, and the sum of all the drag will constitute a force resisting the torque, or turning moment, of the engine.

But this statement is not quite true unless we resolve parallel and perpendicular to the direction of motion of the aircraft instead of to the relative airflow. Fig. 47 shows a section of the airscrew, known as a blade element, with the resultant force resolved in both these ways. Instead of the usual components *L* and *D*, shown dotted, *T*, the thrust, and *Z*, the resisting force, are used in airscrew theory.

Let us first consider the factors which affect the relative direction of the airflow.

If the aircraft has a forward velocity of V ft. per sec., and the airscrew is of diameter D ft., and is revolving at N revs. per sec., then in 1 sec. a point on the tip of the blade goes forward V ft. whilst going a distance πDN ft. in a direction perpendicular to the path of flight. In Fig. 48, BC represents the forward velocity of the airscrew, and AB represents the velocity of the tip perpendicular to the flight path. The resultant AC of these two velocities, gives the velocity of the blade tip in magnitude and direction. Note that if a screw thread could be uncoiled, a triangle like ABC would be obtained.

Now, any other section of the blade rotates on a circle of smaller diameter, so its resultant velocity would be given by a line such as DC. The relative airflow will, at all places, be equal and opposite to the direction of motion of the point on the blade being considered, as shown by the long arrows in the figure, and if each section of the blade is set at an angle of attack α, to the airflow it will produce thrust. We can see that the airscrew will be twisted like the spiral staircase referred to earlier.

Now suppose that the forward velocity is reduced to v without any change in the speed of rotation. The angle α will be increased to β as shown in Fig. 48. Similarly, an increase in the speed of revolution of the airscrew without change in velocity of the aircraft would increase the angle α, because the horizontal component AB would be increased, thus flattening the triangle ABC.

It is now obvious that the twisting of the airscrew blade depends primarily upon the ratio of V to ND. This is an important quantity in airscrew design, known as J. By testing its units, it will be found to be non-dimensional.

Defining Airscrew Pitch

There are two ways of defining the pitch of an airscrew. If the section or element at A moves forward, not along the line AC, but along its chord line AE, the advance per revolution in the direction of flight, from B to some point beyond C, will be a fixed quantity, measurable from the shape, or geometry, of the airscrew. Therefore, it is known as the geometric pitch. This length is the one used in specifying the airscrew.

Since the angle of attack for zero thrust is negative, if the element moves along the zero thrust line AF, it will advance still further per revolution. This distance, which is found experimentally, is known as the experimental pitch. There will, of course, be no thrust and the air will not be accelerated ($V_2 = V_1$), but there will be a torque due to the drag of the blade.

When the element moves along the line AC, at an angle of attack α, the actual advance per revolution, the effective pitch, will be less than the experimental pitch. The difference between these two quantities is known as the slip. The ratio of effective pitch to experimental pitch must not be confused with the efficiency of the airscrew. Efficiency is obtained from the ratio of thrust to torque, and so depends upon the ratio $\frac{T}{Z}$.

So now the similarity between airscrew and wing theory can be seen. The designer will choose an

airscrew such that, if V is the cruising speed of the aircraft, and N the most economical speed for the engine, each section of the airscrew blade will be inclined to the direction of the resultant air velocity at the angle of attack which gives the maximum $\frac{T}{Z}$ ratio. Similarly, thrust and torque formulæ, involving thrust and torque coefficients C_T and C_Q, are in use, and just as the ratio of C_L to C_D gives the efficiency of a wing, so the ratio of C_T to C_Q gives the efficiency of an airscrew.

Static Thrust

Since the thrust curve will be similar to the lift curve (Fig. 18(a)), it is obvious that the angle of attack, and, therefore, the thrust, will increase at low velocities, and will be greatest at zero velocity, viz., at the start of the take-off run. This thrust is known as the static thrust. Now, if the airscrew is designed for high speeds, V will be great compared with πND, so the airscrew will have considerable twist. This may cause the angle of attack when stationary to exceed the stalling angle, with a consequent loss of static thrust and ability to take off. The solution to this problem is the variable-pitch airscrew, with which the pitch can be varied to suit conditions.

Thus, if the drag of each element of the blade is such that the total resisting moment of the airscrew is less than the torque of the engine, its rate of revolution will increase. This will lengthen the base of the triangle in Fig. 48, increasing the angle of attack, and with it, the drag. The resisting torque will thus increase, and we shall attain equilibrium at a higher engine speed.

At high altitudes, where the air is rare, and the drag low, this may not occur until the engine is racing. In other words, when the engine is revving at normal speed, the airscrew cannot absorb all the torque produced by it, unless we increase the pitch. This can be done automatically by means of a governor fitted to the engine, a device known as the constant-speed airscrew. The speed is not quite constant though, but we try to get the best compromise between optimum engine conditions and optimum airscrew conditions.

Now there is a saving grace to be found in the ordinary engine. Its power falls off at altitude, and this will balance the lower torque absorption of an ordinary airscrew. On the other hand, if we fit a supercharged engine, we shall require a variable-pitch or constant-speed airscrew to absorb the higher torque at high altitude, and the ceiling will be very much increased.

Blade Area

The decreased drag at high altitudes, with the consequent tendency to racing of the engine and airscrew, can also be overcome by increased blade area. This may be done either by increasing (1) the diameter of the airscrew, or (2) the chord of the blades, or (3) the number of blades. There are three factors limiting airscrew diameter, (1) strength at root, (2) high undercarriage necessary for ground clearance, and (3) blade-tip speed. If the blade-tip speed approaches the speed of sound (about 1,000 ft. per sec.), the drag increases enormously. The number of blades possible is also limited due to aerodynamic interference between them. We can, however, increase the number

of blades by providing two airscrews on the same axis rotating in opposite directions—contra-rotation. This also straightens out the slipstream, and improves the propulsive efficiency.

Jet Propulsion

The production of a thrust by means of the reaction to the force due to the rapid change of momentum of a rearwardly directed jet of gas, produced inside the aircraft, has been advocated for many years; it is only recently that the idea has practically challenged the airscrew method of producing the thrust. The idea has great possibilities, but it has certain disadvantages.

Jet propulsion, for reasons that will be seen, is associated with the development of a new kind of power unit, the gas turbine, but the two ideas are not necessarily connected. We may get the one without the other, although, unlike airscrew propulsion, the production of a jet is intimately connected with the type of power unit. The essential point is, that the normal power unit converts as much as possible of the heat energy of the fuel used into mechanical energy, in the sense that it causes a shaft to rotate; whereas the jet power unit is merely required to give kinetic energy to a large mass of gas, from which the desired mechanical movement, that of the aircraft, results: the gas turbine is best suited to do this.

The power unit is, in fact, required to impart as much acceleration as possible to as large a mass of gas as possible. We have said a mass of gas because the jet will include the combustion products of the engine as well as the air drawn in at the front of the aircraft. It does not matter what is accelerated, so long as it has plenty of mass. So the object of the jet engine is to convert heat energy into fluid kinetic energy.

Perhaps the simplest way to start thinking about the jet-propulsion unit is to imagine an engine coupled to a fan which will draw a large mass of air through the aircraft, a sort of internal airscrew, with the added advantage that all this air can be used to cool the engine, and that the heat thus abstracted from the engine, can be used to expand the air, and thus increase its velocity. Remember also, that if an aircraft carries 10,000 lb. of fuel, this will weigh just the same after combustion, because its mass cannot be destroyed, and the mass of this fuel might just as well be utilized to provide thrust by throwing the products of combustion backward as fast as possible with the jet of air.

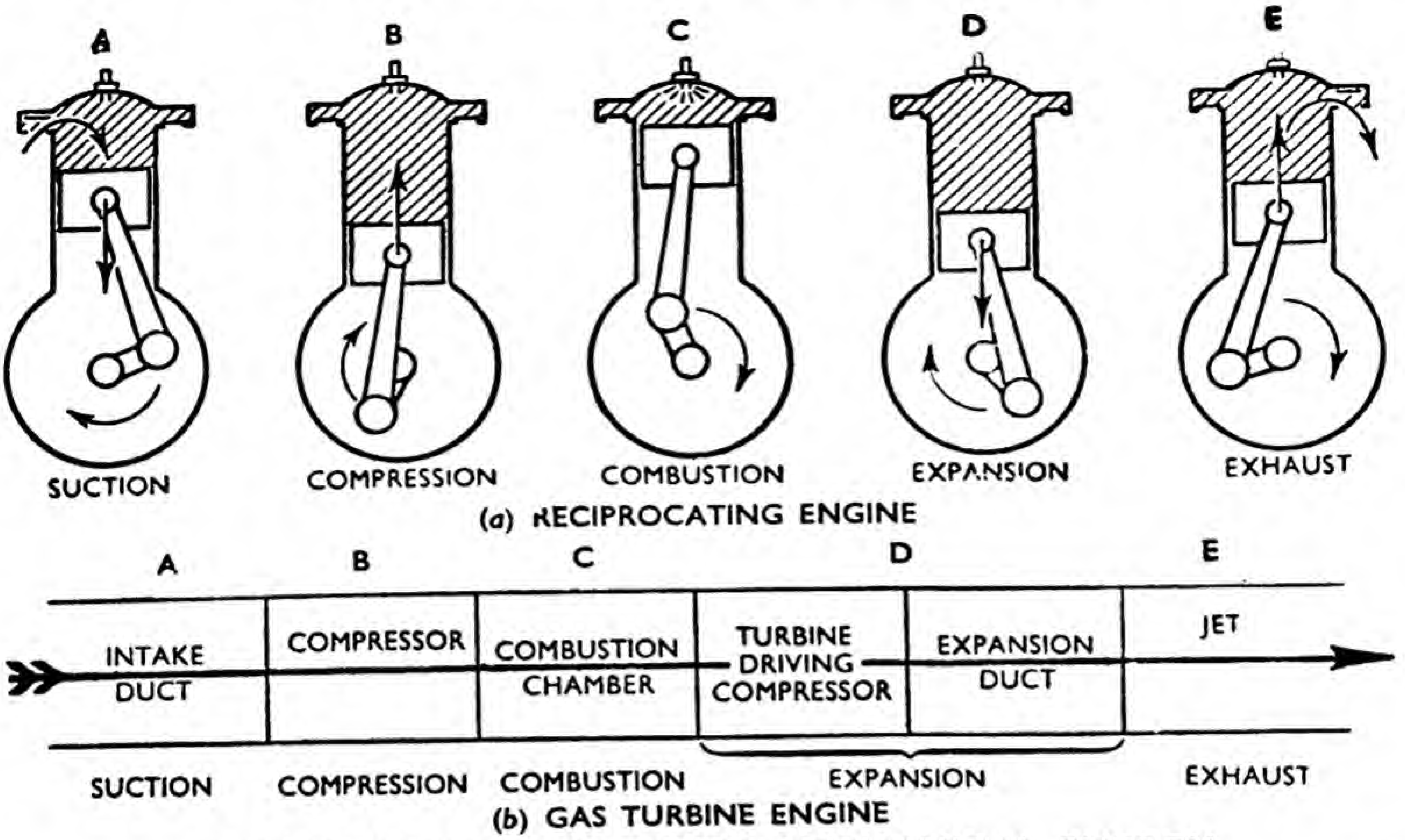
But this simple engine-fan idea would be extremely inefficient. It could not deal with a sufficiently large mass of air, and it would not utilize enough of the heat energy of the fuel. We must not waste heat by throwing it back with the mass of ejected gas, but must endeavour to use it all to increase the velocity of the gas, and this can best be done by designing the engine specially for this purpose.

Let us first consider how the heat energy of a fuel is converted into mechanical energy in a reciprocating internal-combustion engine. There are five stages (Fig. 49(a)), thus :—

A. Suction; air is drawn in.

B. Compression; this air is compressed.

C. Combustion; fuel, which may be drawn in with the air, or



CONVERSION OF HEAT INTO MECHANICAL ENERGY

Fig. 49. (*a*) Five operations essential for the conversion of heat into mechanical energy follow each other, one at a time, in the cylinder of an internal combustion engine. (*b*) Operations in the gas turbine engine, applied to aircraft propulsion, all take place continuously in their own special part of the gas turbine.

else injected at this stage, is burnt in this air, causing a rise in its temperature and pressure.

D. Expansion; this hot compressed mixture is expanded, doing work on the piston by moving it a certain distance, the stroke of the engine, against a resisting force. During expansion, the temperature, as well as the pressure, decreases. The opposite reaction occurs when the end of a cycle pump gets hot as we increase the pressure by pumping. This stroke is the working stroke, and, during it the heat energy of the fuel is converted into mechanical energy. The greater the temperature change during this stroke, the greater the proportion of the heat energy of the fuel which is converted into mechanical energy.

Thermal Efficiency

It can be shown that, in an ideal heat cycle, the thermal efficiency, viz., the proportion of the heat turned into mechanical work is equal to

$$\frac{T_1 - T_2}{T_1}.$$

where T_1 is the absolute temperature before expansion,
and, T_2 is the absolute temperature after expansion.

The absolute temperature is the temperature above that at which there is no heat whatever in a body. The temperature at which there is no heat in a body, the absolute zero, is hundreds of degrees (491 deg. Fahrenheit) below the freezing point of water, so, unless the final temperature is very low, we cannot convert a very large proportion of the heat in a fuel into mechanical energy.

E. Exhaust; the expanded and cooled gases are exhausted.

Compression of the air is essential, because, only by compressing it can the heat energy of the fuel be converted into mechanical work

during the temperature drop which occurs in the subsequent expansion. Moreover, whatever means is considered for converting the heat energy of a fuel into work (it will be called a heat engine) the working fluid (air, steam, etc.) must go through the above-mentioned stages. Only, if the heat engine is designed specifically for jet propulsion, (1) instead of using the expansion stroke to convert heat energy into mechanical energy, it is converted into fluid kinetic energy ; and (2) instead of subjecting given portions of air to these operations successively, but in the same place (the cylinder in Fig. 49(a)), each operation is performed continuously, but in successive places, as in Fig. 49(b), thus producing a continuous jet.

Now, there are two big disadvantages of the reciprocating engine ; (1) its weight-to-power ratio is low because of the heavy reciprocating parts, and because it is doing useful work during only one stroke in four, and (2) it causes annoying vibrations. These difficulties can be overcome by the continuous-operation principle just mentioned, because (i) each component is working continuously, and (ii) the compression can be more conveniently done by means of rotating machinery, the expansion taking place in a turbine at *D* (Fig. 49(b)) which supplies power to a rotary compressor at *B*.

Construction of Turbine

A turbine consists of alternate rows of fixed and moving blades. Each row of fixed blades is arranged radially round the inside of the turbine casing as at *A* (Fig. 50(a)), and the moving blades are attached radially to the revolving shaft as at *B*. The fixed portion is called the stator, and the moving portion the rotor.

Each blade is of aerofoil section, and a streamline flow of gas is obtained, as shown in Fig. 50(b). Our knowledge of aerodynamics will tell us that lift will be obtained in the directions of the arrows, but the details of the problem are a little different, because the gas expands as it passes through the blading, with consequent temperature reduction. The study of these problems, associated with the conversion of heat energy into mechanical energy, is called thermodynamics, and here, as in other jet-propulsion problems, the sciences of thermodynamics and aerodynamics go hand in hand.

Blade Arrangement

The type of rotary compressor usually advocated is really the converse of the turbine ; that is, it bears the same relation to the turbine as the dynamo does to the electric motor. In this case, the blading is arranged in such a way that the relative motion given to the blades creates an aerodynamic force which builds up pressure, and also temperature, as the air flows through the blades.

Thus, in the complete unit, a compressor is fitted in front of the turbine on a common shaft, the outer part of the space between them being used for combustion. A motor must be added, driven from an accumulator to start it up. Fig. 51 shows the scheme in a conception of the aeroplane of the future.

Let us now consider what improvements may be made in (1) the thermodynamic, and (2) the aerodynamic performance of the engine.

(1) It has been pointed out that

the thermal efficiency of a heat engine depends upon the ratio of the temperature drop during expansion to the absolute maximum temperature, viz., $\frac{T_1 - T_2}{T_1}$, so we must endeavour to increase T_1 and decrease T_2. Obviously, then, a high maximum temperature is required, and T_1, a low exhaust temperature T_2, but a decrease in the exhaust temperature will have more effect than an increase in the maximum temperature.

High Altitude Effect

Now, temperature decreases steadily with increasing altitude until −70 deg. F. is reached at 40,000 ft. As a result of this, we can, at high altitudes, abstract more heat from the gas during expansion in the turbine, and thereby obtain more energy from the fuel.

Now, we are at present limited to a maximum temperature of about 2,000 deg. F., because of lack of metals which, at this temperature, have sufficient strength and endurance to withstand the high stresses encountered in the turbine blading. It is partly this fact that has prevented the extensive use of the gas turbine up to the present, although, amongst other applications, gas turbine locomotives have been running successfully on the Swiss Federal Railways for some years.

In all gas turbines it is necessary to compress an excess of air into the combustion chamber to keep the temperature within working limits. The energy expended in doing this is normally wasted, but the compression of a large mass of air is just what we are aiming at in a jet-propulsion unit, so the gas turbine appears to be specially adaptable for aircraft.

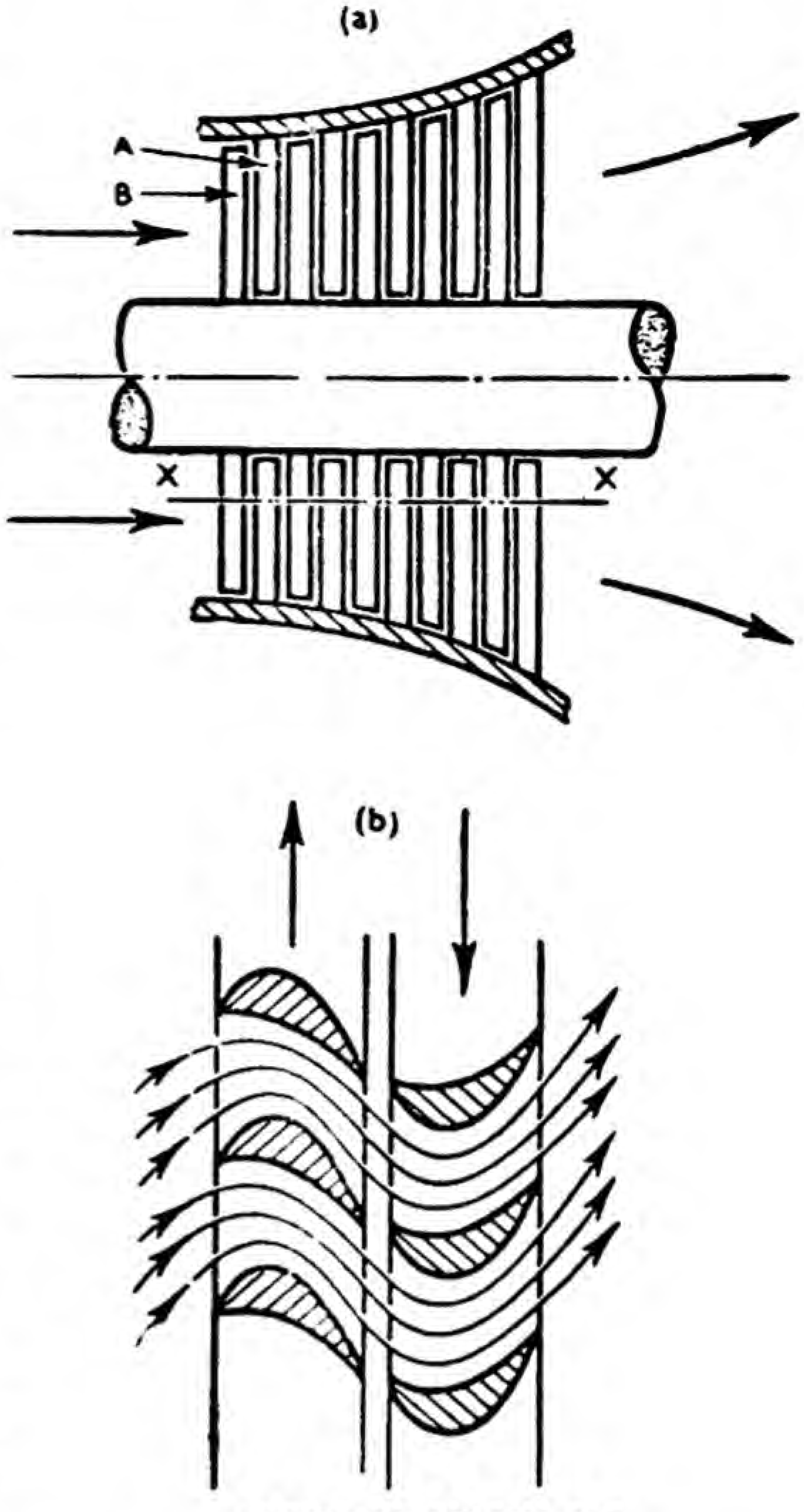


Fig. 50. In a turbine, half of the blades are fixed to the casing as at *A*. They do not rotate, and the complete assembly is called the stator. The other half of the blades is fixed to a central revolving shaft as at *B*. This assembly is called the rotor. The blades are shaped as shown in (*b*) so that the passage of gases through the blades creates ' lift ' in the directions of the vertical arrows. This lift force causes rotation of one set of blades relative to the other.

(2) The trouble with the jet, as we have seen before, is that its propulsive efficiency is low at low speeds. One very inefficient method has been proposed for increasing the momentum of the

jet at take-off. That is, to burn additional fuel in the expansion duct, thereby increasing the mass and velocity of the jet, but lowering both the thermal and propulsive efficiencies. But a more promising proposal is to fit a smaller compressor which will not absorb all the power of the turbine, and to utilize the excess power to drive an airscrew. To go further, we shall soon see gas-turbine units in which the compressor is only just large enough to compress sufficient air to burn the fuel, subject to the temperature limitation mentioned above, and then all the useful power will be used to drive an airscrew.

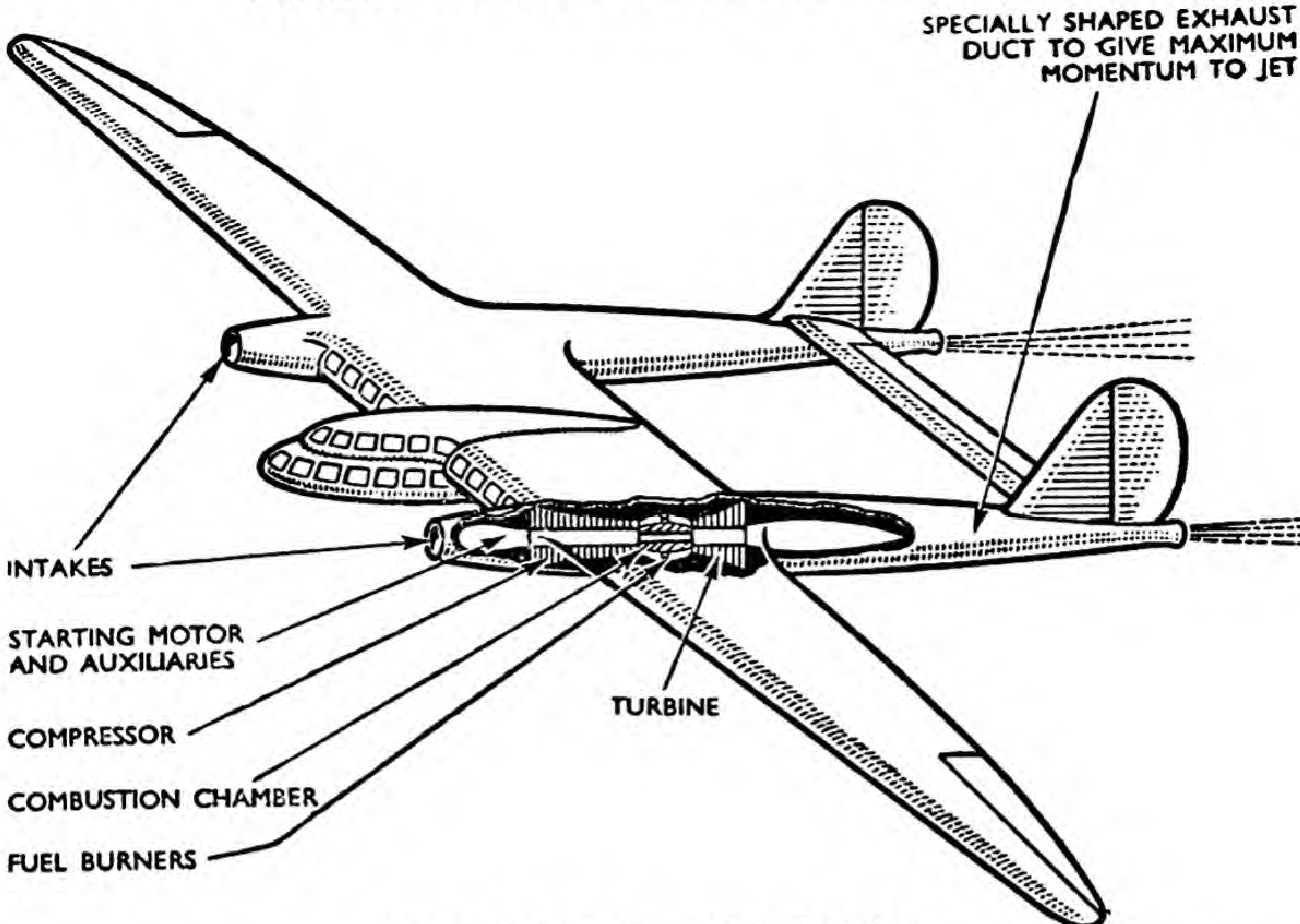
On the other hand, everything works in favour of the jet as the speed is increased. Not only do the jet and aircraft velocities approach one another, improving propulsive efficiency, but, owing to the forward velocity V_1 of the aircraft, we are taking in air which is already at a pressure of $\frac{1}{2}\rho V_1^2$ above the atmosphere. At high velocities, this ram effect becomes appreciable, so less power is absorbed by the compressor, further increasing the efficiency of the plant.

Contra-Rotation

Gas turbines must necessarily revolve at very high speeds of 16,000 to 18,000 r.p.m., which do not admit of direct coupling to an airscrew, so the above schemes would necessitate a reduction gear, which involves extra weight and complication. But, the idea of contra-rotation has been proposed, which may eliminate the necessity for a gearbox with large units, where the rotational speed is lower. Contra-rotation may be applied both to the airscrew, and to the power unit. It has been seen that it is merely necessary to provide relative motion between stator and rotor, in both the turbine and the compressor ; so, all we propose to do is to attach the case of the power unit, the stator, to the inner airscrew, and pass the rotor shaft through to the outer airscrew. This involves mechanical difficulties, but a promising design has been put forward which overcomes them.

This, incidentally, eliminates the gyroscopic effect due to large masses rotating at very high speeds provided the moments of inertia, not the speeds, of the rotating masses are balanced. The gyroscopic effect causes trouble whenever the direction of motion of the aircraft is changed, so we shall undoubtedly hear a good deal about contra-rotation in the future.

Even if future research does not appreciably improve the thermal efficiency of the gas turbine, this will undoubtedly be offset by its peculiar adaptability to aircraft. The major reasons for this adaptability are : (1) it possesses high power for a given weight, (2) the smaller size and convenient shape fit well into a streamline shape (Fig. 51), thus reducing drag, the arch-enemy of high-speed flight, (3) it has few moving parts to go wrong, and no reciprocating parts to cause vibration, (4) it has better performance at high speed and altitude, (5) it will burn cheaper fuel such as diesel oil or paraffin, which does not give off an inflammable vapour, an important point for aircraft, (6) a portion of the compressed air may be by-passed to warm the leading edge of the wings, thus preventing icing, to heat the cabin, or to charge a pressurized cabin, and, (7) intakes



AEROPLANE OF THE FUTURE

Fig. 51. Two jet propulsion units are here shown installed in twin fuselages. There are many other possibilities ; for example, with very large aeroplanes the units may be completely submerged in the wings.

and jets may be placed at suitable points on the aerofoil surface to improve the airflow with the object of increasing lift and reducing drag.

We must not expect jet propulsion immediately to supersede airscrew propulsion. Advance will be slow, and, maybe, along unexpected lines, and it may never replace the airscrew as a means of propulsion at average altitudes and speeds. We have seen that it holds much promise as a means of propulsion at high speeds and altitudes, particularly for the large aircraft proposed for commercial aviation.

Specimen Calculations

Here is a set of specimen calculations for the Tiger Moth, for the problems given on page 428.

The aspect ratio is,

$$\frac{s^2}{S} = \frac{(29{\cdot}3)^2 \times 2}{239} = 7{\cdot}2.$$

To find the wing loading,

$$\frac{W}{S} = \frac{1{,}825}{239} = 7{\cdot}6 \text{ lb. per sq. ft.}$$

Now the C_L at cruising speed

$$= \frac{\frac{W}{S}}{\frac{1}{2}\rho V^2} = \frac{7{\cdot}64}{0{\cdot}00256 \times (93)^2} = 0{\cdot}345.$$

The next problem was to find the maximum C_L. This is equal to

$$\frac{\frac{W}{S}}{\frac{1}{2}\rho V^2}$$

$$= \frac{0{\cdot}8 \times 7{\cdot}64}{0{\cdot}00256 \times (46{\cdot}5)^2} = 1{\cdot}1.$$

Problem five was to find the drag at cruising speed. This is,

$$\frac{550 \times \text{thrust h.p.}}{V}$$

$$= \frac{550 \times 0{\cdot}7 \times 80 \times 60}{93 \times 88} = 230 \text{ lb.}$$

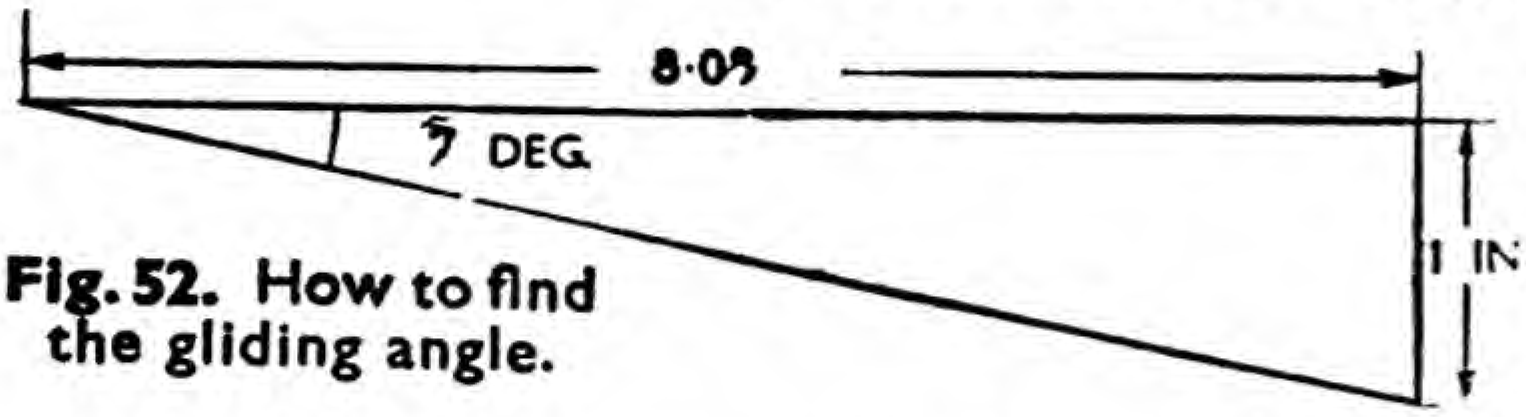


Fig. 52. How to find the gliding angle.

For the C_D at cruising speed

$$\frac{D}{\frac{1}{2}\rho V^2 S} = \frac{226}{0{\cdot}00256 \times (93)^2 \times 239}$$

$= 0{\cdot}043$, and the maximum L/D is

$$\frac{1{,}825}{226} = 8{\cdot}1$$

To find the gliding angle by drawing a triangle with a horizontal side proportional to L, draw this side 8·08 in. long, and another side, at right angles to this, 1 in. long. Draw the third side, and the angle will be found to be 7 deg. (Fig. 52).

The h.p. for climbing is,

$$\frac{W \times \text{rate of climb}}{33{,}000} = \frac{1{,}825 \times 673}{33{,}000}$$

$= 37.$

Answers to Problems

Answers to the problems are below, and appear for the following aircraft in this order; the Tiger Moth, Auster, Sea Otter, Dakota and the Shetland.

Aspect ratio: 7·2, 7·0, 6·93, 9·14, 8·54. Wing loading, lb. per sq. ft.: 7·64, 9·19, 15·0, 25·5, 49·3. C_L at cruising speed: 0·345, 0·332, 0·596, 0·233, 0·568. Maximum C_L: 1·1, 1·8, 1·6, 1·8, 1·9. Drag at cruising speed, lb.: 230, 180, 970, 2,600, 6,700. C_D at cruising speed: 0·043, 0·036, 0·062, 0·024, 0·029. Maximum L/D: 8·1, 9·6, 9·5, 9·7, 19·4. Gliding angle, deg.: 7, 6, 6, 6, 3. H.p. available for climbing: 37, 40, 240, 820, 2,600.

Analysis of Results

If the above results are analysed, it will be found that a number of factors have been omitted in obtaining them. For instance, the effect of altitude has not been considered, and the quoted speeds are usually those given by the airspeed indicator, which are usually found to be lower than the true speed of the aircraft. Again, an aircraft is always getting lighter owing to the consumption of fuel, so that the landing weight is always lower than that at the take-off apart from the fact that the landing speed is usually quoted at a load that is much below the maximum. To allow for this, the landing weight has been taken at 80 per cent of the loaded weight.

The low figure that is quoted for the cruising speed of the Sea Otter is due to the fact that it is a reconnaissance plane, and the normal load at this speed would be less than the loaded weight that is given. Consequently, it should be noted that the value for C_L is too high.

It will be recollected from the discussion that took place on propulsive efficiency (page 430), that no airscrew is capable of converting all the torque into thrust, so an average value for the thrust horsepower has been taken. For the present purposes, 80 per cent is a reasonable figure.

A figure of 80 per cent is not, of course, a very close approximation for the varied types of aircraft selected, so most of the results are quoted to two figures only, but they are sufficiently reliable for useful comparisons to be made.

Wing loading and L/D ratio increase with larger and more modern machines, and the drag coefficient and gliding angle go down. All but the Tiger Moth are fitted with flaps. Note the effect on C_L.

INDEX

Reprinted 1960
Printed in Great Britain by Richard Clay & Co., Ltd., Bungay, Suffolk.